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Formation UAV Flight Control using Virtual Structure and Motion Synchronization

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Abstract— **In this paper, the synchronized position tracking controller is incorporated in formation flight control for multiple flying wings. With this technology, the performance and effectiveness of the formation controller are improved when the virtual structure approach is utilized to maintain formation geometry. Simulations are conducted on the nonlinear model of two flying wings to verify the proposed controller.**

I. Introduction

Formation flight control of multiple Unmanned Aircraft Vehicles (UAVs) has been an active research topic for many years [1], [2], [3], [4] since it promises many practical applications, such as reconnaissance, surveillance, atmospheric study, communication relaying and search and rescue. Some of these tasks may be dangerous and will not be recommended for human pilots, thus making them ideal for autonomous unmanned vehicles.

There are basically three approaches to the formation control problem for multiple vehicles, namely leader following, behavioral, and virtual structure [5], [6], [7]. In the virtual structure approach used in this paper, the entire formation is treated as a single entity. It can evolve as a rigid body in a given direction with some given orientation and maintain the geometric relationship among multiple vehicles base on a reference point in the virtual structure. As for the more specific formation flight problem for aircraft, a consensus is that the vehicles should be able to achieve tracking for given velocity, heading, and altitude commands ([1], [2], [8], [9]) in order for a formation controller to be developed for the aircraft. An autopilot model that provides tracking capabilities for the three commands and the method of trajectory command modifications based on relative position errors is used as part of the control algorithm in our paper. This method is coupled with the virtual structure approach to achieve formation flights for our research.

In this paper, we present that the performance of a formation controller using certain virtual structure definitions can be enhanced by applying the synchronization technology developed by UTIAS [4], [10]. The technology synchronizes the relative position tracking motion between multiple aircraft. Simulation results demonstrate the effectiveness and performance improvement with the proposed control method.

The remainder of this paper is organized as follows. In Section II, the virtual structure is defined followed by the formation algorithm development in Section III. In Section IV, the synchronization technology is incorporated into the controller. Simulation results on formation control of multiple flying wings are given in Section V. Finally, Section VI offers conclusions and future research possibilities.

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II. Virtual Structure

In the virtual structure approach, the entire formation is treated as a rigid body. The positions of the vehicles in the structure are usually defined in a frame with respect to a reference point in the structure. As a trajectory is given for the reference point, the desired position for each vehicle can be calculated as the virtual structure evolves in time. In our formation controller, we define the centroid of the desired formation as the reference point in the virtual structure.

From [7], defining frame O is an inertial frame and frame F is the formation reference frame located at the reference point. The formation is a rigid body with inertial position P_F, velocity V_F, heading ψ_F, and angular velocity ω_F. We also have the reference frame i imbedded in each agent. Each agent can be represented by position P_i, velocity V_i, heading ψ_i and angular velocity ω_i with respect to the inertial frame O or by P_{iF}, V_{iF}, ψ_{iF} and ω_{iF} with respect to the formation reference frame F. Then the equations for the position and velocity dynamics for each agent in the inertial frame in the virtual structure are

$$\begin{aligned} P_i^d(t) &= P_F(t) + C_{OF}(t) P_{iF}^d(t) \\ V_i^d(t) &= V_F(t) + C_{OF}(t) V_{iF}^d(t) \\ &\quad + \omega_F(t) \times \left(C_{OF}(t) P_{iF}^d(t) \right) \end{aligned} \tag{1}$$

where $C_{OF}(t)$ denotes the rotation matrix from frame F to frame O and variables with superscript ^{d} represent the desired values of the variables for the corresponding vehicle.

Consider the case with two vehicles UAV1 and UAV2, assuming each vehicle is fixed in the virtual structure, then $V_{iF}^d(t) = 0$ in (1). In the two vehicles case, $P_{2F}^d(t) = -P_{1F}^d(t)$ if the reference point is defined to be the centroid of the entity. Therefore, defining $\Gamma(t) = C_{OF}(t) P_{1F}^d(t)$, equation (1) for UAV1 becomes

$$\begin{aligned} P_1^d(t) &= P_F(t) + \Gamma(t) \\ V_1^d(t) &= V_F(t) + \omega_F(t) \times \Gamma(t) \end{aligned} \tag{2}$$

and for UAV2, by substitutions of $P_{2F}^d(t)$ and $\Gamma(t)$,

$$\begin{aligned} P_2^d(t) &= P_F(t) - \Gamma(t) \\ V_2^d(t) &= V_F(t) - \omega_F(t) \times \Gamma(t) \end{aligned} \tag{3}$$

Also, at the time instant $t = t_0$ where there is a non-zero $\omega_F(t_0)$, the instantaneous attitude changes required for the

aircraft in order to maintain the geometry (not the attitude of the vehicles alone) will be opposite, as shown in Fig. 1 with $\omega_F(t_0)$ being about the z-axis for an example.

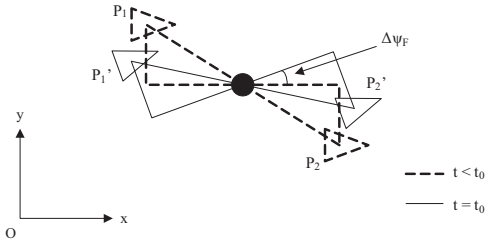

Fig. 1: Change in UAVs' headings

For the UAVs to go from P_i to $P_i^{'}$, UAV1's instantaneous heading change will be in the direction of $-\Delta\psi_F(t_0)$ with a larger magnitude; while UAV2's heading change will be in the direction of $\Delta\psi_F(t_0)$ with a larger magnitude. Therefore,

$$\begin{array}{l}\Delta\psi_1^d(t_0) = -(1+\kappa_1)\Delta\psi_F(t_0)\\ \Delta\psi_2^d(t_0) = (1+\kappa_2)\Delta\psi_F(t_0)\end{array} \tag{4}$$

with $\kappa_i > 0$ as a constant. The same analogy can be applied to the roll ($\Delta\phi$) and pitch ($\Delta\theta$) attitude changes.

Base on the desired positions P_i^d, velocities V_i^d and heading changes $\Delta\psi_i^d$ of the two vehicles in (2-4), it can be observed that the relative values for each vehicle are opposite in magnitude with respect to the reference point P_F when it is the centroid of the virtual structure. Thus, it should be expected that the relative position errors, if there are any, between the actual vehicles' positions and the reference point will be opposing in nature as well when the same trajectory command designated for the reference point is given to both vehicles. Simulation results in Section V verify this hypothesis. Therefore, a controller should be developed to eliminate these position errors for the vehicles to maintain formation.

III. Formation Control Algorithm

To eliminate the relative position errors and to keep the vehicles in formation geometry during flight, a formation controller is needed. It is typically implemented by a two loop scheme where the inner control allows tracking of commanded velocity (V), heading (ψ) and altitude (H). In the outer loop, the formation controller generates the reference commands for the inner controller. An autopilot that is capable of tracking velocity, altitude and heading is used in our paper, so we focus the development on the outer loop formation controller.

Base on the reference trajectory commands $T_r = [V_r, \psi_r, H_r]^T$ for the reference point $P_r = [x_r, y_r, z_r]^T$ and the defined relative distances in the virtual structure, the desired position for each vehicle during the flight $P_i^d = [x_i^d, y_i^d, z_i^d]^T$ can be calculated, whereas the actual position of each aircraft $P_i = [x_i, y_i, z_i]^T$ can be obtained from the GPS system located on the vehicle. Using the reference trajectory and the actual/desired positions for the vehicles as input, the formation controller generates the modified trajectories for each UAV to maintain the geometry of the formation during the flight. We define the relative position errors for the i^{th} vehicle during the flight in the inertial frame as

$$\begin{bmatrix} e_{xi} \\ e_{yi} \\ e_{zi} \end{bmatrix} = \begin{bmatrix} x_i^d - x_i \\ y_i^d - y_i \\ z_i^d - z_i \end{bmatrix} \tag{5}$$

To utilize these relative errors in (5), they need to be converted into errors in the formation frame using the rotation matrix $C_{FO}(t) = C_{OF}(t)^{-1}$. Therefore,

$$\begin{bmatrix} e_{xiF} \\ e_{yiF} \\ e_{ziF} \end{bmatrix} = C_{FO}(t) \begin{bmatrix} e_{xi} \\ e_{yi} \\ e_{zi} \end{bmatrix} \tag{6}$$

The modified trajectory command for the inner loop controller is $T_{ci} = T_r + \Delta T_i$, where ΔT_i is calculated through a PI controller based on the relative position errors in (6).

$$\begin{array}{l}\Delta V_i(t) = K_{px} e_{xiF}(t) + K_{ix}\int_0^t e_{xiF}(t)\,dt\\ \Delta\psi_i(t) = K_{py} e_{yiF}(t) + K_{iy}\int_0^t e_{yiF}(t)\,dt\\ \Delta H_i(t) = K_{pz} e_{ziF}(t) + K_{iz}\int_0^t e_{ziF}(t)\,dt\end{array} \tag{7}$$

$$\Delta T_i = \begin{bmatrix} \Delta V_i(t) \\ \Delta\psi_i(t) \\ \Delta H_i(t) \end{bmatrix} \tag{8}$$

The formation controller applies the corrected trajectory commands T_{ci} based on T_r and ΔT_i in (8) to the aircraft's autopilots. These corrections account for the changes required to maintain the geometry of the formation. Simulation results of the controller are shown in Section V in this paper.

IV. Relative Position Synchronization

One of the objectives in our project is to incorporate the motion synchronization technology in our design to achieve coordinated control of the UAVs. Thus, the next step is to apply this technology in our controller.

The strategy uses the cross coupling concept to synchronize the relative position tracking motion of the aircraft. It utilizes synchronization errors ε_i, which incorporate error information from different agents in the system, to identify the performance of the synchronization. The cross coupled error e_i^* then couples the error e_i and synchronization error ε_i through a positive synchronization gain β_i.

$$e_i^* = e_i + \beta_i \varepsilon_i \tag{9}$$

The objective of the synchronization strategy is to drive e_i^* of each agent in (9) to 0 by choosing the proper gain values, implying that both e_i and ε_i are driven to 0 as well. In order words, the vehicles are using information from each other to eliminate the errors synchronously.

In our case of two vehicles, the position synchronization errors are defined as

$$\begin{aligned} \varepsilon_{x1F} &= e_{x1F} - e_{x2F}, \qquad \varepsilon_{x2F} = e_{x2F} - e_{x1F} \\ \varepsilon_{y1F} &= e_{y1F} - e_{y2F}, \qquad \varepsilon_{y2F} = e_{y2F} - e_{y1F} \\ \varepsilon_{z1F} &= e_{z1F} - e_{z2F}, \qquad \varepsilon_{z2F} = e_{z2F} - e_{z1F} \end{aligned} \tag{10}$$

Then the coupled position errors are formed to include both the position tracking errors e_i and the position synchronization errors ε_i from (10)

$$\begin{aligned} e^*_{xiF} &= e_{xiF} + \beta_{xi}\varepsilon_{xiF} \\ e^*_{yiF} &= e_{yiF} + \beta_{yi}\varepsilon_{yiF} \\ e^*_{ziF} &= e_{ziF} + \beta_{zi}\varepsilon_{ziF} \end{aligned} \tag{11}$$

where β_{xi}, β_{yi}, and β_{zi} in (11) are positive synchronization gains for the x, y, and z channels of the i^{th} aircraft. The method of coupling the errors in (10) can vary when more than two vehicles are involved to couple errors from more than one aircraft, as introduced in [10].

The coupled relative position errors from (11) are used to calculate the trajectory modification ΔT^*_i and the new modified trajectory command that will be passed to the inner controller of the vehicles is $T^*_{ci} = T_r + \Delta T^*_i$ where

$$\begin{aligned} \Delta V^*_i(t) &= K_{px} e^*_{xiF}(t) + K_{ix} \int_0^t e^*_{xiF}(t)\, dt \\ \Delta \psi^*_i(t) &= K_{py} e^*_{yiF}(t) + K_{iy} \int_0^t e^*_{yiF}(t)\, dt \\ \Delta Z^*_i(t) &= K_{pz} e^*_{ziF}(t) + K_{iz} \int_0^t e^*_{ziF}(t)\, dt \end{aligned} \tag{12}$$

$$\Delta T^*_i = \begin{bmatrix} \Delta V^*_i(t) \\ \Delta \psi^*_i(t) \\ \Delta Z^*_i(t) \end{bmatrix} \tag{13}$$

The synchronization strategy is implemented as proposed with the same setup as previous models. Simulation results are shown in Section V in this paper.

To show that the concept of synchronized position will improve the performance of the formation controller, consider the coupled errors e^*_{x1F} and e^*_{x2F} in two cases.

$Case$ 1: The relative position errors are opposite in magnitude as proposed in Section II, then

$$e_{x1F} = -\eta e_{x2F} \quad \eta > 0 \tag{14}$$

Substituting (14) in the calculation of the coupled errors (10), the synchronization error terms become the following.

$$\varepsilon_{x1F} = e_{x1F} - e_{x2F} = \left(1 + \frac{1}{\eta}\right) e_{x1F} \tag{15}$$

$$\varepsilon_{x2F} = e_{x2F} - e_{x1F} = (1 + \eta)\, e_{x2F} \tag{16}$$

Using (15) and (16), the coupled position errors of the vehicles can be derived from (11).

$$e^*_{x1F} = \left[1 + \left(1 + \frac{1}{\eta}\right)\beta_{x1}\right] e_{x1F} \tag{17}$$

$$e^*_{x2F} = \left[1 + (1 + \eta)\,\beta_{x2}\right] e_{x2F} \tag{18}$$

Since $\beta_{xi} > 0$ by definition and $\eta > 0$, therefore $|e^*_{xiF}| > |e_{xiF}|$ from (17) and (18). This causes the controller to correct the velocity command in (12) in a faster manner with the same PI gains as $|\Delta V^*_i| > |\Delta V_i|$, thus eliminating e_{xiF} faster and in turn eliminates the synchronized error ε_{xiF} as well. In conclusion, the vehicles correct their trajectories quicker by using the coupled errors to improve the performance of the controller.

$Case$ 2: The relative position errors has the same sign as each other, then (14) becomes

$$e_{x1F} = -\eta e_{x2F} \quad \eta < 0 \tag{19}$$

From $\beta_{xi} > 0$ and $\eta < 0$, derivations from (17), (18) will result in $|e^*_{x1F}| > |e_{x1F}|$ and $|e^*_{x2F}| < |e_{x2F}|$ or vice versa by having η in reciprocal terms in the equations.

In the case of $|e^*_{xiF}| > |e_{xiF}|$, the coupling strategy again has the effect of eliminating e_{xiF} faster. However, in the case of $|e^*_{xiF}| < |e_{xiF}|$, the correction of e_{xiF} will be slower. This is the scenario where one of the vehicles is sacrificing its own performance to maintain the formation geometry better with other vehicles in the system as a whole. Therefore, the robustness of the formation flight is increased by the coupling and synchronizing behavior between the aircraft.

One possible worst case scenario from Case 2 is when $|e^*_{xiF}| = -\delta_1 |e_{xiF}|$ and $\delta_1 > 0$, resulting in opposite correction taken by one vehicle when calculating trajectory modification, but this scenario will eventually evolve to $|e^*_{xiF}| = \delta_2 |e_{xiF}|$ and $\delta_2 > 0$ as the other vehicle is eliminating its own error in the proper way.

Concluding results from both cases, the performance of the formation controller will improve with the incorporation of the synchronization technology. Simulations have been conducted with both cases and improvements are evident.

V. Simulations

Simulations are performed on a circular maneuver followed by straight line travel for two UAVs in a level flight formation, thus we focus on the commands and responses related to the xy-plane. An autopilot model, using the same architecture as the one in [11], developed for a MAGICC lab flying wing UAV, is used in the simulations. The desired relative distances from UAV1 and UAV2 to the reference point in the virtual structure are defined as [-4, 4, -1]$^T m$ and [4, 4, 1]$^T m$ respectively. The UAVs start at the desired positions at the beginning of the flight. A velocity of $14 m/s$ and a heading rate of 2 $^\circ/s$ are applied to the reference point. No collision avoidance algorithm is implemented.

Fig. 2(a) and Fig. 3(a) illustrate the relative position errors and the trajectories of the aircraft and the reference point with no formation control. It can be seen that the formation flight is not achieved and the relative position errors are opposite in magnitude when the reference point is defined as the centroid in the virtual structure as proposed in Section II.

Formation control is then implemented for the UAVs based on virtual structure approach and trajectory modifications

as proposed in Section III. The setup for the simulation is the same as before and the gains used for the trajectory PI controllers are given in Table I.

TABLE I: Control Gains of PI Controller

Parameters	Values	Parameters	Values
K_{px}	5	K_{ix}	0.5
K_{py}	0.005	K_{iy}	0.0005
K_{pz}	1	K_{iz}	0.005

Fig. 2(b) and Fig. 3(b) show the simulation results with formation controller implemented. It can be seen that the vehicles now maintain formation during flight and the relative position errors are stabilizing around zero, although there are obvious differences between the errors from the two aircraft.

Finally, the synchronization technology in Section IV is incorporated in the controller. The synchronization gains β_{xi}, β_{yi}, and β_{zi} for both vehicles are being set to 1. In this case, simulation results in Fig. 2(c) and Fig. 3(c) show that formation flight is again achieved between the two vehicles and the relative position errors in the virtual structure stabilize around zero. In comparison to Fig. 2(b), the relative position errors for the UAVs have been reduced.

A quantitative index that can be used to measure the performance of the controller is the root mean square (rms) value of the errors. Using the relative position error data from the two vehicles, their error root mean square values (E_{rms}) in the x and y directions for the three simulation scenarios (No. 1, 2 and 3) are summarized into Table II.

TABLE II: Relative position error rms (E_{rms}) of UAVs

UAV1

No.	Formation control method	E_{rms} in x	E_{rms} in y
1	None (using reference commands)	$7.5308m$	$5.5214m$
2	Trajectory commands modification	$0.0951m$	$1.0931m$
3	No.2 with synchronization	$0.0427m$	$0.4322m$

UAV2

No.	Formation control method	E_{rms} in x	E_{rms} in y
1	None (using reference commands)	$7.5000m$	$5.5445m$
2	Trajectory commands modification	$0.0888m$	$1.0016m$
3	No.2 with synchronization	$0.0354m$	$0.5552m$

The large E_{rms} values for the two vehicles in the x and y directions with no formation algorithm corresponds to the formation being broken. When the formation controller is used, the E_{rms} values in the x and y directions decrease by a significant amount. The decrease in the error rms values indicates that the vehicles are trying to maintain formation. The values further decrease when the synchronization strategy is incorporated. As an example, take the UAV1 E_{rms} value in x from Table II, it is shown that the E_{rms} from using the trajectory commands modification controller without synchronization strategy is $0.0951m$, however, the error is reduced to $0.0427m$ with synchronization strategy. Using the error without synchronization as the nominal value, this corresponds to a (0.0951 - 0.0427)/0.0951*100% = 55.1% decrease. Similar calculations reveal the same improvements in the E_{rms} value in y and also the E_{rms} values for UAV2 (appoximately 50%). This demonstrates that the strategy improves the performance of the formation controller. In other words, the vehicles maintain the virtual structure formation in a more "synchronized" pattern.

VI. Conclusions and Future Work

In conclusion, by adapting motion synchronization to the relative distance errors from the UAVs, the formation is being kept better by the vehicles. Simulations have also been conducted on non-level flights and UAVs starting in different initial positions, the results are acceptable as formation is again kept with the controller. This technique can be easily expanded to more than two vehicles as long as the relative distances of each UAV from the reference point are defined.

Future work includes reconfiguration of the virtual structure if the relative distances are defined as functions of time during formation flight. Investigation can also be performed on centralized/decentralized approaches for more than two aircraft. At UTIAS, implementation on real vehicles to verify the proposed controller in this paper is currently being undertaken and flight tests will be performed to study the effectiveness and performance of the controller.

VII. Acknowledgments

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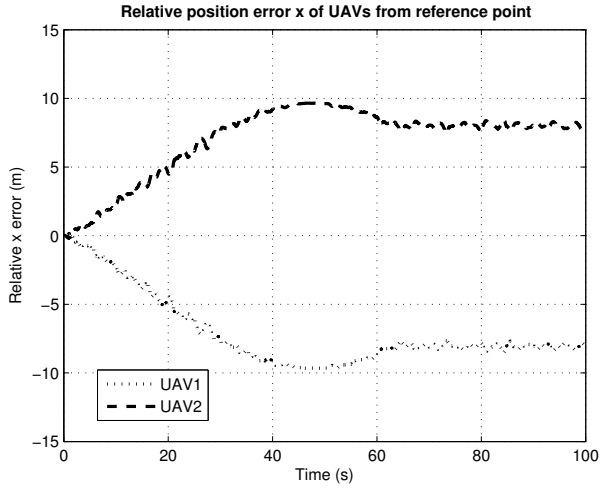
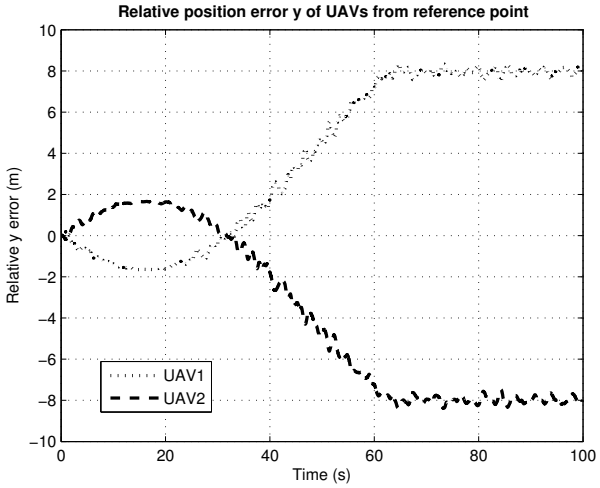
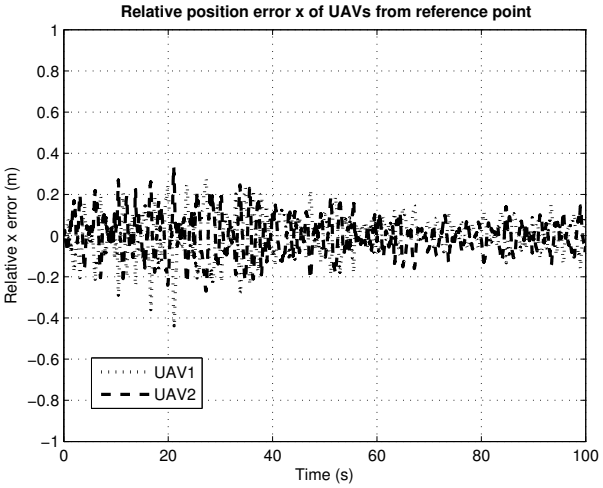
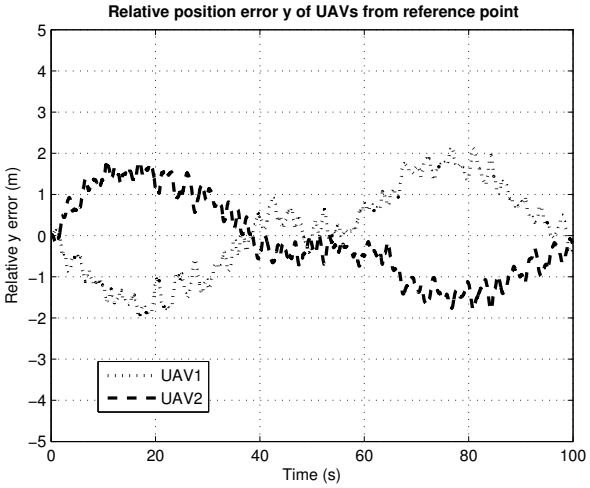


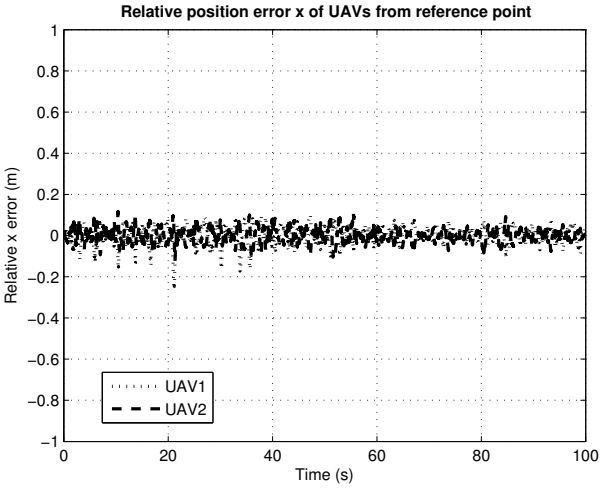


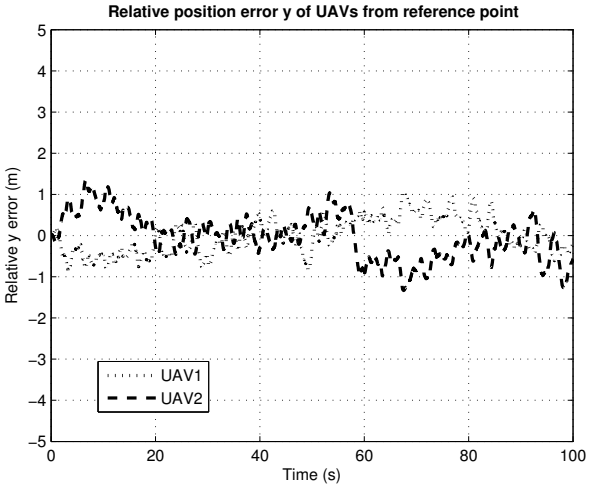
(a) Relative positions errors of UAVs - No formation algorithm implemented





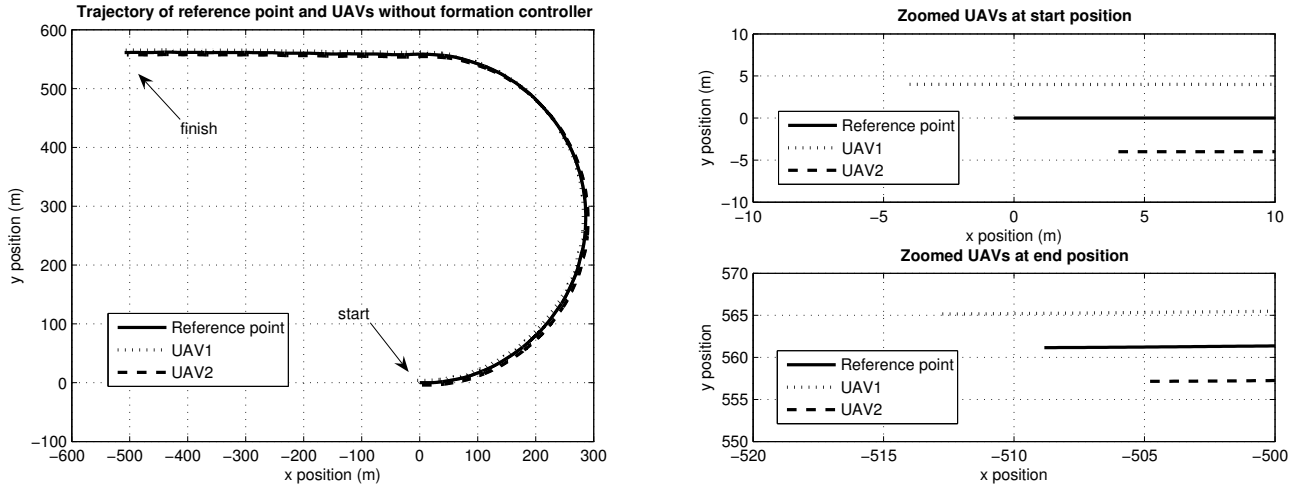
(b) Relative positions errors of UAVs - Trajectory commands modification



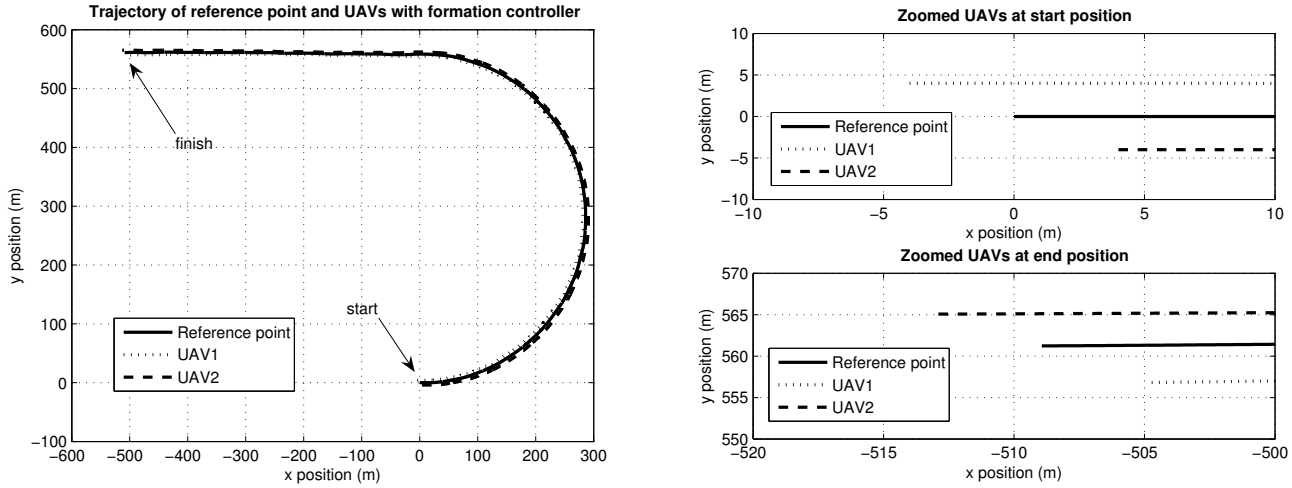


(c) Relative positions errors of UAVs - Trajectory commands modification with synchronization

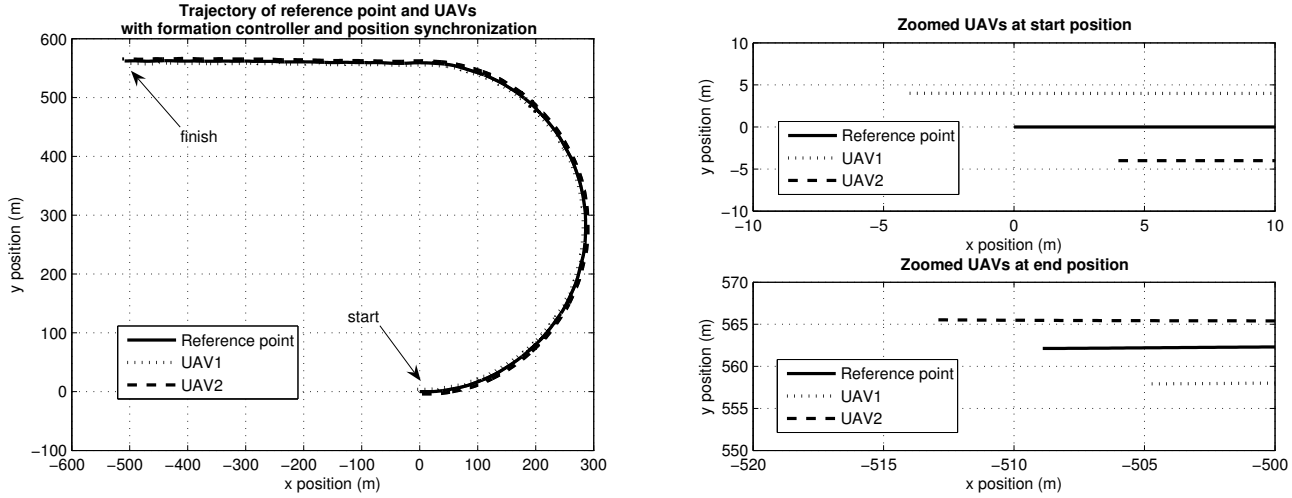
Fig. 2: Relative position errors of 2 flying wings in simulations



(a) Trajectory responses of UAVs - No formation algorithm implemented



(b) Trajectory responses of UAVs - Trajectory commands modification



(c) Trajectory responses of UAVs - Trajectory commands modification with synchronization

Fig. 3: Trajectory responses of 2 flying wings in simulations