Progress of Theoretical Physics, Vol. 64, No. 2, August 1980

Formation of the Giant Planets

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(Received March 14, 1980)

The structure of a gaseous envelope surrounding a protoplanet has been investigated in connection with the formation of the giant planets. Under the assumptions of spherical symmetry and hydrostatic equilibrium, the structure has been calculated for the regions of Jupiter, Saturn, Uranus and Neptune. Energy transfer in the envelope has been taken into account precisely.

When the core mass increases beyond some critical value, the envelope cannot be in hydrostatic equilibrium and collapses onto the core. The most remarkable result is that a common relation between the core mass and the total mass holds irrespectively of the regions in the solar nebula. Therefore, at the collapse, the core mass becomes almost the same regardless of the regions in the nebula. This is consistent with the conclusion obtained from the theory of internal structure of the present giant planets. The grain opacity in the envelope should be about $1\,\mathrm{cm}^2/\mathrm{g}$ in order to explain the estimated core mass (about 10 Earth's mass) of the giant planets. This value of the grain opacity is larger than that expected before.

§ 1. Introduction

The internal structure of the giant planets has been investigated in detail recently owing to improvements in the equation of H_2 gas and developments in observations. A common feature to the four giant planets, namely, Jupiter, Saturn, Uranus and Neptune, is that each planet has a core inside it and a gaseous envelope surrounding the core. What we call the core here is composed of metal/rock and ice such as H_2O, CH_4 and NH_3. Meanwhile the main components of the envelope are H_2 and He. A remarkable conclusion in the current theory is that all the giant planets have almost the same core masses of about $10M_E$ (M_E: the Earth's mass).[1),2)]

Two ideas of the formation of the giant planets have been proposed until now. Firstly, a protoplanet grows through accretion of planetesimals and when its mass reaches some critical value the gaseous envelope surrounding the protoplanet becomes gravitationally unstable to collapse onto it.[3),4)] Secondly, the solar nebula itself fragments directly because of gravitational instability to form a lot of giant gaseous protoplanets.[5),6)] In the second idea, the core of a giant planet is considered to be formed in the giant gaseous protoplanet through sedimentation of grains toward its center.[6)] This idea premises the assumption that the solar nebula is massive (about 1 solar mass). However, it is apprehended that some difficulties appear in this model. For example, it seems to be difficult to dissipate the large quantity of the gaseous envelope of the inner planets. In the present paper, we will adopt the solar nebula model which is less massive (about 0.04 solar mass[7]) and gravitationally stable, and will discuss the formation of the giant planets on the basis of the first idea.

Two studies have been made from the same standpoint as ours. Perri and Cameron[3] have discussed the equilibrium solutions of the envelope, making an assumption that the envelope is wholly adiabatic. They have concluded that when the mass of the core (we call a protoplanet a core hereafter) becomes greater than about $70M_E$ for the Jupiter's region, the envelope collapses onto the core. In this case, however, the core mass is too large in comparison with that of the present Jupiter estimated in the recent theory.[1] On the other hand, Mizuno, Nakazawa and Hayashi[4] (hereafter referred to as Paper I) have investigated the structures of the envelope, approximating it to consist of isothermal layers and adiabatic layers. The isothermal layers have been introduced because the outer region of the envelope is optically thin. They have pointed out that the core mass at the time of the collapse of the envelope is considerably smaller than the value found by Perri and Cameron,[3] though its value depends on the grain opacity in the envelope. However, their core mass at the time of the collapse depends on the distance from the Sun, which makes it difficult to theoretically understand the common core masses to the four giant planets. This is due to their oversimplification in the thermal structure of the envelope, namely, in the energy transfer.

The purpose of the present paper is to investigate the structure of the envelope in precise consideration of the energy transfer and examine the formation processes of the giant planets. At the same time, we will discuss the influence of outer boundary conditions on its formation processes. In § 2, we will describe the assumptions, the basic equations and the boundary conditions. The opacity, which is essential in the energy transfer, will be described in § 2 and Appendix. In § 3, it will be found from the numerical results that the relation between the total mass (core mass plus envelope mass) and the core mass is identical in all the regions of the giant planets. An analytical argument for this identity will be given in § 4. In § 5, we will discuss the history of the formation of the giant planets.

§ 2. Basic equations and assumptions

The situation considered here is as follows. A protoplanet grows in the solar nebula through accretion of planetesimals. Hereafter we call the protoplanet a core. Since the gravity becomes strong with the increase of the core mass, the nebula gas is attracted more and more inside the Hill sphere and the core-envelope structure is formed. Our present purpose is to investigate the structure of the envelope in order to know the relation between the core mass M_c and the total mass M_t inside the Hill sphere; M_t is the sum of the core mass and the envelope mass. In this section, we will describe the basic equations, the assumptions, the boundary conditions and the opacity. However, considerable part has already been described in Paper I and we will only touch on such a part briefly. If necessary, the reader should refer to Paper I.

a) *Assumptions*

The following assumptions are made for simplicity.

1) The envelope is spherically-symmetric and in hydrostatic equilibrium.
2) The growth rate of the core mass, namely, the mass accretion rate $\dot{M}_c$ is constant with the value of $10M_E/10^7$ yr.[8]
3) The luminosity L is supplied by the gravitational energy which accreting planetesimals release. Also L is constant throughout the envelope.
4) The mean density $\bar{\rho}$ of the core is $5.5\,\mathrm{g/cm^3}$, i.e., the mean density of the Earth.
5) The equation of state for ideal gas is used ($X=0.73$, $Y=0.25$).

Among these assumptions, assumptions (1), (4) and (5) are the same as in Paper I. Against assumption (2), in actuality the mass accretion rate may be a function of not only time but also distance from the Sun, but it has not been determined precisely yet. Hence, we simply regard it as a constant. We will discuss the effect of $\dot{M}_c$ on the results in § 3. As to assumption (4), we have confirmed that the effect of the value of $\bar{\rho}$ is very small.

Next, we will comment on assumption (3). There are two possible ways for planetesimals to dissipate the gravitational energy into the thermal energy. One is dissipation through gas drag force acting on the planetesimals. The gas drag force is proportional to the gas density and, hence, the dissipation dominates in the regions near the core. The other one is dissipation at the time of the collision of planetesimals with the core surface. Anyway, dissipation near the core surface is greater and it is expected that the luminosity is constant throughout the envelope. As an alternative source of the luminosity, the gravitational energy due to the gravitational contraction of the envelope can be considered besides the sources mentioned above; we will argue the effect of the additional source in § 3.

b) *Basic equations*

The structure of the envelope is determined by the following equations under assumption (1):

$$\frac{1}{\rho}\frac{dP}{dr} = -\frac{GM_r}{r^2} \tag{1}$$

and

$$\frac{dM_r}{dr} = 4\pi r^2 \rho\,, \tag{2}$$

where ρ and P are the density and the pressure, respectively. The mass inside a sphere of radius r (including the core mass) is denoted by M_r and G is the gravitational constant.

Next, we will express the equations governing the thermal structure of the envelope. Let us divide the envelope into two regions according to the optical depth τ measured from the Hill radius r_0, where $\tau=\int_r^{r_0}\kappa\rho dr$ and κ is the opacity. In the region where $\tau<2/3$, that is, outside the photosphere which is defined by $\tau=2/3$, radiation transports energy. According to Hayashi et al.[9] the temperature distribution in such a region is given by

$$T^4=T_0{}^4+\frac{L}{8\pi r^2}\frac{1+\frac{3}{2}\tau}{2-\frac{3}{2}\tau}. \tag{3}$$

In Eq. (3), T and T_0 are the temperatures at radius r and the Hill radius r_0, respectively. Inside the photosphere, the energy transfer is dealt with as follows. Let us define the radiative and adiabatic temperature gradients by $\nabla_{\rm rad}=(3\kappa L/4\pi a_s cGM_r)\times(P/T^4)$ and $\nabla_{\rm ad}=(\partial\ln T/\partial\ln P)_s$, respectively. In these equations, a_s, c and s are the radiation density constant, the speed of light and the specific entropy of the gas, respectively. If $\nabla_{\rm ad}<\nabla_{\rm rad}$, the energy is transported by convection and we have

$$\frac{ds}{dr}=0. \tag{4}$$

Inversely, when $\nabla_{\rm rad}<\nabla_{\rm ad}$, radiation carries energy and the temperature gradient is given by

$$-\frac{4a_s cT^3}{3\kappa\rho}\frac{dT}{dr}=\frac{L}{4\pi r^2}. \tag{5}$$

Since the radiation pressure is negligible, the equation of state is simply given by

$$P=\frac{k}{\mu H}\rho T, \tag{6}$$

where k and H are the Boltzmann constant and the mass of a hydrogen atom, respectively. The mean molecular weight, μ, is a function of ρ and T because we take account of dissociation of H_2 molecule and ionization of H atom.

c) *Boundary conditions*

The boundary conditions at the Hill radius r_0 are given by

$$\rho=\rho_0,\quad T=T_0,\quad M_r=M_t \quad \text{at} \quad r=r_0\equiv\left(\frac{M_t}{3M_\odot}\right)^{1/3}a, \tag{7}$$

where ρ_0 and T_0 are the density and the temperature of the solar nebula at the distance a from the Sun, respectively, and $M_\odot$ is the solar mass. As for the model of the solar nebula we adopt the model given by Kusaka et al.;[7] ρ_0, T_0

Table I. Physical quantities of the solar nebula given by Kusaka et al.[7]

Region	Jupiter	Saturn	Uranus	Neptune
Distance from the Sun a (A. U.)	5.20	9.54	14.6	17.2
Density ρ_0 (g/cm^3)	1.5×10^{-10}	6.2×10^{-12}	6.9×10^{-13}	3.0×10^{-13}
Temperature T_0 (K)	97	73	54	45

and a are tabulated in Table I. Furthermore the total mass M_t is given prior to the calculation.

The inner boundary conditions are obtained as follows. From assumption (4), the core mass M_c is defined by

$$M_r(r=r_b)=\frac{4\pi r_b^3}{3}\bar{\rho}(\equiv M_c), \tag{8}$$

where r_b is the core radius. The luminosity must satisfy with the following equation according to assumption (3), i.e.,

$$L=[-\phi(r_b)]\dot{M}_c, \tag{9}$$

where $\phi(r_b)$ is the gravitational potential at r_b and given by

$$\phi(r_b)=-\int_{r_b}^{r_0}\frac{GM_r}{r^2}dr. \tag{10}$$

Since there are five boundary conditions for three equations, it is necessary to find the suitable values for M_c and L in order to fulfill the boundary conditions. That is, the problem we have to solve is mathematically an eigenvalue problem.

d) *Opacity*

There are two opacity sources. One is due to grains and the other one is due to molecules and atoms in gas phase.

If the wave length of radiation is larger than the size of grains, the grain opacity is proportional to the mass of floating grains.[10] The mass depends on temperature owing to evaporation of grains. First, let us consider the outermost region of the envelope where the temperature T is lower than 170 K. If the abundance of grains relative to hydrogen (by weight) in this region is f times that in interstellar clouds, the grain opacity κ_g in this region is given by

$$\kappa_g=f\cdot(\kappa_g)_{\mathrm{I.C.}}, \tag{11}$$

where $(\kappa_g)_{\mathrm{I.C.}}$ is the grain opacity in interstellar clouds and about 1 cm^2/g. We call f simply a grain depletion factor hereafter. Owing to the low temperature of the solar nebula in the regions of the giant planets, the grains in the outermost region of the envelope consist of not only metallic/rocky grains but also icy grains

(H_2O, CH_4 and NH_3). According to Podolack and Cameron,[11] the mass ratio of metallic/rocky to icy grains is 0.22 : 0.78. In the region where $170\,K<T<1600\,K$, icy grains evaporate and the grain opacity drops to $0.22\,f(\kappa_g)_{\rm I.C.}$. If $T>1600$ K, metallic/rocky grains evaporate and the grain opacity vanishes.[12]

Therefore, if the grain depletion factor f in the outermost region of the envelope is given, the grain opacity can be determined throughout the envelope. However, f has not been evaluated precisely yet, for some complicated processes govern the factor f as will be mentioned in § 5. Accordingly we regard f as a parameter and change it over the range from zero to unity.

As to the opacity due to molecules and atoms in gas phase, the following sources are taken into account; H_2 pressure-induced absorption, molecular absorption due to H_2O, CO and OH, Rayleigh scattering of H_2 and H, electron scattering, and atomic absorption. The argument about the gaseous opacity will be summarized in Appendix. The gaseous opacity for the case $f=1\times10^{-2}$ is shown in Fig. 3.

§ 3. Numerical results

a) *Total mass* (M_t) *and core mass* (M_c) *relation*

In Fig. 1 the M_t-M_c curves are shown for Jupiter's region. Let us consider first the curve with the fixed grain depletion factor f. When M_t is small, the mass of the nebula gas attracted within the Hill sphere, namely, the envelope mass is very small and M_t nearly equals M_c. As M_t increases, the M_t-M_c curve deviates from the line of $M_t=M_c$. Then M_c reaches the maximum and finally decreases. In the case $f=1$ and 1×10^{-1}, M_c has the second maximum and subsequently M_c decreases (hereafter we call the maximum of M_c—the first maximum if there are two maxima—the critical core mass and also call the corresponding model the critical model). The qualitative character resembles those of Perri and Cameron[3] and Paper I, but quantitatively speaking, M_c derived in the present calculations is smaller than those of their results. This is because of the difference in the treatment of energy transfer. In other words, the reason is that the entropy in the envelope is smaller because of the existence of the radiative region.

Next, let us consider the models with the same M_t. As seen in Fig. 1, M_c becomes greater with the increase of f. This is because the larger f gives the larger entropy in the convective region which contains most of the envelope mass. Since the larger entropy increases the thermal energy of the envelope, the gas density becomes smaller. This leads to the increase of the core mass.

In comparison with the M_t-M_c relations computed for the regions of Jupiter, Saturn, Uranus and Neptune, the following remarkable feature is found. The M_t-M_c relations for the four regions are almost identical while the densities and the temperatures of the solar nebula are considerably different from region to region. The M_t-M_c relation for Neptune's region where the outer boundary con-

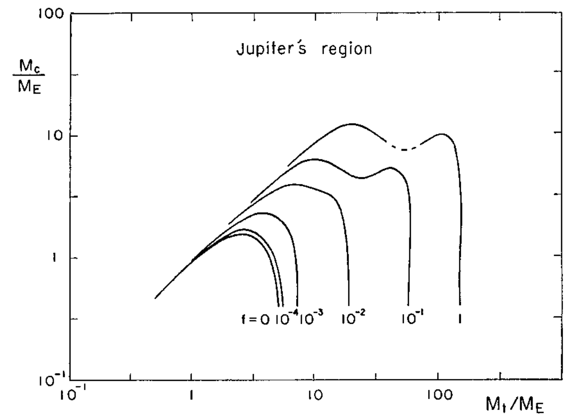

Fig. 1. The relation between the total mass M_t (abscissa) and the core mass M_c (ordinate) for Jupiter's region. The masses are measured in units of M_E (Earth's mass). Figures attached to the curves denote the grain depletion factor f. The dashed portion of the curves shows the models whose envelopes are unstable against small adiabatic perturbation.

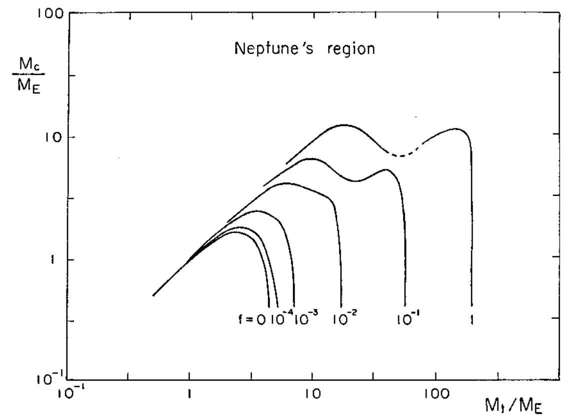

Fig. 2. Same as Fig. 1, but for Neptune's region.

ditions are most different from those for Jupiter's region is shown in Fig. 2. The identity can be seen in comparison of Fig. 1 with Fig. 2. As an example let us compare the four critical models for the case $f=1\times10^{-2}$. In units of M_E, the critical core masses are 3.9, 4.0, 4.0 and 4.0 for the regions of Jupiter, Saturn, Uranus and Neptune, respectively. Moreover, the luminosities are nearly equal to each other. For the above critical models, they are 3.44×10^{26}, 3.55×10^{26}, 3.55×10^{26} and 3.54×10^{26} erg/sec, respectively. We will discuss this identity in more detail in § 4.

b) *Structure of envelope*

The envelope is divided into three regions. They are, inward from outside, the optically thin and nearly isothermal region, the radiative region and the convective region. For example, Fig. 3 shows the structures on the density-temperature plane for the critical models in the regions of Jupiter and Neptune for the case $f=1\times10^{-2}$ as well as the contours of the gaseous opacity. In Fig. 4 the temperature and mass distributions are shown for the same models.

The following important point is found from Fig. 3. The structures on the ρ-T plane for the models are considerably different outside the photosphere, reflecting the difference of the outer boundary conditions. However, as one goes into the radiative region, the structure lines converge to one structure line, which depends only on M_t and f, within the accuracy of about one percent. The same behavior is found for T-distribution (therefore also ρ-distribution) in Fig. 4; the accuracy is within about ten percent. Furthermore, as noticed from M_r-distribution in Fig. 4, most of the envelope mass is contained in the inner part of the radiative region and in the convective region. This means that almost all of the envelope mass is contributed from the regions where the structure lines converge. As a result of it, the core mass becomes almost constant independently of the region in the nebula when M_t and f are fixed.

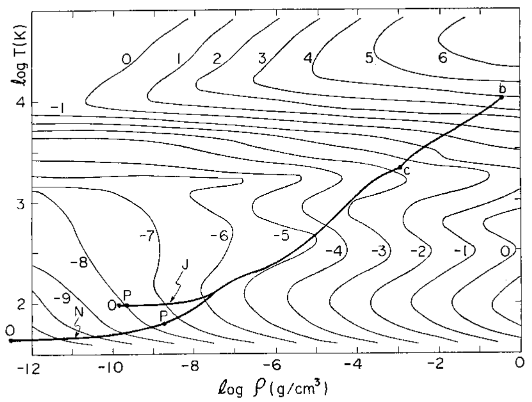

Fig. 3. The structure lines of the critical models on the density-temperature diagram for the case $f=1\times10^{-2}$. The models for the regions of Jupiter (denoted by J) and Neptune (denoted by N) are shown by the bold curves. The subscripts, 0, p, c and b denote the Hill sphere, the photosphere, the outer boundary of the convective region and the bottom of the envelope, respectively. Contours of gaseous opacity for the same value of f are also represented by solid curves. Figures by the solid curves are the common logarithm of gaseous opacity.

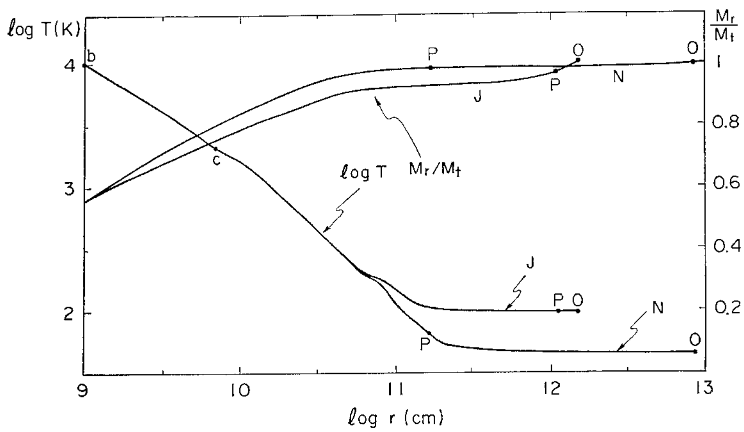

Fig. 4. The temperature and mass distributions versus radius r for the same models as in Fig. 3. The left and right ordinates are temperature T and mass fraction (M_r/M_t), respectively. The other notations are the same as in Fig. 3.

By the way, we can see in Figs. 3 and 4 that there exist the regions where the temperature gradient becomes small abruptly at $T=170$ K and 1,600 K. This is because the evaporation of grains reduces the grain opacity and, hence, the total opacity

Finally the data of the critical models for Jupiter's region are tabulated in Table II for the cases $f=0$, 1×10^{-4}, 1×10^{-2} and 1. As mentioned above, the quantities at a point inside the convective region are nearly independent of the regions in the nebula. Therefore we can use these values tabulated in Table II for the cases of the other regions in the solar nebula within sufficient accuracy (say, ten percent).

Table II. The critical models with various value of f for Jupiter's region. Figures in parenthesis represents powers of 10. In the left column, "convection" and "bottom" represent "outer boundary of convective region" and "bottom of the envelope", respectively.

Grain depletion factor	0	1×10^{-4}	1×10^{-2}	1
Total mass (M_E)	2.5	2.5	7.0	20
Core mass (M_E)	1.5	1.6	3.9	12
Luminosity (erg/sec)	1.70 (26)	1.85 (26)	3.44 (26)	7.32 (26)
Density (g/cm^3)				
photosphere	1.95 (−6)	8.46 (−7)	1.83 (−10)	1.50 (−10)
convection	5.28 (−5)	4.64 (−5)	1.02 (−3)	7.66 (−5)
bottom	1.87 (−1)	1.74 (−1)	3.24 (−1)	2.17 (−1)
Temperature (K)				
photosphere	148	146	97.1	97.0
convection	317	326	2100	2060
bottom	5980	6120	10100	19800
Radius (cm)				
Hill sphere	1.06 (12)	1.06 (12)	1.49 (12)	2.11 (12)
photosphere	2.46 (10)	2.66 (10)	1.08 (12)	2.11 (12)
convection	1.35 (10)	1.30 (10)	6.69 (9)	1.68 (10)
bottom	7.34 (8)	7.55 (8)	1.01 (9)	1.46 (9)

c) *Examination of assumptions*

In order to see the sensitivity of the results to the accretion rate, we have calculated the M_t-M_c relations for $f=1\times10^{-4}$ and 1, changing $\dot{M}_c$ by a factor of ten. The resultant change of the critical core mass has been only within a factor of two in the same direction as the change of $\dot{M}_c$.

Now, let us evaluate the effect of the other source of luminosity, that is, the gravitational energy due to the contraction of the envelope. To do so, we pick up two models with the same value of f but with the different core masses. We can roughly estimate the energy generation rate from the difference of the gravitational energy of the envelope and the time interval between the two models (the time interval can be computed from the mass differences and $\dot{M}_c$). For example, the estimated energy generation rate has been about a half of the luminosity due to the accretion of planetesimals near the critical model for $f=1\times10^{-2}$. In view of the dependence of the solution on $\dot{M}_c$ mentioned in the preceding paragraph, the effect of the additional luminosity on the critical core mass is expected to be very small.

§ 4. Identity of the M_t-M_c relation

In this section we will concentrate on the identity of the M_t-M_c relation obtained in § 3.

As mentioned in § 3 b), when M_t and f are fixed, the solutions in the radiative region approach to an asymptotic solution independent of the outer bundary condition. This can be understood as follows. First, it is to be noticed that there are two features in the outer part of the radiative region. One is that M_r approximately equals M_t, i.e., $M_r \approx M_t$. The other is that the opacity is nearly constant. When $f>1\times10^{-4}$, the gaseous opacity in the region is smaller than the grain opacity κ_g (see Fig. 3). Hence, the total opacity κ is nearly equal to κ_g and constant as long as we are concerned with the region where $T>170$K. Even in the case $f<1\times10^{-4}$, the structure line in the region runs almost parallel with the contour of gaseous opacity and we have again the constant opacity. In view of these two features, we can obtain the following asymptotic solution for large T using Eqs. (1) and (5):

$$T^4 = \frac{3\kappa L}{4\pi a_s c G M_t} P. \tag{12}$$

This is what is called radiative zero solution and indicates the structure of polytrope with index 3.[13] It is also found from Eqs. (1) and (12) that for enough large T, T-distribution can be approximated by

$$T = \frac{\mu H G M_t}{4k} \frac{1}{r}. \tag{13}$$

These asymptotic equations do not depend on the outer boundary conditions (i.e., ρ_0 and T_0) but only on the luminosity. Even when the conditions that $M_r \approx M_t$ and $\kappa \approx$constant are broken in the inner region, it is clear that the solution in the inner region does not depend on ρ_0 and T_0 and can be expressed as a function of only r and L; $M_r = M_r(r, L)$, etc.

Then the asymptotic solutions must satisfy the inner boundary conditions (8) and (9), i.e., we have

$$M_r(r_b, L) = \frac{4\pi r_b^3}{3} \bar{\rho} \quad (=M_c) \tag{14}$$

and

$$L=\dot{M}_c\int_{r_b}\frac{GM_r(r,L)}{r^2}dr\,. \tag{15}$$

In Eq. (15), M_r is expressed explicitly by $M_r(r,L)$ and the upper limit of the integration is omitted because the inner region of the envelope contributes mainly to the gravitational potential. Since Eqs. (14) and (15) do not include the quantities related to the solar nebula, the equations determine r_b or M_c as well as L regardless of the regions in the nebula. This is the identity of the M_t-M_c relation. Of course, the distribution of the density and temperature of the envelope do not depend on the regions in the solar nebula, as long as we are concerned with the inner region of the envelope.

Clearly, the identity discussed here is not true in the case of Perri and Cameron[3] and Paper I. The structure is directly influenced by the boundary conditions because the temperature is drawn with the adiabatic temperature gradient from the outer boundary or if the adiabatic solution is joined with the outer isothermal one. In our case, the radiative region exists between the nearly isothermal and the convective regions, and M_r and κ do not change so much there. These are essential for us to have the identity.

§ 5. Conclusions

Let us consider the situation where a protoplanet is growing in the solar nebula. The M_t-M_c relations shown in Figs. 1 and 2 indicate that when the protoplanet grows beyond the critical core mass, the gaseous envelope in the Hill sphere cannot be in hydrostatic equilibrium. Then the gaseous envelope collapses onto the protoplanet to form a compact and tightly bound object, which is to be called a proto-giant-planet. The qualitative description of the formation process of the giant planets is the same as those in Perri and Cameron[3] and Paper I. However, in quantitative respect, an important point can be concluded from the present calculations. That is, if $\dot{M}_c$ and f do not change very much throughout the solar nebula, the protoplanetary mass at the time of the collapse of the enve-

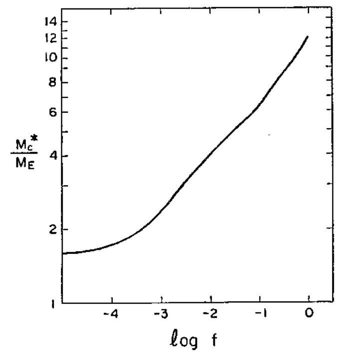

Fig. 5. The critical core mass, M_c^*, as a function of the grain depletion factor f. The mass is measured in the unit of the Earth's mass M_E.

lope does not depend on the regions in the solar nebula. Of course, this is because of the identity of the M_t-M_c relation. This conclusion is in good agreement with those obtained by Slattery[1] and Hubbard and MacFarlane.[2] They showed recently that the four giant planets have almost the same core mass regardless of the differences of the total masses.

In Fig. 5, the critical core mass is shown as a function of f. As seen in Fig. 5, when $f<1\times10^{-4}$, the critical core mass does not change very much with f. When f changes from zero to unity, the critical core mass changes from about $1M_E$ to about $10M_E$ monotonically. A limiting case with $f=\infty$ corresponds to a wholly adiabatic envelope investigated by Perri and Cameron.[3] It is to be noticed that the critical core masses calculated in the present paper become smaller than those in Paper I. For example, the critical core masses for the case of $f=1\times10^{-4}$ and 1×10^{-2} are 4.8 and $54M_E$ in Paper I, respectively, but they decrease to 1.6 and $3.9M_E$ in our present case.

It is found from Fig. 5 that the grain depletion factor in the envelope should be about unity, which corresponds to the grain opacity of about 1 cm^2/g, in order to fit the critical core mass to the core masses (about $10M_E$) of the present giant planets. This means that the concentration of grains in the envelope was considerably large during the growth of the protoplanet. In contrast, the theory on planetary formation[4] assumes that the concentration of grains in the envelope has been very small as compared with that in interstellar clouds, because almost all of the grains in the solar nebula has already been contained in protoplanets and planetesimals.[4] However, this simple consideration is not sufficient to determine the actual concentration. Because grains can be supplied through some mechanisms; for example, accreting planetesimals sprinkle grains into the envelope and materials which evaporate from the core surface recondense in the upper region of the envelope.[14] It is probable that the concentration of grains increases considerably in the envelope more than that expected earlier.

As mentioned previously, the core mass is about $10M_E$ at the time of the collapse of the envelope. At that time the total mass does not necessarily reach the present mass of the giant planets as seen from Figs. 1 and 2, especially for the case of Jupiter and Saturn. Hence it is expected that the nebula gas continues to accrete onto the protoplanet even after the collapse. It is a future problem to examine how far the protoplanet gathers the nebula gas and how long it takes to reach the present mass of the giant planets.

Finally we will remark on the stability of the envelope for the regions of the inner planets. It is expected that the identity of the M_t-M_c relation is also valid for these regions. In fact, we have ascertained from the calculation for the case $f=1\times10^{-2}$ for Earth's region that the M_t-M_c curve agrees with that for Jupiter's region. As seen in Fig. 5, the critical core mass is always greater than $1M_E$. This means that the growth of the inner planets stopped before the collapse of the envelope. Therefore the inner planets did not become the giant planets.

Acknowledgements

The author wishes to thank Professor C. Hayashi and Dr. K. Nakazawa for their useful advice and continual encouragement. The numerical calculations were performed by FACOM M-200 at the Data Processing Center of Kyoto University.

Appendix

——Gaseous Opacity in the Envelope Surrounding a Protoplanet——

As mentioned in § 2 d), we have taken account of the following sources of gaseous opacity; H_2-pressure induced absorption, absorption due to H_2O, CO and OH, Rayleigh scattering of H_2 and H, electron scattering and absorption due to atoms. The abundances of molecules, atoms and electrons are computed on the assumption of chemical equilibruim and ionization equilibrium.[15] The abundance of heavy elements in gas phase has been determined as follows. For the case $T<170$ K, heavy elements do not exist in gas phase because they all condense into grains. Since evaporated icy grains supply C, N and O when $T>170$ K, their relative abundance to solar abundance is f, where f is the grain depletion factor (relative to interstellar clouds) in the region where $T<170$ K. Moreover, the other heavy elements are supplied by metallic/rocky grains only in the region where $T>1600$ K and their abundance is f.

As to the H_2-pressure induced absorption, we have calculated its opacity by the use of the band profile of Linsky when $T>600$ K and by the use of the line profile of Linsky and Trafton when $T<600$ K.[16),17)] As to absorption of H_2O molecule, we have followed Cameron and Pine[18)] when $T<1000$ K and have used the approximate formula of Keeley when $T>1600$ K.[19)] In the region where $1000\,K<T<1600$ K, linear interpolation has been performed. Absorptions due to other molecules and atoms have been calculated according to the approximate formula of Keeley[19)] only in the region where $T>1600$ K. When $T<1600$ K, absorption due to molecules other than H_2 and H_2O is negligible.

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