

# Formation and evaporation of regular black holes

Sean A. Hayward

*Paris, 16th July 2009*

[Black holes](#), predicted by Einstein gravity, appear to exist in the universe.

[Singularities](#) were also predicted to form inside them by singularity theorems

[Penrose 1965, Hawking 1967, Penrose & Hawking 1970. . .].

However, they are generally regarded as indicating the breakdown of the classical theory.

Quantum field theory on a stationary black-hole background predicted [Hawking radiation](#)

[Hawking 1975].

The ingoing radiation has [negative energy flux](#) which contradicts the assumptions of the theorems and, in a semi-classical approximation, causes the black hole to [shrink](#).

In the usual picture, the black hole shrinks until the singularity is reached [Hawking 1976], leading to the information puzzle.

However, if the singularity does not exist, such a picture cannot be correct.

1. Regular black holes: static, asymptotically flat, with regular centres, satisfying the weak energy condition
2. A minimal model: almost-Schwarzschild exterior with de Sitter core
3. Adding radiation: Vaidya-like regions
4. Ingoing radiation: appearance and disappearance of outer and inner trapping horizons
5. Outgoing radiation: pair-creation surface
6. Remarks: no event horizon, no paradox

Based on Phys. Rev. Lett. 96, 031103 (2006) / gr-qc/0506126.

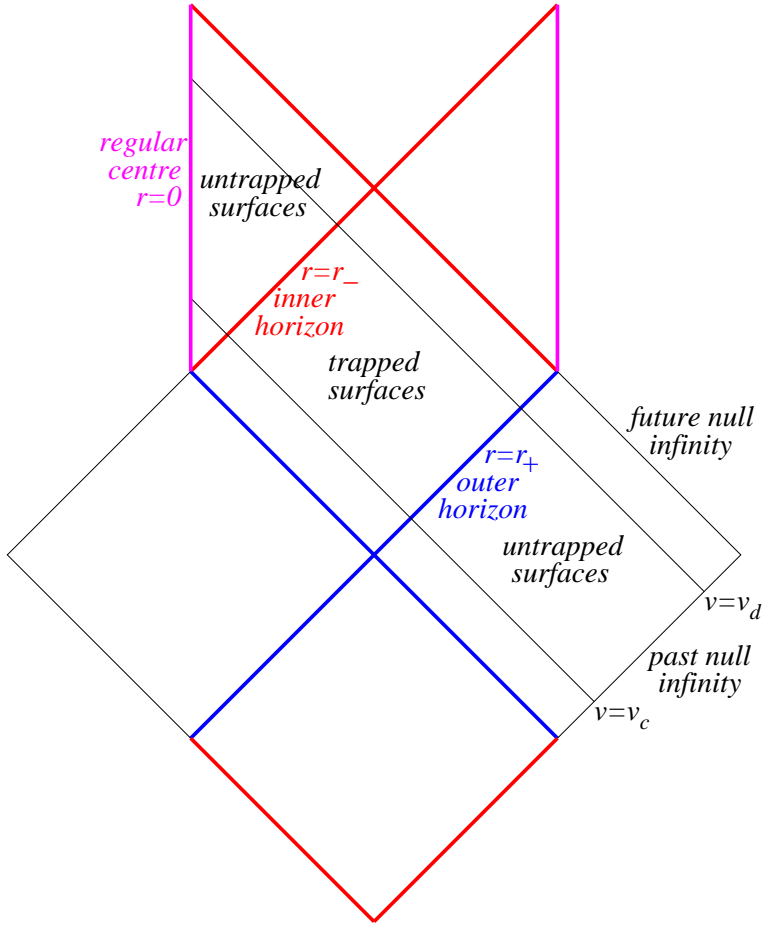
## 1. Regular black holes

**Regular** (i.e. non-singular) black holes have been considered by many authors [Bardeen 1968...].

One can find metrics which are spherically symmetric, static, asymptotically flat, have regular centres, and for which the resulting Einstein tensor is physically reasonable, satisfying the weak energy condition and having components which are bounded and fall off appropriately at large distance.

The simplest causal structure is similar to that of a Reissner-Nordström black hole, with the internal singularities replaced by regular centres.

Penrose diagram of a regular non-extreme black hole:



Consider static, spherically symmetric metrics of the form

$$ds^2 = r^2 dS^2 + dr^2/F(r) - F(r) dt^2, \text{ where } t \text{ is the static time, } r \text{ the area radius}$$

and  $dS^2 = d\theta^2 + d\phi^2 \sin^2 \theta$ .

A surface has area  $4\pi r^2$ , is **trapped** if  $F(r) < 0$  and **untrapped** if  $F(r) > 0$ .

**Trapping horizons**, in this case also Killing horizons, are located at the zeros  $F(r) = 0$ .

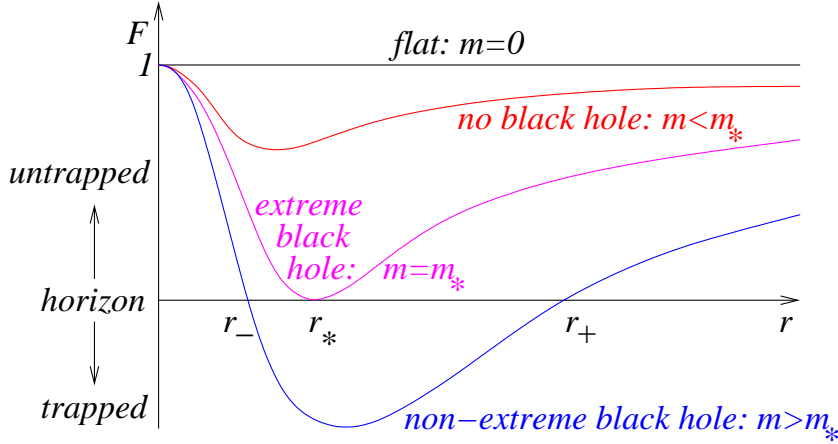
For an [asymptotically flat space-time](#) with total mass  $m$ ,  $F(r) \sim 1 - 2m/r$  as  $r \rightarrow \infty$ .

Similarly, [flatness at the centre](#) requires  $F(r) \sim 1 - r^2/l^2$  as  $r \rightarrow 0$ ,

where  $l$  is a convenient encoding of the central energy density  $3/8\pi l^2$ , assumed positive.

A sketch of  $F(r)$  indicating where it might dip below zero shows that there will be a range of parameters for which there is no black hole, and that the simplest black-hole cases will generically have an inner and outer Killing horizon, the two cases separated by an extreme black hole with degenerate Killing horizon.

The metric function  $F$  for fixed core radius  $l$  and different total masses  $m$ :



It can be shown that, for a metric  $g$  of the above form, the Einstein tensor has the cosmological-constant form  $G \sim -\Lambda g$  as  $r \rightarrow 0$ , with  $\Lambda = 3/l^2$ .

Thus there is an [effective cosmological constant](#) at small distances, with Hubble length  $l$ . Such behaviour has been proposed previously as the equation of state of matter at high density [Sakharov 1966], and based on an upper limit on density or curvature [Markov 1982...].

Since  $l$  gives the approximate length scale below which such effects dominate,

one might expect  $l$  to be the Planck length or of the same order,

though larger length scales are not excluded.

## 2. A minimal model

For definiteness, take a particularly simple metric satisfying the above conditions:

$$F(r) = 1 - \frac{2mr^2}{r^3 + 2l^2m}$$

where  $(l, m)$  are constants which will be assumed positive.

This is similar to the Bardeen black hole, reduces to the Schwarzschild solution for  $l = 0$  and is flat for  $m = 0$ .

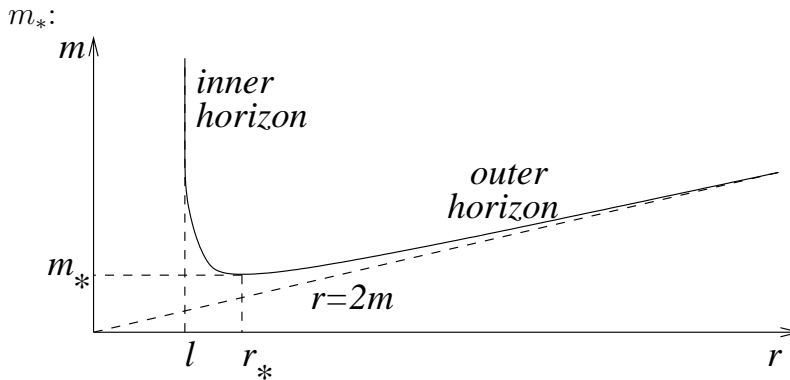
Elementary analysis of the zeros of  $F(r)$  reveals a critical mass  $m_* = (3\sqrt{3}/4)l$  and radius  $r_* = \sqrt{3}l$  such that, for  $r > 0$ ,  $F(r)$  has no zeros if  $m < m_*$ , one double zero at  $r = r_*$  if  $m = m_*$ , and two simple zeros at  $r = r_{\pm}$  if  $m > m_*$ .

These cases therefore describe, respectively, a regular space-time with the same causal structure as flat space-time, a regular extreme black hole with degenerate Killing horizon, and a regular non-extreme black hole with both outer and inner Killing horizons, located at  $r_+ \approx 2m$  and  $r_- \approx l$  for  $m \gg m_*$ .

The horizon radii  $r_{\pm}$  determine the mass

$$m(r_{\pm}) = \frac{\frac{1}{2}r_{\pm}^3}{r_{\pm}^2 - l^2}.$$

Horizon mass-radius relation: a pair of horizons appears when mass  $m$  exceeds critical mass



Note the existence of a **mass gap**: such black holes cannot form with mass  $m < m_*$ .

Also, the inner horizon has radius  $r_- > l$  which is very close to  $l$  for all but the smallest masses. In this sense, the **black-hole core** has a universal structure.

If the Einstein equation  $G = 8\pi T$  is used to interpret components of the energy tensor  $T$ , these metrics are supported by density  $-T_t^t$ , radial pressure  $T_r^r$  and transverse pressure  $T_\theta^\theta = T_\phi^\phi$  given by

$$\begin{aligned} G_t^t = G_r^r &= -\frac{12l^2m^2}{(r^3 + 2l^2m)^2} \\ G_\theta^\theta = G_\phi^\phi &= \frac{24(r^3 - l^2m)l^2m^2}{(r^3 + 2l^2m)^3}. \end{aligned}$$

They fall off very rapidly,  $O(r^{-6})$ , at large distance.

In terms of the energy  $E$  defined by  $g^{rr} = 1 - 2E/r$ , one finds the energy density

$$-T_t^t = (3l^2/2\pi)(E/r^3)^2, \text{ proportional to the square of the curvature } E/r^3$$

[cf. Poisson & Israel 1988].

### 3. Adding radiation

Next rewrite the static space-times in terms of advanced time

$$v = t + \int \frac{dr}{F(r)}$$

$$\text{so that } ds^2 = r^2 dS^2 + 2dvdr - Fdv^2.$$

Now allow the mass to depend on advanced time,  $m(v)$ , defining  $F(r, v)$  as above.

Then the density  $-T_v^v$ , radial pressure  $T_r^r$  and transverse pressure  $T_\theta^\theta$  have the same form, but there is now an additional independent component, radially [ingoing energy flux](#) (or energy-momentum density)  $T_v^r$  given by

$$G_v^r = \frac{2r^4m'}{(r^3 + 2l^2m)^2} \tag{1}$$

where  $m' = dm/dv$ .

This describes [pure radiation](#), recovering the Vaidya solutions for  $l = 0$  and at large radius.

In the Vaidya solutions [Vaidya 1951], the ingoing radiation creates a central singularity,

but in these models, the centre remains regular, with the same central energy density.

It seems that the effective cosmological constant protects the core.

The ingoing energy flux is [positive if  \$m\$  is increasing](#) and [negative if  \$m\$  is decreasing](#).

A key point is that trapping horizons still occur where the invariant  $g^{rr} = F(r, v)$  vanishes.

Then one can apply the previous analysis to locate the trapping horizons in  $(v, r)$  coordinates parameterized by  $m$ , given by  $m(r_\pm)$  above and a mass profile  $m(v)$ .

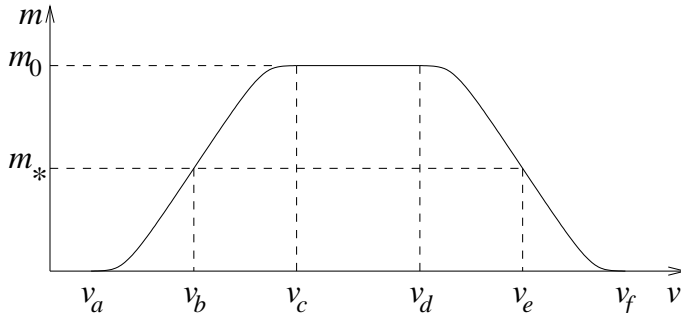
#### 4. Ingoing radiation

One can now model formation and evaporation of a static black-hole region.

Introduce six consecutive advanced times  $v_a < v_b < \dots < v_f$  and consider smooth profiles of  $m(v)$ , meaning  $m'(v)$  at least continuous, such that

$$\begin{aligned} \forall v \in (-\infty, v_a) & : m(v) = 0 \\ \forall v \in (v_a, v_c) & : m'(v) > 0 \\ \forall v \in (v_c, v_d) & : m(v) = m_0 > m_* \\ \forall v \in (v_d, v_f) & : m'(v) < 0 \\ \forall v \in (v_f, \infty) & : m(v) = 0. \end{aligned}$$

A mass profile  $m(v)$  in advanced time  $v$ :



Then

$$\begin{aligned} \exists v_b \in (v_a, v_c) & : m(v_b) = m_* \\ \exists v_e \in (v_d, v_f) & : m(v_e) = m_*. \end{aligned}$$

These transition times mark the appearance and disappearance of a pair of trapping horizons:

for  $v < v_b$  and  $v > v_e$ , there is no trapping horizon,

while for  $v_b < v < v_e$ , there are outer and inner trapping horizons.

These horizons join smoothly at the transitions and therefore unite as a

[single smooth trapping horizon](#) enclosing a [compact region of trapped surfaces](#).

## 5. Outgoing radiation

Thus far, only the ingoing Hawking radiation has been modelled,

since outgoing radiation does not enter the equation of motion of the trapping horizon; in terms of retarded time  $u$ ,  $T_{vv}$  and  $T_{uv}$  enter, but  $T_{uu}$  does not.

Outgoing Hawking radiation will now be modelled: select a certain radius  $r_0 > 2m_0$  outside the black hole, and adopt the above negative-energy radiation only inside that radius, balanced by outgoing positive-energy radiation outside that radius, with the same mass profile.

This is an idealized model of pair creation of ingoing particles with negative energy and outgoing particles with positive energy, locally conserving energy.

In more detail, consider an outgoing Vaidya-like region  $ds^2 = r^2 dS^2 - 2dudr - Fdu^2$  with  $F(r, u)$  as before, with  $m$  replaced by a mass function  $n(u)$ .

Fix the zero point of the retarded time  $u$  so that  $r = r_0$  corresponds to  $u = v$ .

Now take the above model only for  $v < v_d$ .

For  $v > v_d$ , keep the profiles for  $r < r_0$ , but for  $r > r_0$ , take an outgoing Vaidya-like region with

$$\begin{aligned} \forall u < v_d : n(u) &= m_0 \\ \forall u > v_d : n(u) &= m(u). \end{aligned}$$

Then there is a static region with total mass  $m_0$  for  $v > v_d$ ,  $u < v_d$ ,

and a flat region for  $v > v_f$ ,  $u > v_f$ .

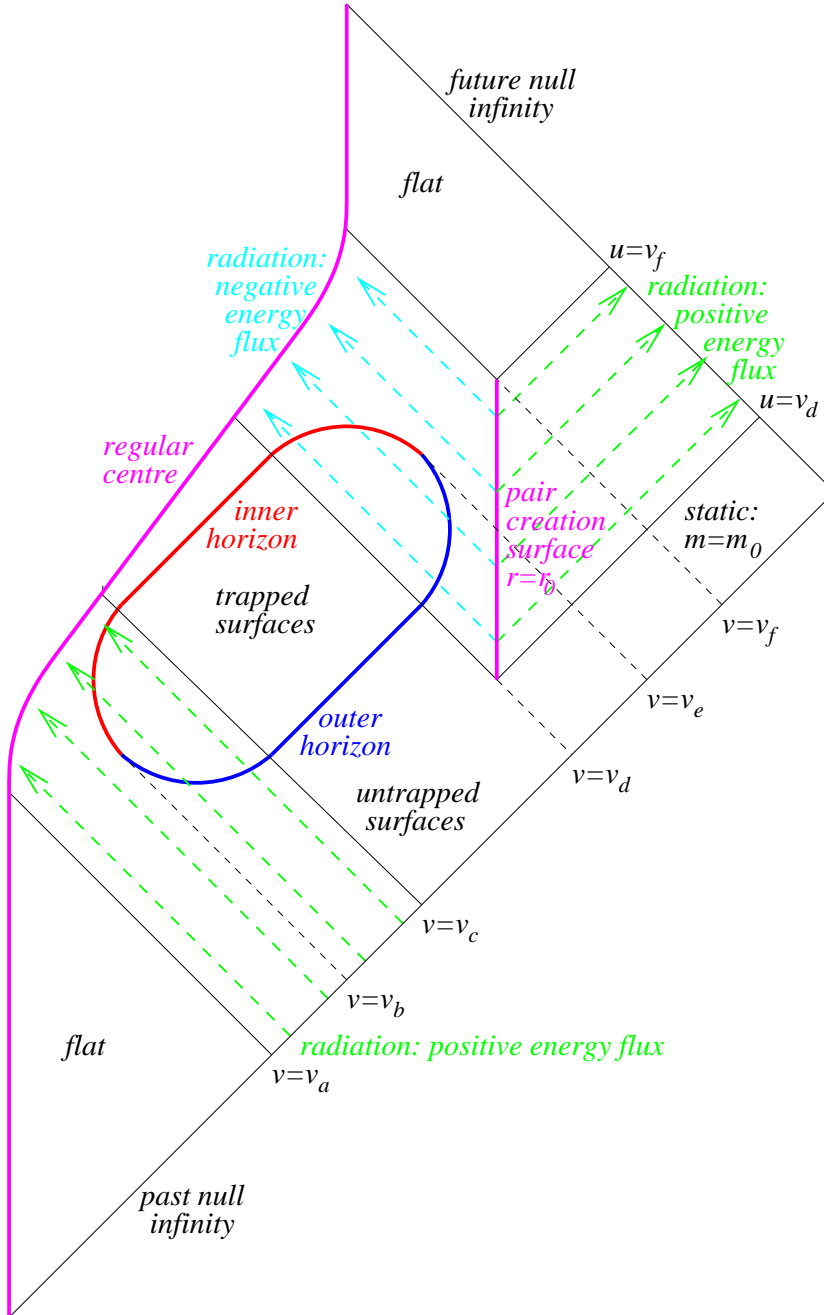
Since the ingoing and outgoing radiation has no net energy but a net outward momentum, one might expect the pair creation surface  $r = r_0$  to have a surface layer with no surface energy density but surface tension  $\tau < 0$ .

This is confirmed using the Israel formalism [Israel 1967], yielding

$$-\frac{16\pi(g^{rr})^{3/2}}{r}\tau = [G^{rr}] = -\frac{4r^4 m'}{(r^3 + 2l^2 m)^2}$$

at  $r = r_0$ ,  $v_d < v < v_f$ .

Penrose diagram of formation and evaporation of a regular black hole in the given models:



The whole picture is given above.

Action begins at  $v = v_a$ , a black hole begins to form at  $v = v_b$ ,

has collapsed completely at  $v = v_c$  to a static state with mass  $m_0$ ,

begins to deflate at  $v = v_d$  and eventually evaporates at  $v = v_e$ ,

leaving flat space finally after  $v = v_f$ ,  $u = v_f$ .

There is **no singularity** and **no event horizon**.



## 6. Remarks

A trapping horizon with both inner and outer sections typically develops in numerical simulations of binary black-hole coalescence, in analytical examples of gravitational collapse such as Oppenheimer-Snyder collapse and according to general arguments.

A key point here is that the [inner horizon never reaches the centre](#), where a singularity would form.

This is compatible with the classical singularity theorems, which make assumptions that are already not satisfied by a Bardeen black hole, such as the strong energy condition.

The negative-energy nature of ingoing Hawking radiation shows that such theorems do not apply to a black hole that might someday begin to evaporate.

In contrast to the usual picture, the [endpoint of evaporation](#)  $v = v_e$ , defined locally by the [disappearance of trapped surfaces](#), occurs when the [outer and inner sections of the trapping horizon reunite](#).

The subsequent timescale until the effective cessation of particle production at  $v = v_f$  can be expected to be of the same order as  $l$ .

Another logical possibility is that the inner and outer horizons approach each other asymptotically, forming the horizon of an extreme black hole with  $m = m_*$ , but such a delicately balanced situation would require justification.

The possibility of a circular trapping horizon has been conjectured before

[Frolov & Vilkovisky 1981, Roman & Bergmann 1983...].

Since there is no event horizon, long accepted as the defining property of a black hole, it seems necessary to stress that the static region looks just like a black hole over timescales that can be arbitrarily long.

Thus it should be regarded as a black hole by any practical definition,

as in the local, dynamical paradigm for black holes in terms of trapping horizons

[Hayward 1994–2009, Ashtekar et al. 1999–2004, Booth & Fairhurst 2004–2007, Gourgoulhon & Jaramillo 2006...].

As the global structure is the same as for Minkowski space-time,

there is [no information paradox](#).

Put another way, the [resolution](#) to the paradox is that black holes are not usefully defined by event horizons.