Koponen and Nousiainen
Complex Adapt Syst Model (2016) 4:24
DOI 10.1186/s40294-016-0035-6

Complex Adaptive Systems Modeling

RESEARCH

Open Access

Formation of reciprocal appreciation patterns in small groups: an agent-based model

Ismo T. Koponen* and Maija Nousiainen

*Correspondence: ismo.koponen@helsinki.fi
Department of Physics, University of Helsinki, Gustav Hallstromin katu 2, P.O. Box 64, 00014 Helsinki, Finland

Abstract

Purpose: In small cooperative and collaborative groups, patterns of interaction, discourse and dialogue are often strongly bidirectional; ties are reciprocal and reciprocated. This reciprocation of ties leads to the formation of interaction patterns that are reciprocated dyads (two individuals connected reciprocally) and triads (three individuals connected reciprocally). In this study, we use an agent-based model to explore how such reciprocated dyadic and triadic patterns emerge from self-reinforced appreciation between peers in a small group.

Methods: The model assumes that the agents' decisions to interact depend on how their self-appreciation compares to their appreciations of their peers (peer-appreciation). These comparisons are competitive in that an agent seek to increase its appreciation in relation to its peers. As a consequence, agents change their self-appreciation and appreciation towards their peers depending on their sensitivity to the competitive comparison.

Results: When agents' sensitivity to competitive comparisons is low, the most common patterns of appreciation are egalitarian triads (all three agents appreciate each other), while for moderate sensitivity, leadership-type patterns emerge (one agent connected strongly to two other unconnected agents). When sensitivity is high, strong reciprocally connected dyads emerge. The model thus predicts thus a transition from egalitarian triads to strong dyads as agents' sensitivity to competitive comparisons increases.

Conclusions: The structural similarity between patterns emerging as model results and patterns reported in empirical research suggests that: (1) reciprocation based on appreciation is a strong candidate for explaining the formation of such patterns, and (2) individual sensitivity to competitive comparisons of appreciation may be a key factor that can be used to the tune dynamics of interaction in small groups.

Keywords: Social interaction patterns, Appreciation, Agent-based model, Dyad and triad consensus

Background

Social relationships and interactions are largely based on mutual affection, trust and appreciation, as with friendship formation (Simmel 1964). When social relationships are based on mutual benefit and exchange, affection, trust and appreciation may play a significant role in shaping the relationships within a group (Lawler et al. 2000, 2008; Lawler 2001). In this case, the common task mediates the positive and satisfying emotions.

Engagement in a common task and the presence of a goal orientation generates positive feedback within a group, which enhances the group's mutual ties when the group members find the interaction satisfying (Lawler et al. 2000, 2008; Lawler 2001). On the other hand, mutual relationships built on trust and appreciation also create expectations for the future behavior of group members that they may fulfill or may not fulfill, thus transforming the existing relationship (Skvoretz and Fararo 1996; Lusher et al. 2014).

Affection, trust or appreciation are mutual, bi-directional and reciprocal; ties thus formed tend to be strongly reciprocated. The reciprocation of ties, however, is thought to be the origin of certain very special features of social patterns such as reciprocal strongly connected dyads (two individuals connected reciprocally) and triads (three individuals connected reciprocally). Strong reciprocation also means that social patterns based only on one-directional ties are rare when ties are based on affection, trust or appreciation (Yoon et al. 2013; Block 2015; Rivera et al. 2010). Patterns similar to affection and trust based interaction are also found in small groups in information sharing and communication (Hogan et al. 1999; Enyedy 2003; Barron 2003), task-related collaboration (van Boxtel et al. 2000; Schwarz and Linchevski 2007; Sangin et al. 2011; Stahl et al. 2014), group performance (Lusher et al. 2014), and co-regulation in group learning (Volet et al. 2009). Studies that focus on co-regulation and peer-to-peer interaction in small groups indicate that in these cases, strongly reciprocated dyadic and triadic patterns of interaction are characteristics of successful and high achieving groups (Hogan et al. 1999; Barron 2003; Volet et al. 2009).

The affect theory of social exchange (Lawler et al. 2000, 2008; Lawler 2001) illuminates the role of affective ties like trust and appreciation in shaping the social relations of task performing groups. The theory attributes the strong reciprocation of social ties and thus the formation of the reciprocated dyads and triads to the positive affective factors that emerge through emotionally satisfying relations and which are thus self-enhanced through reciprocation (Lawler et al. 2008; Yoon et al. 2013). According to this view, engagement in a common task and goal orientation in a group generates positive feedback, which enhances the reciprocation of ties when the interaction is found to be satisfying by the interacting partners. The peers in a group have expectations for their performance and roles in completing the task, and when these expectations are felt to be fulfilled, that increases satisfaction and future engagement and exchange. Consequently, peer-to-peer comparisons and competence evaluations are the mechanisms for the formation of groups in the affect theory of social exchange (Lawler et al. 2000, 2008; Lawler 2001). These mechanisms are thus similar to those found in the social learning theory (Bandura 1997, 2006). In both views, it is recognized that individuals' conceptions of how they are appreciated in social groups essentially determine how much effort they put in the performance, or the others are expected to put in the collaboration. Thus, the formation of interaction patterns is essentially affected by peer-to-peer comparisons aimed to assess one's own performance in relation to one's peers' performance or potential to perform the task (Skvoretz and Fararo 1996; Lusher et al. 2014).

Social learning theory, and the affect theory of social exchange following it, both assume that peer influence functions through constant social comparisons, validations and appraisals of one's position in a group and how one is appreciated as part of the group in comparison to other group members. The self- and peer-appreciations are developed through constant mutual comparisons. The more similar an individual assumes he or she is to an appreciated peer and the more the appreciated peer appreciates the individual, the more the self-appreciation of the individual increases. Peer comparison is thus one of the most elementary forms of conceptualizing reciprocal social relationships. We assume here that to understand how social ties are formed in task-related contexts, we must pay attention to:

- Individuals' conception of how they are appreciated (called self-appreciation here),
- How self-appreciation compares to a peer's appreciation of the individual (peer-appreciation) and,
- How the peer-appreciations are reciprocated.

From the point of view of the affect theory of social exchange (Lawler 2001; Lawler et al. 2008) as well as of the social learning theory (Bandura 1997, 2006), self- and peer-appreciations are candidates for the properties governing the social interactions and formation of interaction patterns in a task performing group. Similarly, the mutual comparisons and appraisals of self- and peer-appreciations are the basic processes that affect the evolution of mutual affiliations and trust, and can thus be expected to become visible in discourse, communication and verbal interactions.

The assumption of the mutual comparisons of self- and peer-appreciations and the reciprocation of peer-appreciations on the formation of mutual patterns of appreciation in small groups can be tested using an agent-based model. Ideally, the results of the agent-based model also guide attention to what kinds of stable structural features can be taken as hallmarks of the reciprocation of peer-appreciations and how the sensitivity of individuals to their peers' assesments may shape these patterns. In this role, the agent-based models may serve as tools to guide the empirical research and data collection to be more sensitive to features that emerge from the underlying mechanisms of tie formation (Epstein 2008).

Methods

In this study, we present an agent-based model to simulate the development of reciprocated ties and the formation of reciprocated dyadic and triadic patterns of appreciation. The central questions are how self- and peer-appreciation influence the interacting partner's decision to interact and how the interaction, when realized, affects the interacting partners' self- and peer-appreciations. The model assumes that the agents' decision to interact depends on how their self-appreciation relates to their appreciations of their peers. In an interaction, agents compare their self-appreciation to the appreciation of their peers towards themselves, and as a consequence of this comparison, change both their self-appreciation and peer-appreciation of other agents. Positive differences in these comparisons increase the agent's self- and peer-appreciations, whereas negative difference decreases both. The agent-based model developed in what follows is based on a variant of the bounded confidence model (Deffuant et al. 2013; Castellano et al. 2009) and closely resembles the so-called Leviathan model of social interactions, where agents' mutual appraisals and the "vanity" effect are taken into account (Deffuant et al. 2013). The implementation of the model is fully stochastic.

Computational model of formation of appreciation patterns

The agent-based model that is introduced here assumes that agents' self-appreciation and peer-appreciations are mutually affected in pair-wise interactions. The following assumptions form the basis of the "minimal" model:

- An agent i holds a self-appreciation κ_{ii}.
- An agent i holds a peer-appreciation κ_{ij} of peer j.
- An agent i is aware that an agent j holds peer-appreciation κ_{ji} of i.

The model fulfills the minimal requirements for agent-based modelling of social learning, namely a rudimentary "theory of mind", so that i's beliefs about what j thinks of i affect i's self-appreciation, and vice versa. These kinds of reciprocal anticipations of each others' beliefs are assumed to be an indispensable part of any model of social dynamics (Sun 2006).

The dynamic evolution of self- and peer-appreciations is implemented as an agent-based model, in which agent i's self-appreciation κ_{ii} and the reciprocated peer-appreciations κ_{ij} and κ_{ji} are assumed to change as a consequence of pair-wise comparisons. The pair-wise comparison of appreciations and how they change self- and peer-appreciations are as follows.

- Change in agent's self-appreciation κ_{ii}: If in the comparative interaction event agent i finds that j's peer-appreciation κ_{ji} is higher than self-appreciation κ_{ii}, then i's self-appreciation increases. If the difference in appreciations is the other way around, then i's self-appreciation decreases.
- Change in the peer-appreciation that agent i holds of j is affected by two contributions. First, the peer-appreciation κ_{ij} that i holds of j is increased if i finds that j holds a peer-appreciation κ_{ji} exceeding the self-appreciation κ_{ii}. Second, the peer-appreciation κ_{ij} is decreased if i's peer-appreciation of j is higher than j's self-appreciation. The first effect is a kind of *competitive* comparison, in which the agent competes for appreciation and increases appreciation towards peers that overly appreciate it (i.e. more than its self-appreciation); this increases the future changes for high appreciation. The latter effect balances the overestimated appreciations.

This kind of comparison is competitive in the sense that an agent, through comparison, seeks to increse its appreciation in relation to other agents. The agents' sensitivity to the competitive comparison is regulated by the parameter $\alpha \in [0, 1]$, referred to as the *competitiveness* in what follows. If $\alpha = 0$, there is no effect. If $\alpha = 1$, the effect is maximal. The competitiveness is similar to vanity -effects due to appraisal in social and political elite group formation (Deffuant et al. 2013). The most central parameter of the model is the agents' sensitivity to competitive comparisons of appreciations described by the *competitiveness* α.

The probability that the agent's self- and peer-appreciations change depends on how the agent posits itself in regard to other agents. If the agent's i self-appreciation is higher than the peer-appreciation towards agent j, it is not likely that i's interaction with j leads to changes in appreciations, whereas in the opposite case, it is very likely that change will take place. The probability that changes in appreciations will take place can be thus modelled as a sigmoidal function given by (Deffuant et al. 2013; Castellano et al. 2009)

$$p_{ij} = \frac{1}{1 + \exp[-\delta_{ij}/\sigma]} \tag{1}$$

where $\delta_{ij} = \kappa_{ij} - \kappa_{ii}$ is the difference between self- and peer-appreciations; the higher the agent's i peer-appreciation of j is in relation to the agent's self-appreciation, the higher the probability is that an effect will take place and that the self- and peer-appreciations will be affected (i.e. the effect is propagated). The parameter $\sigma = 2\sigma'$ controls how big the difference between appreciations can be for the competitive comparisons to affect the appreciations (i.e. the effect is propagated) (Deffuant et al. 2013; Castellano et al. 2009). The parameter σ describes agents' tolerance to diversity in appreciations when they interact with other agents, and it is thus referred to the *diversity* in what follows. Technically, the diversity σ regulates the wideness of the distribution of differences in appreciations that affects the probability of an agent to change its appreciations. For a very small σ an agent is always affected by the peers that it appreciates highly but never by peers it appreciates liitle. The diversity is closely related to homophily, with a large diversity indicating low homophily and a low diversity indicating high homophily.

The changes in appreciations are described here stochastically, where the probability p_{ij} is central in deciding whether the change takes place. The update rules for the agents' i and j self-appreciations and their mutual peer-appreciations are given by

$$\kappa_{ii} \leftarrow \kappa_{ii} + \pi_o \, p_{ij} \, (\kappa_{ji} - \kappa_{ii}) \, \kappa_{ii} \, (1 - \kappa_{ii}) \tag{2}$$

$$\kappa_{jj} \leftarrow \kappa_{jj} + \pi_o \, p_{ij} \, (\kappa_{ij} - \kappa_{jj}) \, \kappa_{jj} \, (1 - \kappa_{jj}) \tag{3}$$

$$\kappa_{ij} \leftarrow \kappa_{ij} + \pi_o \, p_{ij} \, [\alpha \, (\kappa_{ji} - \kappa_{ii}) + (1 - \alpha) \, (\kappa_{jj} - \kappa_{ij})] \, \kappa_{ij} \, (1 - \kappa_{ij}) \tag{4}$$

$$\kappa_{ji} \leftarrow \kappa_{ji} + \pi_o \, p_{ij} \, [\alpha \, (\kappa_{ij} - \kappa_{jj}) + (1 - \alpha) \, (\kappa_{ii} - \kappa_{ji})] \, \kappa_{ji} \, (1 - \kappa_{ji}) \tag{5}$$

where the sensitivity of the agent to competitive comparisons of appreciations is governed by parameter α. In all incremental changes, the model takes into account the fact that appreciations do not increase without limit and are constrained to the maximum value, which is here 1. Similarly, the lowest possible appreciation has a value of 0. In order to take these constraints into account, the equations are of the logistic type and contain the term $\kappa(1 - \kappa)$. The parameter π_o is the overall sensitivity of agents to changing their appreciations. This parameter is treated stochastically, thus simulating the randomness in strengths of individual decisions. In practice, we have assumed that π_o is normally distributed with an average value of $\langle \pi_o \rangle < 0.3$ and a standard deviation of $0.3\pi_o$. These values are small enough to prevent instabilities in iteration and to ensure convergence to results that do not depend on exact values of $\langle \pi_o \rangle$. Then the exact value affects only the number of steps needed so that the iterations converge to dynamically stable values of self- and peer-appreciations.

Equations. (2)–(5) are otherwise symmetrical with respect to i and j (modelling the symmetric reciprocation), but asymmetry prevails in the probability p_{ij} taking into account that agent i is assumed to have a special role in initiating the update of appreciations. In practice, this means that when agents i and j interact, the probability that j changes its self-appreciation depends on agent i's self-appreciation, through p_{ij}, which determines whether or not an initiated event leads to any change. This simulates the effect of initiating a discourse, which provides an advantage for the appreciated agent and increases its appreciation. This asymmetry plays a role in how leadership-type positions are built up [see (Skvoretz and Fararo 1996) for a similar effect].

The model of agents' interactions in Eqs. (2)–(5) is dyadic (denoted by D) in that only a pair of agents is involved. This is the most common interaction studied in the context of communication, information sharing and collaboration [see e.g. ref. (Hogan et al. 1999; Enyedy 2003; Barron 2003; van Boxtel et al. 2000; Schwarz and Linchevski 2007; Sangin et al. 2011; Stahl et al. 2014) and references therein]. For completeness, however, indirect triadic interaction (denoted as T) is studied by including a third agent k so that peer-appreciation κ_{ik} is updated following the rule in Eq. (4) when i and j interact with a probability of p_{ij}. The agent k, however, is treated as a *collateral* one, so that there is no reciprocation of the appreciations. In addition, we study the model with an equal proportion of dyadic and triadic updates ($\mathrm{D} + \mathrm{T}$).

Finally, two notions concerning the realism of the model must be made. First, the implicit assumption in constructing the model is that agents perceive not only their own self-appreciation and their peer-appreciation towards other agents but also the other agents' appreciation towards them. Though the two first assumptions are plausible, the last one may raise doubts. However, the possible randomness (if no bias is assumed) in perceiving the other agents' peer-appreciations is in the model of similar type, and with similar effects, as randomness described by π_o, i.e. adding noise in individual decision events. Second, the parameters α and σ are taken to be same for all agents. The obvious step to a more realistic model is to treat α and σ as having different values for different agents (i.e. as agents' attributes). In the model presented here all agents are "psychologically" identical, whereas real agents are certainly not. This idealization and restriction must be kept in mind when interpreting the results.

Simulation method

The simulations are carried out by selecting stochastically the agents that interact (two agents for dyadic model D and three agents for triadic model D). The initial values of appreciations κ are assigned randomly from a uniform distribution in the range $0.333 < \kappa < 0.667$. This is a practical choice and a wider/narrower distribution of initial values does not affect the distribution of stable patterns; stabilization of patterns just takes more/less iterations. The paramater α is varied from 0 to 1, with steps of 0.1, thus spanning the effect of competiveness from a nonexistent effect to a maximal effect. The tolerance to diversity is explored for $\sigma = 0.05$, 0.10, 0.20 and 0.30, where the lowest and highest values are chosen so that there are no essential changes in results beyond these limits.

The so-called roulette wheel method (Lipowski and Lipowska 2012; Lipowski et al. 2014) applied to probability p_{ij} in Eq. (1) is used to select whether the self- and peer-appreciations are changed or not. In the roulette wheel method a discrete set of N possible events k with probabilities p_k are arranged with cumulative probability $\Phi_k = \sum_{i=1}^{k} p_i / \sum_{i=1}^{N} p_i$. The event k is selected if a random number $0 < r < 1$ falls in the slot $\Phi_{k-1} < r < \Phi_k$. The roulette wheel method (Lipowski and Lipowska 2012) has been used in similar cases of stochastic modelling of emergent social structures and interaction patterns associated with preferential selection of connecting links (Lipowski et al. 2014). In addition to p_{ij}, π_o in Eqs. (2)–(5) is treated as a stochastic variable and selected from a uniform distribution between values $\pi_o \pm \delta\pi_o$, where $\delta\pi_o = 0.33\pi_o$. All the simulations are carried out for 2000 repetitions of each parameter combinations. Some simulation runs were done with different choices of initial values as well as different choices of π_o to test the robustness of results in regard to the choice of parameters regulating the computation.

Patterns of appreciations

Many of the important characteristics of the relationships can be understood by focusing on different dyadic and triadic patterns and how the ties are reciprocated. Given the background assumptions, the expectation is that reciprocated dyads and triads, as well as reciprocated dyads embedded as parts of different triads, are important as expected on the basis of trust or affect based interactions [c.f. refs. (Lawler et al. 2000, 2008; Lawler 2001)]. One can also expect to find patterns of the leadership type containing directed ties in which one agent is more central than others [see e.g. (Skvoretz and Fararo 1996)]. Cyclic (i.e. three agents A, B and C connected cyclically as A → B → C → C), transitive (A → B, C and B → C) and completely non-reciprocated triadic patterns are expected to be rare (Yoon et al. 2013; Block 2015). In this case, nine dyadic and triadic patterns are of interest, out of which six eventually changed most with changes of competitiveness, diversity and collaterality, thus providing a window with which to monitor essential changes in patterns of peer-appreciations caused by these factors. These patterns are shown in Fig. 1. In simulations, we were looking for these patterns and counting their abundances and intensities at the stage of simulation where patterns were stabilized (roughly the last 30 % of update events in the simulations). Such "triadic census" of the patterns acts as a "fingerprint" of the group dynamics and structure (Moody 1998; Itzkovitz et al. 2003). We indexed the patterns according to the common structural indexing (Moody 1998) (see the caption of Fig. 1 for details). The short-hand names of the patterns in Fig. 1 and their indexing are given in Table 1. Here, the patterns are also named on the basis of their role in social dynamics based on peer appreciation, and they are roughly of three different types; triadic, dyadic and leadership-type patterns:

- *Triadic patterns* are groups in which all three agents are linked to each other in different ways. In the *egalitarian triad* (300), all links are reciprocated and of equal strength. In the *collateral triad* (120D), only two agents are connected by a strong reciprocated link and the third agent is collaterally connected to them both by a strong incoming but non-reciprocated link. By contrast, in the *broker triad* (120U), the two agents are connected to the third one by a strong outgoing link [a pattern known as *tertius gaudens* (Simmel 1964)]. This is a broker's position, where a single agent is appreciated by a dyad and can thus influence the dyad.
- *Dyadic patterns* are groups in which a reciprocated dyad is the most dominant structural element. In an *egalitarian dyad* (102), both agents are in similar positions, appreciating each other but not connected to a third party at all. In a *collateral dyad*

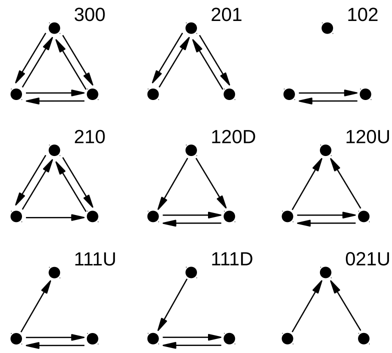

Fig. 1 Patterns and their indexing. Pattern indexing (ind) (Moody 1998) and the formulas to count their abundances N_{ind} and average intensities W_{ind}. These nine patterns are of interest, especially the patterns in the two upper rows, in monitoring change in social dynamics. In the indexing, the first digit indicates the number of reciprocated pairs, the second digit the number of non-reciprocated pairs, and the last digit the number of unconnected pairs. The last character, if given, differentiates between directedness (*U* for upward, *D* for downward) of otherwise similar patterns

Table 1 The number N_{ind} and intensities W_{ind} of peer-appreciation patterns (ind) based on peer-appreciation strengths κ_{ij}

Pattern	ind	Number N_{ind}	Intensity W_{ind}
triad	300	$\sum(S^3)/6$	$N_{300}^{-1}\sum(S'^3)^{\frac{1}{6}}/6$
dyadic L	201	$\sum(S^2 \circ \tilde{E})/2$	$N_{201}^{-1}\sum(S'^2 \circ \tilde{E})^{\frac{1}{4}}/2$
dyad	102	$\sum(\tilde{E}^2 \circ S)/2$	$N_{102}^{-1}\sum(\tilde{E}^2 \circ S')^{\frac{1}{2}}/2$
triadic L	210	$\sum(AA^T \circ S)/2$	$N_{210}^{-1}\sum(A'A'^T \circ S')^{\frac{1}{4}}/2$
C triad	120D	$\sum(A^T A \circ S)/2$	$N_{120D}^{-1}\sum(A'^T A' \circ S')^{\frac{1}{4}}/2$
B triad	120U	$\sum(AA^T \circ S)/2$	$N_{120U}^{-1}\sum(A'A'^T \circ S')^{\frac{1}{4}}/2$
C dyad	111U	$\sum SA^T \circ K \circ \widetilde{K^T}$	$N_{111U}^{-1}\sum(S'A'^T \circ K \circ \widetilde{K^T})^{\frac{1}{3}}$
B dyad	111D	$\sum SA \circ K \circ \widetilde{K^T}$	$N_{111D}^{-1}\sum(S'A' \circ K \circ \widetilde{K^T})^{\frac{1}{3}}$
endorsed L	021U	$\sum(AA^T \circ \tilde{E})/2$	$N_{021U}^{-1}\sum(A'A'^T \circ \tilde{E})^{\frac{1}{2}}/2$

The following descriptive names of patterns in shorthand notations are used: *L* leadership, *C* collateral, *B* broker. The number N_{ind} of different patterns can be counted from the adjacency matrix K′, which has elements $[K']_{ij} = \kappa_{ij}$ and $[K']_{ii} = 0$ (excluding self-appreciations) providing all the information to count the number N_{ind} and intensities W_{ind} of patterns of interest. For this, six other auxiliary matrices derived from K′ are introduced: the symmetric part of K′, denoted by S′, and the asymmetric part, $A' = K' - S'$; matrices K, S and A where entries are 1 for all non-zero entries in S′ and A′, respectively; and the symmetric matrix E constructed so that if $K_{ij} \neq 0$ or $K_{ji} \neq 0$, then $E_{ij} = E_{ji} = 1$. Standard matrix operations are used so that T denotes transpose, Tr trace and ∘ is the element-wise multiplication (Hadamard product). The logical inverse (complement) of the matrix is denoted by ~ e.g. $\tilde{K}$ as the complement of K

(111D), one of the agents in the strong dyad is appreciated collaterally by another agent, whereas in a *broker dyad* (111U), one of the agents in the strong dyad also appreciates a third party. These two latter structures are often transitory structures not only in forming corresponding balanced triadic patterns but also in situations in which balanced triadic patterns fall apart.

- *Leadership patterns* are groups in which one agent attains a special position as a leader. The simplest of such structures is the *endorsed leader* (021U), in which one agent is appreciated by others but which does not appreciate any of the other agents.

> Such a position is expected in cases of strong appreciation or competition. A more egalitarian type of leadership is *dyadic leader* (201), in which two strong dyads share a common agent, which then have a leader-type position. This pattern can evolve to or from a *triadic leader* (210), in which the agents, which are not in a leader-positions, are non-reciprocally connected.

The simulations provide the matrix K' of all connections in terms of peer-appreciations, and from this matrix, the number N of patterns (Moody 1998; Itzkovitz et al. 2003) can be obtained (see Table 1). The intensities W of the patterns (Onnela et al. 2005) can also be obtained from matrix K' as geometric means of the links (peer-appreciations) that constitute the pattern. Details and proofs of the formulas provided in Table 1 to count the patterns and their intensities are given in refs. (Moody 1998; Itzkovitz et al. 2003; Onnela et al. 2005).

Results

The formation of the peer-appreciation patterns is examined here for different groups of 3, 4, 5 and 6 agents. Most results are shown for a five-agent group, G5. This is due to the fact that G5 is large enough that its geometry does not essentially constrain the formation of patterns. For example, in a three-agent group, G3, the pattern formation is essentially constrained by the geometry, and only egalitarian triads, leadership dyads and dyads are observed. The four-agent group, G4, is more interesting because it easily splits into two independent dyads. However, G4 also poses a strong geometric constraint and makes the pattern formation rather predictable qualitatively. The five-agent group, G5, is more interesting because in this case, richer combinations of dyadic and triadic patterns become possible. The six-agent group, G6, and higher are in many cases expected to be very similar to G5 as far as our interest is in the basic units of dyads and triads. Consequently, G5, which is the transitory case of richer pattern formation, is the most interesting case.

The different patterns are counted throughout the simulations, and average values over 2000 steps in each case and over 1000 repetitions (ensembles) are reported in what follows. In all simulations, the competitiveness α is the most important parameter that affects how the sociodynamics change and what patterns are dominant. First, we show the average intensity W of patterns for G5 (five agents) and study the effect of diversity for $\sigma = 0.05$, 0.10, 0.20 and 0.30. Second, the patterns that have the greatest intensities and that are thus the most significant ones for monitoring the changes are shown for G4 and G5. In this case, the effect of collaterality on pattern formation in terms of models D, D+T and T is explored. Third, and last, the effect of group size on pattern formation is explored for G3, G4, G5 and G6.

In all cases, the pattern intensities are based on the strengths of peer-appreciation. However, an agent's self-appreciation is strongly correlated with peer-appreciations. The correlation of an agent's self-appreciation with the highest peer-appreciation towards it is shown in Fig. 2 for G5 with three different diversities: $\sigma = 0.05$, 01.0 and 0.30. In addition, the correlations of reciprocation of peer-appreciations are shown. In all cases, strong correlations exists, shown as the dark regions, for the self-appreciation and maximum peer-appreciation, especially for very high appreciations close to 1 that are typical for strong dyads (dark regions in the upper right corner in the figures) and for average appreciations close to 0.5 typical for egalitarian patterns (dark regions in the center of the figures). However, with increasing diversity, the correlations become weaker, as shown in Fig. 2. This is as expected because low diversity indicates strong homophily, which also enhances formation of the strong dyads, as will be shown later on.

Effects of competitiveness and diversity

The peer-appreciation patterns are best monitored through the pattern intensities W, which also contain information about the average strength of the patterns. In what follows, at low competitiveness ($\alpha < 0.3$), the egalitarian triadic patterns (300) always have an average strength of 0.5 (corresponding to the average initial value distribution, which is uniform from 0 to 1), whereas dyads 102 and leadership dyads 210 have a maximal average strength of 1. In this case, the W value for triad means that in the triad, the agent has a reciprocated peer-appreciation of strength W with a probability of 1, whereas for a dyad and leadership dyad, $W = 0.5$ means that the agent has reciprocated peer-appreciation of strength 1 with a probability W. A similar interpretation holds for embedded dyads.

As the results in Fig. 3 show, for low competitiveness $\alpha < 0.3$, there is a strong tendency to fully reciprocated and completely balanced links so that all agents have equal peer-appreciations. The peer-appreciations and self-appreciations are strongly correlated, so each agent appreciates all other agents as much as it appreciates itself. In this case, as shown in Fig. 3, all appreciation patterns are egalitarian triads (300). In the intermediate range of competitiveness $0.3 < \alpha < 0.6$ (the exact boundaries depending

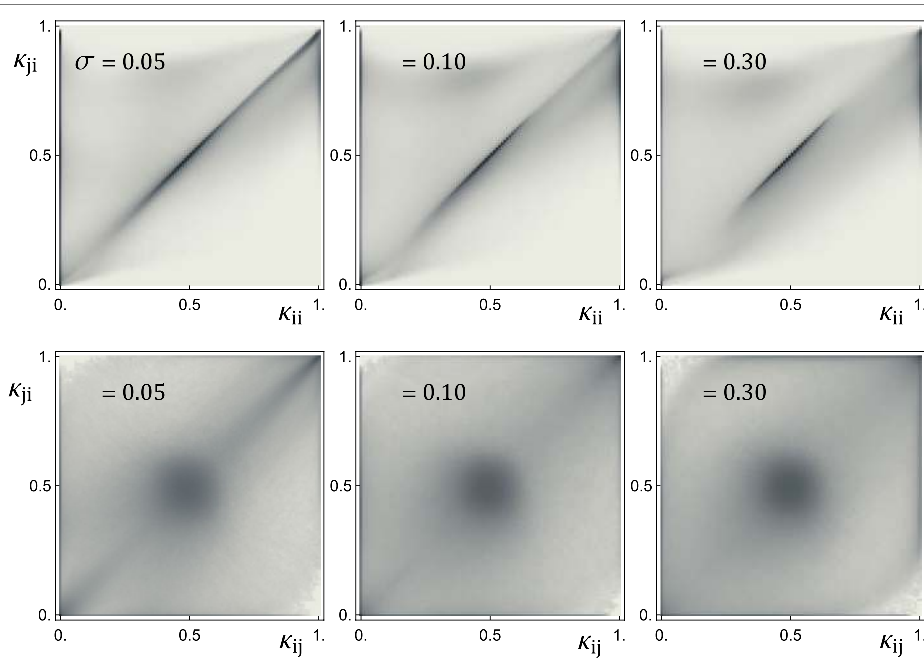

Fig. 2 Correlation of agents' self-appreciation with maximum peer-appreciation. Self-appreciation κ_{ii} correlated with maximum peer-appreciation κ_{ji} in G5 (*upper row*) and the correlation of reciprocation of peer-appreciations κ_{ij} and κ_{ji} (*lower row*). Results are for diversity $\sigma = 0.05$, 0.10 and 0.30 and with collaterality T

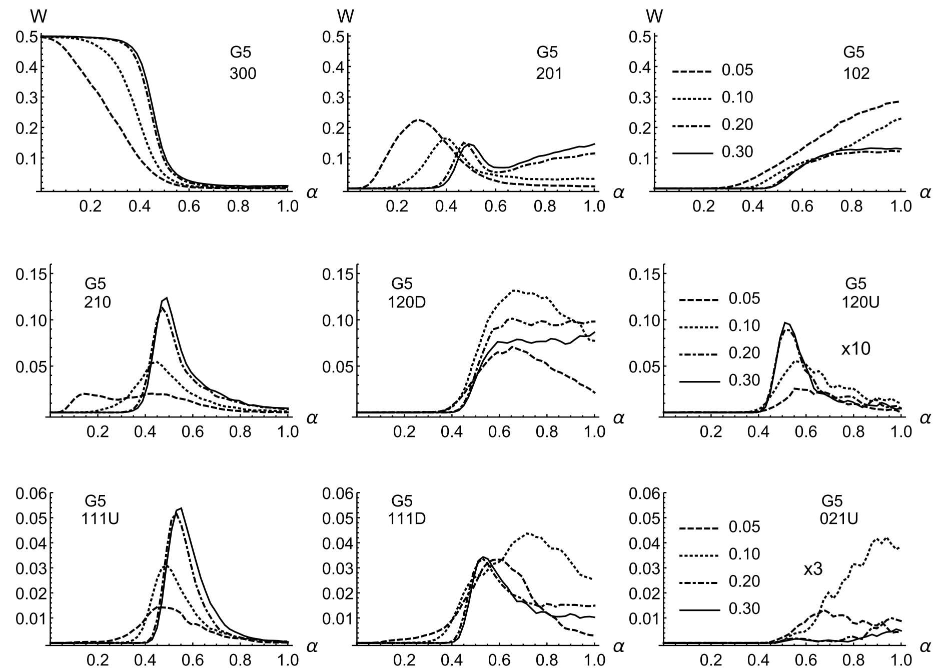

Fig. 3 The average intensities W of the nine most important peer-appreciation patterns. The intensities W for G5 are shown as a function of the competitiveness α. Results are for diversities $\sigma = 0.05, 0.10, 0.20$ and 0.30. Note that the intensities of patterns 021U are multiplied by a factor of 3 for better visibility

on diversity), leadership dyads (201) begin to dominate, as is shown in Fig. 3. In addition, triadic leadership patterns (210) also become abundant in this region. Finally, with increased competitiveness, $\alpha > 0.6$, it becomes rewarding for agents to form even stronger mutually reciprocated links. Thus, single dyads (102) begin to dominate. In addition, collateral triads (120D) become abundant. When the competitiveness increases, the average strength of self- and peer-appreciations increase to values close to the maximum of 1. This means that in the present model, the increased competitiveness leads to stronger self- and peer-appreciations than in situations of low competitiveness, but at the cost of breaking balanced egalitarian triads in favor of very strong isolated dyads. This kind of transition from egalitarian and balanced patterns to isolated strong dyads is probably not an uncommon situation in a collaborative but competitive environment (Lusher et al. 2014), and it is a well known phenomenon in social relations where mutual reciprocated ties become very strong (Simmel 1964; Yoon et al. 2013). However, such patterns, while strong in absence of other ties, are fragile if ties begin to form to a third party; stability is achieved only through isolation. A similar behavior in pattern intensities is typical for all similar ranges of competitiveness. Thus, the competitiveness α is the most important feature of agents' interaction in deciding the formation of appreciation patterns.

In addition to the competitiveness, the diversity σ also affects the formation of peer-appreciation patterns, but not to the same degree as the competitiveness. In Fig. 3, we show the intensities W of the nine most abundant peer-appreciation patterns for G5 with different diversities $\sigma = 0.05, 0.10, 0.20, 0.30$. If the diversity is low, an agent only interacts with agents whose peer-appreciation of it are greater than its self-appreciation.

The diversity, as was shown in Fig. 2, weakens the correlation between high self- and peer-appreciations. Consequently, increasing diversity lowers the formation of dyads and the patterns dependent on them, whereas lowering the diversity enhances dyad formation and lowers the formation of egalitarian patterns. This is easily interpreted as being a consequence of increased homophily and the reciprocation of appreciation of agents when competitiveness is high and diversity is low.

In addition to the patterns shown in Fig. 3, we also checked the cyclic patterns, but their abundance was so low that they were not of interest. The absence of cyclic patterns is known to be a typical feature of social dynamics in which the formation of reciprocated ties is common, e.g. when social ties are based on friendship or trust (Lawler et al. 2008; Yoon et al. 2013). In what follows, we focus on the six patterns that are the most abundant in all cases: egalitarian triads (300), dyadic leaders (201) and single dyads (102), which are the most abundant ones, and triadic leader (210) and collateral and broker dyads (120D and 120U), which are three next most abundant patterns.

Effects of collaterality

Collaterality refers to relationships in which one agent in a reciprocal dyadic pair of agents connects to a third. Fig. 4 shows the effect of collaterality for G4 and G5 in the case of the most abundant patterns. The results show that an increase in collaterality increases the abundance of triadic leaders (210) and collateral and broker triads (120D and 120U, respectively), while abundances of dyads (102) and dyadic leaders (201) are decreased. This is as expected because collaterality tends to add a third agent to the reciprocated dyads. Similarly, increased collaterality causes the egalitarian triads (300) to survive in the region of higher competitiveness.

The effect of collaterality on the abundance of different patterns is qualitatively similar in G4 and G5, although the intensities of patterns differ based on the competitiveness. In G4, the abundance of dyads (102) and dyad-dependent patterns (e.g. 120D and 120U) is higher than in G5, which is obviously due to fact that a group with an odd number of agents can be split into mono-dyadic units. Therefore, dyads and dyad-dependent collateral and broker patterns are more abundant in G4. For dyadic and triadic leadership patterns, group size dependent differences are insignificant in comparison to the differences caused by collaterality.

In summary, if the collaterality of social interactions can be controlled in real groups (as presumably it can be), it provides opportunities to tune the reciprocated dyadic patterns of peer-appreciations; by increasing collaterality we can suppress the formation of isolated dyadic and dyadic leader patterns and the group can be maintained in balanced egalitarian mode. However, whether or not the egalitarian mode is more preferred than the dyadic mode is, of course, a decision that depends on the goals of the group. In addition, as can be expected, in a small group, it matters whether the group consists of an even or odd number of agents. If egalitarian patterns are of interest, the best option is a group consisting of an odd number of agents and high collaterality, in which case "broker" positions are abundant and lead to the stabilization of egalitarian triads and connected dyads. If strong dyads are favored, the group should have an even number of agents and zero collaterality.

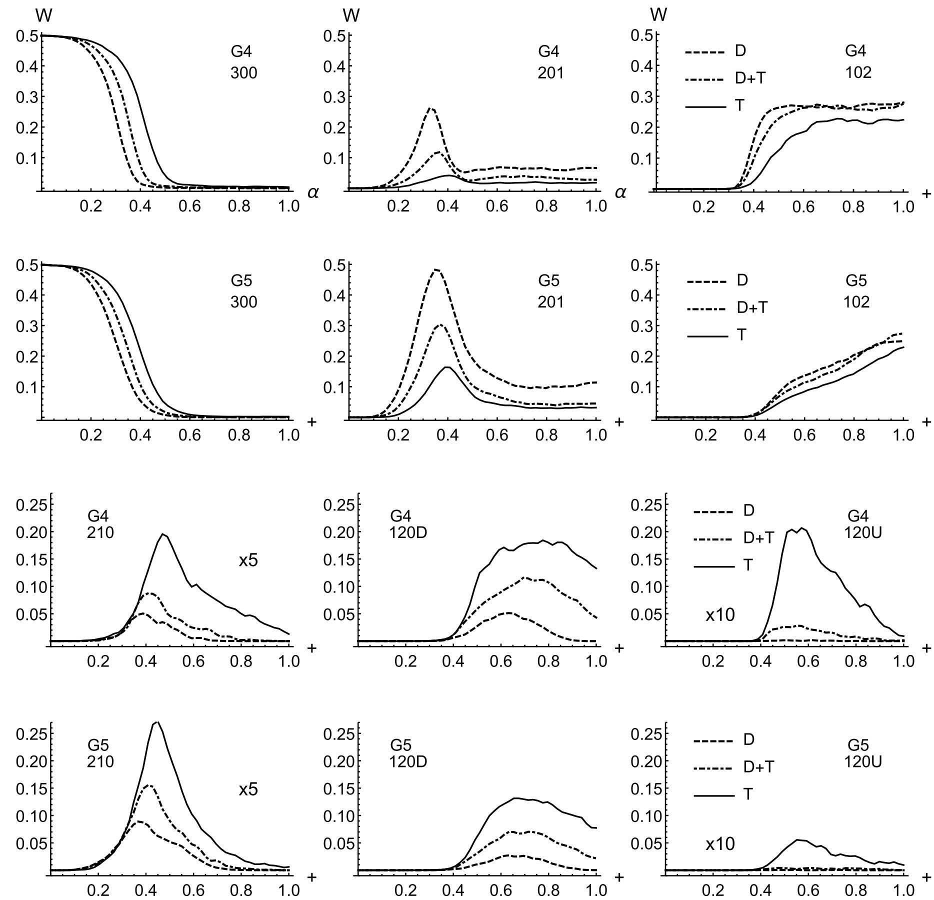

Fig. 4 The intensities W_{pats} of peer-appreciation patterns for different collateralities. The average intensities W_{pats} are shown for dyadic (D), dyadic-triadic (D+T) and triadic (T) collateralities as a function of the competitiveness α .The results of the pattern counts are averages of 2000 repetitions for each data point. Note that the intensities of patterns 210 and 021U are multiplied by factors of 5 and 10, respectively, for better visibility

Effects of group size

The dyadic and triadic patterns are the elementary blocks of higher-order social patterns. As is evident in the case of collaterality, group size greatly affects what kinds of patterns emerge. It is of interest to see how small groups size affects the relative proportions of these elementary units, so simulations were also carried out for groups of sizes 3, 4 and 6 (groups G3, G4 and G6).

The results in Fig. 5 show that the smallest group, G3, has the highest relative abundance of egalitarian triads (300) and collateral triads (120D), whereas G4 has the highest abundance of dyads (102) and broker triads (120U). On the other hand, the larger the group, the larger the relative abundance of dyadic and triadic leaders (201 and 210, respectively). Large group size seems to enhance the effect of reciprocation, thereby allowing easy splitting to dyadic, reciprocated patterns and patterns containing these units. The large abundance of dyads and broker triads (containing a dyad) seems to be a special feature of G4 that is enhanced by increased diversity (related to the decreased homophily) contrary to the tendency of dyadic features in most cases to be enhanced by decreased diversity (related to the decreased effect of homophily). The detailed origin of

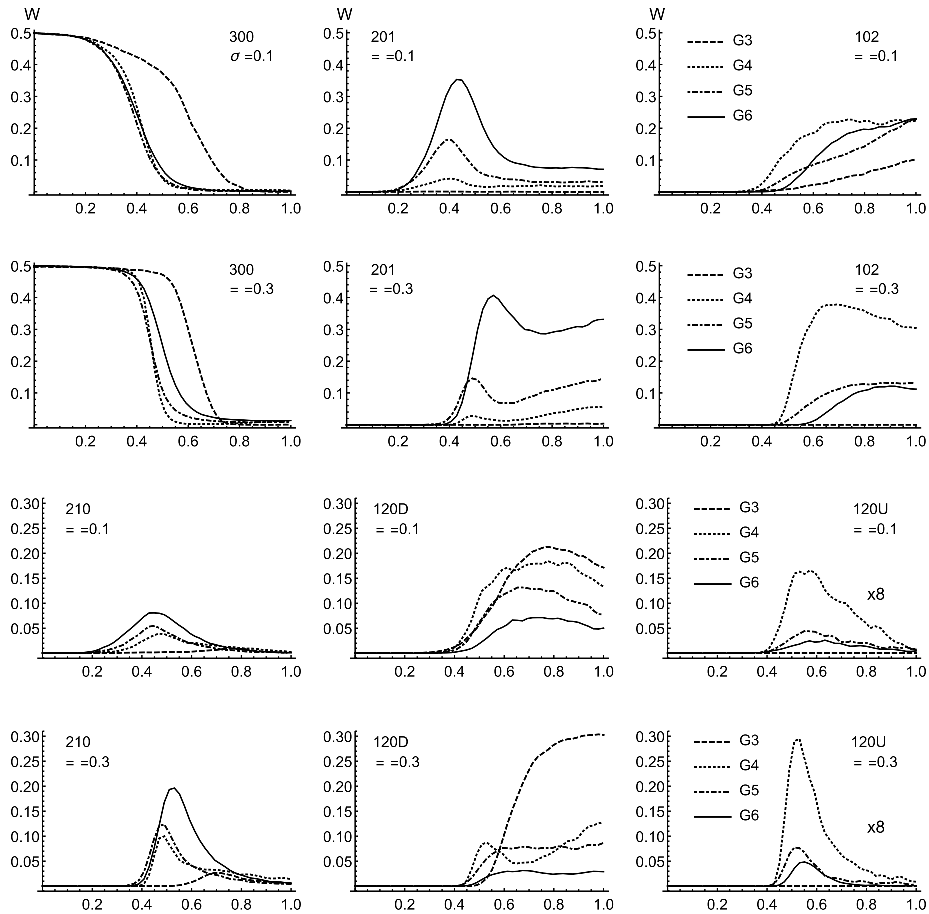

Fig. 5 The patterns of the highest intensities *W* for groups G3–G6. Results are shown for diversity $\sigma =$ 0.1 and 0.3 and collaterality D. For pattern 120U, values are multiplied by a factor of 8 for better visibility

this effect is presumably related to the possibility that when a triad is broken, the likelihood of which is enhanced by diversity, a second dyad is easily formed in G4.

In G6 groups, the dyadic leaders are also quite common in situations of high competitiveness, which is as expected because the larger the group, the more it provides combinatorial freedom to form connected pairs of dyads when dyads become more abundant. This means that in large groups, in case of high competitiveness, sub-groups of dyadic leaderships become common.

In summary, on the basis of the results for different group sizes, it is evident that the most egalitarian peer-appreciations patterns are always obtained for low competitiveness independent of group size. The larger the group, the higher competitiveness it can tolerate while maintaining the egalitarian mode in which all members benefit (transition to dyads and dyadic leadership patterns occur at larger values of the competitiveness α the larger the group size). However, with increasing group size, the relative amount of dyads increases and larger groups break up into dyadic groups more easily after the transition has taken place. This is expected when reciprocation is high at high competitiveness.

Discussion

We have here demonstrated that many patterns of interaction that are found to be typical in collaboration or discourse in task-engaged small groups are reproduced by the agent-based model in which the interaction of agents is based on their self- and peer-appreciations. In the model, the self- and peer-appreciations evolve dynamically through constant peer-to-peer comparisons. The model introduced here is meant to be a kind of minimal model in which only agents' self-conception, conception of peers and very basic comparative evaluations are taken into account. Similar agent-based models have been introduced, particularly, in the context of opinion dynamics [for reviews, see (Castellano et al. 2009; Lorenz 2007; Fortunato et al. 2005) and references therein] and the formation of social structures and hierarchies [see (Snijders et al. 2010; Carletti et al. 2011; Gallos et al. 2012; Murase et al. 2014) and references therein]. Many of these kinds of agent-based models address general and universal features of large networks. Also in these cases, the strength of reinforcement and reciprocation of ties affect the network structure in fundamental ways (Carletti et al. 2011; Murase et al. 2014). The results for large groups and the formation of their structures, however, go well beyond the scope of the present work. Here the focus is on small groups, with potential applications on groups from three to seven members, which is usually the size of collaborative learning and task-performing groups. In what follows, we briefly discuss some agent-based models that have similar focuses and goals and which are thus more closely related to the model presented here than the agent-based models for network formation and the dynamics of very large groups.

Some previous agent-based models for cooperative and competitive small groups have modelled the effect of status comparisons as direct comparisons of the statuses of individual agents (Caram et al. 2010, 2015). Also, in these cases, agents of similar statuses form sub-groups or cliques. These models, like ours, also include the notion that such a status is often bounded and cannot increase or decrease without limits, thus requiring the introduction of logistic growth type terms in update rules. This restriction clearly has an effect on how the groups are formed (Caram et al. 2015). More refined agent-based models of sub-group formation in small groups take into account the fact that the dynamics and status comparisons depend not only on agents' status but also on other agents' beliefs or expectations about other agents' statuses. This effect is taken into account in agent-based models for small groups in which agent-to-agent relations are reinforcing (Lipowski et al. 2014). Such models lead to the formation of different types of leadership patterns, and depending on the strength of the reinforcement, the leadership can be absolute, symmetric (all links of equal strength) or asymmetric (one link stronger than others). These patterns are very similar to the leadership patterns found in the present study.

Some agent-based models of the formation of social ties take a further step by taking into account both the agents' status and the reinforcement of agent-to-agent links so that the comparisons are between the agent's status and the other agents' expectations of that status. Comparison to models where the statuses of agents are compared directly or where only agent-to-agent connections are the basis of comparison, such models add an important sociological component: how an individual's self-conception is affected by peer evaluations. Many social learning and sociological theories claim that such comparisons are essential to shaping social relations (Lawler 2001; Lawler et al. 2008; Bandura 1997, 2006). Also, on the cognitive level, awareness of how our peer's view us (often referred to as theory of the mind) is seen as fundamentally guiding our decisions and actions (Sun 2006). These key notions are included in recent agent-based models of the formation of elite and egalitarian groups, within the so-called Leviathan model (designed to follow Thomas Hobbes's view and thus named Leviathan) of the role of vanity in human social and political life (Deffuant et al. 2013). The Leviathan model has proven quite successful in illuminating how self-appraisal and vanity, when very strong, leads to the formation and consolidation of elite groups, and simultaneously, to the marginalisation of those outside the elite. When vanity is strong, absolute dominance results (Deffuant et al. 2013). Similar results are produced by agent-based models in which agents' statuses in discourse depends on their (epistemic) credits and how credits become reinforced through participation and interaction (discourse) in small groups (Zollman 2012, 2013). In all these cases, status, status expectations and their comparisons dynamically generate the structures of social groups. When sensitivity to such comparisons is strong enough, elite groups are formed. This kind of dynamics is quite convincingly demonstrated by a recent agent-based model tailored to describe the formation of status hierarchies (Grow et al. 2015) following the sociological model of the creation of status hierarchies (Skvoretz and Fararo 1996). In that model, the hierarchical structure is entirely an outcome of internal comparisons (Grow et al. 2015). The agent-based model presented here is, in its spirit and aim, very closely related to the Leviathan model (Deffuant et al. 2013) and the status construction model (Grow et al. 2015) in that it also identifies the relevant micro-level mechanism of how individual relations are formed.

An additional feature of the agent-based model introduced here is how it utilizes graph counting to monitor the development of stable patterns. Approaches in which complex patterns of interaction are broken down in more elementary units of simpler patterns, so-called motifs, is common for network models of sociological, economical and biological systems (Boccaletti et al. 2006; Milo et al. 2004, 2002). Motifs act as fingerprints of more complex systems, and although it is in practice difficult to unambiguously decompose large systems into a collection of motifs, the distribution of motifs is unique enough to differentiate structurally and dynamically different networks (Milo et al. 2004, 2002; McDonnell et al. 2014). In our work, we have followed this approach and used the counting of differently connected triads.

In regard to the empirical research, the similarity of simulation results to empirical findings suggests that in empirical research of small group collaboration, learning and performance, closer attention should be paid to the individuals' expectations of their own and their peers' competencies and performances, and on how sensitive the individuals are to the outcomes of these comparisons. The model discussed here is developed to guide our ongoing empirical research on discourse and dialogue in small, task-oriented learning groups. The preliminary empirical results indicate that the most common patterns in these cases are reciprocated dyads and triads, as expected on the basis of trust-based social relations, or affect theory of exchange, whereas patterns indicating leadership or hierarchical structure are not found. In this ongoing research, the agent-based model has served as a valuable tool for conceptualisation and reasoning because it helps to clarify how group size, reciprocation of query-response sequences and individuals' psychological factors (sensitivity to comparisons, competitiveness, etc.) may affect the formation of discourse and dialogue patterns. The model parameters are partly related to factors that are intrinsic, and partly to factors that can be manipulated. The competitiveness is supposedly related to spontaneous formation of appreciative relationships, but it can nevertheless be enhanced or moderated by the external conditions and instructional strategies that regulate group dynamics. The collaterality and the diversity, however, depend more on the practical arrangements and can be altered according to how the group work is organized. In addition, the group size is of much importance. For example, the specific nature of groups of four members should be recognised. All these factors may be essential in determining how the discourse and dialogue are shaped and what kinds of interaction patterns emerge. Although the results are inconclusive, the model and its results have led to new designs for the empirical approach. We believe that it will ultimately lead us to further understanding why certain patterns of discourse correlate with better learning outcomes.

Conclusions

We have modelled the formation of patterns of mutual appreciation in small groups by using an agent-based model. The patterns of appreciation describe the relationships involved in how individuals appreciate each other in collaborative or task-oriented groups in which mutual trust or benefit is assumed to regulate social interactions. In such patterns, the group members' appreciations of peers are strongly correlated with self-appreciation. The competitive comparisons of appreciations and the (sensitivity to) competitiveness, however, essentially affect the type of patterns that develop in the group. In the present model, the most common peer-appreciation patterns are egalitarian triadic patterns in situations of low and moderate competitiveness, dyadic leadership type patterns in situations of intermediate competitiveness, and dyadic patterns in the situations of high competitiveness. A typical feature of the model is strong reciprocation, which leads to strong dyads in regions of high competitiveness, as in trust- or exchange-based social relations (Block 2015; Yoon et al. 2013; Lusher et al. 2014). As expected in case of strong reciprocation, cyclic and transitive triadic forms are nearly absent. These results correspond well with what can be expected on the basis of the affect theory of social exchange (Lawler et al. 2008; Lawler 2001) and social learning theory (Bandura 1997, 2006). In this regard, the model developed here seems to be successful in modelling the effects of social comparisons on mutual reciprocal appreciation patterns in small groups.

In addition to the competitiveness, (tolerance to) the diversity of group members in regards to their responses to differences in self- and peer-appreciations also affects the formation of appreciation patterns. The weaker the diversity, the more homophilic the relations tend to be and the more enhanced the formation of strong dyads becomes. In such strong dyads, the agents support and enhance each others, thereby making self-appreciations and peer-appreciations high. The cost of forming of strong dyads, however, is the marginalisation of agents outside of these dyads and low self-appreciations of marginalised agents.

The model results suggest that many aspects of forming dyadic and triadic patterns that are recognized by research on collaboration, information sharing and communication patterns (Hogan et al. 1999; Enyedy 2003; Barron 2003; van Boxtel et al. 2000; Schwarz and Linchevski 2007; Sangin et al. 2011; Stahl et al. 2014) can be traced back to strong reciprocation, which in the case of low or moderate competitive comparisons of appreciation lead to egalitarian triads but in the presence of strong competitive comparisons lead to strong dyadic patterns.The notion that cyclic patterns are absent and transitive patterns are only featured weakly in the present model is due to strong reciprocation, which suppresses their formation. The fact that such patterns have not been reported in studies of collaborative discourse and communication suggests that reciprocation based on appreciation is a strong candidate for explaining the formation of the reciprocated dyadic and triadic patterns reported here.

This model study, though highly idealised, suggests several factors which can be used to tune group dynamics. Many of these factors can be readily recognized as having practical value, and, for example, group size and collaterality are well-known factors from practice. In addition, the model highlights the possibility that affecting (tolerance to) diversity and (sensitivity to) competitiveness may result in significant changes in self- and peer-appreciation patterns. In this way the model and results based on it help to conceptualise the different modes of interaction and their role in group dynamics in a more systematic way and help to design controlled experiments in which the effect of different factors resolved here can be empirically tested.

Authors' contributions

ITK was the lead author of the manuscript, conceptualising the problem and designing and implementing the agent-based model. MN made key contributions to conceptualising the problem, analysis of simulation data and recognising the practical applications of the model. All the authors read and approved the final manuscript.

Acknowledgements

We thank the Department of Physics, University of Helsinki, Finland, for making this research possible.

Competing interests

The authors declare that they have no competing interests.

Funding

This research has received no external funding.

Received: 30 June 2016 Accepted: 12 October 2016
Published online: 18 October 2016

References

Bandura A (1997) Self-efficacy: the exercise of control. Freeman, New York
Bandura A (2006) Toward a psychology of human agency. Perspect Psychol Sci 1:164–180
Barron B (2003) When smart groups fail. J Learn Sci 12:307–359
Block P (2015) Reciprocity, transitivity, and the mysterious three-cycle. Soc Netw 40:163–173
Boccaletti S, Latora V, Moreno Y, Chavez M, Hwang D-U (2006) Complex networks: structure and dynamics. Phys Rep 424:175–308
Caram LF, Caiafa CF, Ausloos M, Proto AN (2015) Cooperative peer-to-peer multiagent-based systems. Phys Rev E 92:022805
Caram LF, Caiafa CF, Proto AN, Ausloos M (2010) Dynamic peer-to-peer competition. Physica A 389:2628–2636
Carletti T, Righi S, Fanelli D (2011) Emerging structures in social networks guided by opinion exchanges. Adv Compl Syst 14:13–30
Castellano C, Fortunato S, Loreto V (2009) Statistical physics of social dynamics. Rev Mod Phys 81:591–646
Deffuant G, Carletti T, Huet S (2013) The Leviathan model: absolute dominance, generalised distrust, small worlds and other patterns emerging from combining vanity with opinion propagation. J Artif Soc Soc Simul 16:1–28
Enyedy N (2003) Knowledge construction and collective practice: at the intersection of learning, talk, and social configurations in a computer-mediated mathematics classroom. J Learn Sci 12:361–407
Epstein JM (2008) Why model? J Artif Soc Soc Simul 11:1–12

Fortunato S, Latora V, Pluchino A, Repisarda A (2005) Vector opinion dynamics in a bounded confidence consensus model. Int J Mod Phys C 16:1535–1551
Gallos LK, Rybski D, Liljeros F, Havlin S, Makse HA (2012) How people interact in evolving online affiliation networks. Phys Rev X 2:031014
Grow A, Flache A, Wittek R (2015) An agent-based model of status construction in task focused groups. J Artif Soc Soc Simul 18:4–22
Hogan K, Nastasi BK, Pressley M (1999) Discourse patterns and collaborative scientific reasoning in peer and teacher-guided discussions. Cognit Instr 17:379–432
Itzkovitz S, Milo R, Kashtan N, Ziv G, Alon U (2003) Subgraphs in random networks. Phys Rev E 68:026127
Lawler EJ (2001) An affect theory of social exchange. Am J Sociol 107:321–352
Lawler EJ, Thye SR, Yoon J (2000) Emotion and group cohesion in productive exchange. Am J Sociol 106:611–657
Lawler EJ, Thye SR, Yoon J (2008) Social exchange and micro social order. Am Sociol Rev 73:519–542
Lipowski A, Lipowska D (2012) Roulette-wheel selection via stochastic acceptance. Physica A 391:2193–2196
Lipowski A, Lipowska D, Ferreira AL (2014) Emergence of social structures via preferential selection. Phys Rev E 90:032817
Lorenz J (2007) Continuous opinion dynamics under bounded confidence: a survey. Int J Mod Phys C 18:1819–1838
Lusher D, Kremer P, Robins G (2014) Cooperative and competitive structures of trust relations in teams. Small Gr Res 45:3–36
McDonnell MD, Yaveroglu OM, Schmerl BA, Iannella N, Ward LM (2014) Motif-role-fingerprints: the building-blocks of motifs, clustering-coefficients and transitivities in directed networks. PLoS ONE 9:e114503
Milo R, Itzkovitz S, Kashtan N, Levitt R, Shen-Orr S, Ayzenshtat I, Sheffer M, Alon U (2004) Superfamilies of evolved and designed networks. Science 303:1538–1542
Milo R, Shen-Orr S, Itzkovitz S, Kashtan N, Chklovskii D, Alon U (2002) Network motifs: simple building blocks of complex networks. Science 298:824–827
Moody J (1998) Matrix methods for calculating the triad census. Soc Netw 20:291–299
Murase Y, Torok J, Jo H-H, Kaski K, Kertesz J (2014) Multilayer weighted social network model. Phys Rev E 90:052810
Onnela J-P, Saramaki J, Kertesz J, Kaski K (2005) Intensity and coherence of motifs in weighted complex networks. Phys Rev E 71:065103
Rivera MT, Soderstrom SB, Uzzi B (2010) Dynamics of dyads in social networks: assortative, relational, and proximity mechanisms. Annu Rev Sociol 36:91–115
Sangin M, Molinari G, Nussli M-A, Dillenbourg P (2011) Facilittaing peer knowledge modelling: effects of a knowledge awareness tool on collaborative learning outcomes and processes. Comput Hum Behav 27:1059–1067
Schwarz BB, Linchevski L (2007) The role of task design and argumentation in cognitive development during peer interaction: the case of proportional reasoning. Learn Instr 17:510–531
Simmel G (1964) Quantitative aspects of the group. In: Wolfff KHW (ed) The sociology of georg simmel. The Free Press, Glencoe, pp 108–169
Skvoretz J, Fararo TJ (1996) Status and participation in task groups: a dynamic network model. Am J Sociol 101:1366–1414
Snijders TAB, van de Bunt GG, Steglich CEG (2010) Introduction to stochastic actor-based models for network dynamics. Soc Netw 32:44–60
Stahl G, Law N, Cress U, Ludvigsen S (2014) Analyzing roles of individuals in small-group collaboration processes. Int J Comp-Supp Coll Learn 9:365–370
Sun R (2006) Cognition and multi-agent interaction. Cambridge University Press, Cambridge
van Boxtel C, van der Linden J, Kanselaar G (2000) Collaborative learning tasks and the elaboration of conceptual knowledge. Learn Instr 10:311–330
Volet S, Summers M, Thurman J (2009) High-level co-regulation in collaborative learning: how does it emerge and how is it sustained? Learn Instr 19:128–143
Yoon J, Thye SR, Lawler EJ (2013) Exchange and cohesion in dyads and triads: a test of simmelian hypothesis. Soc Sci Res 42:1457–1466
Zollman KJS (2012) Social network structure and the achievement of consensus. Politics Philos Econ 11:26–44
Zollman KJS (2013) Network epistemology: communication in epistemic communities. Philos Compass 8:15–27