V. Invitation to India.

Telephone: Simla 2487-676
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By air mail.

No. 3719/7.44 (6).
GOVERNMENT OF INDIA.
Central Board of Irrigation.
"Kennedy Hous"
Simla S. W.
Dated, the 25th May, 1948.

Prof. W. Fellenius,
President,
International Association of Hydraulic Structures Research,
c/o Kungl. Tekniska Högskolan, Stockholm.

Dear Sir,

You may have learnt that an invitation sent on behalf of the Government of India and the Indian National Committee to the International Commission on High Dams, to hold a meeting in India has been accepted and it has been decided to hold the Fourth Plenary Session of the International Commission on High Dams in India in February, 1951.

2. The Central Board of Irrigation and the Government of India have had under consideration the desirability of holding at the same time a meeting of the International Association of Hydraulic Structures Research at that time in India and it has now been decided to issue a formal invitation to the Association.

3. On behalf of the Government of India and the Central Board of Irrigation, I am now directed to convey a formal invitation to the International Association of Hydraulic Structures Research to hold its meeting in 1951 in Delhi (India). Government of India trust that this invitation will be acceptable to the Association, and I shall be glad to be informed accordingly at a very early date.

Yours faithfully,
N. D. Gulhati,
Secretary,
Central Board of Irrigation.

D. A.—Nil
"V"
25/5

IVWBV
IAHSR
AIRTH

Anlage
Appendix
Annexe
2.

Internationaler Verband für WasserBauliches Versuchswesen
International Association for Hydraulic Structures Research
Association Internationale de Recherches pour Travaux Hydrauliques

ZWEITE TAGUNG - SECOND MEETING - DEUXIEME REUNION
STOCKHOLM 7—9. VI. 1948

MEYER-PETER, E. and MÜLLER, R., Professors, Zurich.

Formulas for Bed-Load Transport*).

In the following paper, a brief summary is first of all given of the results and interpretation of tests already made known in former publications of the Laboratory for Hydraulic Research and Soil Mechanics at the Federal Institute of Technology, Zurich. After that, an attempt is made to derive an empirical law of bed-load transport based on recent experimental data.

We desire to state expressly that by bed-load transport is meant the movement of the solid material rolling or jumping along the bed of a river; transport of matter in suspension is not included.

I. The former test results.

a) Bed-load with uniform size of grain, natural specific gravity $\gamma_s = 2{,}68$ t/m³.

It was in 1934 that the Laboratory for Hydraulic Research at Zurich published for the first time a formula for bed-load transport based on transport experiments made with bed-load material of uniform grain size. According to that [I], bed-load transport followed a law of the form (Fig. 1):

*) Summary of the results of tests on transport of bedloads made in the Laboratory for Hydraulic Research and Soil Mechanics at the Swiss Federal Institute of Technology, Zurich.

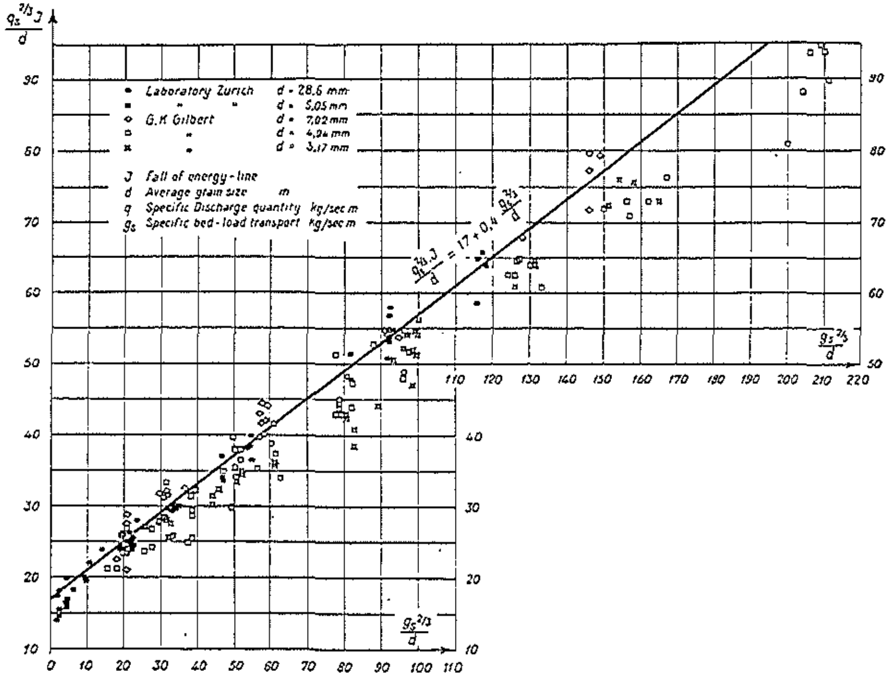


Fig. 1.

$$\left.\begin{aligned} \frac{q_s^{2/3}\,J}{d} &= a + b\,\frac{g_s^{2/3}}{d} \\ \frac{q_s^{2/3}\cdot J}{d} &= a + b'\,\frac{g_s''^{\,2/3}}{d} \end{aligned}\right\}\text{resp.} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

with the limiting value of the "transport magnitude" $\left(\frac{q_s^{2/3}\cdot J}{d}\right)_0$ for the beginning of the moving of the material ($g_s = 0$):

$$\left(\frac{q_s^{2/3}\cdot J}{d}\right)_0 = a \qquad \ldots\ldots\ldots\ldots\ldots \quad (1a)$$

The symbols in these equations signify:

J = the fall (slope) of the energy line;

q_s = the discharge quantity determining the bed-load transport, per unit width of bed and per second (specific discharge quantity);

g_s = the weight of the transported quantity of bed-load per unit width of bed and per second (specific bed-load transport) weighed dry;

g''_s = the specific bed-load transport weighed under water;

d = diameter of the particles of the bed-load material, determined by a square-meshed sieve.

a, b and b′ are constants, $\underline{a}$ not no-dimensional.

For q_s in kg/m.sec., g_s and g''_s in kg/m.sec., d in m, their value is a = 17. b = 0,4 and b′ = 0,547.

b) Test conditions and evaluation of the results.

Each separate test was carried out for some lenght of time with bed-load transport in a steady state. Further, an endeavour was made to have normal flow, i. e. parallelism between bed and water-surface and thus also of the energy line $J_s = J_w = J$ or uniformity of slope of the bed (J_s), of the water-surface (J_w) and of the energy line (J). This condition was not absolutely reached in every test, so that occasionally the average slope of the energy line had to be calculated from the slopes J_s and J_w measured during the test, the mean depth h and the mean velocity

$$v = \frac{Q}{B.h} \qquad \begin{matrix} (Q = \text{total quantity of water}) \\ (B = \text{width of channel}) \\ (v = \text{mean velocity}) \end{matrix} \qquad (2)$$

With the slight deviations between the three slopes, the approximate equation

$$J = J_w - \frac{v^2}{g.h}\left(J_w - J_s\right) \qquad \ldots\ldots\ldots\ldots \quad (3)$$

could be adopted [II], where g is the acceleration due to gravity.

An endeavour was made through the tests to determine the specific bed-load transport per unit of bed width — where G_s or G''_s respectively signifies the total transport —

$$g_s = \frac{G_s}{B} \quad \text{and} \quad g''_s = \frac{G''_s}{B} \qquad \ldots\ldots\ldots\ldots \quad (4)$$

i. e., transferred to natural conditions, the bed-load transport for a longitudinal strip 1 m vide. This intention was rendered difficult in each model channel by the more or less rough sides of the channel. Therefore, when determining the specific discharge quantity q_s — under the simplified assumption of uniformly distributed velocity and turbulence over the whole wetted cross-section — the only part taken into consideration of the total discharge quantity Q was that part Q_s whose energy was converted into eddying on the bed [III, also II]).

The application of Strickler's formula for velocity [IV] to the total discharge in the test channel:

$$v = k_m \cdot R^{2/3} \cdot J^{1/2} \quad \text{or} \quad k_m = \frac{v}{R^{2/3} \cdot J^{1/2}} \qquad \ldots\ldots\ldots\ldots \quad (5)$$

$$\text{with} \quad R = \frac{F}{P} = \frac{B \cdot h}{B + 2h} \qquad (h = \text{water depth}) \qquad \ldots\ldots\ldots\ldots \quad (6)$$

gave for each test a mean coefficient of roughness k_m Further, by preliminary tests it was possible to determine the coefficient of roughness k_w of the sides, and with these magnitudes, under the assumption of a uniform distribution of velocity, the bed roughness k_s and the specific discharge quantity q_s could be calculated. We had, namely:

$$k_s = \frac{k_m \cdot k_w \cdot B^{2/3}}{\left[B \cdot h^{3/2} + 2h\left(k_w^{3/2} - k_m^{3/2}\right)\right]^{2/3}} \qquad \ldots\ldots\ldots\ldots \quad (7)$$

*)

$$\text{and} \quad q_s = Q \cdot \frac{k_w^{3/2}}{2h \cdot k_s^{3/2} + B \cdot k_w^{3/2}} \qquad \ldots\ldots\ldots\ldots \quad (8)$$

$$\text{with} \quad Q_s = q_s \cdot B \qquad \ldots\ldots\ldots\ldots \quad (9)$$

Under the same assumptions, also an hydraulic radius of the bed discharge can be defined. If F_s and R_s are respectively the cross-section and profile radius apertaining to the water quantity Q_s, then:

$$F_s = \frac{Q_s}{v} \quad \text{and} \quad P_s = B$$

$$\text{so that} \quad R_s = \frac{F_s}{P_s} = \frac{Q_s}{v \cdot B}$$

On the other hand, in the rectangular experimental channel

$$B \cdot h \cdot v = Q \quad \text{or} \quad B \cdot v = \frac{Q}{h}, \text{ so that } R_s = \frac{Q_s}{Q} \cdot h \qquad \ldots\ldots\ldots\ldots \quad (10)$$

c) Bed-load with material particles of uniform size but of various gravities.

In connection with the tests mentioned under a), series of tests were carried with baryta ($\gamma_s = 4{,}2$ t/m³) and also with lignite breeze ($\gamma_s = 1{,}25$ t/m³). The results confirmed the form (1) of the law, in that only the constants $\underline{a}$ and b′ showed themselves as magnitudes dependent on the specific gravity. Accordingly the law when generalised [V] (Fig. 2) is:

*) Corr: $Bk_w^{3/2}$ in stead of $Bh^{3/2}$.

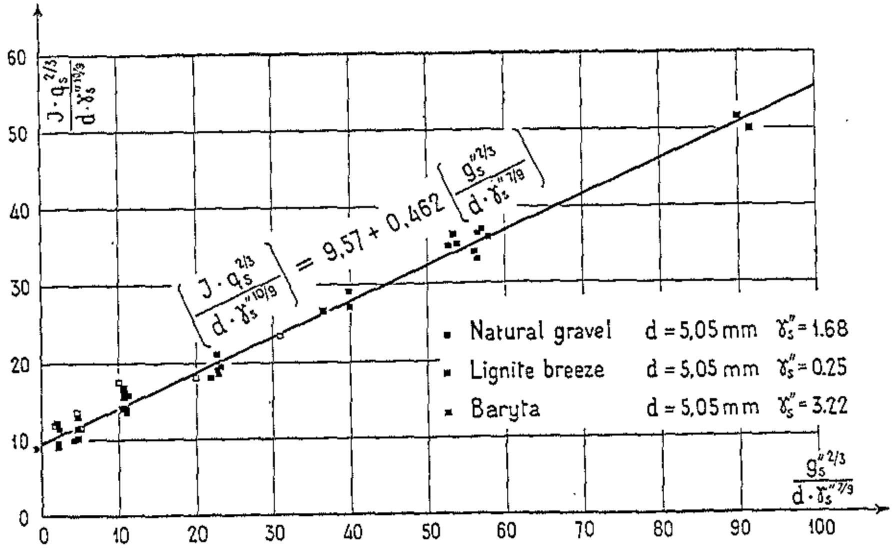


Fig. 2.

$$\frac{q_s^{2/3} \cdot J}{d} = a'' \cdot \gamma_s''^{10/9} + b'' \cdot \gamma_s''^{1/3} \cdot \frac{g_s''^{2/3}}{d} \qquad \ldots\ldots\ldots\ldots \quad (11)$$

with the limiting value of the transport magnitude for the beginning of the bed-load transport ($g_s'' = 0$):

$$\frac{q_s^{2/3}\, J}{d} = a'' \cdot \gamma_s''^{10/9} \qquad \ldots\ldots\ldots\ldots \quad (11a)$$

New in the two equations is:

$$\gamma_s'' = (\gamma_s - \gamma_w) = \gamma_s - 1$$

the specific gravity of the bed-load weighed under water.

Also the new constants a″ and b″ are not no-dimensional. For q_s and g_s'' in kg/m.sec. and d in m, they amount to:

a″ = 9,57 and b″ = 0,462.

d) Bed-load mixtures of natural specific gravity $\gamma_s = 2{,}68$ t/m³.

A first series of tests was made with a bed-load mixture with particles measuring from 0—10 mm [II]. The mixture was analysed with square-meshed sieves, and the result is shown as mixing or summation line of the percentage share of the weight of each fraction lying between two neighbouring sieve sizes (Fig. 3).

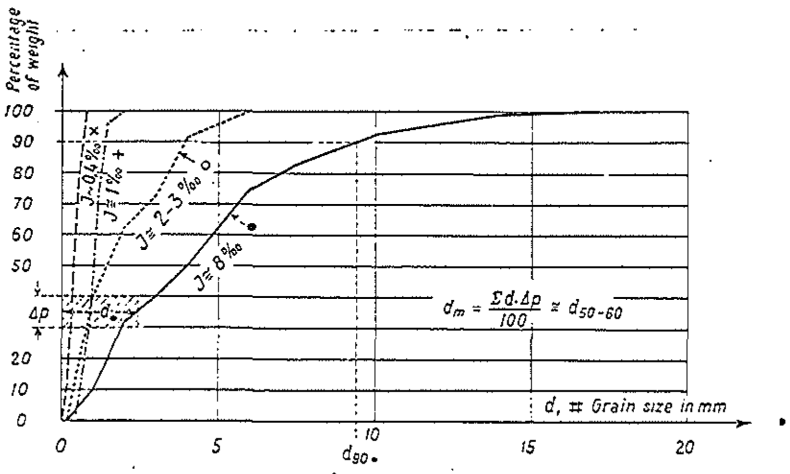


Fig. 3.

When evaluating the results on the basis af the law obtained for particles of uniform size (equation (1)), an attempt was made to introduce a single "effective diameter" to charactérise the mixture. The best solution for the definition of the "effective diameter" was found to be the expression

$$d_m = \frac{\Sigma\, d\, \Delta p}{100} \qquad \text{..........} \quad (12)$$

d is the average size of the particles in a fraction.

The representation of the results, evaluated according to equation (1) (Fig. 4) with $d = d_m$, showed an essential deviation in the run of the function from that with particles of uniform size. Variable pairs of constants (a, b') had to be introduced into equation (1) for different regions of the strength of the bed-load transport. In any case the series of tests

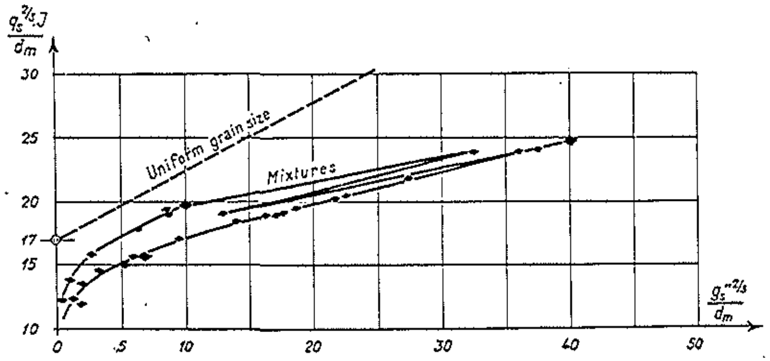


Fig. 4.

showed that a definite conclusion could only be reached with a large number of tests over a big range of variables.

2. New test results and their evaluation.

a) Tests with mixtures of natural specific gravity. Since 1938 the tests with mixtures were continued. An endeavour was made to extend the range of investigation above all with respect to the fall. In order not to obtain any unnatural combination of the variables, natural examples were chosen as basis for the arrangement of the test series and built up, simplified two-dimensionally to a suitable scale in the big measuring channel with a width of 2 m. Whole series of tests were thus carried out with the following falls and mixtures (Fig. 3):

Falls:	*Effective grain diam.*
J = 0,4—0,5 $^0/_{00}$	d_m = 0,4 mm
2 —3 $^0/_{00}$	1,7—2 mm
8 $^0/_{00}$	4,4 mm

Further, individual tests were made in a smaller channel with falls up to 20 $^0/_{00}$.

The described evaluation in the representation

$$\frac{q_s^{2/3} \cdot J}{d_m} \text{ as a function of } \frac{q_s''^{2/3}}{d_m}$$

according to equation (1) gave a very unfavorable picture (Fig. 5). To

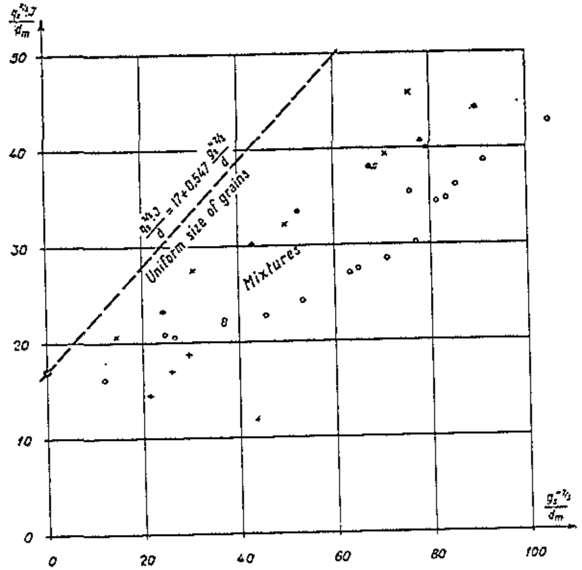


Fig. 5.
(For signatures see Fig. 3 and Table 2.)

be sure, the test points of each series lie approximately on a straight line, but these lines do not coincide with each other, which means that the "constants" prove to be variable. There is also a big deviation from the results obtained with bed-load consisting of particles of uniform size.

In parallel with these bed-load transport measurements, special tests concerning the commencement of the bed-load transport were carried out in the same fall region, and also with bed-load of natural specific gravity but consisting of **particles of uniform size.** The representation of the limiting transport magnitudes

$$\left(\frac{q_s^{2/3}\cdot J}{d}\right)_0$$

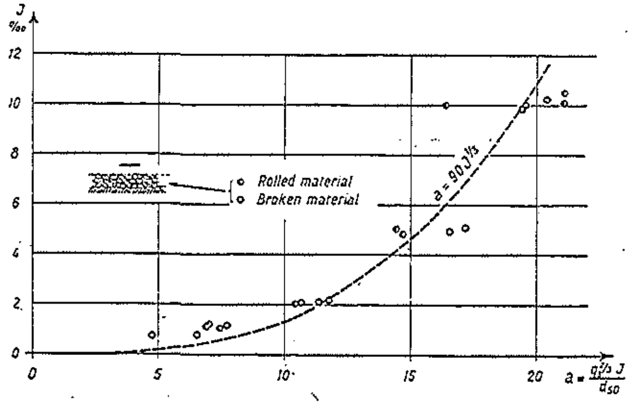


Fig. 6.

as a function of the fall J (Fig. 6) shows without doubt a dependence on the fall. From the tests it follows approximately:

$$\left(\frac{q_s^{2/3}\cdot J}{d}\right)_0 = f\cdot J^{1/3} \quad \text{or} \quad \left(\frac{q_s^{2/3}\cdot J^{2/3}}{d}\right)_0 = f = \text{constant} \quad .. \ (13a)$$

The expression $q_s^{2/3}\ J^{2/3}$ may now be transformed with the help of Strickler's law of friction [IV]. His formula for the mean flow velocity is:

$$v = k_s \cdot R_s^{2/3} \cdot J^{1/2} \quad \ldots\ldots\ldots\ldots \ (5a)$$

where k_s is the coefficient of roughness for the bed in question.

For k_s Strickler proposes the formula: $k_s = \dfrac{c}{d^{1/6}}$ where d is the diameter of the particles in m.

Thus we have:

$$v = c\left(\frac{R_s}{d}\right)^{1/6} \cdot R_s^{1/2} \cdot J^{1/2} \quad \ldots\ldots\ldots\ldots \ (5b)$$

If we now — in contrast to what was done in the former tests — express q_s in t/m.sec. and designate the specific gravity of the water with γ_w in t/m³, we have:

$$q_s = \gamma_w \cdot v \cdot R_s = \gamma_w \cdot c\left(\frac{R_s}{d}\right)^{1/6} \cdot R_s^{3/2} \cdot J^{1/2}$$

or

$$q_s^{2/3} = \gamma_w^{2/3} \cdot c^{2/3}\left(\frac{R_s}{d}\right)^{1/9} \cdot R_s \cdot J^{1/3} \quad \ldots\ldots\ldots\ldots \ (15)$$

Substituting in equation (13a), this gives:

$$\frac{q_s^{2/3}\cdot J^{2/3}}{d} = \gamma_w^{2/3}\cdot c^{2/3}\left(\frac{R_s}{d}\right)^{1/9} \cdot \frac{R_s \cdot J}{d} = f$$

or

$$\left(\gamma_w \cdot R_s \cdot J\right)_0 = \gamma_w^{1/3}\ \frac{f}{c^{2/3}} \cdot \left(\frac{d}{R_s}\right)^{1/9} \cdot d$$

Finally, if we put

$$\frac{\gamma_w^{1/3}\cdot f}{c^{2/3}} = K$$

we have:

$$\left(\frac{\gamma_w \cdot R_s \cdot J}{d}\right)_0 = K\left(\frac{d}{R_s}\right)^{1/9} \quad \ldots\ldots\ldots\ldots \ (14a)$$

The limiting shearing stress $(\gamma_w \cdot R \cdot J)_0$ in t/m² consequently appears with known value of c to be proportional to the particle diameter, but is in addition also dependent on the relative roughness $\frac{d}{R_s}$. The constant K has the dimension of a specific gravity. Even with big fluctuations of $\frac{R_s}{d}$, the factor $\left(\frac{R_s}{d}\right)^{1/9}$ varies within fairly narrow limits. In particular with a **given** river, with a definite size of particles, and with relatively little variation in water depth, it is practically constant.

Essential, however, is the ascertained fact that the commencement of bed-load transport is dependent on a limiting shearing stress. This fact makes it appear probable that also the bed-load transport follows in some form or other, the shearing stress and induces one to evaluate the bed-load transport tests anew in this sense.

Starting with equation (13a), which could be used in place of equation (1a), it appears evident to replace the original expression (equation (1)) tentatively by the following form:

$$\frac{q_s^{2/3}\cdot J^{2/3}}{d_m} = c_1 + c_2\ \frac{q_s''^{2/3}}{d_m} \quad (13)\ (\text{Fig. 7}).$$

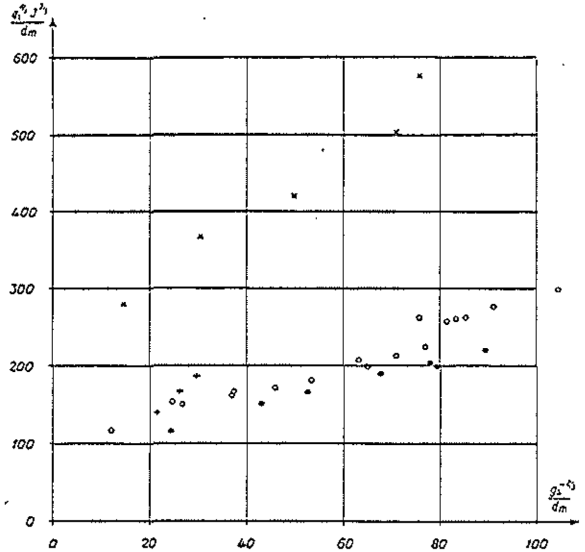


Fig. 7.

or also, corresponding to equation (14a), by the relation

$$\gamma_w \cdot \frac{R_s \cdot J}{d_m} = c_3 + c_4 \frac{g_s''^{2/3}}{d_m} \qquad \text{(14) (Fig. 8).}$$

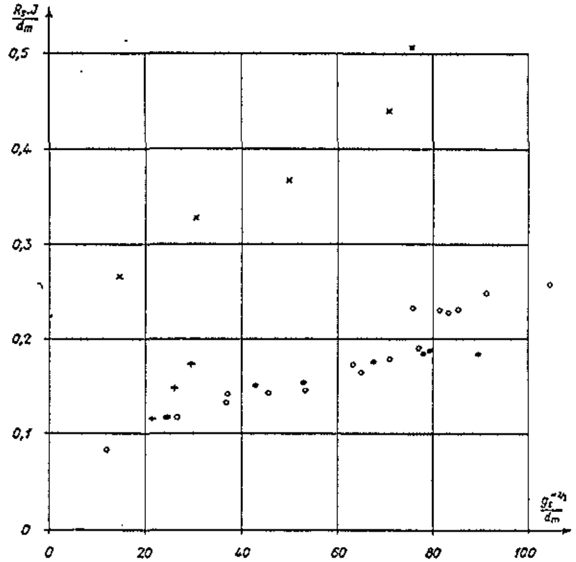


Fig. 8.
(For signatures see Fig. 3 and Table 2.)

Naturally, the lefthand term in (14) does not exactly replace the lefthand term in (13), as can at once be seen by comparing equations (13a) and (14a). A trial should only be made to see which of the two expressions agrees better with the test results. However, neither of them is yet satisfactory.

In both cases it is particularly the tests with the fine bed-load (0,4 mm) and those with great uneveness of the bed (ripples and banks) that do not conform to any of the two laws (13) and (14). In these tests the bed-load transport is essentially smaller and points therefore to the necessity of considering the uneven shape of the bed as a factor hindering bed-load transport. The total fall

$$J = \frac{v^2}{k_s^2 \cdot R_s^{4/3}} \qquad (16)$$

as expression fort the total loss of energy (energy conversion), may be considered as consisting of two partial falls [VI], namely of a first part as expression for the loss occuring through the uneveness of the bed (roughness of shape), and of a second part, the pure frictional fall J_r, as expression for the energy converted into swirling by actual rubbing on the bed (particle roughness). With a given discharge operation (v, R_s), this pure frictional fall amounts to:

$$J_r = \frac{v^2}{k_r^2 \; R_s^{4/3}} \quad \ldots\ldots\ldots\ldots \quad (17)$$

with k_r as coefficient of particle friction with smooth bed. On the other hand a discharge operation (v, R_s) formed by roughness of shape and of the particles will show the total fall of the energy line.

The ratio between the pure frictional fall and the total fall consequently amounts, according to (16) and (17), to:

$$\frac{J_r}{J} = \left(\frac{k_s}{k_r}\right)^2$$

and the pure frictional fall is: $J_r = \left(\frac{k_s}{k_r}\right)^2 \cdot J$ .... (18)

If it is now assumed that the bed-load transport depends only on the energy converted into swirl at the bed-load particle, the pure frictional fall J_r must be inserted in the bed-load transport equations. Equation (14) thus becomes:

$$\gamma_w \left(\frac{k_s}{k_r}\right)^2 \frac{R_s \; J}{d_m} = c_5 + c_6 \frac{g_s''^{2/3}}{d_m} \quad \ldots\ldots\ldots\ldots \quad (19)$$

On the other hand equation (14a) was derived from equation (13a). In the latter, according to equation (15), the term $q_s^{2/3}$ contains the value $J^{1/3}$

(total fall). The expression $R_s \cdot J$ contained in equation (14a) is therefore composed of the value $J^{1/3}$ of the total fall multiplied by another fall value $J^{2/3}$, for which the frictional fall $J_r^{2/3}$ was tentatively inserted.

In this way equation (14) becomes changed to the form:

$$\gamma_w \frac{R_s \cdot J^{1/3} \cdot J_r^{2/3}}{d_m} = c_3 + c_4 \frac{g_s''^{2/3}}{d_m}$$

or with $J_r^{2/3} = \left(\frac{k_s}{k_r}\right)^{4/3} \cdot J^{2/3}$ — according to equation (18) —

$$\gamma_w \left(\frac{k_s}{k_r}\right)^{4/3} \cdot \frac{R_s \cdot J}{d_m} = c_7 + c_8 \cdot \frac{g_s''^{2/3}}{d_m} \quad \text{............} \quad (20)$$

Therefore the value of the exponent of $\frac{k_s}{k_r}$ must lie between 2 (equation (19)) and $4/3$ (equation (20)). The evaluation of the test results has now shown quite clearly that the exponent $3/2$ gives the best interpretation, so that the expression

$$\boxed{\gamma_w \cdot \frac{Q_s}{Q} \left(\frac{k_s}{k_r}\right)^{3/2} \frac{h \cdot J}{d_m} = A + B' \cdot \frac{g_s''^{2/3}}{d_m}} \quad \text{............} \quad (21)$$

in which te factor R_s is expressed by $R_s = \frac{Q_s}{Q} \cdot h$, ... (10)

represents the best form of the law for bed-load of natural specific gravity.

The operation of evaluating the test results is summarised in Table 1. The determination of the coefficient k_r of particle roughness for the smooth bed offers some difficulties. This may, however, be calculated from the λ-values measured by Nikuradse [VII] in the circular pipe in depence on the Reynolds' number and the relative roughness. In the region of fully developed turbulence, Nikuradse's measurements correspond to Strickler's simple formula [IV]:

$$k_r = \frac{c}{d^{1/6}} \quad \text{............} \quad (22)$$

where c = 26 [II]. For the evaluating of any desired crossection, Nikuradse's diagram is drawn in Fig. 9 with the hydraulic radius R_s instead of the pipe dimension. The diagram makes it possible to judge the degree of turbulence from tests and, according to the relation

$$k_r = \left(\frac{8g}{\lambda}\right)^{1/2} \cdot \frac{1}{R_s^{1/6}} \quad \text{............} \quad (23)$$

gives the coefficient of particle roughness k_r when the Reynolds' number

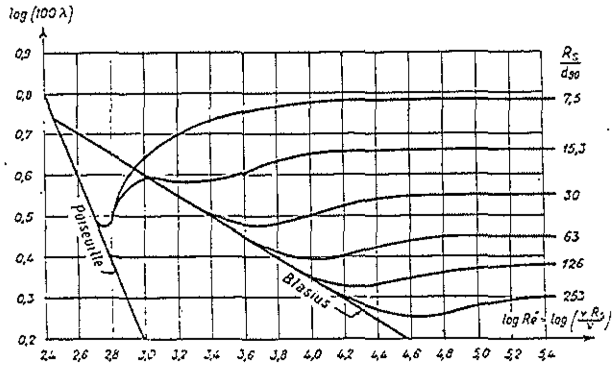


Fig. 9.

$Re' = \frac{v \cdot R_s}{\nu}$ and the relative roughness $\frac{d}{R_s}$ are given.

In the case of mixtures it is first of all not known what value of d must be inserted in equation (22). Nevertheless the calculated k-values give a clue. With smooth bed and slight bed-load transport, k_s becomes k_r, and various series of tests contain measurements with approximately smooth bed. The evaluation of the test results shows clearly that, in the case of mixtures and adopting Nikuradse's diagram, the relative roughness, calculated from the coarser categories of the mixture as about

$$\text{relative roughness} = \frac{d_{90}}{R_s} \quad \text{............} \quad (24)$$

— with d_{90} as diameter at about 90 % by weight of the bed-mixture line — is the decisive factor. Therefore, also when using equation (22), it is necessary to calculate with

$$k_r = \frac{c}{d_{90}^{1/6}} \quad (c = 26 \text{ in metric units, } \frac{m^{1/2}}{sec}) \quad \text{.....} \quad (25)$$

It is therefore the coarse particles of the mixture which condition the roughness decisive for water and bed-load transport. This is also in agreement with the known observation that the average diameter of the surface covering layer in a natural river bed coincides nearly with the diameter at 90 % (d_{90}) of the actual bed mixture [II]. Also Strickler's measurements [IV] confirm this result. According to his data it is necessary to calculate with c = 21 with introducing of d_{50}. This corresponds in the case of the natural mixture to the values d_{90} and c = 26. With the natural mixtures tested in the Laboratory it amounts approximately [II] to

$$\frac{d_{90}}{d_{50}} \cong \left(\frac{90}{50}\right)^2 = \sim 3{,}24$$

From this we have, since:

$$k_r = \frac{21}{d_{50}^{1/6}} = \frac{c}{d_{90}^{1/6}}$$

$$c = \left(\frac{d_{90}}{d_{50}}\right)^{1/6} \cdot 21 = 21 \cdot 1{,}22 = 25{,}6 \qquad |c| = \frac{m^{1/2}}{sec}$$

or approximately $= 26$.

The evaluation of the measurements of the Laboratory confirms for all tests the fully developed turbulence, so that the coefficient of particle roughness may be calculated with sufficient accuracy from

$$k_r = \frac{26}{d_{90}^{1/6}} \qquad (25) \qquad |k_r| = \frac{m^{1/3}}{sec} \qquad |d| = m$$

All tests with material of natural specific gravity may therefore be evaluated with k_s according to equation (7), and q_s and Q_s according to equations (8) and (9) respectively. The representation of equation (21) in the form:

$$\gamma_w \frac{Q_s}{Q} \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \frac{h\,J}{d_m} \quad \text{as a function of} \quad \frac{g_s''^{2/3}}{d_m}$$

actually gives a straight line round which the results of the tests are grouped, wheter the particles are of uniform size or of mixed sizes (Fig. 10). Table 2 gives the principal dates of all tests and their signatures for the Figs. 10 and 11.

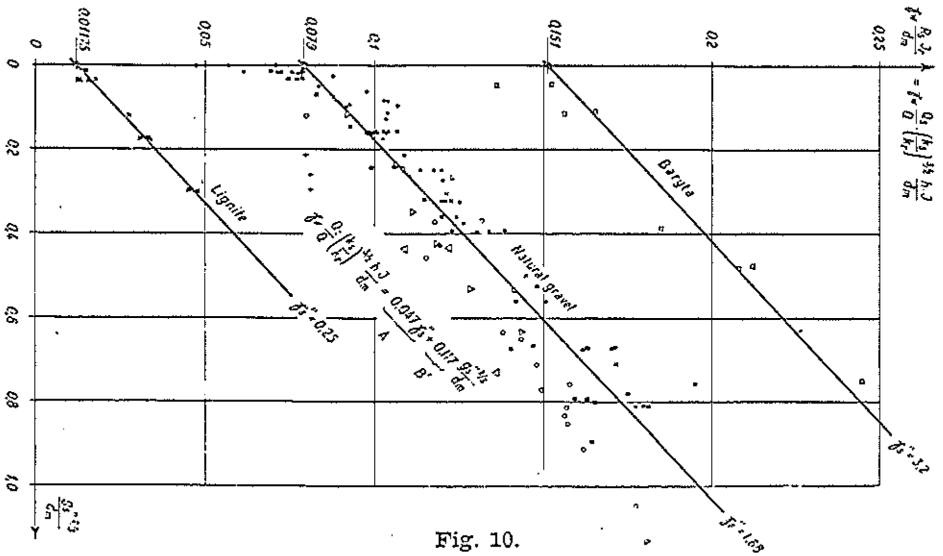


Fig. 10.

b) Extension of the law to bed-load of any desired specific gravity.

The evaluation of the test results with coal breeze and baryta as bed-load according to equation (21) gives in the usual representation (Fig. 10)

$$\gamma_w \cdot \frac{Q_s}{Q} \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot \frac{h \cdot J}{d_m} \quad \text{as a function of} \quad \frac{g_s''^{2/3}}{d_m}$$

straight lines parallel to the tests with natural gravity. From this it follows that only the constant A (equation (21)) depends on the specific gravity, and in fact the direct proportionality of A with $\gamma_s'' = \gamma_s - \gamma_w$ to the specific gravity under water of the bed-load is confirmed. It was therefore at once possible to formulate a general law for bed-load transport in which the shearing tension was introduced in the left side member of equation (1) instead of the discharge quantity, q_s.

$$\boxed{\gamma_w \cdot \frac{Q_s}{Q} \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot h \cdot J = A'' \cdot \gamma_s'' \cdot d_m + B'' \left(\frac{\gamma_w}{g}\right)^{1/3} \cdot g_s''^{2/3}} \quad \ldots\ldots \quad (26)$$

The value of the contants is:

$A'' = 0{,}047$ and $B'' = 0{,}25$

g = acceleration due to gravity, $\frac{m}{sec^2}$

A'' is no-dimensional. As the specific bed-load transport is not a linear function of the shearing tension, the constant B'' of the second righthand member is multiplied with the factor $\left(\frac{\gamma_w}{g}\right)^{1/3}$ in order to get it no-dimensional.

It has not yet been stated experimentally whether equation (26) is still valid for a liquid with a specific gravity different from one.

The method of writing the two constants A'' and B'' is to indicate that in equation (26) the bed-load transport g_s'' is measured under water. The limiting shearing tension for the beginning of the bed-load transport thus amounts to:

$$\boxed{\left[\gamma_w \cdot \frac{Q_s}{Q} \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot h \cdot J\right]_0 = A'' \cdot \gamma_s'' \cdot d_m} \quad \ldots\ldots \quad (26a)$$

In the special case of the smooth bed ($k_s = k_r$) and in wide channels ($Q_s = Q$) with a cover of particles of uniform size d, equation (26a) becomes:

$$\left(\gamma_w \cdot h \cdot J\right)_0 = A'' \cdot \gamma_s'' \cdot d \quad \ldots\ldots\ldots\ldots \quad (27)$$

for the simplest form of the limiting value of the shearing force.

In Fig. 11 are represented all the magnitudes determined from the measurements according to equation (26) for all bed-load transport tests of the Laboratory.

$$\gamma_w \cdot \frac{Q_s}{Q} \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot \frac{h\,J}{d_m} \cdot \frac{1}{\gamma_s''} \quad \text{as a function of} \quad \left(\frac{\gamma_w}{g}\right)^{1/3} \cdot \frac{g_s''^{2/3}}{d_m} \cdot \frac{1}{\gamma_s''}$$

The straight line drawn in as good average value of the measured points, gives for $g_s = 0$ the constant

$$A'' \cong 0{,}047$$

and the inclination of the straight line corresponds to the constant

$$B'' \cong 0{,}25$$

The diagram contains the measurements with particles of uniform size, with mixtures, and with the three kinds of bed-load: natural bed-load, lignite breeze, and baryta.

When comparing the equations (11) and (26) extended to a bed-load of any desired specific gravity, it is interesting to find that, according to the new equation (26), the specific gravity affects only the beginning of the bed-load transport. In the region with transport, the bed-load transport g_s'' by weight is only dependent on the difference

$$\gamma_w \cdot \frac{Q_s}{Q} \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot h \cdot J - A'' \cdot \gamma_s'' \cdot d_m$$

between the effective and the limiting shearing stress.

More careful observation of the run of the function in Fig. 11 shows a fall in the measured points towards $g_s'' = 0$. In cases where only the beginning of the bed-load transport is to be determined, the constant A'' must accordingly be assumed smaller, about

$$A'' \cong 0{,}03,$$

a result which agrees well with the measurements obtained by Shields [VIII]. In addition, it must here be remembered that the observation of the beginning of bed-load transport is always subjective. Table Nr. 3 shows the results of the evaluation — according to equation (26a) — of direct observations (Fig. 6) on the commencement of bed-load transport. The constant A'' is varying between 0,03 and 0,05.

From the nature of things, the measured results (Fig. 11) are widely scattered. When forming a judgement on them, however, the periodic very great fluctuations in intensity of the bed-load transport must be kept in mind, as well as the small fall, which it is very difficult to determine during the test. The Figure 11 contains a very great range of investigations, namely:

Falls from 0,4 to 20 ‰
Size of particles (d & d_m) from 0,4 to 30 mm
Water depths h from 1 to 120 cm

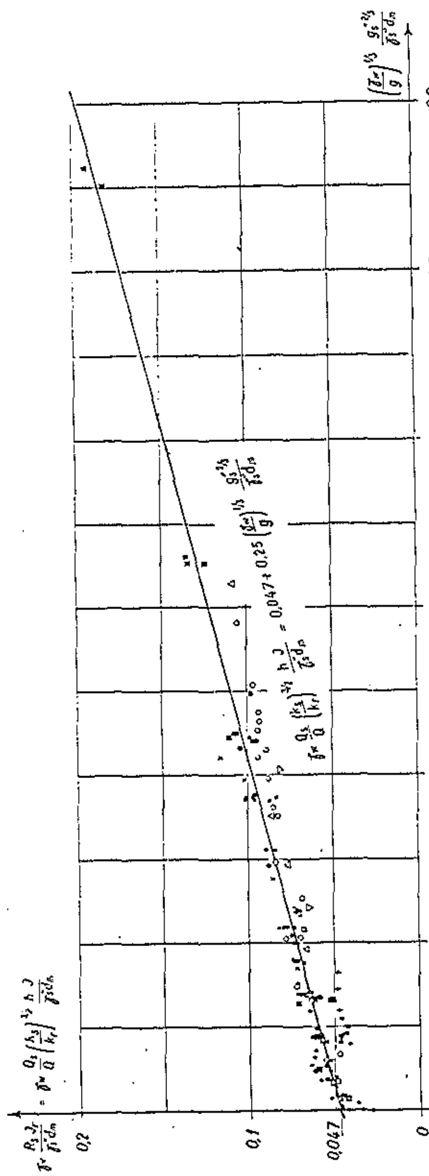


Fig. 11.

Discharge quantities from 2 lit/sec. to 4 m³/sec. (from 0,002 to 2 m³/m.sec.)

Specific gravities (under water γ_s'') from 0,25 to 3,2 tons/m³.

so that the result should be regarded as satisfactory.

3. Summary, and practical instructions for evaluating tests and applying them to natural examples.

Based on the tests carried out for many years on bed-load transport in the region of fully developed turbulence, it is found that a general law of bed-load transport includes the shearing stress as decisive magnitude. The tests comprise

Falls fron 0,4 to 20 ‰

Size of particles (d & d_m) from 0,4 to 30 mm

Water depths h from 1 to 120 cm

Discharge quantities from 2 lit/sec. to 4 m³/sec. (from 0,002 to 2 m³/m.sec.)

Specific gravities (under water $\gamma_s = \gamma_s - 1$) from 0,25 to 3,2 tons/m³.

Both the form of the empirical law of bed-load transport according to equations (21) and/or (26), as also the tests carried out on various scales, show the applicability of the Froude law of similarity and thereby the possibility of utilising the law of bed-load transport under natural conditions with an accuracy sufficient for practical purposes.

Starting with this, the tests comprise a big region, namely from the beginning of bed-load transport up to the big transport capacities of rolling bed-load occuring in nature in streams in full flood. All separate tests were carried out for a length of time with conditions in a steady state, and from these the periodical mean values were obtained. The only assumption remaining is the requirement of good agreement between the particle composition of the moving bed-load and that of the bed, i. e. the movability of the bed as occuring in nature in alluvial stretches.

By the tests an endeavour was made to determine the specific bed-load transport per unit of bed width withouth the influence of the sides, i. e. — passing over to nature — the bed-load transport in a longitunal strip 1 m wide. This intention was rendered difficult in each model channel by the roughness of the sides of the channel. The way in wich the results of the tests were evaluated, i. e. the elimination of the effect of the sides, is therefore of great importance. It can be carried out under the simplified assumption that the distribution of velocity and turbulence is uniform over the whole wetted cross-section [III]. Table 1 shows the process of evaluating the results of the tests with final result of the law of bed-load transport obtained according to equation (26) (Fig. 11):

$$\gamma_w \frac{Q_s}{Q} \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot h \cdot J = 0{,}047 \cdot \gamma_s'' \cdot d_m + 0{,}25 \left(\frac{\gamma_w}{g}\right)^{1/3} \cdot g_s''^{\,2/3} \quad \ldots\ldots \quad (26)$$

and with the limiting case for the beginning of the bed-load transport as confirmed by special tests:

$$\left[\gamma_w \cdot h \cdot J\right]_0 = 0{,}047 \frac{Q}{Q_s} \cdot \left(\frac{k_r}{k_s}\right)^{3/2} \cdot \gamma_s'' \cdot d_m \quad \ldots\ldots\ldots\ldots \quad (26a)$$

In very wide channels without influence from the sides, $\frac{Q_s}{Q} = 1$, and in the case of a smooth bed without any essential gravel-bank or ripple formations, at least at the beginning of the bed-load transport, $\frac{k_r}{k_s} = 1$, so that under these two assumptions the simplified equation

$$\left[\gamma_w \cdot h \cdot J\right]_0 \cong 0{,}047 \, \gamma_s'' \cdot d_m \quad \ldots\ldots\ldots\ldots \quad (27)$$

is obtained for the beginning of bed-load transport. It represents the extrapolation of the law of bed-load transport towards $g_s = 0$, and thus an upper limit. For absolute rest, it is necessary to calculate with

$$\left[\gamma_w \cdot h \cdot J\right]_0 \cong 0{,}03 \, \gamma_s'' \, d \quad \ldots\ldots\ldots\ldots \quad (27a)$$

With increasing bed-load transport $\frac{k_s}{k_r}$ decreases even without any formation of gravel-banks or ripples, — according to the tests solely because of the existence of the bed-load transport. Evidently the stones in movement and raised above the bed form a rough covering moving slower than the water and protecting the bed lying under it. With the formation of gravel-banks and ripples the ratio $\frac{k_s}{k_r}$ becomes still smaller. This form roughness thus reduces the bed-load transport still more.

In the experimental channel the considering of these factors offered no difficulty, since k_s and k_r can be determined from the test measurements. In general also in nature the ratio $\frac{k_s}{k_r}$ can be determined from given conditions and measurements as in the experimental channel. Further, to assist in making an approximate assumption, the ratios $\frac{k_s}{k_r}$ as determined from the tests are given.

These values are plotted in Fig. 12 against $\frac{\gamma_w^{1/3} g_s''^{\,2/3}}{\gamma_s'' g^{1/3} d}$ for different shapes of the bed. The Figs. 13—17 give these shapes for 5 typical test results represented in Fig. 12.

Also in wide channels ($Q_s = Q$) without any gravel-bank or ripple formation in the region of very great bed-load transport, the factor $\frac{k_s}{k_r} < 1$,

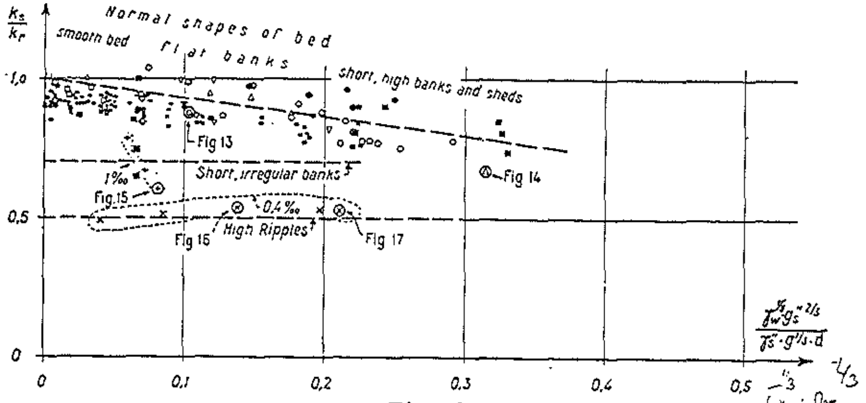


Fig. 12.

so that for wide channels it is necessary to reckon with the equation of bed-load transport:

$$\gamma_w \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot h \cdot J = 0{,}047\, \gamma_s'' \cdot d_m + 0{,}25 \left(\frac{\gamma_w}{g}\right)^{1/3} \cdot g_s''^{2/3} \quad \ldots\ldots \quad (28)$$

The investigations are now being continued with respect to the adoption in nature of the obtained law of bed-load transport. It would be desirable if other institutes, who have carried out bed-load transport tests, would also have their measurements evaluated in the method shown here. In the case of tests which do not lie in the region of fully developed turbulence, k_r is to be determined from Nikuradse's diagram and according to equation (23). Decisive then are the relative roughness $\frac{d_{90}}{R_s}$ and the Reynolds' number $Re' = \frac{v \cdot R_s}{\nu}$.

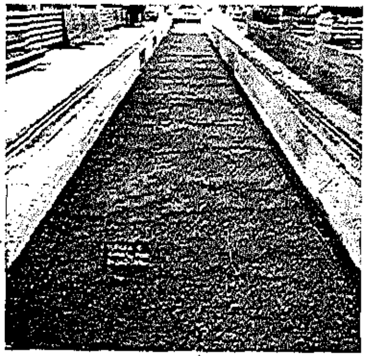


Fig. 13.

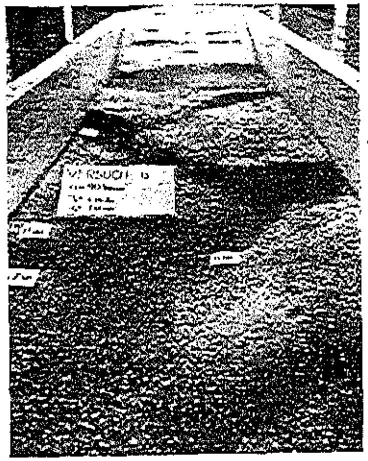


Fig. 14.

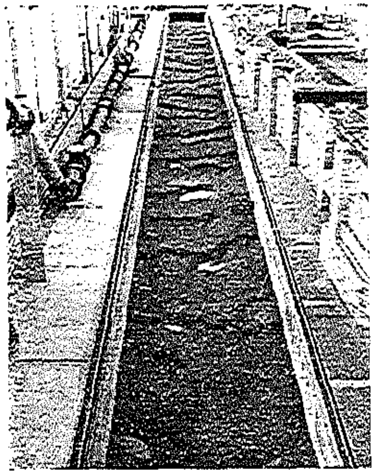


Fig. 15.

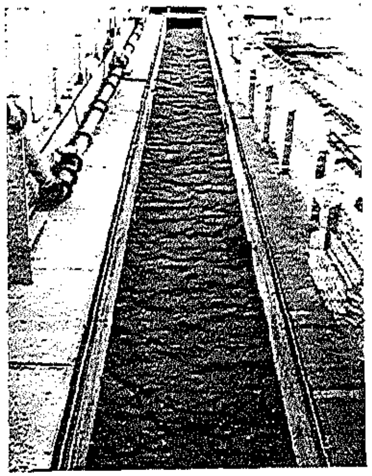


Fig. 16.

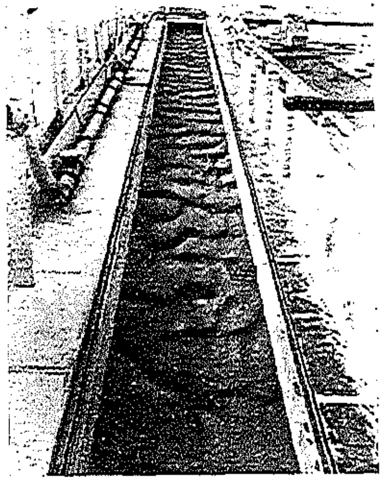
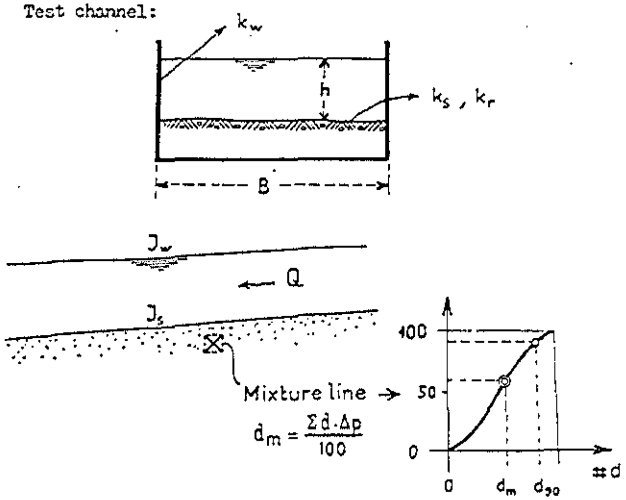


Fig. 17.

Table 1: Evaluation of the results of bed-load transport tests.

Test channel:



Test under steady conditions give:

Given: Q , mixture line (d_m , d_{90}) , B, k_w, $\gamma_s'' = \gamma_s - \gamma_w$

Measured: J_s , J_w , h , $g_s'' = \frac{G''}{B}$ (weighed under water !)

It is possible to calculate: $v = \frac{Q}{B \cdot h}$, $R = \frac{B \cdot h}{B + 2h}$

$$J = J_w - \frac{v^2}{g \cdot h}\,(J_w - J_s)$$

$$k_m = \frac{v}{R^{2/3} \cdot J^{1/2}} \qquad \text{(Strickler)}$$

*) Further: $$k_s = \frac{k_m\, k_w \cdot B^{2/3}}{\left[B \cdot h^{3/2} + 2h\left(k_w^{3/2} - k_m^{3/2}\right)\right]^{2/3}}$$

and $$q_s = Q \cdot \frac{k_w^{3/2}}{2h \cdot k_s^{3/2} + B \cdot k_w^{3/2}}$$

$$Q_s = q_s \cdot B \quad \text{and} \quad \frac{Q_s}{Q} \quad \text{or} \quad R_s = \frac{Q_s}{Q} \cdot h$$

*) Corr: $Bk_w^{3/2}$ in stead of $Bh^{3/2}$.

The mixture line gives d_m and d_{90}

and we have :

in the region of fully developed turbulence: $k_r = \frac{26}{d_{90}^{1/6}}$

consequently given: $\frac{k_s}{k_r}$

With turbulence not fully developed:

Relative roughness $\frac{R_s}{d_{90}}$

Reynolds'number $Re' = \frac{v \cdot R_s}{\nu}$ ($\nu = 1{,}3 \cdot 10^{-6}$ metric units, $m^2 sec^{-1}$)

and Nikuradse's diagram gives λ

From $k_r = \left(\frac{8g}{\lambda}\right)^{1/2} \cdot \frac{1}{R_s^{1/6}}$ Calculation of k_r

thus $\frac{k_s}{k_r}$ given.

Finally the expression:

$$\frac{\gamma_w}{\gamma_s''} \cdot \left(\frac{Q_s}{Q}\right) \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot \frac{h \cdot J}{d_m} \quad \text{as a function of} \quad \left(\frac{\gamma_w}{g}\right)^{1/3} \cdot \frac{g_s''^{2/3}}{d_m} \cdot \frac{1}{\gamma_s''}$$

gives a point of the law of bed-load transport:

$$\frac{\gamma_w}{\gamma_s''} \cdot \left(\frac{Q_s}{Q}\right) \left(\frac{k_s}{k_r}\right)^{3/2} \cdot \frac{h \cdot J}{d_m} = A'' + B'' \cdot \left(\frac{\gamma_w}{g}\right)^{1/3} \cdot \frac{g_s''^{2/3}}{d_m} \cdot \frac{1}{\gamma_s''}$$

According to our tests: $A'' = 0{,}047$
$B'' = 0{,}25$.

γ_w and γ_s'' in $t.m^{-3}$
Q_s and Q in $m^3.sec^{-1}$ or $t.sec^{-1}$
k_s and k_r in $m^{1/3}.sec^{-1}$
h and d_m in m
g in $m.sec^{-2}$
g_s'' in $t.m^{-1}sec^{-1}$

Table 2: Principal dates of all tests and their signatures for the Figures 10, 11 and 12.

Signature	B m	J ‰	Q lit/sec	q_s lit/m.sec.	d_m mm	γ_s'' t/m^3	Characteristic of bed-load	Number of single-tests
●	2	8	100 – 250	48,5 – 120	~ 4,5	1,68	Mixture	7
○	2	2 – 3	70 – 330	34 – 150	1,7 ÷ 2,0	1,68	"	17
+	2	1	100 – 180	46 – 84	1	1,68	"	3
×	2	0,4	200 – 550	90 – 230	0,4	1,68	"	5
△	0,65	2 – 4	15 – 60	20 – 76	1,3 – 1,4	1,68	"	6
▵	0,65	16	2,5	3,8	1,5	1,68	"	1
▽	0,3	16,5	1,2	3,9	1,4	1,68	"	1
▽	0,15	20	0,6 – 1	4 – 6,7	1,4 – 1,6	1,68	"	2
◆	0,5	2,7	15 – 93	26 – 132	2,7 ÷ 4	1,68	"	24
✱	2	3 – 17,7	1600 – 4600	700 – 2000	28,65	1,68	uniform grain size	33
■	0,354	3 – 23	22 – 82	53 – 180	5,2	1,68	"	20
⧆	0,354	1,3 – 10,6	0,8 – 21,6	2,1 – 52	5,2	0,25	" (lignite breeze)	12
⌑	0,354	5,8 – 23	21 – 82	54 – 180	5,2	3,22	uniform grain size (Baryta)	8

Table 3

Determination of the limiting shearing stress by direct observation. Uniform grain size (width of channel 1 m).

$$A'' = \frac{\gamma_w}{\gamma_s'' d_m} \cdot \frac{Q_s}{Q} \left(\frac{k_s}{k_r}\right)^{3/2} \cdot h \cdot J = \frac{\gamma_w}{\gamma_s'' d_m} \cdot \left(\frac{k_s}{k_r}\right)^{3/2} \cdot R_s \cdot J \qquad (26a)$$

d_m mm	d_{90-95} mm	J ‰	Q l/sec	Q_s l/sec	h m	R_s m	k_s	k_r	$\left(\frac{k_s}{k_r}\right)^{3/2}$	A"
Rolled material, $\gamma_s'' = 1,68\ t/m^3$										
3,20	3,48	0,78	210,0	138,0	0,313	0,205	70,2	66,5	1,09	0,0327
2,70	3,00	1,08	88,9	72,2	0,154	0,125	71,5	68,2	1,06	0,0316
3,80	4,24	1,20	152,1	131,0	0,229	0,197	64,0	64,8	1,00	0,0370
3,60	4,00	1,03	183,2	132,0	0,263	0,189	62,0	65,2	0,925	0,0296
3,66	4,00	2,02	99,0	81,8	0,138	0,114	68,2	65,2	1,06	0,0392
8,50	9,70	4,88	150,0	129,0	0,154	0,132	53,2	56,0	0,92	0,0367
3,66	4,00	5,03	35,8	33,7	0,059	0,055	59,8	65,2	0,87	0,0395
7,40	8,96	10,10	65,2	60,7	0,072	0,067	54,1	57,0	0,92	0,0495
8,50	10,00	10,49	76,0	71,0	0,084	0,078	49,0	56,0	0,82	0,0465
3,14	3,48	10,00	11,9	11,6	0,024	0,024	60,5	66,5	0,86	0,0395
									$A''_{average}$ = 0,0382	
Broken material, $\gamma_s'' = 1,68\ t/m^3$										
1,86	2,34	0,77	73,2	62,0	0,148	0,126	74,0	71,5	1,06	0,0328
3,14	3,48	0,72	131,2	95,0	0,224	0,163	72,0	66,5	1,12	0,0250
3,10	3,48	1,10	130,8	100,2	0,204	0,156	66,7	66,5	1,00	0,0330
3,66	4,00	2,07	96,0	82,5	0,137	0,118	64,0	65,1	0,97	0,0382
3,06	3,48	2,16	80,0	68,2	0,123	0,105	62,5	66,0	0,91	0,0400
2,58	2,92	2,09	61,0	53,1	0,105	0,092	64,1	68,5	0,91	0,0400
2,61	3,00	5,09	27,8	26,0	0,048	0,045	64,8	66,2	0,925	0,0480
4,04	4,70	4,86	56,3	51,0	0,077	0,070	63,0	63,5	1,00	0,0500
4,34	5,04	10,25	26,5	25,3	0,042	0,040	52,2	62,8	0,76	0,0430
2,10	2,34	9,86	8,65	8,4	0,020	0,019	61,5	71,5	0,795	0,0430
3,14	3,48	10,00	16,0	15,1	0,030	0,028	57,0	66,5	0,795	0,0425
									$A''_{average}$ = 0,0396	

The following gentlemen have collaborated in the test work carried out since 1932: Dr. H. A. Einstein, F. Brändle, engineer †, Dr. E. Escher, geologist, E. Röthlisberger, engineer, E. Müller, engineer, J. Morf, engineer, E. Bisaz, engineer.

We would express our best thanks to our collaborators for their willing and conscientious work.

Zurich, in February 1948.

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IVWBV
IAHSR
AIRTH

Anlage
Appendix
Annexe
3.

Internationaler Verband für WasserBauliches Versuchswesen
International Association for Hydraulic Structures Research
Association Internationale de Recherches pour Travaux Hydrauliques

ZWEITE TAGUNG - SECOND MEETING - DEUXIEME REUNION
STOCKHOLM 7—9. VI. 1948

TISON L. J., professeur à l'Université de Gand, Directeur de laboratoire d'hydraulique.

Transport de materiaux de fond, et Erosion à l'aval de barrages.

I. Transport de materiaux de fond.

A) Recherches sur la tension limite d'entrainement des matériaux constitutifs du lit.

La présente note résume les résultats de nos recherches dans ce domaine depuis la dernière réunion de notre Association. On pourra se reporter pour les détails à notre publication de même titre parue dans les Annales de la Société Scientifique de Bruxelles de 1947.

a) Les recherches de Shields l'ont conduit à admettre que le rapport $\frac{\tau_0}{(\gamma_1-\gamma)d}$ est une fonction du nombre de Reynolds réduit $Re_* = \frac{u_* d}{\nu}$: τ_0 est la tension limite d'entraînement, γ_1 est le poids spécifique des materiaux de diamètre d et γ est celui du fluide dont la densité est ϱ. u_* vaut $\sqrt{\frac{\tau_0}{\varrho}}$.

Pour Re $<$ 2, Shields trouve que $\frac{\tau_0}{(\gamma_1-\gamma)d} = \frac{0,1}{Re_*}$;

Pour $2 < Re_* < 10$, il obtient $\frac{\tau_0}{(\gamma_1-\gamma)d} = 0,03$;

Et quand Re croît de 10 à 1.000, il estime que le second membre croît de 0,03 à 0.06 et reste ensuite constant.

b) Par contre, les études de White le conduisent à une valeur de τ_0 donnée par $\tau_0 = 0,18\,(\gamma_1-\gamma)\,d\,.\,tg\varphi$, φ étant l'angle de la verticale avec le talus naturel des matériaux (sous eau). Les essais de White vérifient cette formule tant que le mouvement est laminaire avec R_* inférieur à 3,5.