FORMATION OF THE STARS AND DEVELOPMENT OF THE UNIVERSE

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Introduction

In a fascinating article which appeared in *Nature* of February 6, under the title "Stellar Evolution and the Expanding Universe", Mr. F. Hoyle has brought forward convincing arguments for the permanent creation of matter in space. Many physicists will find it difficult to accept this hypothesis. For if there is any law which has withstood all changes and revolutions in physics, it is the law of conservation of energy, which according to Einstein's formula $E = mc^2$ is equivalent to the conservation of mass. The same strange conclusion has, during recent years, been formulated by Prof. Pascual Jordan, but with an important modification, whereby the conservation law is not violated. This is achieved by taking account of the loss of gravitational energy connected with the creation of particles. As Jordan's papers do not seem to be known to many English-speaking physicists, I have asked him to write a short report of his work, and the following article is a translation of his article made by my collaborator, Dr. H. S. Green. MAX BORN

Since the formulation by Einstein of the general theory of relativity, many theoretical considerations have been advanced in which models of the universe are derived *deductively* from certain physical principles (expressed in the form of field equations), which are adopted as hypotheses. The treatment here explained is an attempt to approach the problem more cautiously. It is necessary to reckon with the possibility that processes and natural laws are involved which lie outside our past experience, and which must be learnt by the examination of the universe: an essentially *inductive* approach to the cosmological problem must be sought. The dimensional analysis of the empirical facts presents itself as the method appropriate to this task. This method proceeds by the combination of the experimental data to form dimensionless quantities, and the comparison of these dimensionless numbers with regard to their order of magnitude.

Dimensionless constants, the significance of which remains to be explained, are, as is well known, already apparent in atomic physics. Many attempts have been made to elucidate the significance of Sommerfeld's fine-structure constant and the ratio of the masses of the proton and electron. Though Eddington's well-known considerations have not led to the solution of these problems, he has very usefully made apparent their great urgency and importance. The discovery of different kinds of mesons has intensified these problems: a theory which could explain the mass ratios of the different kinds of mesons could probably account also for the two constants already mentioned. The reciprocity theory of Born and Green promises some interesting progress in this direction.

There is, however, an even more significant dimensionless constant in microphysics. If the ratio of the electrostatic and gravitational attractions of an electron and a proton is formed, the number 2×10^{39} results.

Can it reasonably be hoped that this value can ever be established by theoretical considerations? Eddington cherished the optimistic view that it could; but his speculations really only served to make one more acutely conscious of how improbable it is, on the basis of such natural laws as are known, that an explanation could be given of why a dimensionless constant should have just this enormous value, and not some other. It is not so much a question of the particular natural laws known to us to-day, as of the *mathematical type* common to all the known laws of physics. The mathematical type which characterizes physical laws allows the explanation of such natural constants as π, or 4π, or $\sqrt{2}$; but a number of the order of magnitude 10^{40} can never arise from the mathematically simple, fundamental laws of Nature. But should it now be believed that there are, in the laws of Nature, numerical constants which by a meaningless chance have their actual values—which could equally well assume other arbitrary values, without producing a fundamental discord in the harmony of the natural laws? It is an outstanding merit of Eddington that he emphasized that we should never believe in meaningless coincidences.

There are six distinct quantities which comprise the sum of our knowledge of the structure of the universe on a large scale. After the velocity of light, c, and the gravitational constant, $k = 8\pi f/c^2$, where f is the Newtonian gravitational constant, comes thirdly the maximum age of the oldest celestial bodies known to us. From a variety of data and considerations, one arrives at the conclusion that none of all the known stars and systems of stars can have existed for longer than a certain maximum time, which is approximately 4×10^9 years. Thus one has a quantity of the order of magnitude $A = 10^{17}$ sec.

Three constants additional to those already mentioned are furnished by the statistics of the spiral nebulæ. From the enumeration of the nebulæ, and the determination of the masses of a cross-section of typical nebulæ, one obtains the value of the mean mass-density μ throughout the universe; this comes to about 10^{-28} gm. cm.$^{-3}$. The second of the three 'constants of the spiral nebulæ' is the constant α of the Hubble effect. The spectral lines of the very distant nebulæ show a displacement to the red, which may be described simply by the empirical fact that if a spectral line has the wave-length λ, then the displacement $\Delta\lambda$ is proportional to λ, and also to the distance r of the nebula from us, so that one obtains always the same value for $\alpha = c\Delta\lambda/\lambda r$. The third constant of the spiral nebulæ is empirically the worst determined. It seems, however, that the number of those spiral nebulæ the separation of which from us is in the interval r, $r + \Delta r$, is rather less than proportional to $4\pi r^2 \Delta r$, that is, approximately proportional to $4\pi r^2(1 - \text{const.} r^2)\Delta r$. Here the constant defines the radius of the universe R, to the square of which it is inversely proportional.

The assembly of cosmological data is now complete; and it may be inquired what dimensionless constants can be constructed from c, k, A, μ, α and R. Such are αA; R/cA; $k\mu c^2 A^2$; and it is an extraordinary thing that all three are of the order 1. This is the justification for an attempt to interpret these constants by means of the following simple and illuminating picture. The 'radius of curvature' R is interpreted—and is for that reason given such a name—in the sense of Riemannian geometry: instead of the infinite Euclidean space, one has thus a closed, finite, Riemannian space, the volume of which is of the order of magnitude R^3. In the same intuitive way the Hubble effect is interpreted as a Doppler effect. Different interpretations of the Hubble effect have, indeed, often been attempted, but here the intuitive concept of an expanding space is retained without modification. The empirical relation $R = cA$ then means that the radius of the universe increases at a rate which is just of the magnitude of the velocity of light—an attractive and revealing result. Also $\alpha A = 1$ means that this space, which has been expanding with the velocity of light ever since that time which we recognize to be most remote in the history of the universe, must once have been very small.

But what is meant by the fact that the last of the three dimensionless constants mentioned is of the order of magnitude unity? This relation appears already in the well-known model of the universe which was Einstein's bold attempt, in framing the general theory of relativity, to realize the idea of a closed, Riemannian, physical space. Indeed, if M is understood to be the total mass of the universe—which is then of the order of magnitude μR^3—and cA is replaced as previously by R, the equation in question can be written in the form $kM \simeq R$; and Einstein had previously obtained the relation $kM = 4\pi^2 R$, for a closed space with a *time-independent* radius R, on the basis of the relativistic theory of gravitation.

From a consideration of the Hubble effect, however, the concept of a growing universe has been reached; and then the empirical relation $kM \simeq R$ gives a disturbing conclusion. If R is not fixed, but is always growing, then neither can kM be regarded any longer as invariable; and, of the two factors, the gravitational constant k, and the mass M of the universe, at least one must vary likewise with time. An important contribution to the elucidation of this situation is contained in an observation by A. Haas: the relation $kM \simeq R$ can also be written in the form $fM^2/R \simeq Mc^2$, which means that the negative potential energy of gravitation for the whole universe is equal to the sum of the rest-energies of the masses of the stars. This provides a surprising solution of the problem of the universe: it is possible for the total energy of the universe to have the value zero exactly—through the cancelling out of the positive and negative contributions to the energy. The relation $kM \simeq R$ would then appear as a direct consequence of the conservation of energy, which requires that the evolving universe should continue in a sequence of states the total energy of which always has the value zero.

To proceed further, another dimensionless constant is required; those which can be produced from the six cosmological quantities c, k, A, μ, α and R are exhausted by the three dimensionless constants already mentioned. More are obtained if the cosmological quantities are compared with those derived from microphysics. Hitherto the radius of the universe R and the mass of the universe $M \simeq \mu R^3$ have been expressed in centimetres and grams; now they will be expressed in terms of the fundamental units of microphysics. The choice of these units is, of course, made uncertain by the appearance of the unexplained dimensionless constants of microphysics: it is questionable whether one should take, as the natural unit of mass, the mass of the proton, or that of the meson or electron, and whether the natural unit of length should be Bohr's radius of the hydrogen atom, or the electronic radius e^2/m_0c^2, the ratio of which is essentially the square of the fine-structure constant. However, the dimensional ratios with which one has to do are so enormously large that these distinctions are of little consequence; moreover, there is sufficient reason to give preference to the mass m_M of the meson, and to the electronic radius or 'elementary length' $l = 2 \times 10^{-13}$ cm. If the ratio $M : m_M$, which is then of the order of magnitude of the number of elementary particles in the whole universe, is formed, then a number of the colossal magnitude 10^{80} is obtained. Eddington had the boldness to claim that this number should not be simply accepted as something incapable or without need of explanation; he sought, by means of some curious considerations, to give theoretical grounds for supposing that the number of protons in the universe must have the value 2^{256}. The true solution would appear to lie in another direction: an idea which Dirac has put forward in quite a different connexion will now be introduced.

The ratio R/l is of the same order of magnitude, 10^{40}, already encountered in the comparison of the gravitational and electromagnetic attractions between two elementary particles; if these two dimensionless numbers are divided, therefore, a new dimensionless quantity of the gratifying order of magnitude 1 is obtained. The ratio of the two attractive forces is thereby compared, however, with a number which, as is already known, is not constant: the ratio R/l increases proportionally to the age of the universe; it is, in fact, equal to the age of the universe, expressed in terms of the 'elementary time' $\tau = l/c \simeq 10^{-23}$ sec. Also, from the significant discovery of a quotient of the order of magnitude unity, it must be concluded, with Dirac, that the constant of gravitation is in reality not a constant, but inversely proportional to the age A of the universe.

What is now the obstacle to applying this idea also to the ratio M/m_M, and asserting that the number of elementary particles in the universe obviously increases as the square of the age of the universe? Dirac, who was led away from this application by a special train of thought (concerning the formulation of a cosmology with an infinite Euclidean space and an infinite total mass of the universe), was probably influenced by a fear of contradicting the principle of conservation of energy. However, the foregoing consideration has cleared the way in this respect: with the perception that $kM \simeq R$ guarantees the conservation of energy, the complete harmony of all the statements concerning the proportionality of k to A^{-1}, M to A^2, and R to A is attained.

It is, therefore, accepted that there is a continual creation of matter in the space of the universe, and the question arises, how and where this generation of matter, connected with the growth of the universe, occurs. To answer this, the individual stars, instead of the universe as a whole, are now taken as the subject for careful consideration in the light of dimensional analysis. The mass of the sun amounts to 2×10^{33} gm.; there are certainly many stars with still smaller masses, and an established lower limit scarcely exists. There are, however, on the other hand, few stars with very large masses: 10^{35} gm. = 50 suns may be taken as an approximate value for an upper limit to stellar masses which is very seldom exceeded. It may be asked how many protons such a star would contain: the answer is of the order of magnitude 10^{60}.

Here is a fresh opportunity to apply Dirac's principle, from which it may be concluded that this upper limit, too, must be a function of A; indeed, it must then be interpreted as $A^{3/2}$. According to this hypothesis, the mass of a star (on the average) depends on the age attained by the universe at its formation; this formation being settled, the mass of the star (the number of its nucleons) will remain unchanged (apart from secondary processes, as occasional gathering of dust). Therefore, the normal mass of a star which was generated at an age A_0 of the world must be proportional to $A_0^{3/2}$. (In consequence of this, to-day not only the average mass of recently created stars, but also the average mass of now existing stars has an order of magnitude 10^{60}, proportional to $A^{3/2}$.) Naturally, this dependence of the average of stellar masses on the age of the universe necessarily implies a corresponding dependence of the *gravitational constant*, which is then itself a function of the age of the universe; and, in fact, the proportionality $M_{\mathrm{st}} = A_0^{3/2}$, inferred in the first instance from a comparison of the empirical numbers 10^{40} (present age of the world) and 10^{60} (present mass of the normal star), is in agreement with and corroborated by the fact that the present theories of the internal constitution of the stars continually presuppose stellar quantities which are proportional to $k^{3/2}$.

It has been made clear above that the requirement of conservation of energy on a large scale is satisfied, if a creation of matter is assumed which leads to an increase of M proportional to A^2. Nothing is said there, however, of the way in which this creation of matter can occur. Though the possibility of a localization of energy, like that encountered in Maxwell's theory, can no longer be maintained in Einstein's theory of gravitation, it is certain that in some way a rigorous formulation of the principle of conservation of energy must hold such that, not only for the universe as a whole, but also for a finite region, where the creation takes place, a balance of energy is maintained. Consider now the rough model of a homogeneous sphere with mass M_0 and radius R_0 in an approximately flat, Euclidean space. If it satisfies the relation $kM_0 = R_0$ (apart from a certain numerical factor of the order of magnitude unity) then this space has zero total energy: if it were required to effect a transformation to a cloud of gas of virtually infinite radius, then it would be necessary, in order to overcome the gravitational force between the atoms, to expend just as much energy as would be present in the final state as the sum of the rest-energies. The conjecture suggests itself that the cosmic creation of matter does not take place as a diffuse creation of protons, but by the sudden appearance of whole *drops* of matter, for which M_0 and R_0 are chosen so that each drop, even taken by itself, remains true to the principle of conservation of energy from the instant of its formation—in such a way that the total energy has the value zero.

The critical question at once arises, What density must be ascribed to the drop of newly created matter? It can certainly take only the density of the atomic nucleus, which is of the order of the proton mass, contained in a space of the dimensions of the elementary length. Hence the size of the drop is determined as a function of the gravitational constant at any given time: drops are obtained which contain a number of elementary particles to-day of the order of magnitude 10^{60}, and in general proportional to $k^{-3/2}$ or $A^{3/2}$. Each drop which appears in the space of our universe is therefore a star in its own right.

This picture can be worked out in rather more detail. The universe, regarded as a four-dimensional space-time manifold, is, according to the above, a cone: its apex, $A = 0$, marks the starting-point in time, at which also R and M are zero. It may be imagined, however, that this cone stage does not necessarily possess only one apex, but may have a large number of subsidiary apices. A 'cut' t = const. may then in certain circumstances cut the space-time manifold into several unconnected three-dimensional parts: another isolated, smaller space may exist apart from the large universe, which, by the gradual unfolding of time, and also by its own expansion, may begin to coalesce with the large universe. Supposing that in this small 'world' the mass-density remains constant, and the gravitational constant decreases as the inverse square of its age, then the conservation of energy holds there also. If it is supposed further that when, by such a process, the little world reaches the same value of k as attained in the large universe, it can coalesce with the universe, then it will assume a mass of exactly the same value, at the instant of coalescence, as has been recognized to be characteristic of a newly formed star. Such a world may therefore be regarded as the embryonic state of a star.

A careful discussion of the astrophysical facts seems to indicate that the ideas suggested above are well adapted to give a better understanding of many known empirical facts. Instances will not be developed here; the last two of my papers cited below contain a full discussion. I mention only that I suspect that the supernovæ I (which appear to belong to the star population II, and which, one may venture to say, probably account also for the planetary nebulæ) are stars created in this way.

Finally, it may be noted that the theory, sketched here in a purely inductive manner, has received also a quantitative mathematical treatment. For this, of course, a certain generalization of Einstein's theory of gravitation was required, as this theory treats the constant of gravitation essentially as a genuine constant; and the substitution of a variable k (in Dirac's sense), which must then be treated as a scalar field quantity, requires a fundamental generalization of the theory. For this purpose I found, to all appearances, a very natural starting-point in the five-dimensional (or projective) theory of relativity. The generalization in question has also been studied by Einstein and Bergmann, and by Lichnerowicz and Thiry. In a paper by Ludwig and Mueller, it is shown that, with this new and more general form of the theory of relativity, a deductive foundation and quantitative precision can be given to the model sketched above. This applies to the model of the universe on a large scale as well as to the model of a star in its embryonic state. In this way also the correctness and practicability of the present interpretation of the problem of cosmological energy are confirmed.

After having written this article, I had the opportunity of studying the paper by Mr. F. Hoyle mentioned above by Prof. Max Born, and which will be sure to give a new great impetus to cosmological discussion. Several decisive ideas of Hoyle's are in full harmony with my own theory; especially his very convincing arguments against oscillatory models, and concerning the irreversible conversion of hydrogen. But by far the greatest encouragement which I gain from his very interesting discussion is given by his hypothesis of creation of matter. When I put forward in 1939, following Dirac, the idea of an increasing mass M of the universe, I myself believed this to be a rather astonishing hypothesis. Surely this hypothesis will now be discussed earnestly. But there are also considerable differences between Hoyle's theory and my own. The principal point has already been emphasized by Prof. Born. Will Hoyle's thesis (F), namely, the existence of internebular matter, really be accepted by empirical astronomers? I learn from Dr. Baade that he does not believe it.

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FORESTRY AND HILL FARMING

IN the face of present economic difficulties, the right use of the limited area of productive land in Great Britain, shrinking through the demands of housing and industry and through diversion for use by the Services, becomes of prime importance. When the agricultural improver lifts his eyes to the hills in the hope of increasing food production, he finds that the forester has also marked the hills for the development of his forests. The case for increasing home-grown timber supplies is strong and finds expression in an afforestation policy that has been approved in principle by all parties. On the other hand, we must have more home-grown meat, and in this connexion the importance of hill lands is not simply in the actual mutton and lamb produced, but also in their value as a source of the foundation stocks of lowland sheep.

In view of the existence of these conflicting claims on hill land, the holding of a joint discussion by Sections K* (Forestry) and M (Agriculture) of the British Association was opportune, the more so as Newcastle upon Tyne lies on the verge of a great hill area where the issue of sheep versus trees has been actively joined.

In the opening paper, Lord Robinson surveyed the position mainly from the point of view of timber production. He pointed out that hill country forms about thirty per cent of the area of Britain and, except for the highest land, was originally mainly forest. At present it is mainly devoted to grazing. There has been a recession in this type of utilization; apart from the incursions of forestry, the hill areas are less populous, and sheep stocks are lower and less healthy than formerly. The industry supports about one family per 800 acres; the production is about 8 lb. of mutton and 1·9 lb. of wool annually per acre in England and Wales, with lower figures for Scotland. The industry is heavily subsidized, and Lord Robinson doubted if the expenditure on hill farming could be justified unless soil fertility is restored and maintained, and also whether this could be effected except by ploughing and re-seeding.

A conservative estimate of coniferous timber production is $1\frac{1}{2}$ tons of air-dry timber per acre per annum. Employment at first is double that under sheep farming and rises until about one employee is required for 50 acres of forest. The present programme envisages the provision of at least ten thousand new houses and the establishment of new communities.

Lord Robinson held that there is a place both for forestry and grazing in the hills, and both must be planned as economic units. Dollars saved on imported timber can pay several times over for food displaced, but this displacement need not occur if the grazings unoccupied by the new forests are improved.

Prof. R. W. Wheldon, from the agricultural side, emphasized the importance of hill-sheep farming. At present, grassland types of sheep are dominant owing to the decay of arable-sheep farming. Type conservation and the release of lowland areas from rearing require an integration of hill and lowland, and the same applies to a certain degree to cattle. The possibilities of direct food production in hill areas have been enlarged in recent years. In estimating the contribution of the hill areas to our meat supplies, crude estimates of output may be misleading. The hills are important as providing the foundation stocks for lowland meat production. Private estates have successfully combined farming and forestry. The use of hill lands both for sheep and timber is a problem of practical integration.

Mr. A. P. Long dealt with some of the practical aspects of hill forestry in Britain. The ultimate objective of five million acres afforested by the end of fifty years is to be obtained by (1) the maintenance and replanting of existing woodlands, some cleared and some derelict, and (2) the planting of new areas. A proper balance must be maintained between these programmes. In the new plantings some displacement is inevitable; but much of the land is not fully productive, and we cannot afford to maintain land in an unproductive state.

Mr. Long did not consider that the displacement of food would be so great as often imagined. The number of sheep displaced in one year under the present programme is only 0·27 per cent of the total sheep population. The temporary loss could be repaired by a modest increase in production elsewhere. The need for timber is great, particularly for pitwood, which is a major problem in war-time. The general shortage of timber is likely to persist, and we cannot afford to postpone plans for improving our position.

Land for afforestation is acquired in close consultation with the Ministry of Agriculture, and every effort is made to ensure as little disturbance as possible. Planting may occur many years after acquisition, and only rarely is notice given to quit more than part of a farm. Particular attention is paid to the utilization of unplantable land. The main source of forest land is rough pasture, areas that are unproductive being first tackled; for example, bracken land and water catchment areas. Mr. Long also stressed the social implications of the afforestation programme. Employing about ten times as much labour as sheep farming per unit area, it offers a means of checking rural depopulation and building up a vigorous rural community life.