Article 14:

Formulating the theories of gravity/intrinsic gravity and motion/intrinsic motion and their union at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field. Part I.

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The theory of gravitational relativity and intrinsic theory of gravitational relativity (TGR/ϕTGR), the special theory of relativity and intrinsic special theory of relativity (SR/ϕSR), and their union, on flat four-dimensional relativistic spacetime (Σ, ct) and its underlying flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$, at the second stage of evolutions of spacetime/intrinisic spacetime and parameters/intrinsic parameters in a gravitational field of arbitrary strength, isolated in the previous papers, are developed fully in the first two parts of this paper. Mass and other parameter relations in the context of TGR and the implied modification of Newton's law of universal gravity in the context of TGR are derived. Local Lorentz invariance is validated on flat spacetime in the context of TGR. This first part is devoted to the graphical approaches in the four-world picture to these flat spacetime/intrinsic spacetime theories, while analytical approaches shall be developed in the second part to complement the graphical approaches. The other theories isolated at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in every gravitational field namely, the metric theory of absolute intrinsic gravity (ϕMAG) and combined metric theory of absolute intrinsic gravity and absolute intrinsic motion (ϕMAG $\cup$ ϕMAM), on curved 'two-dimensional' absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$; their projective theories into the flat relativistic intrinsic spacetime namely, the Newtonian theory of absolute intrinsic gravity (ϕNAG) and combined Newtonian theory of absolute intrinsic gravity and absolute intrinsic motion (ϕNAG $\cup$ ϕNAM), as well as the outward manifestations of these in the flat four-dimensional relativistic spacetime namely, the non-observable Newtonian theory of absolute gravity (NAG) and combined Newtonian theory of absolute gravity and absolute motion (NAG $\cup$ NAM), shall be developed in the third part of this paper.

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1 Introduction

1.1 On the global spacetime/intrinsic spacetime geometries of theories/intrinsic theories of gravity, motion and other non-gravitational laws at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field

The theory of relativity in spacetime and intrinsic theory of relativity in intrinsic spacetime, due to the presence of a long-range metric force field, developed in [1] and adapted to the gravitational field in section 2 of [2], is a direct pre-requisite to this paper. As has been robustly established in those previous papers, the four-dimensional spacetime, qualified as relativistic spacetime and denoted by (Σ, ct) in our notation, containing the relativistic masses $(m, \varepsilon/c^2)$ of material particles and bodies and the underlying relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ containing the relativistic intrinsic masses $(\phi m, \phi\varepsilon/\phi c^2)$ of particles and bodies, which evolve at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field, are everywhere flat in a gravitational field of arbitrary strength (or in every gravitational field).

There are, in addition, in every gravitational field, the curved two-dimensional proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ with orthogonal curvilinear intrinsic dimension $\phi\rho'$ and $\phi c\phi t'$ and consequently with intrinsic Lorentzian metric tensor at every point of it, containing the intrinsic rest masses $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of material particles and bodies, which projects the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ underneath it. There is also the curved 'two-dimensional' absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$, an absolute intrinsic Riemannian metric space with absolute intrinsic sub-Riemannian metric tensor $\phi\hat{g}_{ik}$, containing the absolute intrinsic rest masses $\phi\hat{m}_0$ and $\phi\hat{M}_0$ of particles and bodies. Finally there is the constantly flat 'two-dimensional' absolute-absolute intrinsic-intrinsic spacetime $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$, isolated in [3], containing the absolute-absolute intrinsic-intrinsic rest masses $(\phi\phi\hat{\hat{m}}_0, \phi\phi\hat{\hat{\varepsilon}}/\phi\phi\hat{\hat{c}})^2$ of particles and bodies in it, in every gravitational field.

The flat relativistic spacetime (Σ, ct) containing the relativistic masses $(m, \varepsilon/c^2)$ or $(M, E/c^2)$ of particles and bodies and the hierarchy of intrinsic spacetimes namely, flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ underlying (Σ, ct), curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$, curved absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ and flat absolute-absolute intrinsic-intrinsic spacetime $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$ underlying the flat $(\phi\rho, \phi c\phi t)$ and the associated hierarchy of intrinsic masses $(\phi m, \phi\varepsilon/\phi c^2)$, $(\phi m_0, \phi\varepsilon'/\phi c^2)$, $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ and $(\phi\phi\hat{\hat{m}}_0, \phi\phi\hat{\hat{\varepsilon}}/\phi\phi\hat{\hat{c}}^2)$ respectively, listed above have been shown to evolve simultaneously at the combined first and second stages of evolutions of spacetime/intrinsic spacetimes and parameters/intrinsic parameters in a gravitational field in sub-section 1.1 of [2] and illustrated graphically as the global spacetime/ intrinsic spacetime geometries of Figs. 7 and 8 and their inverses in Figs. 9 and 10 of that paper.

Fig. 7 of [2] was re-presented as Fig. 9 of [4], where the flat 'two-dimensional' absolute-absolute intrinsic-intrinsic spacetime $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$, isolated in [3] was incorporated into the geometry. Thus Fig. 9 of [4] of combined first and second stages of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field and its complementary diagram and their inverses (not drawn) in [4], constitute the complete set of spacetime/intrinsic spacetime geometries that support the theories of gravity/intrinsic gravity, motion/intrinsic motion and all other non-gravitational laws/intrinsic non-gravitational laws in a gravitational field in our universe and the negative universe.

In order to make this paper as autonomous as possible and also for convenience of reading, Fig. 9 of [4] shall be reproduced as Fig. 1 and its complementary diagram (not drawn) in [4] shall be presented as Fig. 2 of this paper. We only need to incorporate the flat $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$ and $(-\phi\phi\hat{\hat{\rho}}^*, -\phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}}^*)$ into Figs. 7 and 8 of [2] to accomplish these. However the inverse diagrams obtained by incorporating $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$, and $(-\phi\phi\hat{\hat{\rho}}^*, -\phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}}^*)$, into Fig. 9 and 10 of [2], shall not be drawn in order to conserve space.

As finally determined in section 3 of [4], the theories of gravity/intrinsic gravity and theories of combined gravity/intrinsic gravity and motion/intrinsic motion at the first stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field must be formulated within intrinsic local Lorentz frames on the curved '2-dimensional' absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ and curved two-dimensional proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ with respect to intrinsic 1-observers along the curved proper intrinsic space $\phi\rho'$ and 3-observers in the relativistic Euclidean 3-space Σ in Fig. 1. These are ϕMAG, ϕNAG* and ϕMAG+ ϕMAM, ϕNAG*+ϕNAM* on the curved $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ and the primed intrinsic theories ϕNAG′ and ϕNAG′+ϕNAM′ within intrinsic Local Lorentz frames on curved $(\phi\rho', \phi c\phi t')$.

There is also the primed intrinsic classical (or Newton's) theory of (relative) gravity (ϕCG′) within intrinsic local Lorentz frames on the curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$, with essential equations (115) − (118) of [4], formulated with respect to intrinsic 1-observers in the curved proper intrinsic space $\phi\rho'$ and 3-observers in the relativistic Euclidean 3-space Σ in Fig. 1. The CG′/ϕCG′ arise

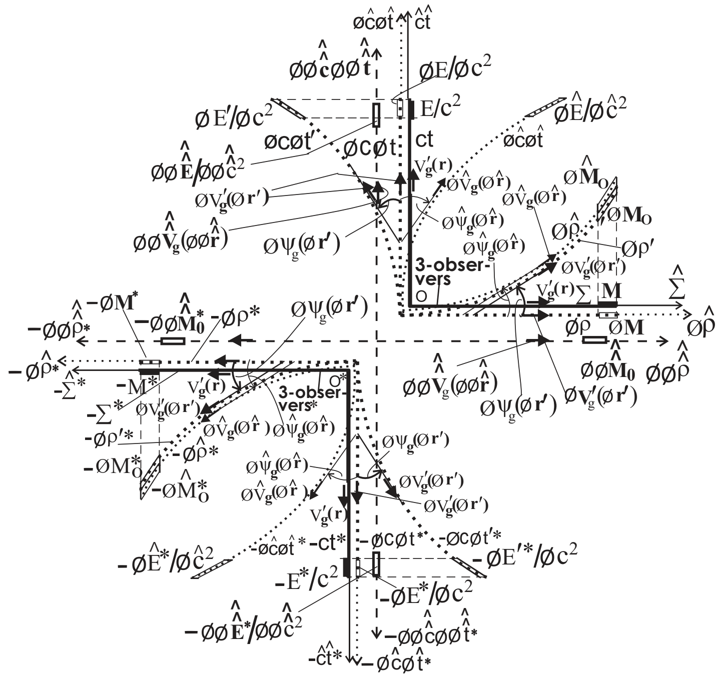

Figure 1: The global spacetime/intrinsic spacetime diagram of combined first and second stages of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field of arbitrary strength that is valid with respect to 3-observers in the relativistic Euclidean 3-spaces in our universe and the negative universe.

from the proper intrinsic gravitational speed $\phi V_g'(\phi r')$, proper intrinsic gravitational potential $\phi\Phi'(\phi r')$ and proper intrinsic gravitational field $\phi g'(\phi r')$ established along the curved $\phi\rho'$ and $\phi c\phi t'$ by ϕM_0 and $\phi E'/\phi c^2$ of the gravitational field source at the origins of the curved $\phi\rho'$ and $\phi c\phi t'$ respectively.

Apart from ϕMAG, ϕNAG*, ϕMAG+ϕMAM, ϕNAG*+ ϕNAM*, ϕNAG′, ϕNAG′+ϕNAM′ and ϕCG′, there are also primed intrinsic spacial theory of relativity (ϕSR′) and other primed intrinsic classical non-gravitational laws ϕCNGL′ and primed intrinsic special-relativistic non-gravitational laws (ϕCNGL′+ϕSR′) within intrinsic local Lorentz frames on curved proper intrinsic spacetime ($\phi\rho'$, $\phi c\phi t'$), formulated with respect to intrinsic 1-observers along the curved proper intrinsic space

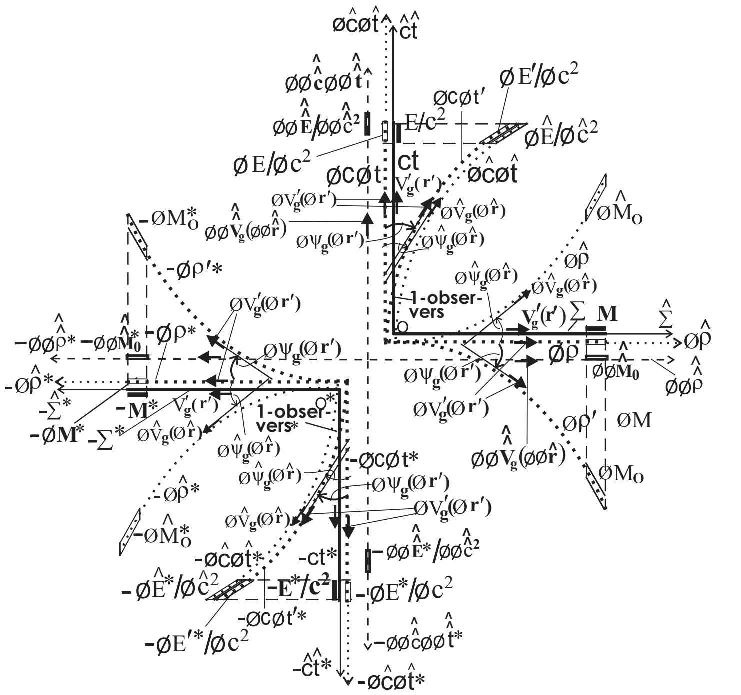

Figure 2: The complementary diagram to Fig. 1 that is valid with respect to 1-observers in the relativistic time dimensions in our universe and the negative universe.

$\phi\rho'$ and 3-observers in the relativistic Euclidean 3-space Σ in Fig. 1. The intrinsic theories on the curved $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ and curved $(\phi\rho', \phi c\phi t')$ listed in this and the foregoing two paragraphs have been brought forward to the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters from the first stage in a gravitational field of arbitrary strength.

The primed intrinsic theories ϕNAG′, ϕNAG′+ϕNAM′, ϕSR′, ϕCNGL′, ϕSR′, ϕSR′+ϕCNGL′ and ϕCG′ within proper (or primed) intrinsic local Lorentz frames on curved two-dimensional proper intrinsic space $(\phi\rho', \phi c\phi t')$, formulated with respect to intrinsic 1-observers along the curved proper intrinsic space $\phi\rho'$ in Fig. 1, at the first stage of evolutions of spacetime/intinsic spacetime and parameters/intrinsic parameters in a gravitational field, project the unprimed intrinsic theories namely,

ϕNAG, ϕNAG+ϕNAM, ϕSR, ϕCNGL, ϕCNGL+ϕSR and ϕCG respectively into the respective unprimed (or relativistic) intrinsic local Lorentz frames on the flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in Fig. 1, at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field, which are valid with respect to 3-observers in the relativistic Euclidean 3-space Σ overlying in Fig. 1.

The projective unprimed (or gravitational-relativistic) intrinsic theories on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ are then made manifest in the respective unprimed (or gravitational-relativistic) theories namely, NAG, NAG+NAM, SR, CNGL, CNGL+SR and CG, within unprimed (or relativistic) local Lorentz frames on flat four-dimensional relativistic spacetime (Σ, ct) with respect to 3-observers in Σ at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field.

There are also the ϕMAG, ϕNAG* and ϕMAG+ϕMAM, ϕNAG*+ϕNAM* on the curved 'two-dimensional' absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ to be formulated with respect to 3-observers in the relativistic Euclidean 3-space Σ, as well as the Newtonian theory of absolute-absolute intrinsic-intrinsic gravity $\phi\phi$NAAG, the Newtonian theory of absolute-absolute intrinsic-intrinsic motion $\phi\phi$NAAM and their union $\phi\phi$NAAG+$\phi\phi$NAAM on the constantly flat absolute-absolute intrinsic-intrinsic spacetime $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$, to be formulated with respect to 3-observers in Σ in Fig. 1, at the second stage of evolutions of spacetime]intrinsic spacetime and parameters/intrinsic parameters in a gravitational field.

The program of this paper in three parts is to formulate the unprimed (or gravitational-relativistic) theories at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field namely, NAG/ϕNAG, NAG/ϕNAG+NAM/ϕNAM, SR/ϕSR, CNGL/ϕCNGL and CG/ϕCG on the flat relativistic spacetime/flat relativistic intrinsic spacetime $(\Sigma, ct)/(\phi\rho, \phi c\phi t)$, as well as ϕMAG, ϕNAG* and ϕMAG+ϕMAM, ϕNAG*+ϕNAM* on curved $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ with respect to 3-observers in the relativistic Euclidean 3-space Σ in Fig.1, while $\phi\phi$NAAG, $\phi\phi$NAAM and $\phi\phi$NAAG+$\phi\phi$NAAM on flat $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$, shall be formulated with respect to 3-observers in the relativistic Euclidean 3-space Σ in Fig. 1 in another paper later in this volume.

The unprimed (or gravitational-relativistic) intrinsic classical (or Newton's) theory of gravity (ϕCG) within unprimed intrinsic local Lorentz frames on the flat relativistic intrinsic spacetime $\phi\rho, \phi c\phi t)$ and its outward manifestation (CG) within unprimed local Lorentz frames on the flat relativistic spacetime (Σ, ct), shall be developed along with the unprimed (or gravitational-relativistic) intrinsic special theory of relativity (ϕSR) within unprimed intrinsic local Lorentz frames on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ and its outward manifestation namely, the unprimed (or gravitational-relativistic) special theory of relativity within unprimed local Lorentz frames on the flat relativistic spacetime (Σ, ct), with respect to 3-observers in Σ in Fig. 1, in the first two parts of this paper.

The ϕMAG, ϕNAG* and ϕMAG+ϕMAM, ϕNAG*+ ϕNAM* on curved absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ and NAG/ϕNAG, NAG/ϕNAG+NAM/ϕNAM on flat relativistic spacetime/flat relativistic intrinsic spacetime $(\Sigma, ct)/(\phi\rho, \phi c\phi t)$, shall be formulated with respect to 3-observers in Σ in Fig. 1 in the third part of this paper, while $\phi\phi$NAAG, $\phi\phi$NAAM and $\phi\phi$NAAG+$\phi\phi$NAAM on flat $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$, shall be formulated with respect to 3-observers in the relativistic Euclidean 3-space Σ in another paper later in this volume, as mentioned above.

The unprimed classical and special-relativistic non-gravitational laws (CNGL and CNGL+SR) on flat relativistic spacetime (Σ, ct) and the unprimed intrinsic classical and intrinsic special-relativistic non-gravitational laws (ϕCNGL and ϕCNGL+ ϕSR) on flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in Fig. 1, shall be considered in the process of validating the principle of equivalence on flat relativistic spacetime (Σ, ct) in a gravitational field of arbitrary strength in another paper following the third part of this paper.

1.2 Further on the concepts of gravitational velocity, gravitational potential and gravitational acceleration in spacetime and the respective intrinsic parameters in intrinsic spacetime

The concept of static speed was derived graphically within a long-range metric force field in section 2 of [5], where it was denoted by $V'_{\rm S}$. It was particularized to the gravitational field, given the alternative name of gravitational speed and re-denoted by $V'_g(r')$ in [2]. There indeed exists the concept of gravitational velocity $\vec{V}'_g(r')$ in the Euclidean 3-space Σ', which corresponds to the concept of gravitational acceleration $\vec{g}'(r')$ in Σ' in the phenomenon of gravity, where $\vec{V}'_g(r')$ and $\vec{g}'(r')$ are related thus

$$|\vec{g}'(r')| = -\frac{GM_{0a}}{r'^2} = \frac{1}{2}\frac{d}{dr'}[V'_g(r')^2] \tag{1}$$

The definition of $V'_g(r')$ that satisfies Eq. (1) is

$$V'_g(r')^2 = \frac{2GM_{0a}}{r'} \tag{2a}$$

or

$$V'_g(r') = -\sqrt{\frac{2GM_{0a}}{r'}} \tag{2b}$$

where $\vec{V}'_g(r')$ is the proper gravitational velocity at radial distance r' from the center of the assumed spherical rest mass M_0 of the gravitational field source in the proper Euclidean 3-space Σ' (in Fig. 2 or 3 of [6]).

The relationship between gravitational speed and gravitational potential also deduced and written as Eq. (17) of [2] is the following

$$\Phi'(r') = -\frac{1}{2}V'_g(r')^2 = -\frac{GM_{0a}}{r'} \tag{3}$$

Except for the replacement of the rest mass M_0 by the active gravitational mass (or gravitational charge) M_{0a}, Eqs. (1) − (3) have been deduced and presented as Eqs. (16a-b) − (18) of [2]. The need to replace the rest mass by the active gravitational mass (or gravitational charge) in the definitions of the proper gravitational velocity $\vec{V}'_g(r')$, proper (or Newtonian) gravitational potential $\Phi'(r')$ and proper (or Newtonian) gravitational acceleration (or field) $\vec{g}'(r')$ was deduced in sub-sub-section 2.1.5 of [6] and sub-section 2.1 of [4], see the discussion leading to Eq. (55) of [4].

The negative root is taken in Eq. (2b) in order to make the gravitational speed (or velocity) attractive like gravitational acceleration and gravitational potential. Indeed $\vec{V}'_g(r')$ and $\vec{g}'(r')$ are collinear vectors, both pointing radially towards the center of the gravitational field source in the case of a spherical gravitational field source. The definition of the gravitational speed along with its negative sign (or its attractive nature) of Eq. (2b) was deduced in sub-sub-section 2.1.5 of [6] and sub-section 2.1 of [4] with respect to 3-observers in the relativistic Euclidean 3-space Σ in Fig. 1; see the discussion leading to Eq. (55) of [4]. However there is yet a final more fundamental justification for the attractive nature of the gravitational speed (or velocity), from which the gravitational potential and gravitational acceleration (or field) inherit their attractive nature, which shall be presented elsewhere with further development, as mentioned in section 1.2 of [4].

One finds from the relation of gravitational potential $\Phi'(r')$ and gravitational acceleration (or field) $\vec{g}'(r')$ to the gravitational velocity $\vec{V}'_g(r')$ in (1) and (3), that the gravitational velocity is the most fundamental of the three gravitational parameters $\vec{V}'_g(r')$, $\Phi'(r')$ and $\vec{g}'(r')$. There could not have been the concepts of gravitational potential and gravitational acceleration without the concept of gravitational velocity as Eqs. (1) and (3) show. As a matter of fact, gravitational potential and gravitational field inherit their attractive natures from the attractive nature of their gravitational velocity progenitor as shall be justified shortly. Recall that absolute intrinsic static speed is a fundamental geometrical parameter isolated in a long range metric force field in general in section 2 of [5]. The concepts of potential and field could not appear at that geometrical foundation.

Now the centrality of the gravitational potential and gravitational field obtains in a spherically symmetric gravitational field only. The gravitational potential and gravitational field are functions of all the spherical coordinates r', $r'\theta'$ and $r'\sin\theta'\varphi'$ that originate from the centroid of a non-spherical gravitational field source as $\Phi'(r',\theta',\varphi')$ and $\vec{g}\,'(r',\theta',\varphi')$. The gravitational field does not point purely radially towards the centroid of a non-spherical gravitational field source.

On the other hand, gravitational velocity is central in both spherically-symmetric and non-spherically-symmetric gravitational fields. Thus gravitational velocity can be function of the radial coordinate only as $\vec{V}'_g(r')$ and point radially towards the center or centroid of every every gravitational field source (spherical or non-spherical). Thus we can write as follows for a non-spherically-symmetric gravitational field

$$\left.\begin{aligned}
\vec{V}'_g &= \vec{V}'(r') = V'_g(r')\hat{r}\,'; \\
\vec{g}\,' &= \vec{g}\,'(r',\theta',\varphi') \\
&= g'_r(r',\theta',\varphi')\hat{r}' + g'_\theta(r',\theta',\varphi')\hat{\theta}'; \\
&\quad +g'_\varphi(r',\theta',\varphi')\hat{\varphi}'; \\
\Phi' &= \Phi'(r',\theta',\varphi'); \\
&\quad (\text{non}-\text{spherical grav. field source})
\end{aligned}\right\} \tag{4}$$

The centrality in all gravitational fields is a property of the gravitational velocity to be explained formally elsewhere with further development.

Another important difference among the properties of gravitational potential $\Phi'(r',\theta',\varphi')$ and gravitational field $\vec{g}\,'(r',\theta',\varphi')$ in a non-spherically-symmetric gravitational field (or $\Phi'(r')$ and $\vec{g}\,'(r')$ in a spherically-symmetric gravitational field) and the gravitational velocity $\vec{V}'_g(r')$ in a non-spherically-symmetric or spherically-symmetric gravitational field, is that proper (or classical) gravitational potential $\Phi'(r',\theta',\varphi')$ and proper (or classical) gravitational field $\vec{g}\,'(r',\theta',\varphi')$ in the proper Euclidean 3-space Σ' in the context of the primed classical theory of gravity (CG$'$) on flat proper spacetime (Σ', ct') at the first stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters, transform non-trivially to relativistic gravitational potential $\Phi(r,\theta,\varphi)$ and relativistic gravitational field

$\vec{g}(r, \theta, \varphi)$ on flat relativistic Euclidean 3-space Σ in the context of the theory of gravitational relativity (TGR) on flat relativistic spacetime (Σ, ct) in Fig. 1 at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field as follows

$$\Phi(r, \theta, \varphi) = f_\Phi(V'_g(r'))\Phi'(r', \theta', \varphi') \tag{5}$$

and

$$\vec{g}(r, \theta, \varphi) = f_g(V'_g(r'))\vec{g}'(r', \theta', \varphi') \tag{6}$$

where the functions $f_\Phi(V'_g(r')$ and $f_g(V'_g(r')$ shall be determined in the second part of this paper.

Whereas gravitational velocity is invariant, transforming trivially in the context of the relativistic theory of gravity between flat proper spacetime (Σ', ct') and flat relativistic spacetime (Σ, ct) in every gravitational field as follows

$$\vec{V}_g(r) = \vec{V}'_g(r') \tag{7}$$

This invariance was first stated without proof as invariance of static velocity by Eq. (79b) of [7] and particularized to the gravitational field (still without proof) as Eq. (2b) of [2]. The proof of Eq. (7) can still not be given at this point, but elsewhere with further development, where the peculiar properties of the gravitational velocity namely, its centrality in all gravitational fields and its invariance (7), as well as the mechanism by which a gravitational field source establishes non-uniform gravitational velocity $\vec{V}'_g(r')$ along every radial direction from its centroid, thereby giving rise to gravitational field and gravitational potential as progenies, shall be unraveled.

As prescribed without proof in sub-sub-section 2.1.5 of [6], the non-observable immaterial negative active gravitational mass (or negative gravitational charge) $-M_{0\mathrm{a}}$, hidden within the observable positive physical (or material) rest mass M_0 of a gravitational field source is the source of the proper gravitational velocity, proper (or classical) gravitational potential and proper (or classical) gravitational field. Hence these parameters have been written in terms of the negative gravitational charge $-M_{0\mathrm{a}}$ in Eqs. (1), (2a) or (2b) and (3). Thus the negativity of the gravitational charge is the origin of the attractive nature of gravitational velocity, gravitational potential and gravitational field, as being prescribed for now in the present theory.

An important task to be executed elsewhere with further development is fundamental explanations of the origin of immaterial active gravitational mass (or gravitational charge) and its negative sign, as well as the model of how $-M_{0\mathrm{a}}$ is contained in the rest mass M_0 and the mechanism by which $-M_{0\text{a}}$ hidden in M_0 establishes $\vec{V}_g(r')$, $\Phi'(r',\theta',\varphi')$ and $\vec{g}'(r',\theta',\varphi')$ at every point in the proper Euclidean 3-space Σ' from the centroid of M_0.

Just as gravitational potential $\Phi'(r',\theta',\varphi')$ is a property of space at a position of coordinates $(r', r'\theta', r'\sin\theta'\varphi')$ from the centroid of the rest mass M_0 of the gravitational field source, such that when a test particle arrives at this position, it acquires gravitational potential $\Phi'(r',\theta',\varphi')$, so is gravitational velocity $\vec{V}'_g(r')$ a property of space at at the position of coordinates $(r', r'\theta', r'\sin\theta'\varphi')$ from the centroid of the rest mass M_0 of the gravitational field source, which a test particle acquires upon arriving there.

Unlike the dynamical velocity v of dynamics (or special relativity), gravitational velocity − a static velocity − is not made manifest in actual translation in space of the test particle that acquires it. Thus a test particle at rest relative to an observer at radial distance r' from the centroid of the rest mass M_0 of a gravitational field source in the proper Euclidean 3-space Σ', possesses yet gravitational velocity $\vec{V}_g(r')$ relative to this observer and all other observers.

Gravitational velocity is different from escape velocity v_{esc}, which has the same expression as Eq.(2b) for $V'_g(r')$, in the sense that v_{esc} is a dynamical velocity directed radially away from a gravitational field source, which a particle possesses and escapes the gravitational influence of the field source. Escape velocity, although determined by the gravitational field source, is a property of the particle.

Gravitational velocity $\vec{V}'_g(r')$ is a more appropriate parameter to incorporate into the theory of gravitational relativity (TGR) on flat relativistic spacetime (Σ, ct) in Fig. 1, started in section 2 of [2], than gravitational potential. It has several analogies to the dynamical velocity $\vec{v}$ of dynamics (or special relativity). For instance, the gravitational speed $V'_g(r')$ effects the theory of gravitational relativity (TGR) on flat four-dimensional relativistic spacetime (Σ, ct), just as dynamical speed v effects the special relativity (SR) on the flat four-dimensional relativistic spacetime (Σ, ct) in Fig. 1.

The gravitational velocity $\vec{V}'_g(r')$ of TGR being a property of space, makes TGR possible on the flat relativistic spacetime (Σ, ct) in all finite neighborhood of a gravitational field source in the absence of a test particle. On the other hand, dynamical velocity $\vec{v}$ of SR, being a property of the particle in motion, makes it mandatory for a particle to be in motion relative to the observer for SR to be possible.

The gravitational velocity $\vec{V}'_g(r')$, (like gravitational potential $\Phi'(r',\theta',\varphi')$), is invariant with the observer or frame of reference, whereas the dynamical velocity varies with the observer or frame of reference. The concept of relativity associated with gravitational velocity $\vec{V}'_g(r')$ and the theory of gravitational relativity (TGR) induced by $\vec{V}'_g(r')$, in the absence of SR, is merely relativity with position in space in a gravitational field and not relativity with observer or frame of reference, as discussed in sub-sub-section 2.2.1 of [2]. It refers to variation with gravitational speed $V'_g(r')$ of space and time intervals of events and physical parameters, which implies their variations with position in a gravitational field.

On the other hand, gravitational velocity is an absolute parameter in the context of dynamics (or SR), since the gravitational velocity at a given position in space is not made manifest in motion and is the same relative to all observers of frames of reference. Conversely dynamical velocity $\vec{v}$ is absolute in the context of TGR, since a given dynamical velocity of a particle relative to an observer does not vary with gravitational velocity or with position in a gravitational field. That is, it is invariant in the context of TGR as shall be demonstrated in the second part of this paper.

It may be recalled that the clarification of the concepts of relative static speed and relativity associated with relative static speed in a relative metric force field was done in sub-section 2.3 of [1] and adapted to the clarification of relative gravitational speed and relativity associated with relative gravitational speed in a relative gravitational field in sub-sub-section 2.2.1 of [2].

Now the largest possible kinematic velocity of particles, including photon, in spacetime is the velocity of an electromagnetic wave in vacuum, $c_\gamma = 3 \times 10^8$ m/s. Likewise the largest possible gravitational (or static) velocity that can be established at a point in spacetime by a gravitational field source or combination of gravitational field sources is the velocity of gravitational waves, $c_g = 3\times10^8$ m/s. These velocities of 'signal' were first introduced in [8], see Table III and Table IV of that paper.

While the velocity of light c_γ is made manifest in actual translation through space of electromagnetic waves, the maximum over all gravitational velocities c_g, (like gravitational velocity $V_g(r')$), is not made manifest in actual translation through space of gravitational waves. It is *a priori* in the present theory that gravitational waves possess constant gravitational (or static) speed, $c_g = 3 \times 10^8$ m/s, but are at rest always relative to all observers. This actually implies that gravitational radiation involving energy transfer in spacetime is impossible or does not exist. The fact that gravitational effect propagates through space at the speed of light but not as a wave, such that if a body is suddenly introduced or annihilated at a point in space, the effect propagates away at the speed of light, has a different explanation, which has been started in sub-section 1.1 of [2] and will be completed elsewhere with further development.

The value of gravitational velocity at the surface (or event horizon) of a black hole is c_g. This is so since at the surface (or event horizon) of a black hole of rest mass M_0 and radius r_b, the gravitational speed is given from Eq. (2a) as, $V_g(r_b)/c_g = (2GM_{0\mathrm{a}}/r_b c_g^2)^{1/2}$. But $2GM_{0\mathrm{a}}/r_b c_g^2 = 1$ for a black hole. Hence $V_g(r_b) = c_g$. Thus a particle that falls to the surface (or event horizon) of a black hole acquires the gravitational (or static) speed, $c_g = 3 \times 10^8$ m/s. We shall find in future articles that this event (of fall of a test particle to the event horizon of a black hole) is not allowed for a test particle with non-zero rest mass, just as a particle with non-zero rest mass cannot attain the speed of light in vacuum c_γ in relative motion.

As has been noted in [8] and earlier in this section, we have isolated two different speeds of 'signals' namely, the dynamical speed of electromagnetic waves (or light), usually denoted by c, but which has been re-denoted by c_γ since [8], and the gravitational (or static) speed of gravitational waves, which has been denoted by c_g since [8]. This fact has remained unknown in physics until now. The only speed of signal known in physics until now is the dynamical speed of light c_γ, usually denoted by c, which both electromagnetic and gravitational waves are known to possess.

1.3 Further on the spacetime/intrinsic spacetime geometries of the theory of gravitational relativity/intrinsic theory of gravitational relativity and special theory of relativity/intrinsic special theory of relativity in a gravitational field

As introduced in section 2 of [8], the flat four-dimensional metric spacetime (Σ, ct) is composed of the flat four-dimensional affine spacetime of dynamics and electromagnetism $(\Sigma_d, c_\gamma t)$ and the flat four-dimensional metric spacetime of the theories of gravity $(\Sigma_g, c_g t)$. That is,

$$(\Sigma, ct) \equiv (\Sigma_g, c_g t) \cup (\Sigma_d, c_\gamma t)$$

or

$$(x^1, x^2, x^3, ct) \equiv (x_g^1, x_g^2, x_g^3, c_g t) \cup (\chi^1, \chi^2, \chi^3, c_\gamma t)$$

The affine spacetime of dynamics and electromagnetism $(\Sigma_d, c_\gamma t)$ is inseparably embedded in the metric spacetime of the theories of gravity $(\Sigma_g, c_g t)$, yielding the metric compound spacetime (Σ, ct).

Likewise the metric compound intrinsic spacetime $(\phi\rho, \phi c\phi t)$ is composed of the affine intrinsic spacetime of intrinsic dynamics and intrinsic electromagnetism denoted by $(\phi\chi, \phi c_\gamma \phi t)$ in [8] and the metric intrinsic spacetime of the theories of intrinsic gravity $(\phi\rho_g, \phi c_g \phi t)$. That is,

$$(\phi\rho, \phi c\phi t) \equiv (\phi\rho_g, \phi c_g \phi t) \cup (\phi\chi, \phi c_\gamma \phi t)$$

Again the affine $(\phi\chi, \phi c_\gamma \phi t)$ is inseparably embedded in the metric $(\phi\rho_g, \phi c_g \phi t)$ yielding the metric compound intrinsic spacetime $(\phi\rho.\phi c\phi t)$.

The masses m and intrinsic masses ϕm of every particle or body are likewise composed of the non-ponderable (or affine) dynamical component m_d and ϕm_d and the ponderable (metric) components m_g and ϕm_g. That is,

$$m \equiv m_g \cup m_d$$

and

$$\phi m \equiv \phi m_g \cup \phi m_d$$

Again m_d is inseparably embedded in m_g forming the compound mass m and ϕm_d is inseparably embedded in ϕm_g forming the compound intrinsic mass ϕm in nature. Thus as the non-ponderable (or affine) dynamical mass m_d of a particle moves at a velocity $\vec{v}$ in the affine spacetime of dynamics $(\Sigma_d, c_\gamma t)$ relative to an observer, it drags its ponderable (or metric) gravitational mass m_g along, such that m_g moves at equal velocity $\vec{v}$ in its spacetime of the theories of gravity $(\Sigma_g, c_g t)$ relative to the observer. Consequently the ponderable (or metric) compound mass m is observed to move at velocity $\vec{v}$ in the metric compound spacetime (Σ, ct) relative to the observer.

Now the gravitational velocity $\vec{V}_g'(r')$ is a relative velocity in the context of the theory of gravitational relativity (TGR) on flat relativistic spacetime (Σ, ct) in Fig. 1, started in section 2 of [2] and shall be advanced further in this first part of this paper; the gravitational-relativistic form of the classical (or Newton's) theory of gravity (CG) on flat relativistic spacetime (Σ, ct), shall be developed in the second part of this paper and a Maxwellian theory of gravity (MTG) that describes the 'propagation' at gravitational velocity $\vec{V}_g'(r')$ on the flat relativistic spacetime (Σ, ct), of massless gravitational field $\vec{g}$ and another induced massless partner-gravitational field $\vec{d}$ in the relativistic Euclidean 3-space Σ in a gravitational field, to be developed elsewhere with progress of the present theory. The MTG in the metric spacetime $(\Sigma_g, c_g t)$ of the theories of gravity is the gravitational counterpart of electromagnetism (EM) in the affine spacetime of electromagnetism and dynamics $(\Sigma_d, c_\gamma t)$.

The gravitational velocity $\vec{V}_g'(r')$ must be treated as a relative velocity in the context of the theories of gravity namely, TGR, CG and MTG, on the flat relativistic spacetime (Σ, ct), where, as discussed in the sub-section 2 of [2] and mentioned in the preceding sub-section, the relativity of $\vec{V}_g'(r')$ refers to the variation of its magnitude with position in the gravitational field.

On the other hand, the gravitational velocity is absolute in the context of the dynamical theories namely, the special theory of relativity (SR), the special-relativistic form of classical (or Newton's) theory of motion (CM) and dynamics of non-gravitational fields and parameters, that is, electromagnetism (EM) and other non-gravitational laws. In other words, should the dynamical velocity $\vec{v}$ of relative motion be replaced by the gravitational (or static) velocity $\vec{V}_g'(r')$ in these dynamical laws, then $\vec{V}_g'(r')$ must be treated as absolute and the resulting theories as non-observable, which is so since $\vec{V}_g'(r')$ is not made manifest in motion and since it is the same relative to all observers or frames of reference.

Let us temporarily separate the affine proper intrinsic time dimension $\phi c_\gamma \phi t'$ from the metric proper intrinsic gravitational time dimension $\phi c_g \phi t'$ and combine the metric compound proper intrinsic space $\phi\rho'$ with $\phi c_\gamma \phi t'$ to have flat proper intrinsic spacetime $(\phi\rho', \phi c_\gamma t')$ underlying flat proper spacetime $(\Sigma', c_\gamma t')$ in the assumed absence of relative gravity (or assumed absence of relative gravitational velocity $\vec{V}_g'(r')$).

Then let us introduce non-uniform intrinsic gravitational speed $\phi V_g'(r')$ along the straight line $\phi\rho'$ along the horizontal and straight line $\phi c_\gamma \phi t'$ along the vertical. This will cause $\phi\rho'$ to be curved towards the vertical, while $\phi c_\gamma \phi t'$ will remain not curved from its vertical position. This is so because the intrinsic gravitational speed $\phi V_g'(\phi r')$ being absolute in the context of intrinsic dynamics, it is absolute on the intrinsic dynamical spacetime $(\phi\chi', \phi c_\gamma \phi t')$. Consequently the intrinsic dynamical time dimension $\phi c_\gamma \phi t'$ is unaffected (or is invariant) with the presence of $\phi V_g'(\phi r')$. On the other hand, the presence of $\phi V_g'(\phi r')$ along the compound proper intrinsic space $\phi\rho'$ will cause $\phi\rho'$ to transform non-trivially into the compound relativistic intrinsic space $\phi\rho$. Graphically the foregoing paragraph and this paragraph mean that the presence of non-uniform $\phi V_g'(\phi r')$ along $\phi\rho'$ along the horizontal and along $\phi c_\gamma \phi t'$ along the vertical, will cause $\phi\rho'$ to be curved towards the vertical, thereby projecting $\phi\rho$ along the horizontal, while $\phi c_\gamma \phi t'$ will remain not curved from its vertical position, as illustrated in Fig. 3(a).

On the other hand, let us hypothetically combine the compound proper intrinsic space $\phi\rho'$ with the proper intrinsic gravitational time dimension $\phi c_g \phi t'$ to have a flat $(\phi\rho', \phi c_g \phi t')$ in the assumed absence of relative intrinsic gravitational field (or in the assumed absence of relative gravitational velocity $\vec{V}_g'(r')$). Then let non-uniform

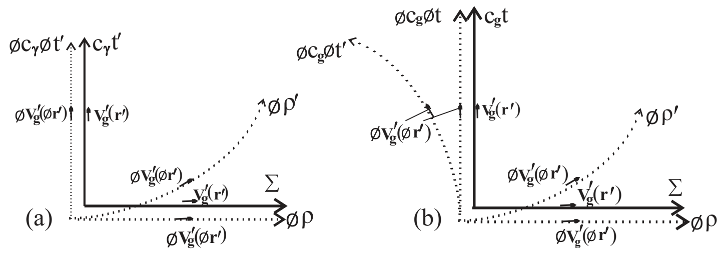

Figure 3:

intrinsic gravitational speed $\phi V_g'(\phi r')$ be introduced along the straight line $\phi\rho'$ along the horizontal and straight line $\phi c_g \phi t'$ along the vertical of the flat $(\phi\rho', \phi c_g\phi t')$. This will cause $\phi\rho'$ to be curved into the first quadrant towards the vertical and $\phi c_g\phi t'$ to be curved into the second quadrant towards the horizontal simultaneously, so that $\phi\rho'$ and $\phi c_g\phi t'$ constitute orthogonal curvilinear intrinsic dimensions, as illustrated in Fig. 3(b).

Fig. 3(b) arises because $\phi V_g'(\phi r')$ being a relative intrinsic speed in the context of the theories of intrinsic gravity, it is relative on the intrinsic gravitational spacetime $(\phi\rho_g', \phi c_g\phi t')$. Consequently the presence of non-uniform $\phi V_g'(\phi r)$ along $\phi c_g\phi t'$ will cause it to transform non-trivially into $\phi c_g\phi t$. Graphically this means that the presence of non-uniform $\phi V_g'(\phi r')$ along $\phi c_g\phi t'$ along the vertical will cause $\phi c_g\phi t'$ to be curved relative to the vertical as in Fig. 3(b).

The curved compound proper intrinsic space − straight line proper intrinsic dynamical time dimension $(\phi\rho', \phi c_\gamma\phi t')$ in Fig. 3(a) possesses non-Lorentzian intrinsic metric tensor of the Gaussian form,

$$d\phi s'^2 = \phi c_\gamma^2 d\phi t'^2 - \phi g_{11} d\phi\rho'^2 \tag{8}$$

On the other hand, the curved compound proper intrinsic space − curved proper intrinsic gravitational time dimension $(\phi\rho', \phi c_g\phi t')$ in Fig. 3(b) possesses the intrinsic Lorentzian metric

$$d\phi s'^2 = \phi c_g^2 d\phi t'^2 - d\phi\rho'^2 \tag{9}$$

This is so because $\phi\rho'$ and $\phi c_g\phi t'$ are orthogonal curvilinear intrinsic dimensions. However Eq. (9) must be derived from the full diagram in the two-world picture of Fig. 1 along with its complementary diagram of Fig. 2, as done in section 2 of [2].

The conclusion that can be drawn from all the foregoing is that if the dynamical time dimension $c_\gamma t$ and the intrinsic time dimension $\phi c_\gamma\phi t$ are the only time dimension and intrinsic time dimension that exist along the with the metric 3-space Σ and intrinsic metric space $\phi\rho$ in nature, then the theories of gravity on flat relativistic spacetime (Σ, ct) namely, TGR, CG and MTG, will be impossible, since flat relativistic spacetime $(\Sigma, c_\gamma t)$ with Lorentzian metric tensor does not exist in Fig. 3(a).

On the other hand, if the gravitational time dimension $c_g t$ and intrinsic gravitational time dimension $\phi c_g \phi t$ are the only time dimension and intrinsic time dimension that exist along with the metric 3-space Σ and intrinsic metric space $\phi\rho$, so that Fig. 3(b), which must be drawn in the two-world picture, obtains in every gravitational field, then TGR, CG and MTG will be possible on the flat relativistic spacetime $(\Sigma, c_g t)$. However the dynamical theories namely, SR, CM, EM and other non-gravitational dynamical laws on flat spacetime in a gravitational field will be impossible on flat spacetime in this case, as shall be discussed shortly.

However the dynamical time dimension and intrinsic dynamical time dimension $(c_\gamma t/\phi c_\gamma \phi t)$ and the gravitational time dimension and intrinsic gravitational time dimension $(c_g t/\phi c_g \phi t)$ are not separated in gravitation as done in Figs. 3(a) and 3(b) in nature. What happens in reality is that although it is the proper intrinsic gravitational time dimension $\phi c_g \phi t'$ that is curved by the presence of non-uniform relative intrinsic gravitational speed $\phi V'_g(\phi r')$ in a gravitational field, since $\phi c_\gamma \phi t'$ is not separated from $\phi c_g \phi t'$, both $\phi c_g \phi t'$ and $\phi c_\gamma \phi t'$ are curved, so that the compound intrinsic time dimension $\phi c \phi t' \equiv \phi c \phi t' \cup \phi c_\gamma \phi t'$ is curved along with the compound intrinsic space $\phi\rho' \equiv \phi\rho'_g \cup \phi\chi'$ in a gravitational field, as illustrated in Fig. 3(c). Fig. 3(c) must actually be presented in the full form within the two-world picture as in Fig. 1.

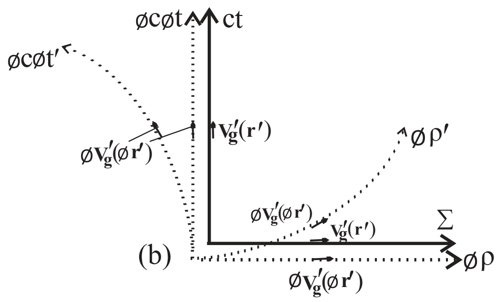

Figure 3: c)

The conclusion then is that it is the metric compound two-dimensional proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ that is curved relative to its projective flat metric compound two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ that underlies flat metric compound four-dimensional relativistic spacetime (Σ, ct) in a gravitational field in the contexts of the theory of gravitational relativity/intrinsic theory of gravitational relativity (TGR/ϕTGR). Consequently the theories of gravity namely, TGR, CG and MTG operate on flat compound four-dimensional metric spacetime (Σ, ct) and the theories of intrinsic gravity namely, ϕTGR, ϕCG and ϕMTG operate on flat compound two-dimensional intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in every gravitational field.

Again let us artificially separate the compound one-dimensional symmetry-partner relativistic mass $\phi\varepsilon/\phi c^2$ of a particle in the straight line compound relativistic intrinsic time dimension $\phi c\phi t$ along the vertical into its affine dynamical component $\phi\varepsilon_d/\phi c_\gamma^2$ and metric gravitational component $\phi\varepsilon_g/\phi c_g^2$, where $\phi\varepsilon_d/\phi c_\gamma^2$ is resident in the the affine relativistic intrinsic dynamical time dimension $\phi c_\gamma\phi t$ and $\phi\varepsilon_g/\phi c_g^2$ is resident in the metric relativistic intrinsic gravitational time dimension $\phi c_g\phi t$.

The special theory of relativity (SR) and intrinsic special theory of relativity (ϕSR) are yet absent in the discussion in the preceding paragraph. The term 'relativistic' in relativistic intrinsic mass $\phi\varepsilon/\phi c^2$ in relativistic intrinsic time dimension $\phi c\phi t$ and relativistic intrinsic mass ϕm in relativistic intrinsic space $\phi\rho$, refers to the presence of the theory of gravitational relativity (TGR) that converts the flat proper spacetime (Σ', ct') containing the rest masses $(m_0, \varepsilon'/c^2)$ and $(M_0, E'/c^2)$ of particles and bodies into flat relativistic spacetime (Σ', ct') containing the relativistic masses $(m, \varepsilon/c^2)$ and $(M, E/c^2)$ of particles and bodies (in the absence of SR) and the presence of the intrinsic theory of gravitational relativity (ϕTGR) that converts the flat proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ containing the intrinsic rest masses $(\phi m_0, \phi\varepsilon'/\phi c^2)$ and $(\phi M_0, \phi E'/\phi c^2)$ of particles and bodies into flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ containing the relativistic intrinsic masses $(\phi m, \phi\varepsilon/\phi c^2)$ and $(\phi M, \phi E/\phi c^2)$ of particles and bodies (in the absence of ϕSR).

As done previously, gravitational-relativistic shall often be used to refer to the presence of TGR, as distinct from special-relativistic that refers to the presence of SR. The adjective relativistic shall be used to refer to either situation whenever possibility of confusion can be ruled out. The relativistic mass m or M in the relativistic Euclidean 3-space Σ in the context of TGR shall be identified as the inertial mass in the second part of this paper.

Let us artificially combine the metric gravitational-relativistic intrinsic gravitational mass $\phi\varepsilon_g/\phi c_g^2$ (artificially separated from $\phi\varepsilon_d/\phi c_\gamma^2$), which is resident in the metric relativistic intrinsic gravitational time dimension $\phi c_g \phi t$ along the vertical with the metric compound relativistic intrinsic mass ϕm in the metric compound relativistic intrinsic space $\phi\rho$ along the horizontal. This gives the relativistic intrinsic mass $(\phi m, \phi\varepsilon_g/\phi c_g^2)$ of the particle in flat relativistic intrinsic spacetime $(\phi\rho, \phi c_g \phi t)$ in the absence of special relativity and intrinsic special relativity yet.

Let us then introduce special relativity/intrinsic special relativity by considering the gravitational-relativistic intrinsic mass $(\phi m, \phi\varepsilon_g/\phi c_g^2)$ of the particle to perform intrinsic motion at intrinsic dynamical speed ϕv on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c_g \phi t)$ relative to an observer. The possession of intrinsic speed ϕv relative to the observer of the compound intrinsic mass ϕm contained in an elementary interval of intrinsic metric space $d\phi\rho$, will cause it to be in intrinsic motion along an affine intrinsic space coordinate $\phi\tilde{x}$ that is inclined at an intrinsic angle $\phi\psi$ relative to $\phi\rho$ along the horizontal. This is so because possession of intrinsic speed ϕv relative to the observer by the affine dynamical mass ϕm_d will cause ϕm_d to undergo intrinsic motion along the inclined affine intrinsic space $\phi\tilde{x}$ and drag the metric gravitational intrinsic mass ϕm_g along, thereby making the metric compound intrinsic mass $\phi m \equiv \phi m_g \cup \phi m_d$ contained in elementary interval $d\phi\rho$ of intrinsic metric space $\phi\rho$ to move at intrinsic speed ϕv along the inclined affine intrinsic space $\phi\tilde{x}$.

On the other hand, the possession of intrinsic dynamical speed ϕv relative to an observer by the metric gravitational mass $\phi\varepsilon_g/\phi c_g^2$ in the metric intrinsic gravitational time dimension $\phi c_g \phi t$ along the vertical, will not cause $\phi\varepsilon_g/\phi c_g^2$ contained in elementary interval $\phi c_g d\phi t$ of the metric intrinsic gravitational time dimension $\phi c_g \phi t$ to be in intrinsic motion along an affine intrinsic coordinate $\phi c_\gamma \phi\tilde{t}$ that is inclined anti-clockwise at an intrinsic angle $\phi\psi$ relative to $\phi c_g \phi t$ along the vertical. Rather $\phi\varepsilon_g/\phi c_g^2$ contained in interval $\phi c_g d\phi t$ will remain not rotated from $\phi c_g \phi t$ along the vertical, but will move at intrinsic speed ϕv along $\phi c_g \phi t$ along the vertical.

The end of the foregoing paragraph is so because the relative intrinsic dynamical speed ϕv in the context of ϕSR (or on the flat affine intrinsic dynamical spacetime $(\phi\chi, \phi c_\gamma \phi\tilde{t})$ of ϕSR), is an absolute intrinsic speed on the flat metric intrinsic gravitational spacetime $(\phi\rho_g, \phi c_g \phi t)$. Hence the possession of ϕv relative to an observer by $\phi\varepsilon_g/\phi c_g^2$ in $\phi c_g \phi t$, will leave both $\phi\varepsilon_g/\phi c_g^2$ and $\phi c_g \phi t$ unchanged (or invariant). Graphically this means that $\phi\varepsilon_g/\phi c_g^2$ contained in interval $\phi c_g d\phi t$ of $\phi c_g \phi t$, cannot be in motion along an affine intrinsic coordinate that is rotated anti-clockwise by an intrinsic angle $\phi\psi$ relative to $\phi c_g \phi t$ along the vertical. Rather $\phi\varepsilon_g/\phi c_g^2$ contained in $\phi c_g d\phi t$ will be moving at the intrinsic speed ϕv in $\phi c_g \phi t$ along the vertical relative to the observer.

Possession of intrinsic dynamical speed ϕv relative to an observer by the metric intrinsic gravitational mass ϕm_g contained in interval $d\phi\rho_g$ of the metric intrinsic gravitational space $\phi\rho_g$ along the horizontal, will likewise leave both ϕm_g and $\phi\rho_g$ unchanged (or invariant). Graphically this means that ϕm_g contained in $d\phi\rho_g$ should not be in motion along an affine intrinsic coordinate that is rotated anti-clockwise by an intrinsic angle $\phi\psi$ relative to $\phi\rho_g$ along the horizontal.

However since ϕm_d contained in $d\phi\chi$ is not separated from ϕm_g contained in $d\phi\rho_g$, thereby giving rise to the compound intrinsic mass ϕm contained in interval $d\phi\rho$ of compound intrinsic space $\phi\rho$ in the artificially prescribed intrinsic mass $(\phi m, \phi\varepsilon/\phi c_g^2)$ of a particle, the intrinsic motion of ϕm_d contained in $d\phi\chi$ along the affine intrinsic space coordinate $\phi\tilde{x}$, which is rotated anti-clockwise by intrinsic angle $\phi\psi$ relative to the horizontal, by virtue of the intrinsic speed ϕv of ϕm_d relative to the observer, will drag ϕm_g contained in $d\phi\rho_g$ in intrinsic motion at intrinsic speed ϕv along the inclined intrinsic affine space coordinate $\phi\tilde{x}$.

Whereas since only $\phi\varepsilon_g/\phi c_g^2$ contained in $\phi c_g d\phi t$ exists in $\phi c_g\phi t$ along the vertical in the artificially prescribed intrinsic mass $\phi m, \phi\varepsilon_g/\phi c_g^2)$ of the particle, there is no rotation of $\phi\varepsilon_d/\phi c_\gamma^2$ contained in $\phi c_\gamma d\phi\tilde{t}$ to cause the rotation of $\phi\varepsilon_g/\phi c_g^2$ contained in $\phi c_g d\phi t$ from its vertical position. This paragraph and the foregoing two paragraphs explain the geometry of Fig. 4(a) for the relative intrinsic motion of the artificially prescribed $\phi m, \phi\varepsilon_g/\phi c_g^2)$.

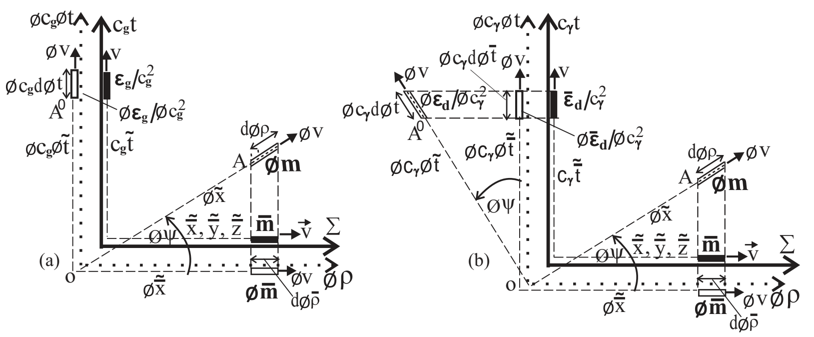

Figure 4:

On the other hand, let us artificially combine the metric compound intrinsic mass ϕm occupying interval $d\phi\rho$ of the metric compound intrinsic space $\phi\rho$ along the horizontal with the affine equivalent intrinsic dynamical mass $\phi\varepsilon_d/\phi c_\gamma^2$ occupying interval $\phi c_\gamma d\phi t$ of affine intrinsic dynamical time dimension $\phi c_\gamma\phi t$ along the vertical (which is artificially separated from the metric intrinsic gravitational mass $\phi\varepsilon_g/\phi c_g^2$), in the assumed absence of intrinsic dynamical speed ϕv relative to an observer (or of ϕSR).

Let us then introduce intrinsic special relativity/special relativity by considering the intrinsic mass $(\phi m, \phi\varepsilon_d/\phi c_\gamma^2)$ of the artificial particle on the artificial flat intrinsic space $(\phi\rho, \phi c_\gamma \phi t)$ to possess intrinsic dynamical speed ϕv relative to the observer. This will cause the compound intrinsic mass ϕm contained in interval $d\phi\rho$ of the metric compound intrinsic space $\phi\rho$ to be in intrinsic motion at intrinsic speed ϕv along an affine intrinsic space coordinate $\phi\tilde{x}$ that is inclined at an intrinsic angle $\phi\psi$ relative to $\phi\rho$ along the horizontal, as in Fig. 4(a).

The possession of intrinsic dynamical speed ϕv relative to the observer of the affine symmetry-partner intrinsic dynamical mass $\phi\varepsilon_d/\phi c_\gamma^2$ occupying interval $\phi c_\gamma d\phi t$ of affine dynamical intrinsic time dimension $\phi c_\gamma \phi t$ along the vertical, will likewise cause $\phi\varepsilon_d/\phi c_\gamma^2$ contained in interval $\phi c_\gamma d\phi t$ to be intrinsic motion along an affine intrinsic time coordinate $\phi c_\gamma \phi\tilde{t}$ that is inclined anti-clockwise into the second quadrant at intrinsic angle $\phi\psi$ relative to $\phi c_\gamma \phi t$ along the vertical. This is so because a relative intrinsic dynamical speed ϕv is a relative intrinsic speed on the flat affine intrinsic dynamical spacetime $(\phi\chi, \phi c_\gamma \phi t)$. Consequently possession of intrinsic dynamical speed ϕv relative to an observer by affine intrinsic dynamical mass $(\phi m_d, \phi\varepsilon_d/c_\gamma^2)$ on flat $(\phi\chi, \phi c_\gamma \phi t)$, will cause rotation of affine intrinsic frame $(\phi\tilde{\chi}, \phi c_\gamma \phi\tilde{t})$ of the particle relative to affine intrinsic frame $(\phi\tilde{\tilde{\chi}}, \phi c_\gamma \phi\tilde{\tilde{t}})$ of the observer.

The foregoing two paragraphs imply that possession of intrinsic dynamical speed ϕv of the artificially prescribed intrinsic mass $(\phi m, \phi\varepsilon_d/\phi c_\gamma^2)$ of a particle on the artificial flat intrinsic spacetime $(\phi\rho, \phi c_\gamma t)$, will give rise to the geometry depicted in Fig. 4(b). However it is the full form within the two-world picture of Fig. 4(b) and its complementary diagram that must be drawn, as shall be done later in this paper.

The inclined affine intrinsic space coordinate space − straight line intrinsic metric time dimension along the vertical $(\phi\tilde{\chi}, \phi c_g \phi t)$ in Fig. 4(b) possesses non-Lorentzian 'metric' tensor of the Gaussian form,

$$d\phi\tilde{s}^2 = \phi c_g^2 d\phi t^2 - \phi g_{11} d\phi\tilde{\chi}^2 \tag{10}$$

On the other hand, the inclined affine intrinsic spacetime $(\phi\tilde{\chi}, \phi c_\gamma \phi\tilde{t})$ in Fig. 4(b) possesses the intrinsic Lorentzian 'metric' tensor

$$d\phi\tilde{s}^2 = \phi c_g^2 d\phi t^2 - d\phi\tilde{\chi}^2 \tag{11}$$

However Eq. (11) must be derived from the full diagram in the two-world picture along with its complementary diagram, as done with Figs. 8(a) and 8(b) of [9] and as shall be re-visited later in this paper.

The conclusion that can be drawn from the above is that if the gravitational time dimension $c_g t$ and intrinsic gravitational time dimension $\phi c_g \phi t$ are the only time dimension and intrinsic time dimension that exist along with the metric Euclidean 3-space Σ and its underlying straight line intrinsic metric space $\phi\rho$ in every gravitational field in nature, then the dynamical theories namely, SR, the special-relativistic classical (or Newtonian) theory of motion (CM), electromagnetism (EM) and other non-gravitational dynamical laws, will be impossible on a flat relativistic spacetime $(\Sigma, c_g t)$ in a gravitational field, since a flat relativistic affine intrinsic spacetime geometry of ϕSR and hence a flat affine spacetime geometry of SR do not exist in Fig. 4(a).

On the other hand, if the affine dynamical time dimension $c_\gamma t$ and affine intrinsic dynamical time dimension $\phi c_\gamma \phi t$ are the only time dimension and intrinsic time dimension that exist along with the metric Euclidean 3-space Σ and straight line metric intrinsic intrinsic space $\phi\rho$ underlying Σ in every gravitational field (as known until now in physics), so that Fig. 4(b), which must be drawn in the two-world picture along with its complementary diagram exists in every gravitational field, then SR, CM, EM and other non-gravitational dynamical laws will be possible on a flat relativistic spacetime $(\Sigma, c_\gamma t)$ in every gravitational field, since Lorentzian affine intrinsic spacetime geometry and hence Lorentzian spcetime geometry obtain in Fig. 4(a). However TGR, CG and MTG will be impossible on the flat relativistic spacetime $(\Sigma, c_\gamma t)$ in this situation.

However the affine dynamical time dimension/affine intrinsic dynamical time dimension $(c_\gamma t/\phi c_\gamma \phi t)$ and metric gravitational time dimension/metric intrinsic gravitational time dimension $(c_g t/\phi c_g \phi t)$ are not separated in dynamics an gravity in reality unlike as done in Figs. 4(a) and 4(b). What happens in reality is that although it is the affine symmetry-partner intrinsic dynamical mass $\phi\varepsilon_d/\phi c_\gamma^2$ contained in interval $\phi c_\gamma d\phi t$ of affine intrinsic dynamical time dimension $\phi c_\gamma \phi t$, which possesses intrinsic dynamical speed ϕv relative to the observer and undergoes intrinsic motion at intrinsic speed ϕv along an affine intrinsic time coordinate $\phi c_\gamma \phi\tilde{t}$ that is inclined anti-clockwise at an intrinsic angle $\phi\psi$ relative to $\phi c_\gamma \phi t$ along the vertical, as illustrated in Fig. 4(b), since $\phi\varepsilon_d/\phi c_\gamma^2$ and $\phi\varepsilon_g/\phi c_g^2$ are not separated in nature, $\phi\varepsilon_d/\phi c_\gamma^2$ drags $\phi\varepsilon_g/\phi c_g^2$ along. Consequently it is the metric compound symmetry-partner intrinsic mass $\phi\varepsilon/\phi c^2 \equiv \phi\varepsilon_g/\phi c_g^2 \cup \phi\varepsilon_d/\phi c_\gamma^2$ that undergoes intrinsic motion at intrinsic speed ϕv along the inclined affine intrinsic time coordinate $\phi c_\gamma \phi\tilde{t}$ in a gravitational field.

Thus the metric compound symmetry-partner intrinsic mass $\phi\varepsilon/\phi c^2 \equiv \phi\varepsilon_g/\phi c_g^2 \cup \phi\varepsilon_d/\phi c_\gamma^2$ occupying interval $\phi cd\phi t$ of the metric compound intrinsic time dimension $\phi c\phi t$, which is in intrinsic motion at intrinsic dynamical speed ϕv along an affine intrinsic time coordinate $\phi c_\gamma \phi\tilde{t}$, which is inclined into the second quadrant at intrinsic angle $\phi\psi$ relative to the metric intrinsic time dimension $\phi c\phi t$ along the vertical, must be combined with the metric compound intrinsic mass $\phi m \equiv \phi m_g \cup \phi m_d$, occupying interval $d\phi\rho$ of the metric compound intrinsic space $\phi\rho$, which is in intrinsic motion at intrinsic dynamical speed ϕv along an affine intrinsic space coordinate $\phi\tilde{\chi}$ that is inclined into the first quadrant at equal intrinsic angle $\phi\psi$ relative to the metric compound intrinsic space $\phi\rho$ along the horizontal. In other words, the artificial diagram of Fig. 4(b) must be replaced with the natural diagram of Fig. 4(c) for SR/ϕSR in every gravitational field.

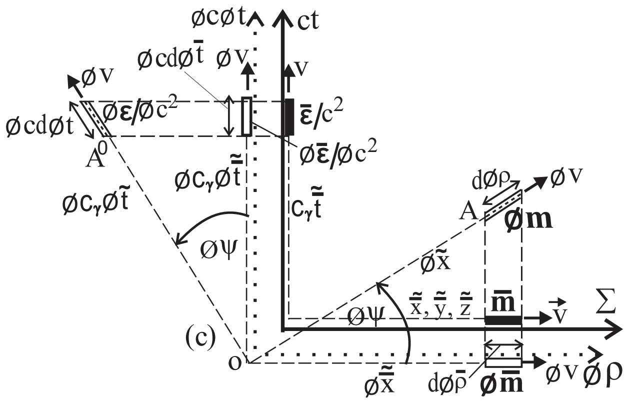

Figure 4: c)

However it is the full form in the two-world picture of Fig. 4(c) and its complementary diagram, along with their inverses, that must be drawn, from which the intrinsic local Lorentz transformation/local Lorentz transformation (ϕLLT/LLT) and their inverses must be derived in every gravitational field, as done in [9] and as shall be re-visited later in this paper.

Again the conclusion that follows from the natural geometry of Fig. 4(c) for SR/ϕSR in a gravitational field, is that the dynamical laws namely, SR, CM, EM and other non-gravitational dynamical laws, operate on the flat metric compound gravitational-relativistic spacetime (Σ, ct) (prescribed in the context of the theory of gravitational relativity (TGR)) and the intrinsic dynamical theories namely, ϕSR, ϕCM, ϕEM and other non-gravitational intrinsic dynamical laws, operate on the flat gravitational-relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ prescribed by ϕTGR.

The TGR, the relativistic form (in the context of TGR) of the classical (or Newtonian) theory of gravity (CG) and the Maxwellian theory of gravity (MTG), involving relative gravitational velocity $\vec{V}_g(r')$ on the flat metric compound gravitational-relativistic spacetime (Σ, ct), are the counterparts of SR, the special-relativistic classical (or Newtonian) theory of motion (CM) and electromagnetism (EM), involving dynamical velocity $\vec{v}$ relative to the observer on the flat relativistic spacetime (Σ, ct) in a gravitational field. The ϕTGR, ϕCG and ϕMTG, involving relative intrinsic gravitational speed $\phi V_g(\phi r')$ on the flat metric compound gravitational-relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$, are likewise the counterparts of ϕSR, ϕCM and ϕEM, involving intrinsic dynamical speed ϕv relative to the observer on the flat relativistic spacetime (Σct) in every gravitational field.

2 The theory of gravitational relativity/intrinsic theory of gravitational relativity by graphical approach

As mentioned towards the end of sub-section 1.1, the first two parts of this paper shall be devoted to the development of the theory of gravitational relativity/intrinsic theory of gravitational relativity (TGR/ϕTGR); the gravitational-relativistic form (in the context of TGR/ϕTGR) of the classical (or Newton's) law of gravity/intrinsic classical (or intrinsic Newton's) law of gravity (CG/ϕCG) and the gravitational-relativistic form (in the context of TGR/ϕTGR) of the special theory of relativity and intrinsic special theory of relativity (SR/ϕSR), on the flat relativistic spacetime (Σ, ct) and its underlying flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in Fig. 1.

Now the absolute intrinsic rest mass $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ in absolute intrinsic motion at absolute dynamical speed $\phi\hat{V}_d$ relative to the curved 'two-dimensional' absolute intrinsic metric spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ within an absolute intrinsic local Lorentz frame on the curved $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$, at 'distance' $\phi\hat{r}$ from the base of the absolute intrinsic rest mass $(\phi\hat{M}_0, \phi\hat{E}/\phi\hat{c}^2)$ of the gravitational field source at the origin of the curved $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ in Fig. 1, acquires the absolute intrinsic gravitational speed $\phi\hat{V}_g(\phi\hat{r})$ established at 'distance' $\phi\hat{r}$ along the curved $\phi\hat{\rho}$ and $\phi\hat{c}\phi\hat{t}$ by $\phi\hat{M}_0$ and $\phi\hat{E}/\phi\hat{c}^2$ respectively.

The absolute intrinsic dynamical speed $\phi\hat{V}_d$ of the absolute intrinsic rest mass $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ of the test particle and the absolute intrinsic gravitational speed $\phi\hat{V}_g(\phi\hat{r})$ within the absolute intrinsic local Lorentz frame on the curved $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$

at 'distance' $\phi\hat{r}$ along the curved $\phi\hat{\rho}$ and $\phi\hat{c}\phi\hat{t}$ from the base of $\phi\hat{M}_0$ in $\phi\hat{\rho}$ and the base of $\phi\hat{E}/\phi\hat{c}^2$ in $\phi\hat{c}\phi\hat{t}$, are then projected invariantly into the corresponding proper intrinsic local Lorentz frame on the curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$, at 'distance' $\phi r'$ along the curved $\phi\rho'$ and $\phi c\phi t'$ from the base of ϕM_0 in $\phi\rho'$ and the base of $\phi E'/\phi c^2$ in $\phi c\phi t'$ in Fig. 1.

Thus the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of the particle 'projected' into the curved $(\phi\rho', \phi c\phi t')$ within the proper intrinsic local Lorentz frame on the curved $(\phi\rho', \phi c\phi t')$ at 'distance' $\phi r'$ along the curved $\phi\rho'$ and curved $\phi c\phi t'$ from the base of ϕM_0 on curved $\phi\rho'$ and base of $\phi E'/\phi c^2$ on curved $\phi c\phi t'$, by the absolute intrinsic rest mass $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ of the particle in absolute intrinsic motion relative to the curved $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ in Fig. 1, possesses the projective absolute intrinsic speeds $\phi\hat{V}_g(\phi\hat{r})$ and $\phi\hat{V}_d$. In addition, the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of the particle possesses proper intrinsic gravitational speed $\phi V'_g(\phi r')$ established at 'distance' $\phi r'$ along the curved $\phi\rho'$ by ϕM_0 at the origin of the curved $\phi\rho'$ and at 'distance' $\phi r'$ along $\phi c\phi t'$ by $\phi E'/\phi c^2$ at the origin of the curved $\phi c\phi t'$ in Fig. 1.

As follows from the foregoing two paragraphs, the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of the particle possesses the intrinsic speeds $\phi\hat{V}_d$, $\phi\hat{V}_g(\phi\hat{r})$ and $\phi V'_g(\phi r')$ within the proper intrinsic local Lorentz frame on the global curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ at 'distance' $\phi r'$ along the curved $\phi\rho'$ from the base of ϕM_0 in $\phi\rho'$ and at 'distance' $\phi r'$ along the curved $\phi c\phi t'$ from the base of $\phi E'/\phi c^2$ in $\phi c\phi t'$, as indicated in Fig. 1. Given that the intrinsic rest mass $(\phi m_{0A}, \phi\varepsilon'_A/\phi c^2)$ of an observer possesses absolute intrinsic dynamical speed $\phi\hat{V}_{dA}$ within this proper intrinsic local Lorentz frame on the global curved $(\phi\rho', \phi c\phi t')$, then the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of the particle will be in intrinsic motion at intrinsic dynamical speed $\phi v = \phi\hat{V}_d - \phi\hat{V}_{dA}$, relative to the intrinsic rest mass $(\phi m_{0A}, \phi\varepsilon'_A/\phi c^2)$ of the observer within this proper intrinsic local Lorentz frame on the curved $(\phi\rho', \phi c\phi t')$.

It follows that primed intrinsic special theory of relativity (ϕSR$'$) can be formulated for the intrinsic motion at intrinsic speed ϕv of the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of the particle relative to the intrinsic rest mass $(\phi m_{0A}, \phi\varepsilon'_A/\phi c^2)$ of the observer within the proper intrinsic local Lorentz frame on the global curved $(\phi\rho', \phi c\phi t')$ at 'distance' $\phi r'$ along the curved $\phi\rho'$ from the base of ϕM_0 in $\phi\rho'$ in Fig. 1. It also follows that the primed intrinsic classical (or intrinsic Newton's) theory of gravity (ϕCG$'$) can be formulated in terms of the proper intrinsic gravitational speed $\phi V'_g(\phi r')$ and the associated proper intrinsic gravitational potential $\phi\Phi'(\phi r')$ and proper intrinsic gravitational acceleration $\phi g'(\phi r')$ within this proper intrinsic local Lorentz frame on the global curved $(\phi\rho', \phi c\phi t')$.

Further more the primed Newtonian theory of absolute intrinsic gravity (ϕNAG′) can be formulated in terms of the absolute intrinsic gravitational speed $\phi\hat{V}_g(\phi\hat{r})$ and the associated absolute intrinsic gravitational potential $\phi\hat{\Phi}(\phi\hat{r})$ and absolute intrinsic gravitational acceleration $\phi\hat{g}(\phi\hat{r})$, which are invariantly projected into the proper intrinsic local Lorentz frame on the global curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ at 'distance' $\phi r'$ along the curved $\phi\rho'$ from the base of ϕM_0 in $\phi\rho'$ by $\phi\hat{V}_g(\phi\hat{r})$, $\phi\hat{\Phi}(\phi\hat{r})$ and $\phi\hat{g}(\phi\hat{r})$ on the curved absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ in Fig. 1. Also the primed Newtonian theory of absolute intrinsic motion (ϕNAM′) can be formulated in terms of the absolute intrinsic dynamical speed $\phi\hat{V}_d$ projected into the proper intrinsic local Lorentz frame on the global curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ at distance $\phi r'$ from the base of ϕM_0 in $\phi\rho'$, as an intrinsic speed possessed by the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of the particle.

Thus the primed intrinsic theories ϕCG′, ϕSR′, ϕNAG′+ ϕNAM′ exist within the proper intrinsic local Lorentz frame on the global curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ at 'distance' $\phi r'$ from the base of ϕM_0 in the curved $\phi\rho'$ in Fig. 1. Let the elementary intervals $d\phi\rho'$ of the curved proper intrinsic space $\phi\rho'$ and elementary interval $\phi cd\phi t'$ of the curved proper intrinsic time dimension $\phi c\phi t'$ be the dimensions of this proper intrinsic local Lorentz frame on the global curved $(\phi\rho', \phi c\phi t')$. Then the intrinsic local Lorentz frame shall be denoted by $(d\phi\rho', \phi cd\phi t')$. It contains the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of the particle and harbors the primed intrinsic theories ϕCG′, ϕSR′, ϕNAG′ and ϕNAM′.

The primed intrinsic local Lorentz frame $(d\phi\rho', \phi cd\phi t')$ on the global curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$, with the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ of the particle and the primed intrinsic theories ϕCG′, ϕSR′, ϕNAG′ and ϕNAM′ within it, is then projected as the unprimed intrinsic local Lorentz frame $(d\phi\rho, \phi cd\phi t)$ on the global flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ with the gravitational-relativistic intrinsic mass $(\phi m, \phi\varepsilon/\phi c^2)$ of the particle and the gravitational-relativistic (or unprimed) intrinsic theories ϕCG, ϕSR, ϕNAG and ϕNAM within it in Fig. 1.

The projective unprimed intrinsic local Lorentz frame $(d\phi\rho, \phi dc\phi t)$ and the gravitational-relativistic intrinsic mass $(\phi m, \phi\varepsilon/\phi c^2)$ of the particle and the gravitational-relativistic intrinsic theories ϕCG, ϕSR, ϕNAG and ϕNAM in it on the global flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$, are then made manifest in the unprimed local Lorentz frame, containing the gravitational-relativistic mass $(m, \varepsilon/c^2)$ of the particle and harboring the unprimed theories CG, SR, NAG and NAM within it on the global flat relativistic spacetime (Σ, ct) in Fig. 1.

The foregoing paragraph is further summarized as the following transformations:

$$(d\phi\rho', \phi cd\phi t') \to (d\phi\rho, \phi cd\phi t);$$

$$(\phi m_0, \phi\varepsilon'/\phi c^2) \to (\phi m, \phi\varepsilon/\phi c^2);$$

$$(\phi \mathrm{CG}', \phi \mathrm{SR}', \phi \mathrm{NAG}', \phi \mathrm{NAM}') \to (\phi \mathrm{CG}, \phi \mathrm{SR}, \phi \mathrm{NAG}, \phi \mathrm{NAM}),$$

in the context of the intrinsic theory of gravitational relativity (ϕTGR), which are made manifest outwardly in the following transformations:

$$(dr', r'd\theta', r'\sin\theta' d\varphi', cdt') \to (dr, rd\theta, r\sin\theta d\varphi, cdt);$$

$$(m_0, \varepsilon'/c^2) \to (m, \varepsilon/c^2);$$

$$(\mathrm{CG}', \mathrm{SR}', \mathrm{NAG}', \mathrm{NAM}') \to (\mathrm{CG},\ \mathrm{SR},\ \mathrm{NAG},\ \mathrm{NAM}),$$

in the context of the theory of gravitational relativity (TGR).

Now global curved four-dimensional proper spacetime (Σ', ct') does not exist along with global curved two-dimensional proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ in Fig. 1. Thus there is nowhere to place the proper local Lorentz frame in Fig. 1. It shall therefore be placed on the global flat proper spacetime (Σ', ct') in Fig. 11 of [6] at the first stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field, which endured for no moment before evolving to the final Fig. 1 at the second stage.

The program of this first part and the second part of this paper is the following

1. Derivation of intrinsic metric spacetime coordinate interval transformations, intrinsic mass and other intrinsic parameter transformations in the context of ϕTGR and derivation of the gravitational-relativistic intrinsic theories ϕGC and ϕSR within intrinsic local Lorentz frames on the global flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$, in terms of the gravitational-relativistic intrinsic parameters ϕm, $\phi\Phi(\phi r)$ and $\phi g(\phi r)$ obtained; and
2. Derivation of metric spacetime coordinate interval transformations, mass and other parameter transformations in the context of TGR and derivation of the gravitational-relativistic theories GC and SR within local Lorentz frames on the global flat relativistic spacetime (Σ, ct), in terms of the gravitational-relativistic parameters m, $\Phi(\phi r)$ and $\vec{g}(\phi r)$ obtained.

There are two approaches towards the accomplishment of items 1 and 2 above namely, a graphical approach to be developed in this first part of this paper and an analytical approach, to complement the graphical approach, to be developed in the second part of this paper.

The rest of this section shall be devoted to the development of TGR/ϕTGR by the graphical approach, while the next section shall be devoted to the development of SR/ϕSR by the graphical approach on flat spacetime in a gravitational field, upon the flat spacetime/intrinsic spacetime $(\Sigma, ct)/(\phi\rho, \phi c\phi t)$ and mass/intrinsic mass $(m/\phi m)$ that evolve in the context of TGR/ϕTGR. Actually the TGR/ϕTGR by the graphical approach has been accomplished to a large extent in section 2 of [2]. We shall be repeating section 2 of [2], while adding some important details in the rest of this section.

2.1 Derivation of intrinsic gravitational local Lorentz transformation graphically and validating intrinsic gravitational local Lorentz invariance in the context of the intrinsic theory of gravitational relativity

The global spacetime/intrinsic spacetime diagrams of combined first and second stages of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field of arbitrary strength of Figs. 7 and 8 and their inverses Figs. 9 and 10 of [2] are required here. Figs. 7 and 8 and [1] have been reproduced as Figs. 1 and 2 of this paper, while incorporating the flat absolute-absolute intrinsic-intrinsic spacetimes $(\phi\phi\hat{\hat{\rho}}, \phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}})$ of our universe and $(-\phi\phi\hat{\hat{\rho}}^{*}, -\phi\phi\hat{\hat{c}}\phi\phi\hat{\hat{t}}^{*})$ of the negative universe isolated in [3], which could not appear in Figs. 7 – 10 of [2]. However the inverses of Figs. 1 and 2 of this paper have not been drawn on order to conserve space.

The local spacetime/intrinsic spacetime diagrams (within a local Lorentz frame) shown as Figs. 11 and 12 and their inverses as Figs. 13 and 14, drawn from the global geometries of Figs. 7 and 8 and their inverses of Figs. 9 and 10 respectively, within a gravitational field of arbitrary strength in [2], shall be reproduced as Figs. 5 – 8 here.

The local spacetime/intrinsic spacetime diagram of Fig. 5 is valid with respect to 3-observers in the relativistic Euclidean 3-spaces Σ and $-\Sigma^{*}$ of our universe and the negative universe. It has been drawn within a proper (or primed) intrinsic local Lorentz frame at 'distance $\phi r'$' along the curved proper intrinsic space $\phi\rho'$ from the base of the intrinsic rest mass ϕM_0 of the gravitational field source located at the origin of the curved proper intrinsic space $\phi\rho'$ in Fig. 1 of this paper, which corresponds to unprimed intrinsic local Lorentz frame at 'distance' ϕr along the straight line relativistic intrinsic space $\phi\rho$ along the horizontal, from the base of the gravitational-relativistic intrinsic mass ϕM of the gravitational field source in

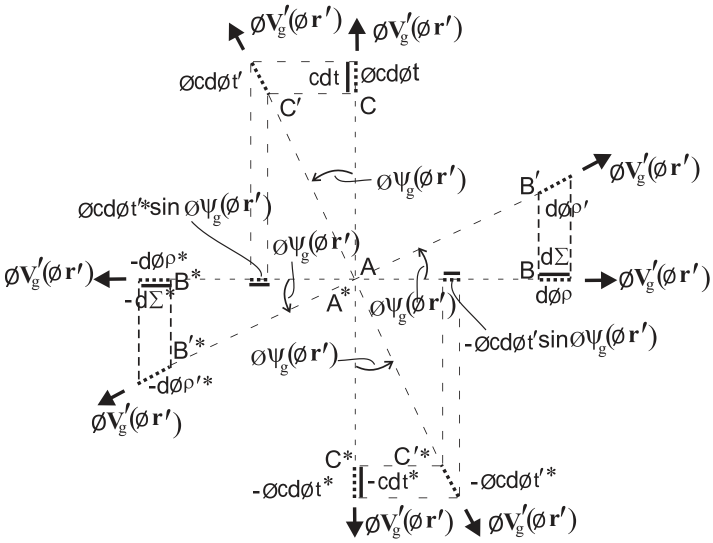

Figure 5:

$\phi\rho$ and unprimed local Lorentz frame at radial distance r from the center of the gravitational-relativistic mass M of the assumed spherical gravitational field source in Σ. Spherical gravitational field sources shall be assumed until such a time when the Maxwellian theory of gravity (MTG) shall be developed when non-spherical gravitational field sources shall be brought in.

The explanation of the derivation of Fig. 5 from Fig. 7 of [2] or from Fig. 1 of this paper is as done for the derivation of Fig. 6 from Fig. 1 in [1]. The partial intrinsic gravitational local Lorentz transformation derivable with respect to 3-observers in the relativistic Euclidean 3-space Σ in our universe from Fig. 5, which has been derived in [2], is the following

$$\left.\begin{aligned} d\phi\rho' &= d\phi\rho \sec\phi\psi_g(\phi r') - \phi c_g d\phi t \tan\phi\psi_g(\phi r'); \\ & \quad (\text{w.r.t. } 3-\text{observers in } \Sigma) \end{aligned}\right\} \qquad (12)$$

The complementary diagram to Fig. 5, which is valid with respect to 1-observers in the time dimensions ct and $-ct^*$ of our universe and the negative universe respectively, is depicted as Fig. 6. Fig. 6 has been drawn within the same local Lorentz frame as Fig. 5, from the global geometry of Fig. 8 of [2] or Fig. 2 of this paper, with the same explanation for drawing Fig. 7 from Fig. 3 in [1].

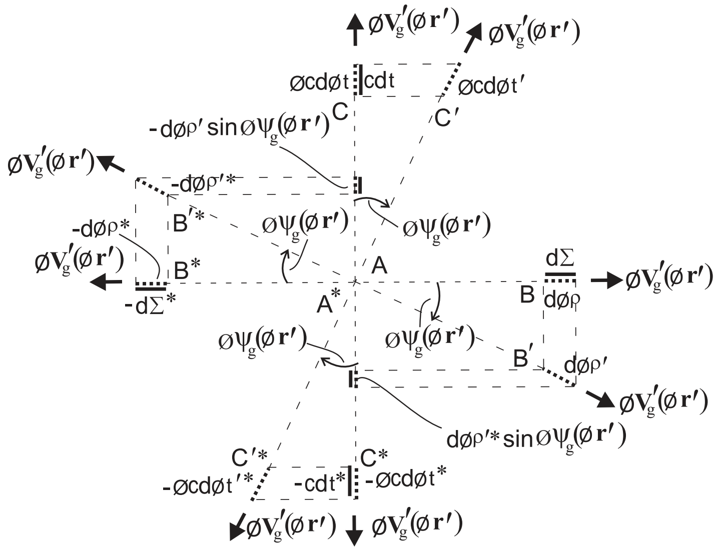

Figure 6:

The partial intrinsic gravitational local Lorentz transformation derivable with respect to 1-observers in the relativistic time dimension *ct* of our universe from Fig. 6, which has been derived in [2] is the following

$$\left.\begin{aligned} \phi cd\phi t' &= \phi cd\phi t \sec\phi\psi_g(\phi r') - d\phi\rho \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 1-\text{observers in } ct) \end{aligned}\right\} \tag{13}$$

By collecting Eqs. (12) and (13) we obtain the full intrinsic gravitational local Lorentz transformation (ϕGLLT) derivable from Figs. 5 and 6 as follows

$$\left.\begin{aligned} d\phi\rho' &= d\phi\rho \sec\phi\psi_g(\phi r') - \phi c_g d\phi t \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 3-\text{observers in } \Sigma); \\ \phi cd\phi t' &= \phi cd\phi t \sec\phi\psi_g(\phi r') - d\phi\rho \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 1-\text{observers in } ct) \end{aligned}\right\} \tag{14}$$

There is an inverse to system (14), which must be derived from the inverses to Figs. 5 and 6. The inverse to Fig. 5 is depicted in Fig. 7. Fig. 7 has been drawn within the same local Lorentz frame as Figs. 5 and 6, from the global geometry of Fig. 9

of [2] or from the undrawn inverse to Fig. 1 of this paper, with same explanation for drawing Fig. 8 from Fig. 4 in [1].

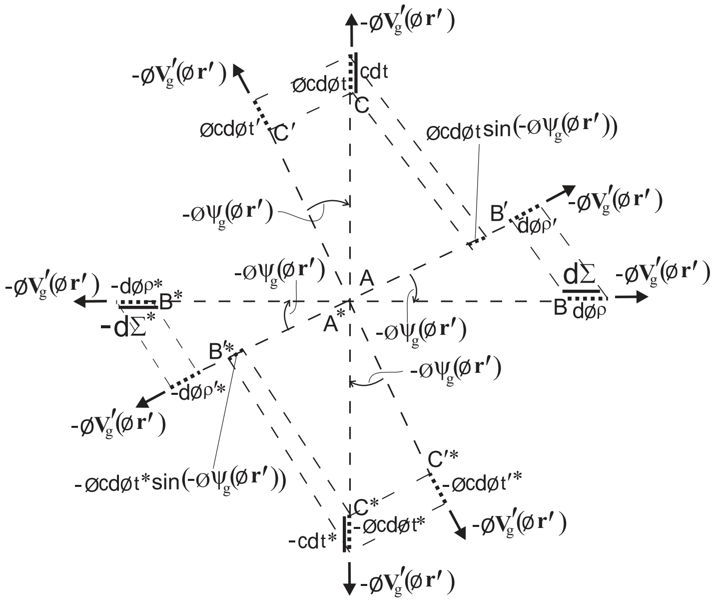

Figure 7: The inverse to the spacetime/intrinsic spacetime geometry to Fig. 5 at the second stage of evolutions of spacetimes/intrinsic spactimes within symmetry-partner gravitational fields in the positive and negative universes that is valid with respect to 1-observers in the relativistic time dimensions in the two universes.

Fig. 7 is valid with respect to 1-observers in the relativistic time dimensions ct and $-ct^*$ of our universe and the negative universe. The explanation of this is the same as given for the validity of Fig. 8 of [1] and Fig. 9 of [2] with respect to 1-observers in ct' and $-ct'^*$ in those diagrams.

The partial inverse intrinsic gravitational local Lorentz transformation that is derivable with respect to 1-observers in ct in our universe from Fig. 7, which has been derived in [2] is the following

$$\left.\begin{aligned} d\phi\rho' &= d\phi\rho' \sec\phi\psi_g(\phi r') + \phi c d\phi t' \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 1-\text{observers in } ct) \end{aligned}\right\} \tag{15}$$

The inverse to Fig. 6 is depicted as Fig. 8. Again Fig. 8 has been drawn within the same local Lorentz frame as Fig. 5 – Fig. 7, from the global geometry of Fig. 10 of [2] or the inverse to Fig. 2 (not drawn) of this paper, with same explanation for drawing Fig. 9 from Fig. 5 in [1].

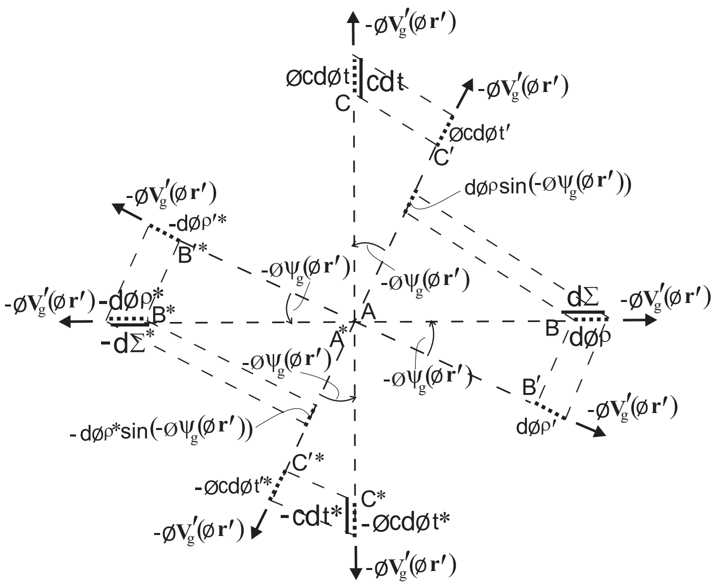

Figure 8: The inverse to the spacetime/intrinsic spacetime geometry to Fig. 6 at the second stage of evolutions of spacetimes/intrinsic spactimes within symmetry-partner gravitational fields in the positive and negative universe that is valid with respect to 3-observers in the relativistic Euclidean 3-spaces in the two universes.

Fig. 8 is valid with respect to 3-observers in the relativistic Euclidean 3-spaces Σ and $-\Sigma^*$ of our universe and the negative universe. the explanation of this is as given for the validity of Fig. 9 of [1] or Fig. 10 of [2] with respect to 3-observers in Σ and $-\Sigma^*$ in those diagrams.

The partial intrinsic gravitational local Lorentz transformation that can be derived with respect to 3-observers in Σ in our universe from Fig. 8, which has been derived in [2] is the following

$$\left.\begin{aligned} \phi c d\phi t &= \phi c d\phi t' \sec\phi\psi_g(\phi r') + d\phi\rho' \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 3-\text{observers in } \Sigma) \end{aligned}\right\} \quad (16)$$

By collecting Eqs. (14) and (15) we obtain the full inverse intrinsic gravitational local Lorentz transformation (inverse ϕGLLT), that is, inverse to system (14), as follows

$$\left.\begin{aligned} d\phi\rho &= d\phi\rho' \sec\phi\psi_g(\phi r') + \phi cd\phi t' \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 1-\text{observers in } ct)\phi cd\phi t; \\ &= \phi cd\phi t' \sec\phi\psi_g(\phi r') + d\phi\rho' \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 3-\text{observers in } \Sigma) \end{aligned}\right\} \tag{17}$$

The elementary indefinitely short intervals $d\phi\rho'$ and $\phi cd\phi t'$ that appear in Figs. 5 – 8 and in systems (14) and (17), have been taken about 'distance' $\phi r'$ along the curved $\phi\rho'$ from the base of ϕM_0 in $\phi\rho'$ and along the curved $\phi c\phi t'$ from the base of $\phi E'/\phi c^2$ in $\phi c\phi t'$ in Figs. 7 and 8 of [2] or Figs. 1 and 2 of this paper. They are the intrinsic dimensions of the proper (or primed) intrinsic local Lorentz frame $(d\phi\rho', \phi cd\phi t')$ on the global curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ at 'distance' $\phi r'$ along the curved $\phi\rho'$ from the base of ϕM_0 in $\phi\rho'$ in those figures, as mentioned earlier. They project elementary intervals $d\phi\rho$ and $\phi cd\phi t$ of relativistic intrinsic space and relativistic intrinsic time dimension at 'distance' ϕr along $\phi\rho$ from the base of ϕM in $\phi\rho$ along the horizontal and at 'distance' ϕr along $\phi c\phi t$ from the base of $\phi E/\phi c^2$ in $\phi c\phi t$ along the vertical.

The projective elementary intrinsic coordinate intervals $d\phi\rho$ and $\phi cd\phi t$ are the intrinsic dimensions of the relativistic (or unprimed) intrinsic local Lorentz frame $(d\phi\rho, \phi cd\phi t)$ on flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ at 'distance' ϕr along $\phi\rho$ from the base of ϕM in $\phi\rho$ in Figs. 7 and 8 of [2] or Figs. 1 and 2 of this paper.

As derived in [2], the relative intrinsic angle $\phi\psi_g(\phi r')$ is related to the relative intrinsic gravitational speed $\phi V'_g(\phi r')$ within the intrinsic local Lorentz frame at 'distance' $\phi r'$ along the curved $\phi\rho'$ from the base of ϕM_0 in $\phi\rho'$ in Fig. 1 as

$$\sin\phi\psi_g(\phi r') = \frac{\phi V'_g(\phi r')}{\phi c_g} \equiv \phi\beta_g(\phi r') \tag{18a}$$

$$\cos\phi\psi_g(\phi r') = \sqrt{1 - \frac{\phi V'_g(\phi r')^2}{\phi c_g^2}} \equiv \phi\gamma_g(\phi r')^{-1} \tag{18b}$$

By using Eqs. (18a) and (18b), the ϕGLLT (14) and its inverse (17) can be written explicitly in terms of intrinsic gravitational speed respectively as follows

$$\left.\begin{aligned} d\phi\rho' &= \phi\gamma_g(\phi r')(d\phi\rho - \phi V_g'(\phi r')d\phi t); \\ &\quad \text{(w.r.t. 3 – observers in } \Sigma); \\ d\phi t' &= \phi\gamma_g(\phi r')(d\phi t - \frac{\phi V_g'(\phi r')}{\phi c_g^2} d\phi\rho); \\ &\quad \text{(w.r.t 1 – observers in } ct) \end{aligned}\right\} \tag{19}$$

and

$$\left.\begin{aligned} d\phi\rho &= \phi\gamma_g(\phi r')(d\phi\rho' + \phi V_g'(\phi r')d\phi t'); \\ &\quad \text{(w.r.t. 1 – observers in } ct); \\ d\phi t &= \phi\gamma_g(\phi r')(d\phi t' + \frac{\phi V_g'(\phi r')}{\phi c_g^2} d\phi\rho'); \\ &\quad \text{(w.r.t 3 – observers in } \Sigma) \end{aligned}\right\} \tag{20}$$

As also derived in [2], the intrinsic gravitational speed $\phi V_g'(\phi r')$ is related to the intrinsic rest mass ϕM_0 of the gravitational field source as

$$\phi V_g'(\phi')^2 = 2G\phi M_0/\phi r'$$

However this relation must now be written in terms of the immaterial intrinsic active gravitational mass (or gravitational charge), after introducing the gravitational charge that is imperceptibly hidden within the rest mass as the source of gravitational speed, gravitational potential and gravitational acceleration in [6], as already done in [4]; see Eq. (119) of [4]. In other words, we must replace the last equation by the following

$$\phi V_g'(\phi')^2 = 2G\phi M_{0\text{a}}/\phi r' \tag{21}$$

Then the relations (18a) and (18b) can be written in terms of $2G\phi M_{0\text{a}}/\phi r'$ as

$$\sin\phi\psi_g(\phi r') = \sqrt{\frac{2G\phi M_{0\text{a}}}{\phi r'}} \equiv \phi\beta_g(\phi r') \tag{22a}$$

$$\cos\phi\psi_g(\phi r') = \sqrt{1 - \frac{2G\phi M_{0\text{a}}}{\phi r'\phi c_g^2}} \equiv \phi\gamma_g(\phi r')^{-1} \tag{22b}$$

By using Eqs. (22a) and (22b), the ϕGLLT (14) or (19) and its inverse (17) or (20) can be written explicitly in terms of the intrinsic gravitational parameter

$2G\phi M_{0\mathrm{a}}/\phi r'$ respectively as follows

$$\left.\begin{aligned} d\phi\rho' &= \phi\gamma_g(\phi r')(d\phi\rho - \sqrt{\frac{2G\phi M_{0\mathrm{a}}}{\phi r'}}d\phi t); \\ &\quad (\text{w.r.t. } 3-\text{observers in } \Sigma); \\ d\phi t' &= \phi\gamma_g(\phi r')(d\phi t - \sqrt{\frac{2G\phi M_{0\mathrm{a}}}{\phi r'\phi c_g^4}}d\phi\rho); \\ &\quad (\text{w.r.t } 1-\text{observers in } ct) \end{aligned}\right\} \tag{23}$$

and

$$\left.\begin{aligned} d\phi\rho &= \phi\gamma_g(\phi r')(d\phi\rho' + \sqrt{\frac{2G\phi M_{0\mathrm{a}}}{\phi r'}}d\phi t'); \\ &\quad (\text{w.r.t. } 1-\text{observers in } ct); \\ d\phi t &= \phi\gamma_g(\phi r')(d\phi t' + \sqrt{\frac{2G\phi M_{0\mathrm{a}}}{\phi r'\phi c_g^4}}d\phi\rho'); \\ &\quad (\text{w.r.t } 3-\text{observers in } \Sigma) \end{aligned}\right\} \tag{24}$$

where $\phi\gamma_g(\phi r')$ is given by Eq. (22b).

As also derived in [2], the ϕGLLT (14), (19) or (23) or its inverse (17), (20) or (24) leads to intrinsic gravitational local Lorentz invariance (ϕGLLI)

$$\phi c^2 d\phi t^2 - d\phi\rho^2 = \phi c^2 d\phi t'^2 - d\phi\rho'^2 \tag{25}$$

This invariance obtains at every point on the curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ and at every point on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in Figs. 1 and 2, showing formally that the relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ is everywhere flat in every gravitational field.

Another results derived in [2] is the intrinsic gravitational length contraction and intrinsic gravitational time dilation implied by the ϕGLLT and its inverse. In order to do this, only the intrinsic coordinate interval transformations derived with respect to 3-observers in the Euclidean 3-space Σ in the ϕGLLT and its inverse are relevant, since these are the observers that observe or measure length contraction and time dilation. By collecting the intrinsic coordinate interval transformations with respect to 3-observers in Σ in systems (14) and (17) we have

$$\left.\begin{aligned} d\phi\rho' &= d\phi\rho \sec\phi\psi_g(\phi r') - \phi c_g d\phi t \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 3-\text{observers in } \Sigma); \\ \phi c d\phi t &= \phi c d\phi t' \sec\phi\psi_g(\phi r') + d\phi\rho' \tan\phi\psi_g(\phi r'); \\ &\quad (\text{w.r.t. } 3-\text{observers in } \Sigma \end{aligned}\right\} \tag{26}$$

Now when a hypothetical intrinsic 1-observer in the relativistic intrinsic space $\phi\rho$ underlying Σ, with respect to whom the first equation of system (26) is also valid, picks his intrinsic laboratory rod to measure the resultant intrinsic coordinate interval projected into the relativistic intrinsic space $\phi\rho$ along the horizontal in Fig. 5, given by the right-hand side of the first equation of system (26), he will be able to measure the term $d\phi\rho \sec\phi\psi_g(\phi r')$ but not the term $\phi c d\phi t \tan\phi\psi_g(\phi r')$. Likewise when the hypothetical intrinsic 1-observer in the intrinsic space $\phi\rho$ underlying Σ, with respect to whom the second equation of system (26) is also valid, picks his laboratory clock to measure the resultant intrinsic coordinate interval projection into the relativistic intrinsic time dimension $\phi c\phi t$ in Fig. 6, expressed by the right-hand side of the second equation of system (26), he will be able to measure the term $\phi c d\phi t' \sec\phi\psi_g(\phi r')$ but not the term $d\phi\rho' \tan\phi\psi_g(\phi r')$.

Thus by collecting the terms that are measurable with intrinsic laboratory rod and intrinsic laboratory clock in system (26) by intrinsic 1-observer in $\phi\rho$ we have

$$d\phi\rho = d\phi\rho' \cos\phi\psi_g(\phi r') \tag{27a}$$

$$\phi t = d\phi t' \sec\phi\psi_g(\phi r') \tag{27b}$$

Equations (27a) and 27(b) are mere intrinsic coordinate interval projections with respect to intrinsic 1-observers in $\phi\rho$ and 3-observers in the relativistic Euclidean 3-space Σ overlying $\phi\rho$.

The forms of Eqs. (27a) and (27b) implied by system (19) and (20) are the following

$$d\phi\rho = \phi\gamma_g(\phi r')^{-1} d\phi\rho' = \left(1 - \frac{\phi V_g'(\phi r')^2}{\phi c_g^2}\right)^{1/2} d\phi\rho' \tag{28a}$$

$$d\phi t = \phi\gamma_g(\phi r') d\phi t' = \left(1 - \frac{\phi V_g'(\phi r')^2}{\phi c_g^2}\right)^{-1/2} d\phi t' \tag{28b}$$

And the form of Eqs. (27a) and (27b) implied by systems (23) and (24) are the following

$$d\phi\rho = \phi\gamma_g(\phi r')^{-1}d\phi\rho' = \left(1 - \frac{2G\phi M_{0a}}{\phi r'\phi c_g^2}\right)^{1/2} d\phi\rho' \tag{29a}$$

$$d\phi t = \phi\gamma_g(\phi r')d\phi t' = \left(1 - \frac{2G\phi M_{0a}}{\phi r'\phi c_g^2}\right)^{-1/2} d\phi t' \tag{29b}$$

Equations (27a) and (27b), Eqs. (28a) and (28b) and Eqs, (29a) and (29b) are alternative forms of intrinsic gravitational length contraction and intrinsic gravitational time dilation formulae in the context of the intrinsic theory of gravitational relativity (ϕTGR). They pertain to the measurable sub-space of the total space of ϕTGR, where the total space of ϕTGR is the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in Fig. 1.

Let us obtain a graphical representation of the measurable sub-space of ϕTGR, to which the intrinsic gravitational length contraction and intrinsic gravitational time dilation formulae pertain. We must simply combine the lower half of the first quadrant of Fig. 5 and the upper half of the first quadrant of Fig. 8, both of which are valid with respect to 3-observers in the relativistic Euclidean 3-space Σ, into one diagram depicted in Fig. 9.

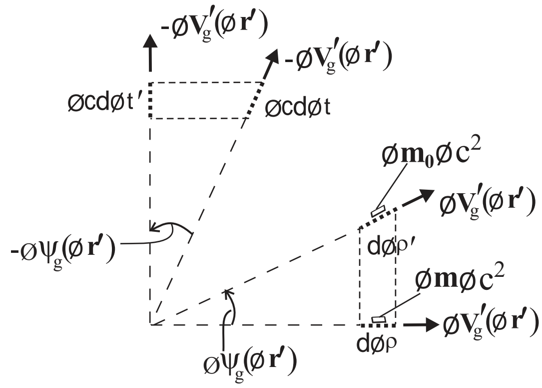

Figure 9: The measurable sub-space of the space of ϕTGR to which intrinsic gravitational length contraction and intrinsic gravitational time dilation formulae in the context of ϕTGR pertain with respect to 3-observers in the Euclidean 3-space Σ.

The intrinsic coordinate interval projection relations that can be derived with respect to 3-observers in the Euclidean 3-space Σ from Fig. 9 are

$$d\phi\rho = d\phi\rho' \cos\phi\psi_g(\phi r') \ ; \ \phi c d\phi t = \phi c d\phi t' \cos\phi\psi_g(\phi r'),$$

which are the same as Eq. (27a) and (27b).

Fig. 9 and the intrinsic gravitational length contraction and intrinsic gravitational time dilation formulae derivable from it are valid with respect to 3-observers in the relativistic Euclidean 3-space Σ overlying the relativistic intrinsic space $\phi\rho$ along the horizontal. It is important to note that there is no projection of the inclined $\phi c d\phi t$ along the horizontal and no projection of the inclined $d\phi\rho'$ along the vertical in the measurable sub-space of ϕTGR of Fig. 9.

2.2 Derivation of intrinsic mass relation in the context of ϕTGR by the graphical approach

Now in the hypothetical situation of the absence of relative intrinsic gravitational speed but the presence of non-uniform absolute intrinsic gravitational speed $\phi\hat{V}_g(\phi\hat{r})$ along the straight line proper intrinsic space $\phi\rho'$ along the horizontal in Fig. 3 or 4 of [6], the intrinsic rest mass ϕm_0 of a particle located at 'distance' $\phi r'$ along the straight line $\phi\rho'$ along the horizontal from the base of the intrinsic rest mass ϕM_0 of the gravitational field source in $\phi\rho'$, is equivalent to intrinsic total energy $m_0\phi c^2$.

If we now allow the intrinsic rest mass ϕM_0 of the gravitational field source to establish relative intrinsic gravitational speed $\phi V'_g(\phi r')$ at 'distance' $\phi r'$ along the straight line $\phi\rho'$ along the horizontal where ϕm_0 is located, then the interval $d\phi\rho'$ of $\phi\rho'$ about this point containing ϕm_0 will be inclined at intrinsic angle $\phi\psi_g(\phi r')$ relative to the horizontal and project interval $d\phi\rho$ of relativistic intrinsic space along the horizontal, as illustrated in Fig. 9. The intrinsic rest mass of the test particle still possessing intrinsic total energy $\phi m_0\phi c^2$ is inclined along $d\phi\rho'$ and 'projects' intrinsic grav- itational-relativistic mass ϕm contained within the projective interval $d\phi\rho$ along the horizontal. The 'projective' gravitational-relativistic intrinsic mass ϕm is equivalent to intrinsic total energy $\phi m\phi c^2$ within $d\phi\rho$ along the horizontal, as illustrated in Fig. 9.

The intrinsic mass relation in the context of ϕTGR is a relationship between the intrinsic rest mass ϕm_0 contained within the inclined $d\phi\rho'$ and the 'projective' gravitational-relativistic intrinsic mass ϕm contained within the projective $d\phi\rho$ along the horizontal in Fig. 9. In other to derive that relationship, let us re-write Eq. (18b) as follows

$$\phi c \cos\phi\psi_g(\phi r') = \phi c\sqrt{1 - \phi V'_g(\phi r')^2/\phi c_g^2} \tag{30}$$

The interpretation of this equation is that the compound intrinsic speed of signals ϕc at every point of the inclined interval of proper intrinsic metric space $\phi\rho'$ projects a component $\phi c \cos\psi_g(\phi r')$ into every point of the projective interval of relativistic intrinsic metric space $d\phi\rho$ along the horizontal. Let us obtain the square of Eq. (30) and multiply the result by ϕm_0 to have

$$\phi m_0 \phi c^2 \cos^2 \phi\psi_g(\phi r') = \phi m_0 \phi c^2 (1 - \phi V_g'(\phi r')^2/\phi c_g^2) \tag{31}$$

The implication of Eq. (31) is that the intrinsic total energy $\phi m_0 \phi c^2$ of the test particle in the inclined proper intrinsic space interval $d\phi\rho'$ 'projects' gravitational-relativistic intrinsic total energy $\phi m_0 \phi c^2 \cos^2 \phi\psi_g(\phi r')$ into the projective relativistic intrinsic space interval $d\phi\rho$ along the horizontal in Fig. 9.

Thus what has been written as $\phi m \phi c^2$ within $d\phi\rho$ in Fig. 9 is the same as $\phi m_0 \phi c^2 \cos^2 \phi\psi_g(\phi r')$, from which we have

$$\phi m \phi c^2 = \phi m_0 \phi c^2 \cos^2 \phi\psi_g(\phi r')$$

Hence

$$\phi m = \phi m_0 \cos^2 \phi\psi_g(\phi r') \tag{32}$$

or

$$\phi m = \phi m_0 \left(1 - \frac{\phi V_g'(\phi r')^2}{\phi c_g^2}\right) \tag{33}$$

or

$$\phi m = \phi m_0 \left(1 - \frac{2G\phi M_{0\mathrm{a}}}{\phi r' \phi c_g^2}\right) \tag{34}$$

Eqs. (32) – (34) are alternative forms of the intrinsic mass relations in the context of ϕTGR, which shall be re-derived by an alternative analytical approach in the second part of this paper. The intrinsic mass relation in the context of ϕTGR is a new result not derived in [2].

As mentioned earlier, the gravitational-relativistic intrinsic mass ϕm ($= \phi m_0 \times \phi\gamma_g(\phi r')^{-2}$) in the context of ϕTGR, shall be referred to as gravitational-relativistic intrinsic mass. Indeed every relativistic (or unprimed) parameter on the flat relativistic spacetime (Σ, ct) in Fig. 1, which evolves from the corresponding proper (or classical) parameter on the flat proper spacetime (Σ', ct') in Fig. 11 of [6] in the context of TGR, shall be referred to as the gravitational-relativistic parameter.

The intrinsic gravitational local Lorentz transformation (ϕGLLT) in the alternative forms of systems (14), (19) and (23) and its inverse in the alternative forms

(17), (20) and (24); the validation of intrinsic gravitational local Lorentz invariance (ϕGLLI) (25); the intrinsic gravitational length contraction and intrinsic gravitational time dilation formulae in the alternative forms of Eqs. (27a-b), (28a-b) and (29a-b) and the intrinsic mass relation in the context of ϕTGR in the alternative forms of Eqs. (32) – (34), all derived graphically in this sub-section and the previous one are sufficient results of ϕTGR for now. Other results shall be added from the analytical approach in the second part of this paper.

3 Intrinsic special theory of relativity (ϕSR) and combined ϕSR and ϕTGR on flat intrinsic spacetime in a gravitational field by the graphical approach

The flat four-dimensional relativistic spacetime (Σ, ct) and its underlying flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in Fig. 1 and 2, which evolve in the context of the theory of gravitational relativity (TGR) and intrinsic theory of gravitational relativity (ϕTGR) respectively, constitute the flat spacetime for the special theory of relativity (SR) and flat intrinsic spacetime for the intrinsic special theory of relativity (ϕSR) in a gravitational field of arbitrary strength.

It is the gravitational-relativistic mass m that evolves in the relativistic Euclidean 3-space Σ in the context TGR that undergoes relative motion on the flat relativistic spacetime (Σ, ct) in the context of SR in every gravitational field and it is the gravitational-relativistic intrinsic mass ϕm that evolves in the relativistic intrinsic space $\phi\rho$, given in the alternative forms of Eqs. (32) – (34), that undergoes relative intrinsic motion on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in the context of ϕSR in every gravitational field.

The unprimed (or gravitational-relativistic) intrinsic special theory of relativity (ϕSR), involving the gravitational-relativistic intrinsic mass ϕm in relative intrinsic motion within an unprimed intrinsic local Lorentz frame on flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$, to be developed in this section, is the projection of the primed intrinsic special theory of relativity (ϕSR$'$), involving the intrinsic rest mass ϕm_0 of the particle or object in relative intrinsic motion within the corresponding proper (or primed) intrinsic local Lorentz frame on the curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ in Fig. 1.

The unprimed (or gravitational-relativistic) special theory of relativity (SR), involving the gravitational-relativistic mass m of the particle or object in relative motion within the corresponding local Lorentz frame on the flat four-dimensional relativistic spacetime (Σ, ct) is the outward (or physical) manifestation of ϕSR.

3.1 Derivation of intrinsic local Lorentz transformation (ϕLLT) and its inverse and validating intrinsic local Lorentz invariance (ϕLLI) of ϕSR on the flat relativistic intrinsic spacetime of ϕTGR by the graphical approach

In order to derive combined ϕTGR and ϕSR, we must simply formulate ϕSR on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in terms of the gravitational-relativistic intrinsic mass ϕm of the test particle, which evolved in the context of ϕTGR, derived in sub-section 2.2, as the intrinsic mass that undergoes intrinsic motion relative to the observer in every gravitational field.

Let the gravitational-relativistic intrinsic mass $(\phi m, \phi\varepsilon/\phi c^2)$ of a particle occupy a little relativistic intrinsic metric spacetime $(d\phi\rho, \phi cd\phi t)$ of the flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ of ϕTGR. Let us denote the intrinsic affine spacetime frame attached to $(\phi m, \phi\varepsilon/\phi c^2)$ by $(\phi\tilde{x}, \phi c\phi\tilde{t})$ – this is the particle's intrinsic affine spacetime frame on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in every gravitational field (denoted by $(\phi\tilde{x}', \phi c\phi\tilde{t}')$ on the flat proper intrinsic metric spacetime $(\phi\rho', \phi c\phi t')$ in the absence of relative gravity at the first stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in every gravitational field in [6] and [4]).

As the gravitational-relativistic intrinsic mass $(\phi m, \phi\varepsilon/\phi c^2)$ moves at intrinsic dynamical speed ϕv relative to the observer within an intrinsic local Lorentz frame on flat intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$, it becomes the special-relativistic cum gravitational-relativistic intrinsic mass in the context of combined ϕSR and ϕTGR (or in the context of ϕSR+ϕTGR) on the flat $(\phi\rho, \phi c\phi t)$. The special-relativistic cum gravitational-relativistic intrinsic mass shall be denoted by $(\phi\overline{m}, \phi\overline{\varepsilon}/\phi c^2)$, where $\phi\overline{m} = \phi\gamma_d(\phi v)\phi m$ and $\phi\overline{\varepsilon}/\phi c^2 = \phi\gamma_d(\phi v)\phi\varepsilon/\phi c^2$. The special-relativistic cum gravitational-relativistic intrinsic mass $(\phi\overline{m}, \phi\overline{\varepsilon}/\phi c^2)$ occupies a little intrinsic metric spacetime interval to be denoted by $(d\phi\bar{\rho}, \phi cd\phi\bar{t})$ of the global flat relativistic intrinsic spacetime $(d\phi\rho, \phi cd\phi t)$.

Let us denote the intrinsic affine spacetime frame attached to $(\phi\overline{m}, \phi\overline{\varepsilon}/\phi c^2)$ by $(\phi\bar{\tilde{x}}, \phi c\phi\bar{\tilde{t}})$ – this is the observer's intrinsic affine spacetime frame on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in a gravitational field of arbitrary strength (denoted by $(\phi\tilde{x}, \phi c\phi\tilde{t})$ on flat proper intrinsic spactime $(\phi\rho', \phi c\phi t')$ in the absence of relative gravity at the first stage of evolutions of spacetime/intrinsic spacetime in a gravitational field in [6] and 13).

As developed in [9] and applied in section 2 of [4], the intrinsic motion of the gravitational-relativistic mass $(\phi m, \phi\varepsilon/\phi c^2)$ of the particle at intrinsic dynamical speed ϕv will give rise to the spacetime/intrinsic spacetime geometry of ϕSR in a gravitational field in the two-world picture depicted in Fig. 10.

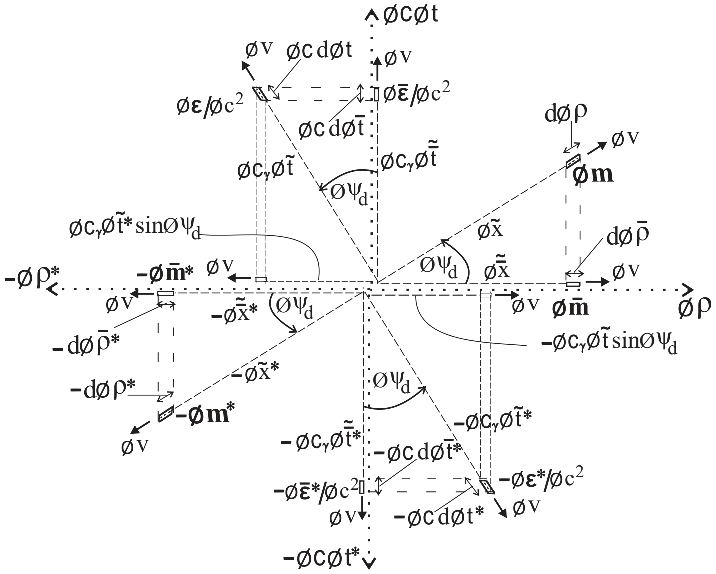

Figure 10:

Fig. 10 is valid relative to symmetry-partner 3-observers (Peter and Peter*) in the relativistic Euclidean 3-spaces Σ and $-\Sigma^*$ in our universe and the negative universe. Fig. 10 is the same as Fig. 8(a) of [9], except that the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ and the intrinsic special-relativistic mass $(\phi m, \phi\varepsilon/\phi c^2)$; $\phi m = \phi\gamma_d(\phi v)\phi m_0$ of the particle in intrinsic motion relative to the observer were not shown in Fig. 8(a) of [9].

Further more, the affine intrinsic space coordinates denoted by $\phi\tilde{x}'$ and $\phi\tilde{x}$ in in the assumed absence of gravitational field [9] are denoted by $\phi\tilde{x}$ and $\phi\tilde{\tilde{x}}$ respectively in a gravitational field in Fig. 10. The affine intrinsic time coordinates denoted by $\phi c\phi\tilde{t}'$ and $\phi c\phi\tilde{t}$ in [9], are denoted by $\phi c_\gamma\phi\tilde{t}$ and $\phi c_\gamma\phi\tilde{\tilde{t}}$ respectively in Fig. 10. Fig. 10 on flat relativistic spacetime (Σ, ct) of TGR in a relative gravitational field, at the second stage of evolutions of spacetime/intrinsic spacetime, corresponds to Fig. 4 of [4] on flat proper spacetime (Σ', ct') in the absence of relative gravity at the first stage.

As first introduced in [8] and discussed further in sub-section 1.3 of this paper, the intrinsic time dimension $\phi c\phi t$ is composed of the affine dynamical component $\phi c_\gamma \phi t$ and the metric static (or gravitational) component $\phi c_g \phi t$. The affine intrinsic dynamical time coordinates must appear in ϕSR, as discussed in sub-section 1.3 of this paper and must be denoted by $\phi c_\gamma \tilde{t}$ and $\phi c_\gamma \phi \tilde{\tilde{t}}$, as done one in Fig. 10. Indeed Fig. 10 is the full two-world form of the partial spacetime/intrinsic spacetime geometry of SR/ϕSR of Fig. 4(c) in a gravitational field field. The concept of time dimension being composed of the affine dynamical and metric gravitational components was unknown in [9], hence the affine intrinsic time coordinates were denoted by $\phi c\phi \tilde{t}'$ and $\phi c\phi \tilde{t}$ in that paper.

The intrinsic affine coordinates are represented by broken lines, while the intrinsic metric space $\phi\rho$ and intrinsic metric time dimension $\phi c\phi t$ are represented by dotted lines as usual in Fig. 10. The little intrinsic metric spacetime interval $(d\phi\rho, \phi c d\phi t)$ contained within the gravitational-relativistic intrinsic mass (ϕm, $\phi\varepsilon/\phi c^2$) is located at the end of the inclined extended affine intrinsic spacetime $(\phi\tilde{x}, \phi c_\gamma \phi \tilde{t})$ of the particle's intrinsic frame and the little intrinsic metric spacetime interval $(d\phi\overline{\rho}, \phi c d\phi \overline{t})$ contained within the gravitational-relativistic cum special-relativistic intrinsic mass $(\phi\overline{m}, \phi\overline{\varepsilon}/\phi c^2)$ is located at the end of the projective extended affine intrinsic spacetime $(\phi\tilde{\tilde{x}}, \phi c_\gamma \phi \tilde{\tilde{t}})$ of the observer's intrinsic frame in Fig. 10. The projective affine intrinsic coordinates $\phi\tilde{\tilde{x}}$ and $\phi c_\gamma \tilde{\tilde{t}}$ lie along the intrinsic metric space $\phi\rho$ and intrinsic metric time dimension $\phi c\phi t$ respectively, but they cannot alter $\phi\rho$ and $\phi c\phi t$.

The global flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ that evolved in the context of ϕTGR is not affected by the intrinsic motion on the flat intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ of the intrinsic mass of a particle relative to an observer. However the little gravitational-relativistic intrinsic metric spacetime interval $(d\phi\rho, \phi c d\phi t)$ contained within the intrinsic gravitational-relativistic mass (ϕm, $\phi\varepsilon/\phi c^2$) of ϕTGR is transformed into little gravitational-relativistic cum special-relativistic intrinsic metric spacetime interval $(d\phi\overline{\rho}, \phi c d\phi \overline{t})$ contained within the intrinsic gravitational-relativistic cum special-relativistic mass $(\phi\overline{m}, \phi\overline{\varepsilon}/\phi c^2)$ that evolves at the top of the intrinsic observer's frame $(\phi\tilde{\tilde{x}}, \phi c_\gamma \phi \tilde{\tilde{t}})$, due to the intrinsic motion of $(\phi m, \phi\varepsilon/\phi c^2)$ relative to the observer (or in the context of combined ϕTGR and ϕSR).

The partial intrinsic local Lorentz transformation (in the context of ϕSR), which can be derived from Fig. 10 with respect to the 3-observer (Peter) in the relativistic

Euclidean 3-space Σ, as done from Fig. 8(a) of [9], is the following

$$\left.\begin{aligned} \phi\tilde{x} &= \phi\tilde{\tilde{x}}\sec\phi\psi_d - \phi c_\gamma\phi\tilde{\tilde{t}}\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \end{aligned}\right\} \tag{35}$$

The complementary diagram to Fig. 10 that co-exists with Fig. 10, which is valid with respect to 1-observers P̃eter and P̃eter* in the time dimensions *ct* and $-ct^*$ of our universe and the negative universe is depicted in Fig. 11.

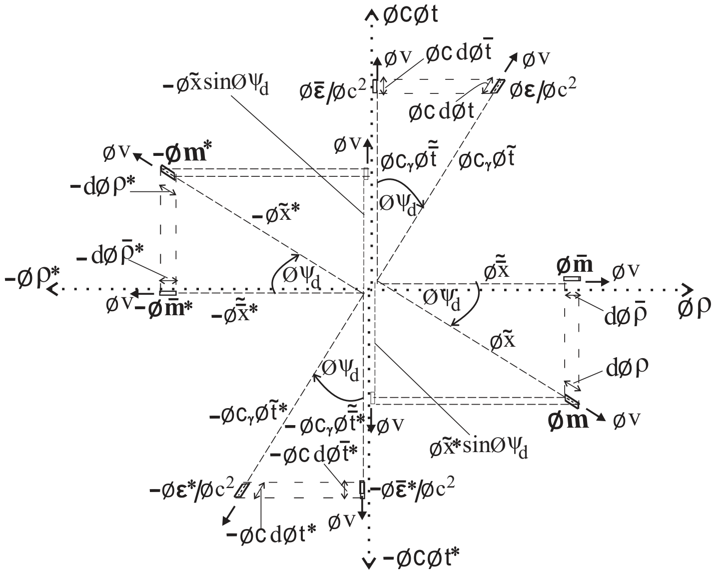

Figure 11:

Fig. 11 is the same as Fig. 8(b) of [9], except for the same differences between Fig. 8(a) of [9] and Fig. 10, discussed above, which also exist between Fig. 8(b) of [9] and Fig. 11.

The partial intrinsic local Lorentz transformation (in the context of ϕSR), which can be derived from Fig. 11 with respect to 1-observer P̃eter in the relativistic metric time dimension *ct*, as done from Fig. 8(b) of [9], is the following

$$\left.\begin{aligned} \phi c_\gamma\phi\tilde{t} &= \phi c_\gamma\phi\tilde{\tilde{t}}\sec\phi\psi_d - \phi\tilde{\tilde{x}}\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 1-\text{observer P̃eter in } ct) \end{aligned}\right\} \tag{36}$$

By collecting Eqs. (35) and (36) we obtain the full intrinsic local Lorentz transformation (ϕLLT) (in the context of ϕSR) as follows

$$\left.\begin{aligned} \phi\tilde{x} &= \phi\tilde{\tilde{x}}\sec\phi\psi_d - \phi c_\gamma \phi\tilde{\tilde{t}}\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \\ \phi c_\gamma \phi\tilde{t} &= \phi c_\gamma \phi\tilde{\tilde{t}}\sec\phi\psi_d - \phi\tilde{\tilde{x}}\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \tag{37}$$

There is an inverse to system (37), which must be derived from the inverses to Figs. 10 and 11. While Figs. 10 and 11 are essentially the same as Figs. 8(a) and 8(b) of [9], as mentioned above, the inverses to Figs. 10 and 11 are essentially the same as the inverses to the inverses of Figs. 8(a) and 8(b) of [9] namely, Figs. 9(a) and 9(b) of that paper. The inverses to Fig. 10 and 11 shall not be drawn here in order to conserve space, while the inverse to system (37) shall just be written as follows

$$\left.\begin{aligned} \phi\tilde{\tilde{x}} &= \phi\tilde{x}\sec\phi\psi_d + \phi c_\gamma \phi\tilde{t}\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{aul in } ct) \\ \phi c_\gamma \phi\tilde{\tilde{t}} &= \phi c_\gamma \phi\tilde{t}\sec\phi\psi_d + \phi\tilde{x}\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 3-\text{observer Paul in } \Sigma) \end{aligned}\right\} \tag{38}$$

The intrinsic local Lorentz transformation (ϕLLT) of system (37) and its inverse of system (38) are described as local because the intrinsic affine coordinates that appear in them and in Figs. 10 and 11 and their inverses (not drawn), are limited in extensions to the interior of the intrinsic local Lorentz frame on the flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$, at an arbitrary 'distance' ϕr from the base of the gravitational-relativistic mass ϕM of the gravitational field source in $\phi\rho$ in Fig. 1.

Now the relative intrinsic angle $\phi\psi_d$ in systems (37) and (38) and in Figs. 10 and 11, is related to the intrinsic dynamical speed ϕv of intrinsic motion relative to the observer within the intrinsic local Lorentz frame as follows

$$\sin\phi\psi_d = \phi v/\phi c_\gamma \equiv \phi\beta_d(\phi v) \tag{39a}$$

$$\cos\phi\psi_d = \sqrt{1 - \phi v^2/\phi c_\gamma^2} \equiv \phi\gamma_d(\phi v) \tag{39b}$$

The formal derivation of Eqs. (39a) and (39b) from systems (37) and (38) has been done in [9]. It must be noted that the dynamical intrinsic speed ϕc_γ of intrinsic electromagnetic waves appears in Eqs. (39a) and (39b) in the context of ϕSR, so that the numerator and the denominator in $\phi v/\phi c_\gamma$ are homogeneous in dynamical intrinsic speeds. It may be recalled that the separation of the speed of 'signals' into the static (or gravitational) speed of gravitational waves c_g and dynamical speed of electromagnetic waves c_γ was first introduced only in [8]. Consequently the intrinsic speed ϕc_γ in Eqs. (39a) and (39b) could only appear as ϕc in the corresponding equations in [9].

By using Eqs. (39a) and (39b), the ϕLLT (37) and its inverse (38) can be written explicitly in terms of the intrinsic speed ϕv respectively as follows

$$\left.\begin{aligned} \phi\tilde{x} &= \phi\gamma_d(\phi v)(\phi\tilde{\tilde{x}} - \phi v\phi\tilde{\tilde{t}}\,); \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \\ \phi\tilde{t} &= \phi\gamma_d(\phi v)(\phi\tilde{\tilde{t}} - \frac{\phi v}{\phi c_\gamma^2}\phi\tilde{\tilde{x}}\,); \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \tag{40}$$

and

$$\left.\begin{aligned} \phi\tilde{\tilde{x}} &= \phi\gamma_d(\phi v)(\phi\tilde{x} + \phi v\phi\tilde{t}\,); \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{aul in } ct) \\ \phi\tilde{\tilde{t}} &= \phi\gamma_d(\phi v)(\phi\tilde{t} + \frac{\phi v}{\phi c_\gamma^2}\phi\tilde{x}); \\ &\quad (\text{w.r.t. } 3-\text{observer Paul in } \Sigma) \end{aligned}\right\} \tag{41}$$

Either system (37) or (38) or the explicit form (40) or (41) leads to intrinsic local Lorentz invariance (ϕLLI) (in the context of ϕSR) on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ that evolved in the context of ϕTGR in every gravitational field,

$$\phi c_\gamma^2\phi\tilde{\tilde{t}}^2 - \phi\tilde{\tilde{x}}^2 = \phi c_\gamma^2\phi\tilde{t}^2 - \phi\tilde{x}^2 \tag{42}$$

This intrinsic local Lorentz invariance is valid within every intrinsic local Lorentz frame on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ of ϕTGR in Fig. 1 in every gravitational field.

The intrinsic affine length contraction and intrinsic affine time dilation formulae in the context of ϕSR on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in a gravitational field, which systems (37) and (38) imply, as derived in [9], are the following

$$\phi\tilde{\tilde{x}} = \phi\tilde{x}\cos\phi\psi_d \tag{43a}$$

$$\phi\tilde{\tilde{t}} = \phi\tilde{t}\sec\phi\psi_d \tag{43b}$$

The alternative forms in terms of the intrinsic speed ϕv of Eqs. (43a) and (43b), which systems (40) and (41) imply are the following

$$\phi\tilde{\tilde{x}} = \phi\gamma_d(\phi v)^{-1}\phi\tilde{x} = (1 - \frac{\phi v^2}{\phi c_\gamma^2})^{1/2} d\phi\tilde{x} \quad (44a)$$

$$\phi\tilde{\tilde{t}} = \phi\gamma_d(\phi v)\phi\tilde{t} = (1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2} d\phi\tilde{t} \quad (44b)$$

The derivations of Eqs. (43a) and (43b) from systems (37) and (38) and the derivations of Eqs. (44a) and (44b) from systems (40) and (41) have been done fully in [9]. It must be mentioned that Eqs. (43a-b) and (44(a-b) are valid with respect to 3-observer (Peter) at rest relative to the observer's frame.

3.1.1 Explicit form of combined ϕTGR and ϕSR

Now as mentioned earlier in this section, the unprimed (or gravitational-relativistic) intrinsic special theory of relativity (ϕSR) within an intrinsic local Lorentz frame on flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ that evolved in the context of ϕTGR in Fig. 1, is the projection of the primed intrinsic special theory of relativity (ϕSR$'$) within the corresponding intrinsic local Lorentz frame on the curved proper intrinsic metric spacetime $(\phi\rho', \phi c\phi t')$ in that figure. Consequently the intrinsic affine coordinates $\phi\tilde{x}$ and $\phi c_\gamma\phi\tilde{t}$ of the intrinsic particle's frame in ϕSR within intrinsic local Lorentz frame on flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$, are projections of the intrinsic affine coordinates $\phi\tilde{x}'$ and $\phi c_\gamma\phi\tilde{t}'$ of intrinsic particle's frame in ϕSR$'$ within the corresponding intrinsic local Lorentz frame on curved $(\phi\rho', \phi c\phi t')$ in Fig. 1.

The transformation of the particle's intrinsic frame $(\phi\tilde{x}', \phi c_\gamma\phi\tilde{t}')$ on the curved $(\phi\rho', \phi c\phi t')$ into particle's intrinsic frame $(\phi\tilde{x}, \phi c_\gamma\phi\tilde{t})$ on the flat $(\phi\rho, \phi c\phi t)$ must be derived in the context of ϕTGR, as the intrinsic gravitational local Lorentz transformation (ϕGLLT) of system (14), (19) or (23) and its inverse of system (17), (20) or (24). That is, we must simply replace $d\phi\rho'$, $\phi cd\phi t'$, $d\phi\rho$ and $\phi cd\phi t$ in those systems by $\phi\tilde{x}'$, $\phi c_\gamma\phi\tilde{t}'$, $\phi\tilde{x}$ and $\phi c_\gamma\phi\tilde{t}$ respectively. The resulting systems shall not be written out explicitly in order to conserve space. However the intrinsic gravitational length contraction and intrinsic gravitational time dilation (in the context of ϕTGR), which they imply are given like Eqs. (27a-b) or (28a-b) or (29a-b) as follows

$$\phi\tilde{x} = \phi\tilde{x}' \cos\phi\psi_g(\phi r') \quad (45a)$$

$$\phi\tilde{t} = \phi\tilde{t}' \sec\phi\psi_g(\phi r') \quad (45b)$$

or

$$\phi\tilde{x} = \phi\gamma_g(\phi r')^{-1}\phi\tilde{x}' = (1 - \frac{\phi V_g(\phi r')^2}{\phi c_g^2})^{1/2}\phi\tilde{x}' \quad (46a)$$

$$\phi\tilde{t} = \phi\gamma_g(\phi r')\phi\tilde{t}' = (1 - \frac{\phi V_g(\phi r')^2}{\phi c_g^2})^{-1/2}\phi\tilde{t}' \quad (46b)$$

or

$$\phi\tilde{x} = \phi\gamma_g(\phi r')^{-1}\phi\tilde{x}' = (1 - \frac{2G\phi M_{0a}}{\phi r'\phi c_g^2})^{1/2}\phi\tilde{x}' \quad (47a)$$

$$\phi\tilde{t} = \phi\gamma_g(\phi r')\phi\tilde{t}' = (1 - \frac{2G\phi M_{0a}}{\phi r'\phi c_g^2})^{-1/2}\phi\tilde{t}' \quad (47b)$$

We shall now incorporate Eqs. (45a-b), (46a-b) and (47a-b) derived int the context of ϕTGR into Eqs. (43a-b),and Eqs. (44a-b) derived in the context of ϕSR to obtain gravitational-relativistic cum special-relativistic intrinsic length contraction and gravitational-relativistic cum special-relativistic intrinsic time dilation in the context of combined ϕTGR and ϕSR in the following alternative forms on flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in a gravitational field of arbitrary strength

$$\phi\tilde{\tilde{x}} = \phi\tilde{x}' \cos\phi\psi_g(\phi r') \cos\phi\psi_d \quad (48a)$$

$$\phi\tilde{\tilde{t}} = \phi\tilde{t}' \sec\phi\psi_g(\phi r') \sec\phi\psi_d \quad (48b)$$

or

$$\phi\tilde{\tilde{x}} = \phi\gamma_g(\phi r')^{-1}\phi\gamma_d(\phi v)^{-1}\phi\tilde{x}' = (1 - \frac{\phi V_g(\phi r')^2}{\phi c_g^2})^{1/2}(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{1/2}\phi\tilde{x}' \quad (49a)$$

$$\phi\tilde{\tilde{t}} = \phi\gamma_g(\phi r')\phi\gamma_d(\phi v)\phi\tilde{t}' = (1 - \frac{\phi V_g(\phi r')^2}{\phi c_g^2})^{-1/2}(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}\phi\tilde{t}' \quad (49b)$$

or

$$\phi\tilde{\tilde{x}} = \phi\gamma_g(\phi r')^{-1}\phi\gamma_d(\phi v)^{-1}\phi\tilde{x}' = (1 - \frac{2G\phi M_{0a}}{\phi r'\phi c_g^2})^{1/2}(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{1/2}\phi\tilde{x}' \quad (50a)$$

$$\begin{aligned} \phi\tilde{\tilde{t}} &= \phi\gamma_g(\phi r')\phi\gamma_d(\phi v)\phi\tilde{t}' \\ &= (1 - \frac{2G\phi M_{0a}}{\phi r'\phi c_g^2})^{-1/2}(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}\phi\tilde{t}' \end{aligned} \qquad (50b)$$

Now the intrinsic local Lorentz transformation (ϕLLT) of system (37) and its inverse of system (38) and their explicit forms in terms of intrinsic dynamical speed ϕv of systems (40) and (41), have been written for the intrinsic affine coordinates $\phi\tilde{x}$, $\phi c_\gamma\phi\tilde{t}$, $\phi\tilde{\tilde{x}}$ and $\phi c_\gamma\phi\tilde{\tilde{t}}$ in Figs. 10 and 11 and their inverses (not drawn). They can equally be written for the little interval of intrinsic metric spacetime $(d\phi\rho, \phi c d\phi t)$ contained within the gravitational-relativistic mass $(\phi m, \phi\varepsilon/\phi c^2)$ and the little interval of intrinsic metric spacetime $(d\phi\bar{\rho}, \phi c d\phi\bar{t})$ contained within the gravitational-relativistic cum special-relativistic mass $(\phi\overline{m}, \phi\overline{\varepsilon}/\phi c^2)$ in respectively as follows

$$\left.\begin{aligned} d\phi\rho &= d\phi\bar{\rho}\sec\phi\psi_d - \phi c d\phi\bar{t}\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \\ \phi c d\phi t &= \phi c d\phi\bar{t}\sec\phi\psi_d - d\phi\bar{\rho}\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \qquad (51)$$

and

$$\left.\begin{aligned} d\phi\bar{\rho} &= d\phi\rho\sec\phi\psi_d + \phi c d\phi t\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{aul in } ct) \\ \phi c d\phi\bar{t} &= \phi c d\phi t\sec\phi\psi_d + d\phi\rho\tan\phi\psi_d; \\ &\quad (\text{w.r.t. } 3-\text{observer Paul in } \Sigma) \end{aligned}\right\} \qquad (52)$$

or

$$\left.\begin{aligned} d\phi\rho &= \phi\gamma_d(\phi v)(d\phi\bar{\rho} - \phi v d\phi\bar{t}\,); \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \\ d\phi t &= \phi\gamma_d(\phi v)(d\phi\bar{t} - \frac{\phi v}{\phi c_\gamma^2}d\phi\bar{\rho}\,); \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \qquad (53)$$

and

$$\left.\begin{aligned} \phi\bar{\rho} &= \phi\gamma_d(\phi v)(d\phi\rho + d\phi v\phi t\,); \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{aul in } ct) \\ d\phi\bar{t} &= \phi\gamma_d(\phi v)(d\phi t + \frac{\phi v}{\phi c_\gamma^2}d\phi\rho); \\ &\quad (\text{w.r.t. } 3-\text{observer Paul in } \Sigma) \end{aligned}\right\} \qquad (54)$$

System (51) or (52) or its explicit form in terms of ϕv (53) or (54) leads to the following invariance

$$\phi c^2 d\phi\bar{t}^2 - d\phi\bar{\rho}^2 = \phi c^2 d\phi t^2 - d\phi\rho^2 \quad (55)$$

This is intrinsic local Lorentz invariance (in the context of ϕSR) within the little intrinsic metric spacetime ($d\phi\rho, \phi c d\phi t$) contained within the intrinsic mass (ϕm, $\phi\varepsilon/\phi c^2$) of the particle in relative intrinsic motion relative to an observer.

The intrinsic length contraction and intrinsic time dilation formulae (43a) and (43b) implied by systems (37) and (38) or their explicit forms in terms of ϕv of Eqs. (44a) and (44b) implied by systems (40) and (41), correspond to the following implied by systems (51) and (52) and systems (53) and (54)

$$d\phi\bar{\rho} = d\phi\rho \cos\phi\psi_d \quad (56a)$$

$$d\phi\bar{t} = d\phi t \sec\phi\psi_d \quad (56b)$$

or

$$d\phi\bar{\rho} = \phi\gamma_d(\phi v)^{-1} d\phi\rho = (1 - \frac{\phi v^2}{\phi c_\gamma^2})^{1/2} d\phi\rho \quad (57a)$$

$$d\phi\bar{t} = \phi\gamma_d(\phi v) d\phi t = (1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2} d\phi t \quad (57b)$$

Equations (56a) and (56b) and Eqs. (57a) and (57b) are alternative form of intrinsic length contraction and intrinsic time dilation of the little intrinsic metric spacetime interval within the intrinsic mass of the particle in intrinsic motion relative to an observer. They are valid with respect to the 3-observer (Peter) at rest relative to the observer's frame as being formulated at present.

The intrinsic length contraction and intrinsic time dilation formulae in terms of intrinsic affine coordinates $\phi\tilde{x}$, $\phi c_\gamma\phi\tilde{t}$, $\phi\tilde{\tilde{x}}$ and $\phi c_\gamma\phi\tilde{\tilde{t}}$ of Eqs. (45a-b), Eqs. (46a-b) and Eqs. (47a-b) in the context of ϕTGR can equally be written in terms of the little intrinsic metric spacetime coordinate intervals $d\phi\rho$, $\phi c d\phi t$, $d\phi\bar{\rho}$ and $\phi c d\phi\bar{t}$ contained within the intrinsic mass of the particle in intrinsic motion relative to the observer. However those equations shall not be written in order to conserve space.

Finally the intrinsic length contraction and intrinsic time dilation formulae in the context of combined ϕTGR and ϕSR of Eqs. (48a-b), (49a-b) and (50a-b) are given terms of the intrinsic metric coordinate intervals $d\phi\rho$, $\phi c d\phi t$, $d\phi\bar{\rho}$ and $\phi c d\phi\bar{t}$ within the particle respectively as follows

$$d\phi\bar{\rho} = d\phi\rho' \cos\phi\psi_g(\phi r')\cos\phi\psi_d \quad (58a)$$
$$d\phi\bar{t} = d\phi t' \sec\phi\psi_g(\phi r')\sec\phi\psi_d \quad (58b)$$

or

$$d\phi\bar{\rho} = \phi\gamma_g(\phi r')^{-1}\phi\gamma_d(\phi v)^{-1}d\phi\rho' = (1 - \frac{\phi V_g(\phi r')^2}{\phi c_g^2})^{1/2}(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{1/2}d\phi\rho' \quad (59a)$$
$$d\phi\bar{t} = \phi\gamma_g(\phi r')\phi\gamma_d(\phi v)d\phi t' = (1 - \frac{\phi V_g(\phi r')^2}{\phi c_g^2})^{-1/2}(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}d\phi t' \quad (59b)$$

or

$$d\phi\bar{\rho} = \phi\gamma_g(\phi r')^{-1}\phi\gamma_d(\phi v)^{-1}d\phi\rho' = (1 - \frac{2G\phi M_{0\mathrm{a}}}{\phi r'\phi c_g^2})^{1/2}(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{1/2}d\phi\rho' \quad (60a)$$
$$d\phi\bar{t} = \phi\gamma_g(\phi r')\phi\gamma_d(\phi v)\phi t' = (1 - \frac{2G\phi M_{0\mathrm{a}}}{\phi r'\phi c_g^2})^{-1/2}(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}\phi t' \quad (60b)$$

Equations (58a-b), (59a-b) and (60a-b) express gravitational-relativistic cum special-relativistic intrinsic length contraction and gravitational-relativistic cum special-relativistic intrinsic time dilation in the context of ϕTGR+ϕSR of the little proper intrinsic metric spacetime intervals $d\phi\rho'$ and $\phi c d\phi t'$ contained within the intrinsic rest mass $(\phi m_0, \phi\varepsilon'/\phi c^2)$ in intrinsic motion at intrinsic dynamical speed ϕv within the proper (or primed) intrinsic local Lorentz frame on the curved proper intrinsic metric spacetime $(\phi\rho', \phi c\phi t')$ relative to the observer in Fig. 1. They are valid relative to the 3-observer (Peter) in the relativistic Euclidean 3-space Σ, who is at rest relative to the observer's frame within the corresponding local Lorentz frame on the flat relativistic spacetime (Σ, ct).

3.1.2 The case of the electron

Now all equations from system (51) through Eqs. (60a-b) have been written for a particle or object with compound rest mass

$$(m_0\,,\,\varepsilon'/c^2) \equiv (m_{0g} \cup m_{0d}\,,\,\varepsilon'_g/c_g^2 \cup \varepsilon'_d/c_\gamma^2),$$

contained in non-zero volume of compound proper metric spacetime

$$(d\Sigma', cdt') \equiv (d\Sigma'_g \cup d\Sigma'_d\, ,\, c_g dt' \cup c_\gamma dt').$$

Consequently such a particle or object has non-zero compound intrinsic rest mass

$$(\phi m_0\, ,\, \phi\varepsilon'/\phi c^2) \equiv (\phi m_{0g} \cup \phi m_{0d}\, ,\, \phi\varepsilon'_g/\phi c_g^2 \cup \phi\varepsilon'_d/\phi c_\gamma^2),$$

contained in a non-zero interval of compound proper intrinsic metric spacetime

$$(d\phi\rho'\, ,\, \phi c d\phi t') \equiv (d\phi\rho'_g \cup d\phi\chi'\, ,\, \phi c_g d\phi t' \cup \phi c_\gamma d\phi t').$$

Consequently the gravitational-relativistic intrinsic mass of the particle or object that evolved in the context of ϕTGR namely,

$$(\phi m\, ,\, \phi\varepsilon/\phi c^2) \equiv (\phi m_g \cup \phi m_d\, ,\, \phi\varepsilon_g/\phi c_g^2 \cup \phi\varepsilon_d/\phi c_\gamma^2),$$

is contained in non-zero interval of compound relativistic intrinsic spacetime of ϕTGR,

$$(d\phi\rho\, ,\, \phi c d\phi t) \equiv (d\phi\rho_g \cup d\phi\chi\, ,\, \phi c_g d\phi t \cup \phi c_\gamma d\phi t)$$

and the gravitational-relativistic cum special-relativistic intrinsic mass,

$$(\phi\overline{m}\, ,\, \phi\bar{\varepsilon}/\phi c^2) \equiv (\phi\overline{m}_g \cup \phi\overline{m}_d\, ,\, \phi\bar{\varepsilon}_g/\phi c_g^2 \cup \phi\bar{\varepsilon}_d/\phi c_\gamma^2),$$

is contained in non-zero interval of gravitational-relativistic cum special-relativistic intrinsic spacetime,

$$(d\phi\overline{\rho}\, ,\, \phi c d\phi\bar{t}) \equiv (d\phi\overline{\rho}_g \cup d\phi\overline{\chi}\, ,\, \phi c_g d\phi\bar{t} \cup \phi c_\gamma d\phi\bar{t}),$$

in Figs. 10 and 11.

On the other hand, let us replace the particle or object with metric compound rest mass $m_0 \equiv m_{0g} \cup m_{0d}$ by the electron with pure affine dynamical rest mass $m_{0\mathrm{e}}$. The rest mass of the electron occupies a spherical volume $d\Sigma'_d$ of radius $r_{0\mathrm{e}}$, of the affine proper dynamical 3-space Σ'_d, which corresponds to a point of zero extension of the metric compound proper Euclidean 3-space Σ'. Consequently the intrinsic rest mass of the electron $(\phi m_{0\mathrm{e}}\, ,\, \phi\varepsilon_{0\mathrm{e}}/\phi c_\gamma^2)$, occupies interval $(d\phi\chi'\, ,\, \phi c_\gamma d\phi t')$ of affine dynamical proper intrinsic spacetime $(\phi\chi'\, ,\, \phi c_\gamma \phi t')$, which corresponds to a point of zero extension in the metric compound proper intrinsic spacetime $(\phi\rho\, ,\, \phi c\phi t')$.

Thus if we replace the particle or object in motion relative to the observer in a gravitational field, considered so far, by the electron, then the little interval of the relativistic intrinsic metric spacetime interval ($d\phi\rho$, $\phi c d\phi t$) containing the metric compound gravitational-relativistic intrinsic mass (ϕm , $\phi\varepsilon/\phi c^2$) and the intrinsic metric spacetime interval ($d\phi\overline{\rho}$, $\phi c d\phi\bar{t}$) containing the metric compound gravitational-relativistic cum special-relativistic intrinsic mass ($\phi\overline{m}$, $\phi\overline{\varepsilon}/\phi c^2$), must be replaced by little interval of pure affine relativistic intrinsic dynamical spacetime coordinate interval ($d\phi\chi$, $\phi c_\gamma d\phi t$) (that evolved in the context of ϕTGR), containing gravitational-relativistic mass of the electron ($\phi m_{\rm e}$, $\phi\varepsilon_{\rm e}/\phi c_\gamma^2$) (that evolved in the context of ϕTGR) and little interval of pure affine relativistic intrinsic dynamical spacetime coordinate interval ($d\phi\overline{\chi}$, $\phi c_\gamma d\phi\bar{t}$) (that evolved in the context of ϕTGR+ϕSR), containing gravitational-relativistic cum special-relativistic intrinsic mass of the electron ($\phi\overline{m}_{\rm e}$, $\phi\overline{\varepsilon}_{\rm e}/\phi c_\gamma^2$) (that evolved in the context of ϕTGR+ϕSR) respectively, in Figs. 10 and 11 and in all equations from system (51) through Eqs. (60a-b). Those equations shall not be written however in order to conserve space.

The unwritten resulting equations obtain for the pure affine dynamical intrinsic spacetime interval ($\phi\chi'$, $\phi c_\gamma d\phi t'$) contained within the intrinsic rest mass of the electron ($\phi m_{0\rm e}$, $\phi\varepsilon_{0\rm e}/\phi c_\gamma^2$), on curved proper intrinsic metric spacetime ($\phi\rho'$, $\phi c\phi t'$), despite the fact that the rest mass of the electron ($m_{0\rm e}$, $\varepsilon_{0\rm e}/c_\gamma^2$) occupies a point of zero extension of the metric compound proper spacetime (Σ', ct') and the intrinsic rest mass of the electron occupies a point of zero extension of the metric compound proper intrinsic spacetime ($\phi\rho'$, $\phi c\phi t'$).

3.2 Graphical approach to the derivation of intrinsic mass relations in the contexts of ϕSR and combined ϕTGR and ϕSR

We recall that like the coordinate 4-vector $\tilde{x}_\lambda = (\tilde{x}_0, \tilde{x}_1, \tilde{x}_2, \tilde{x}_3) = (c_\gamma\tilde{t}, \tilde{x}, \tilde{y}, \tilde{z})$ of SR in the rectangular coordinate system of the Euclidean 3-space Σ, in the flat four-dimensional relativistic metric spacetime (Σ, ct) that evolved in every gravitational field in the context of TGR, the momentum 4-vector p_λ on (Σ, ct) is given in the rectangular coordinate system of the Euclidean 3-space Σ as

$$p_\lambda = (p_0\,,\; p_1\,,\; p_2\,,\; p_3) = (mc_\gamma\,,\; mv'_x\,,\; mv'_y\,,\; mv'_z) \qquad (61)$$

This is the gravitational-relativistic momentum 4-vector that evolved on the flat relativistic spacetime (Σ, ct) in the context of TGR. The velocity $\vec{v}' = v'_x\hat{i} + v'_y\hat{j} + v'_z\hat{k}$ is being assumed to be a non-zero velocity of the gravitational-relativistic mass m of the particle (that evolved in the relativistic Euclidean 3-space Σ in the context of

TGR), relative to its own frame, that is relative to the particle's frame $(c_\gamma\tilde{t}\,,\,\tilde{x}\,,\,\tilde{y}\,,\,\tilde{z})$ on the flat metric spacetime (Σ, ct).

The corresponding gravitational-relativistic cum special-relativistic momentum 4-vector $\overline{p}_\lambda$ that evolved on the flat relativistic spacetime (Σ, ct) in the context of combined TGR and SR is

$$\overline{p}_\lambda = (\overline{p}_0\,,\,\overline{p}_1\,,\,\overline{p}_2\,,\,\overline{p}_3) = (\overline{m}c_\gamma\,,\,\overline{m}v'_x\,,\,\overline{m}v'_y\,,\,\overline{m}v'_z) \qquad (62)$$

where $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ is the velocity of the gravitational-relativistic mass m in the particle's affine frame $(c_\gamma\tilde{t}\,,\,\tilde{x}\,,\,\tilde{y}\,,\tilde{z})$ relative to the observer's affine frame $(c_\gamma\tilde{\tilde{t}}\,,\,\tilde{\tilde{x}}\,,\,\tilde{\tilde{y}}\,,\,\tilde{\tilde{z}})$ on the flat metric spacetime (Σ, ct).

Also corresponding to the intrinsic coordinate 2-vector $\phi\tilde{x}_\lambda = (\phi\tilde{x}_0\,,\,\tilde{x}_1) = (\phi c_\gamma\phi\tilde{t}\,,\,\phi\tilde{x})$ of ϕSR on the flat two-dimensional relativistic intrinsic spacetime $(\phi\rho,$ $\phi c\phi t)$ that evolved in the context of TGR, the intrinsic momentum is a 2-vector ϕp_λ on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ where

$$\phi p_\lambda = (\phi p_0\,,\,\phi p_1) = (\phi m\phi c_\gamma\,,\,\phi m\phi v') \qquad (63)$$

This is the gravitational-relativistic intrinsic momentum 2-vector that evolved in on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in the context of ϕTGR. The intrinsic speed $\phi v'$ is being assumed to be non-zero intrinsic speed of the gravitational-relativistic intrinsic mass ϕm of the particle relative to its own frame, that is, of ϕm relative to the intrinsic particle's frame $(\phi c_\gamma\phi\tilde{t}\,,\,\phi\tilde{x})$ on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ of ϕTGR.

The corresponding gravitational-relativistic cum special-relativistic intrinsic momentum 2-vector $\phi\overline{p}_\lambda$, in the context of combined ϕTGR and ϕSR is

$$\phi\overline{p}_\lambda = (\phi\overline{p}_0\,,\,\phi\overline{p}_1) = (\phi\overline{m}\phi c_\gamma\,,\,\phi\overline{m}\phi v) \qquad (64)$$

where ϕv is the intrinsic speed of the gravitational-relativistic intrinsic mass ϕm in the intrinsic particle's frame $(\phi c_\gamma\phi\tilde{t}\,,\phi\tilde{x})$ relative to the observer's intrinsic frame $(\phi c_\gamma\phi\tilde{\tilde{t}}\,,\,\phi\tilde{\tilde{x}})$ on the flat intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$.

Corresponding to the intrinsic spacetime diagrams of Figs. 10 and 11 in the context of ϕSR , there are intrinsic momentum diagrams, which must be obtained by replacing the affine intrinsic spacetime coordinates $\phi c_\gamma\phi\tilde{t}$ and $\phi\tilde{x}$ of the particle's intrinsic frame by the components $\phi p_0 = \phi m\phi c_\gamma$ and $\phi p_1 = \phi m\phi v'$ respectively of the gravitational-relativistic intrinsic momentum 2-vector ϕp_λ of Eq. (63) and by replacing the affine intrinsic spacetime coordinates $\phi c_\gamma\phi\tilde{\tilde{t}}$ and $\phi\tilde{\tilde{x}}$ of the observer's

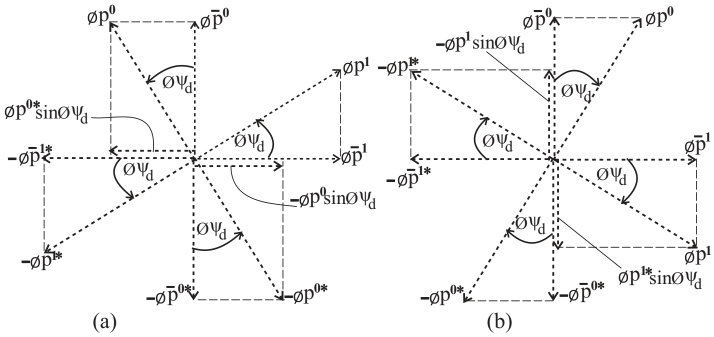

Figure 12:

intrinsic frame by the components $\overline{p}_0 = \phi\overline{m}\phi c_\gamma$ and $\overline{p}_1 = \phi\overline{m}\phi v$ respectively of the gravitational-relativistic cum special-relativistic intrinsic momentum 2-vector $\phi\overline{p}_\lambda$ of Eq. (64). The resulting diagrams are depicted in Figs. 12(a) and 12(b).

Fig. 12(a) is valid with respect to the 3-observer (Peter) in the Euclidean 3-space Σ at rest relative to the observer's frame in our universe and his symmetry-partner (Peter*) in the Euclidean 3-space $-\Sigma^*$ at rest relative to the observer's frame in the negative universe, while Fig. 12(b) is valid with respect to the 1-observer ($\tilde{\text{P}}$eter) the time dimension ct at rest relative to the observer's frame in our universe and his symmetry-partner ($\tilde{\text{P}}$eter*) in the time dimension $-ct^*$ at rest relative to the observer's frame in the negative universe.

The partial intrinsic momentum transformation derivable with respect to 3-observer Peter in Σ in our universe from Fig. 12(a), by following the procedure used to derive partial intrinsic coordinate transformation with respect to Peter in Σ from Fig. 8(a) of [9], is the following

$$\left.\begin{aligned} \phi p^1 \;&=\; \phi\overline{p}^{\,1}\sec\phi\psi_d - \phi\overline{p}^{\,0}\tan\phi\psi_d; \\ &\quad \text{(w.r.t. 3 − observer Peter in } \Sigma) \end{aligned}\right\} \tag{65}$$

And the partial intrinsic momentum transformation derivable with respect to 1-observer $\tilde{\text{P}}$eter in our universe from Fig. 12(b), by following the procedure used to derive partial intrinsic coordinate transformation with respect to $\tilde{\text{P}}$eter in ct from Fig. 8(b) in [9], is the following

$$\left.\begin{aligned} \phi p^0 \;&=\; \phi\overline{p}^{\,0}\sec\phi\psi_d - \phi\overline{p}^{\,1}\tan\phi\psi_d; \\ &\quad \text{(w.r.t. 1 − observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \tag{66}$$

By collecting Eqs. (65) and (66) we obtain the full intrinsic momentum transformation derivable from Figs. 12(a) and 12(b) as follows

$$\left.\begin{aligned} \phi p^1 &= \phi\overline{p}^{\,1} \sec\phi\psi_d - \phi\overline{p}^{\,0} \tan\phi\psi_d; \\ &\quad \text{(w.r.t. 3 − observer Peter in } \Sigma) \\ \phi p^0 &= \phi\overline{p}^{\,0} \sec\phi\psi_d - \phi\overline{p}^{\,1} \tan\phi\psi_d; \\ &\quad \text{(w.r.t. 1 − observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \quad (67)$$

There is an inverse intrinsic momentum transformation, that is, the inverse to system (67), which must be derived from the inverses to Figs. 12(a) and 12(b). The inverse diagrams shall not be drawn however in order to conserve space, while the inverse to system (67) is the following

$$\left.\begin{aligned} \phi\overline{p}^{\,1} &= \phi p^1 \sec\phi\psi_d + \phi p^0 \tan\phi\psi_d; \\ &\quad \text{(w.r.t. 1 − observer } \tilde{\text{P}}\text{eter in } ct) \\ \phi\overline{p}^{\,0} &= \phi p^0 \sec\phi\psi_d + \phi p^1 \tan\phi\psi_d; \\ &\quad \text{(w.r.t. 3 − observer Peter in } \Sigma) \end{aligned}\right\} \quad (68)$$

Either system (67) or (68) leads to the following invariance

$$(\phi\overline{p}^{\,0})^2 - (\phi\overline{p}^1)^2 = (\phi p^0)^2 - (\phi p^1)^2 \quad (69)$$

And by letting $\overline{p}^{\,0} = \phi\overline{m}\phi c_\gamma$; $\phi\overline{p}^{\,1} = \phi\overline{m}\phi v$; $\phi p^0 = \phi m\phi c_\gamma$ and $\phi p^1 = \phi m\phi v'$, along with $\sec\phi\psi_d = \phi\gamma_d(\phi v) = (1 - \phi v^2/\phi c_\gamma^2)^{-1/2}$ and $\tan\phi\psi_d = \phi\gamma_d(\phi v)\phi v/\phi c_\gamma$ in systems (67) and (68) we have

$$\left.\begin{aligned} \phi m\phi v' &= \phi\gamma_d(\phi v)(\phi\overline{m}\phi v - \phi\overline{m}\phi v); \\ &\quad \text{(w.r.t. 3 − observer Peter in } \Sigma) \\ \phi m\phi c_\gamma &= \phi\gamma_d(\phi v)(\phi\overline{m}\phi c_\gamma - \phi\overline{m}\frac{\phi v^2}{\phi c_\gamma}); \\ &\quad \text{(w.r.t. 1 − observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \quad (70)$$

and

$$\left.\begin{aligned} \phi\overline{m}\phi v &= \phi\gamma_d(\phi v)(\phi m\phi v' + \phi m\phi v); \\ &\quad \text{(w.r.t. 1 − observer } \tilde{\text{P}}\text{aul in } ct) \\ \phi\overline{m}\phi c_\gamma &= \phi\gamma_d(\phi v)(\phi m\phi c_\gamma + (\phi m\phi v'\frac{\phi v}{\phi c_\gamma}); \\ &\quad \text{(w.r.t. 3 − observer Paul in } \Sigma) \end{aligned}\right\} \quad (71)$$

And the invariance (69) becomes the following

$$\phi\overline{m}^{\,2}\phi c_\gamma^2 - \phi\overline{m}^{\,2}\phi v^2 = \phi m^2\phi c_\gamma^2 - \phi m\phi v'^2 \tag{72}$$

The vanishing of the right-hand side of the first equation of system (70) implies the vanishing of $\phi p^1 = \phi m\phi v'$. Indeed the gravitational-relativistic intrinsic mass ϕm is at rest relative to its own frame (or particle's intrinsic frame) $(\phi c_\gamma\phi\tilde{t}, \phi\tilde{x})$. Hence it possesses zero intrinsic speed $(\phi v' = 0)$ and zero intrinsic momentum $(\phi m\phi v' = 0)$ relative to its frame. On the other hand, ϕm possesses component of intrinsic momentum $\phi p^0 = \phi m\phi c_\gamma$ along the natural intrinsic geodesic $\phi c_\gamma\phi\tilde{t}$ of its intrinsic frame. Consequently $\phi p^0 = \phi m\phi c_\gamma$ in the particle's frame must be retained. In other words, $\phi v'$ must be allowed to vanish in Eq. (63) to have the correct intrinsic momentum 2-vector in the intrinsic particle frame as $\phi p_\lambda = (\phi m\phi c_\gamma\,, 0)$.

The gravitational-relativistic cum special-relativistic intrinsic mass $\phi\bar{m}$ of the particle actually possesses intrinsic speed ϕv of intrinsic motion relative to its frame (or relative to the observer at rest relative to its frame) $(\phi c_\gamma\phi\tilde{\tilde{t}}\,, \phi\tilde{\tilde{x}})$. Consequently both $\phi\overline{p}^{\,0} = \phi\overline{m}\phi c_\gamma$ and $\phi\overline{p}^1 = \phi\overline{m}\phi v$ must be retained in the observer's frame.

By allowing $\phi m\phi v'$ to vanish, while retaining the other terms in the invariance (72) we have

$$\phi\overline{m}^{\,2}(\phi c_\gamma^2 - \phi v^2) = \phi m\phi c_\gamma^2 \tag{73}$$

Hence

$$\phi\overline{m} = \phi m(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2} = \phi\gamma_d(\phi v)\phi m \tag{74}$$

This is the intrinsic mass relation with respect to 3-observer (Peter) at rest relative to the observer's frame in the context of ϕSR on flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in every gravitational field. It can also be written in terms of the intrinsic angle $\phi\psi_d$ in Figs.10 and 11 and Figs. 12(a) and 12(b) as

$$\phi\overline{m} = \phi\gamma_d(\phi v)\phi m = \phi m \sec\phi\psi_d \tag{75}$$

Now by multiplying through Eq. (73) by ϕc_γ^2, we obtain the following intrinsic energy expression in the context of ϕSR

$$\phi\overline{m}^{\,2}\phi c_\gamma^4 - \phi\overline{m}^{\,2}\phi c_\gamma^2\phi v^2 = \phi m^2\phi c_\gamma^4 \tag{76}$$

The intrinsic special-relativistic kinetic energy $\phi\overline{T}$ is likewise given as follows

$$\begin{aligned}\phi\overline{T} &= \phi\overline{m}\phi c_\gamma^2 - \phi m\phi c_\gamma^2\\ &= \phi\gamma_d(\phi v)\phi m\phi c_\gamma^2 - \phi m\phi c_\gamma^2\\ &= \phi m\phi c_\gamma^2\left((1 - \phi v^2/\phi c_\gamma^2)^{-1/2} - 1\right)\end{aligned} \tag{77}$$

Equations (76) and (77) are valid with respect to the 3-observer (Peter) at rest relative to the observer's affine frame $(c_\gamma\tilde{\tilde{t}}, \tilde{\tilde{x}}, \tilde{\tilde{y}}, \tilde{\tilde{z}})$ on the flat relativistic metric spacetime (Σ, ct).

By incorporating the intrinsic mass re4lation in the context of ϕTGR derived graphically and presented in the alternative forms of Eqs. (32) – (34) in sub-section 2.2 into the intrinsic mass relation in the context of ϕSR in the alternative forms of Eqs. (74) and (75) we obtain the following alternative forms of the intrinsic mass relations in the context of combined ϕTGR and ϕSR

$$\begin{aligned}\phi\overline{m} &= \phi m_0 \cos^2\phi\psi_g(\phi r')\sec\phi\psi_d \\ &= \phi m_0\phi\gamma_g(\phi r')^{-2}\phi\gamma_d(\phi v) &(78)\\ &= \phi m_0(1-\frac{\phi V_g'(\phi r')^2}{\phi c_g^2})(1-\frac{\phi v^2}{\phi c_\gamma^2})^{-1/2} &(79)\\ &= \phi m_0(1-\frac{2G\phi M_{0a}}{\phi r'\phi c_g^2})(1-\frac{\phi v^2}{\phi c_\gamma^2})^{-1/2} &(80)\end{aligned}$$

And the intrinsic special-relativistic kinetic energy in the context of combined ϕTGR and ϕSR is given in the following alternative forms by incorporating Eqs. (32) – (34) into Eq. (77)

$$\begin{aligned}\phi\overline{T} &= \phi m_0\phi c_\gamma^2\cos^2\phi\psi_g(\phi r')[\sec\phi\psi_d - 1] &(81)\\ &= \phi m_0\phi c_\gamma^2(1-\frac{\phi V_g'(\phi r')^2}{\phi c_g^2})[(1-\frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}-1] &(82)\\ &= \phi m_0\phi c_\gamma^2(1-\frac{2G\phi M_{0a}}{\phi r'\phi c_g^2})[(1-\frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}-1] &(83)\end{aligned}$$

Now the gravitational-relativistic cum special-relativistic intrinsic mass $\phi\overline{m}$ in the context of ϕTGR+ϕSR is given in the alternative forms of Eqs. (78), (79) and (80) and the pure gravitational-relativistic intrinsic mass ϕm in the context of ϕTGR is given in the alternative forms of Eqs. (32), (33) and (34). By using Eq. (78) or (79) or (80) and Eq. (32) or (33) or (34) in Eq. (76) we have

$$\phi\tilde{m}_0^2\phi c_\gamma^4 - \phi\tilde{m}_0^2\phi c_\gamma^2\phi v^2 = \phi m_0^2\phi c_\gamma^4 \qquad (84)$$

where $\phi\tilde{m}_0 = \phi\gamma_d(\phi v)\phi m_0$ is the special-relativistic intrinsic mass expression in the context of the primed intrinsic special theory of relativity (ϕSR′) (while retaining the notation in section 2 of [4]), on the within the proper intrinsic local Lorentz frame on the curved proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ at 'distance' $\phi r'$ along the curved $\phi\rho'$ from the base of ϕM_0 in $\phi\rho'$ with respect to an intrinsic 1-observer on the curved $\phi\rho'$ within or outside this intrinsic local Lorentz frame in Fig. 1.

The effect of intrinsic gravitational relativity (ϕTGR) cancels out in the intrinsic gravitational-relativistic cum intrinsic special-relativistic expression (76), thereby making the pure intrinsic special-relativistic expression (84) to remain unchanged with position in a gravitational field.

The intrinsic local Lorentz transformation (ϕLLT) and its inverse in the context of ϕSR on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in the alternative forms of systems (37) and (38) and systems (40) and (41); the intrinsic local Lorentz invariance (ϕLLI) (42) in the context of ϕSR on flat $(\phi\rho, \phi c\phi t)$ in a gravitational field of arbitrary strength; the intrinsic length contraction and intrinsic time dilation formulae on flat $(\phi\rho, \phi c\phi t)$ in the context of ϕSR in a gravitational field of arbitrary strength in the alternative forms of Eqs. (43a-b) and (44a-b) and in the context of combined ϕTGR and ϕSR in the alternative forms of Eqs. (48a-b), (49a-b) and (50a-b); the intrinsic mass relation in the context ϕSR on flat intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in a gravitational field of arbitrary strength in the alternative forms of Eqs. (74) and (75); the intrinsic total energy expression and intrinsic kinetic energy in the context of ϕSR on the flat $(\phi\rho, \phi c\phi t)$ in a gravitational field of arbitrary strength of Eqs. (76) and (77); the intrinsic mass expression in the context of combined ϕTGR and ϕSR of Eqs. (78), (79) or (80) and for intrinsic kinetic energy of Eqs. (81), (82) or (83), are adequate results for the topic of this sub-section. The results have indeed been derived graphically. Other intrinsic parameter relations in the context of combined ϕTGR and ϕSR shall be derived analytically in the second part of this paper.

4 The TGR, SR and combined TGR and SR on flat four-dimensional spacetime as outward manifestations of ϕTGR, ϕSR and combined ϕTGR and ϕSR on flat two-dimensional intrinsic spacetime

4.1 The TGR as outward manifestation on flat spacetime of ϕTGR on flat intrinsic spacetime

The flat four-dimensional relativistic metric spacetime (Σ, ct) that evolved at the combined first and second stages of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field in Fig.1, is the outward manifestation of the underlying flat two-dimensional relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$, where $\phi\rho$ is a one-dimensional isotropic intrinsic space (with no unique orientation in the relativistic Euclidean 3-space Σ) with respect to 3-observers in Σ. The theory of gravitational relativity (TGR) on flat four-dimensional spacetime (Σ, ct) is likewise the outward (or physical) manifestation of the intrinsic theory of gravitational relativity (ϕTGR) on the flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$.

The foregoing implies that the results of TGR on flat spacetime (Σ, ct) can be written directly from the results of ϕTGR on flat intrinsic spacetime $(\phi\rho, \phi c\phi t)$ by simply removing the symbol ϕ from the results of ϕTGR. However in doing this, proper care must be taken of the fact that TGR is a physical four-dimensional theory, while ϕTGR is an intrinsic two-dimensional theory.

Now in converting the two-dimensional intrinsic gravitational local Lorentz transformation (ϕGLLT) and its inverse, written in terms of the intervals $d\phi\rho'$ and $\phi cd\phi t'$ of the two-dimensional proper intrinsic spacetime $(\phi\rho', \phi c\phi t')$ and intervals $d\phi\rho$ and $\phi cd\phi t$ of the flat two-dimensional relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in the alternative forms of systems (14) and (17), systems (19) and (20) and systems (23) and (24), to the four-dimensional gravitational local Lorentz transformation (GLLT) and its inverse, to be written in terms of coordinate intervals dr' , $r'd\theta'$, $r' \sin\theta' d\varphi'$ and cdt' of the flat four-dimensional proper spacetime (Σ', ct') and coordinate intervals dr , $rd\theta$, $r\sin\theta d\varphi$ and cdt of the flat four-dimensional relativistic spacetime (Σ, ct), we must be guided by the following facts:

1. The Euclidean Σ' and Σ are relative spaces (i.e. without hat label unlike the absolute space $\hat{\Sigma}$) and non-isotropic with respect to 3-observers in Σ' or Σ (unlike the absolute space $\hat{\Sigma}$, which is isotropic with respect to 3-observers in the relative Euclidean 3-spaces Σ' or Σ, as properly established in sub-section 4.7 of [6]). Isotropy of a given 3-space relative to an observer in the same or another space, as used here, means that all directions of the given space are identical with respect to the observer, thereby making the isotropic space to contract to a one-dimensional isotropic space (or isotropic dimension) with no unique orientation in the 3-space of the observer. clearly the 3-space Σ is not isotropic with respect to observers in it by this definition.
2. The gravitational velocity $\vec{V}_g'(r')$ is a relative velocity in the context of TGR. This simply means that the magnitude of $\vec{V}_g'(r')$ varies with position of different radial distances r from the center of the gravitational field source in Σ, as discussed in sub-section 2.2.1 of [2].
3. The gravitational velocity $\vec{V}_g'(r')$ is purely radial in every gravitational field, spherically-symmetric or not, as discussed earlier in sub-section 1.2 (see sys-

tem (4)) of this paper, but which is still to be formally established.

The three facts itemized above imply that the ϕGLLT (14) and its inverse (17) must be transformed into GLLT and its inverse on flat four-dimensional spacetime respectively as follows

$$\left.\begin{array}{lcl} dr' & = & dr \sec\psi_g(r') - cdt \tan\psi_g(r'); \\ & & r'd\theta' = rd\theta; \;\; r' \sin\theta' d\varphi' = r\sin\theta d\varphi; \\ & & (\text{w.r.t. } 3-\text{observers in } \Sigma) \\ cdt' & = & cdt \sec\psi_g(r') - dr \tan\psi_g(r') \\ & & (\text{w.r.t. } 1-\text{observers in } ct) \end{array}\right\} \quad (85)$$

and

$$\left.\begin{array}{lcl} dr & = & dr' \sec\psi_g(r') + cdt' \tan\psi_g(r'); \\ & & rd\theta = r'd\theta'; \;\; r\sin\theta d\varphi = r' \sin\theta' d\varphi'; \\ & & (\text{w.r.t. } 1-\text{observers in } ct') \\ cdt & = & cdt' \sec\psi_g(r') + dr' \tan\psi_g(r') \\ & & (\text{w.r.t. } 3-\text{observers in } \Sigma') \end{array}\right\} \quad (86)$$

If the Euclidean 3-spaces Σ' and Σ were absolute and isotropic with respect to 3-observers in them, (like the absolute space $\hat{\Sigma}$ is absolute and isotropic with respect to 3-observers in Σ' or Σ), then the first three equations of system (85) would have been

$$d\Sigma' = d\Sigma \sec\psi_g(r') - cdt \tan\psi_g(r')$$

. And if Σ' and Σ are considered to be relative and non-isotropic with respect to 3-observers in them, which they are, but $\vec{V}'_g(r')$ is not purely radial towards the center of the gravitational field source, then the transformations of $r'd\theta'$ and $r'\sin\theta' d\varphi'$ into $rd\theta$ and $r\sin\theta d\varphi$ would not have taken the trivial forms they take in systems (85) and (86).

The appearance of the angle $\psi_g(r')$ in systems (85) and (86) suggests that the spacetime coordinate intervals dr' and cdt' are inclined at angle $\psi_g(r')$ relative to dr and cdt respectively in a local spacetime geometry, like $d\phi\rho'$ and $\phi cd\phi t'$ are actually inclined at intrinsic angle $\phi\psi_g(\phi r')$ relative to $d\phi\rho$ and $\phi cd\phi t$ respectively in the local intrinsic spacetime geometries of Figs. 7 and 8. The appearance of the angle $\psi_g(r')$ in systems (85) and (86) then suggests further that there are global spacetime geometries in which extended proper radial dimension r' and extended proper time dimension ct' are curved relative to their projective extended straight line relativistic radial dimension r and relativistic time dimension ct respectively, like extended $\phi\rho'$ and $\phi c\phi t'$ are actually curved relative to extended $\phi\rho$ and $\phi c\phi t$ respectively in Figs. 1 and 2.

However local spacetime geometries in which spacetime intervals dr' and cdt' are inclined by angle $\psi_g(r')$ relative to dr and cdt respectively and global spacetime geometries in which extended spacetime dimensions r' and ct' are curved relative to extended r and ct, which system (85) may suggest, is hypothetical; they does not exist in reality. They may be referred to as intrinsic relative rotation of dr' and cdt' relative to dr and cdt and intrinsic curvature of extended r' and ct' relative to extended r and ct. This is what is realized by the actual rotational of intrinsic spacetime intervals $d\phi\rho'$ and $\phi cd\phi t'$ relative to $d\phi\rho$ and $\phi cd\phi t$ by intrinsic angle $\phi\psi_g(\phi r')$ in Figs. 7 and 8 and actual curvature of extended intrinsic spacetime dimensions $\phi\rho'$ and $\phi c\phi t'$ relative to extended $\phi\rho$ and $\phi c\phi t$ in every gravitational field in Figs. 1 and 2.

The outward manifestations of the definition of the intrinsic angle $\phi\psi_g(\phi r')$ in Eqs. (18a) and (18b), obtained by simply removing the symbol ϕ are the following

$$\sin\psi_g(r') = V_g'(r')/c_g \equiv \beta_g(r') \tag{87a}$$

$$\cos\psi_g(r') = \sqrt{1 - V_g'(r')^2/c_g^2} \equiv \gamma_g(r')^{-1} \tag{87b}$$

The outward manifestations on four-dimensional spacetime of systems (19) and (20), which can be obtained by using Eqs. (87a) and (87b) on systems (85) and (86) are the following respectively

$$\left.\begin{aligned} dr' &= \gamma_g(r')(dr - V_g'(r')dt);\\ &\quad r'd\theta' = rd\theta;\;\; r'\sin\theta' d\varphi' = r\sin\theta d\varphi;\\ &\quad (\text{w.r.t. } 3-\text{observers in } \Sigma)\\ dt' &= \gamma_g(r')\left(dt - \frac{V_g'(r')}{c_g^2}dr\right);\\ &\quad (\text{w.r.t. } 1-\text{observers in } ct) \end{aligned}\right\} \tag{88}$$

and

$$\left.\begin{aligned} dr &= \gamma_g(r')(dr' + V_g'(r')dt');\\ &\quad rd\theta = r'd\theta';\;\; r\sin\theta d\varphi = r'\sin\theta' d\varphi';\\ &\quad (\text{w.r.t. } 1-\text{observers in } ct')\\ dt &= \gamma_g(r')\left(dt' + \frac{V_g'(r')}{c_g^2}dr'\right);\\ &\quad (\text{w.r.t. } 3-\text{observers in } \Sigma') \end{aligned}\right\} \tag{89}$$

where $\gamma_g(r')$ is given by Eq. (87b).

The outward manifestations on flat four-dimensional spacetime in the context of TGR of Eqs. (22a) and (22b) in the context of ϕTGR are the following

$$\sin\psi_g(r') = \sqrt{\frac{2GM_{0a}}{r'c_g^2}} \equiv \beta_g(r') \tag{90a}$$

$$\cos\psi_g(r') = \sqrt{1 - \frac{2GM_{0a}}{rc_g^2}} \equiv \gamma_g(r)^{-1} \tag{90b}$$

The outward manifestation on flat four-dimensional spacetime (Σ, ct) of systems (23) and (24) on flat intrinsic spacetime $(\phi\rho, \phi c\phi t)$, which can be obtained by using Eqs. (90a) and (90b) in systems (85) and (86) are the following respectively

$$\left.\begin{aligned} dr' &= \gamma_g(r')\left(dr - \sqrt{\frac{2GM_{0a}}{r'}}dt\right); \\ &\quad r'd\theta' = rd\theta;\ \ r'\sin\theta' d\varphi' = r\sin\theta d\varphi; \\ &\quad (\text{w.r.t. } 3 - \text{observers in } \Sigma) \\ dt' &= \gamma_g(r')\left(dt - \sqrt{\frac{2GM_{0a}}{r'c_g^4}}dr\right); \\ &\quad (\text{w.r.t. } 1 - \text{observers in } ct) \end{aligned}\right\} \tag{91}$$

and

$$\left.\begin{aligned} dr &= \gamma_g(r')\left(dr' + \sqrt{\frac{2GM_{0a}}{r'}}dt'\right); \\ &\quad rd\theta = r'd\theta';\ \ r\sin\theta d\varphi = r'\sin\theta' d\varphi'; \\ &\quad (\text{w.r.t. } 1 - \text{observers in } ct') \\ dt &= \gamma_g(r')\left(dt' + \sqrt{\frac{2GM_{0a}}{r'c_g^4}}dr'\right); \\ &\quad (\text{w.r.t. } 3 - \text{observers in } \Sigma') \end{aligned}\right\} \tag{92}$$

where $\gamma_g(r')$ is given by Eq. (90b).

Systems (85), (88) and (91) are alternative forms of gravitational local Lorentz transformation (GLLT) in the context of TGR and systems (86), (89) and (92) are their inverses. Either the GLLT (85), (88) or (91) or its inverse (86), (89) or (92) leads to gravitational local Lorentz invariance (GLLI) in the context of TGR

$$c^2dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) = c^2dt'^2 - dr'^2 - r'^2(d\theta'^2 + \sin^2\theta' d\varphi'^2) \tag{93}$$

This is the outward manifestation on flat four-dimensional spacetime in the context of TGR of the intrinsic gravitational local Lorentz invariance (ϕGLLI) (25) on flat two-dimensional intrinsic spacetime in the context of ϕTGR.

The validity of Eq. (93) at every point in spacetime in a gravitational field, guarantees formally the flatness everywhere in a gravitational field of the four-dimensional relativistic spacetime (Σ, ct), which evolved in the context of TGR at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field, as illustrated already in the global geometries of Figs. 1 and 2.

The outward manifestations on flat four-dimensional spacetime in the context of TGR of the intrinsic gravitational length contraction and intrinsic gravitational time dilation in the context of ϕTGR, given in the alternative forms of Eqs. (27a-b), (28a-b) and (29a-b) are the following respectively

$$dr = dr' \cos\psi_g(r');\ rd\theta = rd\theta';\ \text{and } r\sin\theta d\varphi = r'\sin\theta' d\varphi' \quad (94a)$$

$$dt = dt' \sec\psi_g(r') \quad (94b)$$

$$dr = \gamma_g(r')^{-1}dr' = (1 - \frac{V'_g(r)^2}{c_g^2})^{1/2}dr';\ rd\theta = rd\theta';$$
$$\text{and } r\sin\theta d\varphi = r'\sin\theta' d\varphi' \quad (95a)$$

$$dt = \gamma_g(r')dt' = (1 - \frac{V'_g(r)^2}{c_g^2})^{-1/2}dt' \quad (95b)$$

and

$$dr = \gamma_g(r')^{-1}dr' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2}dr';\ rd\theta = rd\theta';$$
$$\text{and } r\sin\theta d\varphi = r'\sin\theta' d\varphi' \quad (96a)$$

$$dt = \gamma_g(r')dt' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{-1/2}dt' \quad (96b)$$

Equations (94a-b), (95a-b) and (96a-b) are alternative forms of gravitational length contraction and gravitational time dilation in the context of TGR. It must be noted that the rotation of dr' relative to dr suggested by the first equation of system (94a) and the rotation of cdt' relative to cdt suggested by Eq. (94b) are intrinsic rotations, that is, they are not actual or observable rotations with respect to 3-observers in Σ, as discussed earlier. The non-observable intrinsic rotations are what appear as actual rotations of $d\phi\rho'$ relative to $d\phi\rho$ and $\phi cd\phi t'$ relative to $\phi cd\phi t$ in Fig. 9 of the measurable sub-space of ϕTGR, to which intrinsic gravitational length contraction and intrinsic gravitational time dilation formulae of Eqs. (27a-b), 28(a-b) or (29a-b) in the context of ϕTGR pertain.

Finally the outward manifestations on the flat four-dimensional spacetime in the context of TGR of the intrinsic mass relation in the context of ϕTGR, derived graphically in sub-section 2.2 and presented in the alternative forms of Eqs. (32), (33) and (34), is given in the following alternative forms, obtained by simply removing the symbol ϕ from Eqs. (32) – (34)

$$m = m_0\gamma_g(r')^{-2} = m_0 \cos^2 \psi_g(r') \tag{97}$$

$$= m_0\left(1 - \frac{V_g'(r')^2}{c_g^2}\right) \tag{98}$$

$$= m_0\left(1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2}\right) \tag{99}$$

The gravitational-relativistic mass m that evolved from the rest mass m_0 in the context of TGR shall be identified as the inertial mass and passive gravitational mass in the second part of this paper.

The gravitational local Lorentz transformation (GLLT) in the alternative forms of systems (85), (88) and (91) and its inverse in the alternative forms of systems (86), (89) and (92); the gravitational local Lorentz invariance (GLLI) (93); the gravitational length contraction and gravitational time dilation formulae in the alternative forms of Eqs¿ (94a-b), (95a-b) and (96a-b) and the mass relation in the context of TGR in the alternative forms of Eqs. (97) – (99), are sufficient results of TGR for now. Other results shall be added from the analytical approach to TGR to be developed in the second part of this paper.

Since the results of TGR in this sub-section have been written directly from the results of ϕTGR derived graphically in sub-sections 2.1 and 2.2, we have in effect accomplished the graphical approach to TGR. It must be reiterated however that there are no local spacetime geometries involving relative rotations of physical spacetime intervals and no global spacetime geometries involving curvature of extended physical spacetimes in the context of TGR.

4.2 SR and combined SR and TGR on flat spacetime in a gravitational field of arbitrary strength

Just as done by writing the results of the theory gravitational relativity (TGR) on the flat relativistic spacetime (Σ, ct) in sub-section 4.1 directly from the corresponding results of the intrinsic theory of gravitational relativity (ϕTGR), derived graphically in sub-sections 2.1 and 2.2, the results of SR and combined SR and TGR on the flat relativistic spacetime (Σ, ct), shall be written directly from the results of ϕSR and combined ϕSR and ϕTGR on flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$, derived graphically in sub-sections 3.1 and 3.2. This shall entail the removal of the symbol ϕ from the results of ϕSR and ϕSR+ϕTGR essentially, while taking proper care of the fact that SR and SR+TGR are four-dimensional theories on flat (Σ, ct), while ϕSR and ϕSR+ϕTGR are two-dimensional intrinsic theories on flat $(\phi\rho, \phi c\phi t)$.

The intrinsic local Lorentz transformation (ϕLLT) in the context of ϕSR and its inverse in terms of extended straight line affine intrinsic spacetime coordinates, which are but limited in extensions to the interior of a local Lorentz frame in the external gravitational field, in the alternative forms of systems (37) and (38) and systems (40) and (41), on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ that evolved in the context of ϕTGR in a gravitational field, are made manifest outwardly (or physically) within a local Lorentz from on the flat four-dimensional relativistic metric spacetime (Σ, ct) in the context of SR in the external gravitational field respectively as follows

$$\left.\begin{aligned} \tilde{x} &= \tilde{\tilde{x}}\sec\psi_d - c_\gamma\tilde{\tilde{t}}\tan\psi_d;\ \tilde{y} = \tilde{\tilde{y}};\ \tilde{z} = \tilde{\tilde{z}};\\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma)\\ c_\gamma\tilde{t} &= c_\gamma\tilde{\tilde{t}}\sec\psi_d - \tilde{\tilde{x}}\tan\psi_d;\\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \tag{100}$$

and

$$\left.\begin{aligned} \tilde{\tilde{x}} &= \tilde{x}\sec\psi_d + c_\gamma\tilde{t}\tan\psi_d;\ \tilde{y} = \tilde{\tilde{y}};\ \tilde{z} = \tilde{\tilde{z}};\\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct)\\ c_\gamma\tilde{\tilde{t}} &= c_\gamma\tilde{t}\sec\psi_d + \tilde{x}\tan\psi_d;\\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \end{aligned}\right\} \tag{101}$$

or

$$\left.\begin{aligned}
\tilde{x} &= \gamma_d(v)(\tilde{\tilde{x}} - v\tilde{\tilde{t}}\,);\ \tilde{\tilde{y}} = \tilde{y};\ \tilde{\tilde{z}} = \tilde{z};\\
&\quad \text{(w.r.t. 3 − observer Peter in } \Sigma)\\
\tilde{t} &= \gamma_d(v)(\tilde{\tilde{t}} - \frac{v}{c_\gamma^2}\tilde{\tilde{x}}\,);\\
&\quad \text{(w.r.t. 1 − observer } \tilde{\text{P}}\text{eter in } ct)
\end{aligned}\right\} \tag{102}$$

and

$$\left.\begin{aligned}
\tilde{\tilde{x}} &= \gamma_d(v)(\tilde{x} + v\tilde{t}\,);\ \tilde{\tilde{y}} = \tilde{y};\ \tilde{\tilde{z}} = \tilde{z};\\
&\quad \text{(w.r.t. 1 − observer } \tilde{\text{P}}\text{eter in } ct)\\
\tilde{\tilde{t}} &= \gamma_d(v)(\tilde{t} + \frac{v}{c_\gamma^2}\tilde{x});\\
&\quad \text{(w.r.t. 3 − observer Peter in } \Sigma)
\end{aligned}\right\} \tag{103}$$

where the outward manifestations on the flat spacetime (Σ, ct) of Eqs. (39a) and (39b) on flat intrinsic spacetime $(\phi\rho, \phi c\phi t)$ namely,

$$\sin\psi_d = v/c_\gamma \equiv \beta_d(v) \tag{104a}$$

$$\cos\psi_d = \sqrt{1 - v^2/c_\gamma^2} \equiv \gamma_d(v)^{-1} \tag{104b}$$

have been used in converting systems (100) and (101) to systems (102) and (103). Except for the change of notations of the affine intrinsic coordinates, systems (100) and (101) are the same as systems (28) and (29) of [9] and systems (102) and (103) are the same as systems (33) and (34) of [9].

As discussed in [9], the rotation of the affine spacetime coordinates $\tilde{x}$ and $c_\gamma\tilde{t}$ relative to $\tilde{\tilde{x}}$ and $c_\gamma\tilde{\tilde{t}}$ respectively by angle ψ_d, which system (100) may suggest and the inverse rotation of $\tilde{\tilde{x}}$ and $c_\gamma\tilde{\tilde{t}}$ relative to $\tilde{x}$ and $c_\gamma\tilde{t}$ respectively at negative angle $-\psi_d$, which system (101) may suggest, do not exist in reality, or are fictitious. They may be described as intrinsic rotations, which is formally what the rotations of the intrinsic affine coordinates $\phi\tilde{x}$ and $\phi c_\gamma\phi\tilde{t}$ relative to $\phi\tilde{\tilde{x}}$ and $\phi c_\gamma\phi\tilde{\tilde{t}}$ respectively in Figs. 10 and 11 and the inverse rotation of $\phi\tilde{\tilde{x}}$ and $\phi c_\gamma\phi\tilde{\tilde{t}}$ relative to $\phi\tilde{x}$ and $\phi c_\gamma\phi\tilde{t}$ respectively by negative intrinsic angle $-\phi\psi_d$ in the inverses to Figs. 10 and 11 (not drawn) represent.

The outward (or physical) manifestations on the flat four-dimensional relativistic metric spacetime (Σ, ct) in the context of TGR, of intrinsic gravitational local Lorentz transformation (ϕGLLT) and its inverse on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in the context of intrinsic gravitational theory of relativity (ϕTGR), in the alternative forms of systems (14) and (17), systems (19) and

(20) and systems (23) and (24), take on the alternative forms of systems (85) and (86), systems (88) and (89) and systems (91) and (92) in every gravitational field (spherically-symmetric or not), with respect to 3-observers in Σ. This, as discussed earlier in sub-section 4.1, is due to the fact that the relativistic Euclidean 3-space Σ is not isotropic (i.e. all directions in Σ are not the same) with respect to 3-observers in Σ and the gravitational velocity is purely radial in every gravitational field, as discussed in sub-section 1.2 leading to system (4), to be established formally elsewhere with further development.

The outward manifestations on the flat four-dimensional relativistic spacetime (Σ, ct) in the context of SR in a gravitational field of the intrinsic local Lorentz transformation (ϕLLT) and its inverse on flat two-dimensional intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ in the context of ϕSR in a gravitational field, in the alternative forms of systems (37) and (38) and systems (40) and (41), likewise take the alternative forms of systems (100) and (101) and systems (102) and (103), for every pair of frames of reference in relative motion, as explained hereunder.

Now let the affine spacetime coordinate systems $(c_\gamma\tilde{t}\,,\ \tilde{x}\,, \tilde{y}\,,\ \tilde{z})$ and $(c_\gamma\tilde{\tilde{t}}\,,\ \tilde{\tilde{x}}\,,\ \tilde{\tilde{y}}\,,\ \tilde{\tilde{z}})$ on the flat four-dimensional relativistic metric spacetime (Σ, ct) of TGR be the frames of reference of a particle and the observer respectively within a local Lorentz frame on the flat spacetime (Σ, ct) in a gravitational field of arbitrary strength. The corresponding affine intrinsic spacetime coordinate systems of the intrinsic frames of the particle and observer in the underlying flat two-dimensional relativistic intrinsic metric spacetime $(\phi\rho\,,\ \phi c\phi t)$ of ϕTGR are $(\phi c_\gamma\phi\tilde{t}\,,\ \phi\tilde{x})$ and $(\phi c_\gamma\phi\tilde{\tilde{t}}\,,\ \phi\tilde{\tilde{x}})$ respectively, where $\phi\tilde{x}$ and $\phi\tilde{\tilde{x}}$ are both aligned along the singular one-dimensional universal isotropic relativistic intrinsic space $\phi\rho$.

Let the particle's frame $(c_\gamma\tilde{t}\,,\ \tilde{x}\,,\ \tilde{y}\,,\ \tilde{z})$ be in motion at velocity, $\vec{v}_{OP} = \vec{v}$, relative to the observer's frame $(c_\gamma\tilde{t}\,,\ \tilde{x}\,,\ \tilde{y}\,, \tilde{z})$, which implies that the particle's intrinsic frame $(\phi c\phi t, \phi x)$ is in intrinsic motion at intrinsic speed, $\phi v_{OP} = \phi v$, relative to the observer's intrinsic frame $(c_\gamma\tilde{\tilde{t}}\,,\ \tilde{\tilde{x}}\,,\ \tilde{\tilde{y}}\,,\ \tilde{\tilde{z}})$, where $|\phi v| = |\vec{v}|$. The intrinsic speed ϕv lies along the intrinsic coordinate ϕx, which, itself lies along the singular universal isotropic intrinsic space $\phi\rho$.

The outward (or physical) manifestation of the intrinsic coordinate system $(\phi c_\gamma\phi\tilde{t}, \phi\tilde{x})$ obtained by simply removing the symbol ϕ is $(c_\gamma\tilde{t}\,,\ \tilde{x})$. It then follows that the intrinsic motion at intrinsic speed ϕv along the intrinsic coordinate $\phi\tilde{x}$ of the particle's intrinsic frame, $(\phi c_\gamma\phi\tilde{t}\,,\ \phi\tilde{x})$ relative to the observer's intrinsic frame $(\phi c_\gamma\phi\tilde{\tilde{t}}\,,\ \phi\tilde{\tilde{x}}\,)$ is made manifest outwardly as the motion at speed v along the coordinate $\tilde{x}$ of the partial coordinate system $(c_\gamma\tilde{t}\,,\ \tilde{x})$ of the particle's frame on the flat four-dimensional spacetime $(\Sigma\,,\,ct)$ relative to the observer. When the other coordinates of the particle's frame namely, $\tilde{y}$ and $\tilde{z}$, are incorporated into $(c_\gamma\tilde{t}\,,\,\tilde{x})$ we have a situation where the intrinsic motion at intrinsic speed, $\phi v_{OP} = \phi v$, along the intrinsic coordinate $\phi\tilde{x}$ of the particle's intrinsic frame $(\phi c_\gamma\phi\tilde{t}, \phi\tilde{x})$ relative to the observer's intrinsic frame $(\phi c_\gamma\phi\tilde{\tilde{t}}\,,\,\phi\tilde{\tilde{x}})$ is made manifest outwardly as the motion at velocity, $\vec{v}_{OP} = \vec{v}$, along the coordinate $\tilde{x}$ of the particle's frame $(c_\gamma\tilde{t}\,,\,\tilde{x}\,,\,\tilde{y}\,,\,\tilde{z})$ relative to the observer's frame $(c_\gamma\tilde{\tilde{t}}\,,\,\tilde{\tilde{x}}\,,\,\tilde{\tilde{y}}\,,\,\tilde{\tilde{z}})$ on the flat four-dimensional spacetime $(\Sigma\,,\,ct)$.

Once it is adopted as a convention that the $X-$axis of every frame shall be along the direction of the velocity $\vec{v}$ of relative motion of the frame, then the velocity, $\vec{v}_{OP} = \vec{v}$, is purely along the $X-$axis of every frame. Thus for the present case of a particle's frame $(c_\gamma\tilde{t}\,,\,\tilde{x}\,,\,\tilde{y}\,,\,\tilde{z})$ in motion at velocity, $\vec{v}_{OP} = \vec{v}$, relative to the observer's frame $(c_\gamma\tilde{\tilde{t}}\,,\,\tilde{\tilde{x}}\,,\,\tilde{\tilde{y}}\,,\,\tilde{\tilde{z}})$, $\vec{v}_{OP}$ is purely along the coordinate $\tilde{x}$, which also lies above the isotropic intrinsic space $\phi\rho$. That is,

$$\vec{v}_{OP} = \vec{v} = v_x\hat{i} = v\hat{i} \tag{105a}$$

On the other hand, the observer's frame $(c_\gamma\tilde{\tilde{t}}\,,\,\tilde{\tilde{x}}\,,\,\tilde{\tilde{y}}\,,\,\tilde{\tilde{z}})$ is in motion at velocity, $\vec{v}_{PO} = -\vec{v}$, relative to the particle's frame $(c_\gamma\tilde{t}\,,\,\tilde{x}\,,\,\tilde{y}\,,\,\tilde{z})$ and the observer's intrinsic frame $(\phi c_\gamma\phi\tilde{\tilde{t}}\,,\,\phi\tilde{\tilde{x}})$ is in intrinsic motion at intrinsic speed, $\phi v_{PO} = -\phi v$, relative to the particle's intrinsic frame $(\phi c_\gamma\phi\tilde{t}\,,\,\phi\tilde{x})$ in the above. The intrinsic motion at intrinsic speed $-\phi v$ of the observer's intrinsic frame $(\phi c_\gamma\phi\tilde{\tilde{t}}\,,\,\phi\tilde{\tilde{x}})$ relative to the particle's intrinsic frame $(\phi c_\gamma\phi\tilde{t}, \phi\tilde{x})$, which occurs along the intrinsic space coordinate $\phi\tilde{\tilde{x}}$ that is aligned along the singular universal isotropic intrinsic space $\phi\rho$, is made manifest in the motion of the observer's frame $(c_\gamma\tilde{\tilde{t}}\,,\,\tilde{\tilde{x}}\,,\,\tilde{\tilde{y}}\,,\,\tilde{\tilde{z}})$ at velocity, $\vec{v}_{PO} = -\vec{v}$, along the coordinate $\tilde{\tilde{x}}$ of the observer's frame relative to the particle's frame $(c_\gamma\tilde{t}\,,\,\tilde{x}\,,\,\tilde{y}\,,\,\tilde{z})$ on the flat four-dimensional spacetime $(\Sigma\,,\,ct)$. Again, $\vec{v}_{PO} = -\vec{v}$, is purely along the coordinate $\tilde{\tilde{x}}$ of the observer's frame. That is,

$$\vec{v}_{PO} = -\vec{v} = -v_{\tilde{\tilde{x}}}\hat{i} = -v\hat{i} \tag{105b}$$

The velocities $\vec{v}_{PO}$ and $\vec{v}_{OP}$ lie along the same line but are oppositely directed in the Euclidean 3-space Σ. It then follows that the coordinates $\tilde{x}$ of the particle's frame, along which the velocity $\vec{v}_{OP}$ lies, and the corresponding coordinate $\tilde{\tilde{x}}$ of the observer's frame along which the velocity $\vec{v}_{PO}$ lies, are collinear in Σ.

The conclusion then is that although the coordinate systems $(c_\gamma\tilde{t}\,,\,\tilde{x}\,,\,\tilde{y}\,,\,\tilde{z})$ and $(c_\gamma\tilde{\tilde{t}}\,,\,\tilde{\tilde{x}}\,,\,\tilde{\tilde{y}}\,,\,\tilde{\tilde{z}})$ of two frames in relative motion can be orientated relative to each other in space in an uncountable number of ways, the relative orientation of the coordinate systems in which the coordinates $\tilde{x}$ and $\tilde{\tilde{x}}$ of the two frames are collinear along the direction of their relative velocity is the naturally prescribed orientation for deriving the Lorentz transformation (LT) and its inverse for the two frames. However the corresponding coordinates of the two frames that are collinear with the velocity of their relative motion may be taken as $\tilde{y}$ and $\tilde{\tilde{y}}$ or $\tilde{z}$ and $\tilde{\tilde{z}}$; it is just a matter of convention that they shall be taken as $\tilde{x}$ and $\tilde{\tilde{x}}$. A corollary of this conclusion is that the LT and its inverse take on the forms of systems (100) and (101) or systems (102) and (103) for every pair of frames in relative motion.

The natural orientation of the coordinate systems of two frames in relative motion for deriving the LT and its inverse isolated above is natural because it takes into consideration the fact that the intrinsic motion at intrinsic speed, $\phi v_{OP} = \phi v$, of the intrinsic frame $(\phi c_\gamma \phi\tilde{t}\,,\ \phi\tilde{x})$ relative to the intrinsic frame $(\phi c_\gamma \phi\tilde{\tilde{t}}, \phi\tilde{\tilde{x}}\,)$ and the converse intrinsic motion at intrinsic speed, $\phi v_{PO} = -\phi v$, of the intrinsic frame $(\phi c_\gamma \phi\tilde{\tilde{t}}, \phi\tilde{\tilde{x}}\,)$ relative to the intrinsic frame $(\phi c_\gamma \phi\tilde{\tilde{t}}, \phi\tilde{\tilde{x}}\,)$ take place along the intrinsic coordinates $\phi\tilde{x}$ and $\phi\tilde{\tilde{x}}$ respectively, which are both aligned along the singular universal isotropic intrinsic space $\phi\rho$. The intrinsic coordinates $\phi\tilde{x}$ and $\phi\tilde{\tilde{x}}$ and the intrinsic speeds ϕv_{OP} and ϕv_{PO}, which are aligned along the singular straight line universal isotropic intrinsic space $\phi\rho$ are then made manifest in coordinate $\tilde{x}$ and $\tilde{\tilde{x}}$ and velocities $\vec{v}_{OP}$ and $\vec{v}_{PO}$ that lie along a straight line along the collinear coordinates $\tilde{x}$ and $\tilde{\tilde{x}}$ in Σ.

An arbitrary orientation in space of the coordinates systems $(c_\gamma\tilde{t}\,,\ \tilde{x}\,,\ \tilde{y}\,,\ \tilde{z})$ and $(c_\gamma\tilde{\tilde{t}}\,,\ \tilde{\tilde{x}}\,,\ \tilde{\tilde{y}}\,,\ \tilde{\tilde{z}}\,)$ of two frames in relative motion at a velocity $\vec{v}$ on the flat four-dimensional spacetime $(\Sigma\,,\ ct)$, for the purpose of deriving the LT and its inverse, in which the coordinates $\tilde{x}$ and $\tilde{\tilde{x}}$ and the velocity $\vec{v}$ are not collinear, does not put into consideration the relative intrinsic motion at intrinsic speed ϕv of the intrinsic frames $(\phi c_\gamma\phi\tilde{t}\,,\ \phi\tilde{x})$ and $(\phi c_\gamma\phi\tilde{\tilde{t}}\,,\ \phi\tilde{\tilde{x}}\,)$ in the underlying flat two-dimensional intrinsic spacetime $(\phi\rho\,,\ \phi c\phi t)$. Whereas it is the relative intrinsic motion of the intrinsic frames in intrinsic spacetime that determines the observed relative motion of the frames in spacetime. Such arbitrary orientation of coordinate systems of two fames in relative motion is impossible. On the other hand, the collinearity of the coordinates $\tilde{x}$ and $\tilde{\tilde{x}}$, which is inherent in the Lorentz transformation and its inverse in the familiar forms of systems (102) and (103), is usually considered to be an assumption in the special theory of relativity.

It is crucial to note that ϕSR involves extended intrinsic affine spacetime coordinates $\phi\tilde{\tilde{x}}$ and $\phi c_\gamma\phi\tilde{\tilde{t}}$ and consequently SR involves extended four-dimensional affine spacetime coordinates $\tilde{\tilde{x}}, \tilde{\tilde{y}}, \tilde{\tilde{z}}$ and $c_\gamma\tilde{\tilde{t}}$, (which are but limited to interiors of a local

Lorentz frames in a gravitational field for motion within a gravitational field). The only metric intrinsic spacetme involved in ϕSR is the little intrinsic metric spacetime interval $d\phi\overline{\rho}$ and $\phi c\phi\bar{t}$ contained within the gravitational-relativistic cum special-relativistic intrinsic mass $\phi\overline{m}$ of the test particle in relative motion. Consequently the only metric spacetime involved in SR is the little volume $d\overline{\Sigma}$ contained within $\overline{m}$ moving in Σ and little interval of time dimension $cd\bar{t}$ contained within the the symmetry-partner mass $\overline{\varepsilon}/c^2$ moving along the time dimension ct, of the particle in relative motion. An implication of this is that the motion of a test particle in the extended flat relativistic metric spacetime $(\Sigma\,,\, ct)$ that evolves in the context of TGR can neither alter the Lorentzian metric nor the label of $(\Sigma\,,\, ct)$. In other words, the extended flat spacetime $(\Sigma\,,\, ct)$ of TGR does not transform into another extended flat spacetime $(\overline{\Sigma}\,,\, c\bar{t})$ due to the relative motion of a particle or body in $(\Sigma\,,\, ct)$ in the context of SR.

Similarly it is due to the fact that the isotropic relativistic intrinsic space $\phi\rho$ and the intrinsic gravitational speed $\phi V_g'(\phi r')$ that lies along $\phi\rho$ is naturally orientated along radial directions from the centroid of every gravitational field source (spherical or non-spherical) in the relativistic Euclidean 3-space Σ only that the outward manifestation in Σ of $\phi V_g'(\phi r')$ namely, the gravitational velocity $\vec{V}_g'(r')$ is naturally along radial directions from the centroid of every gravitational field source (spherical or non-spherical) in Σ only, as shall be taken up fully elsewhere with further development. Consequently $\vec{V}_g'(r')$ is radially towards the centroid of every gravitational field source, spherically-symmetric or not, as stated by system (4), and GLLT and its inverse can take on the forms of systems (85) and (86) or systems (88) and (89) or systems (91) and (92), in which the coordinates intervals $r'd\theta'$ and $r'\sin\theta' d\varphi'$ transform into the coordinate intervals $rd\theta$ and $r\sin\theta d\varphi$ trivially only in every gravitational field (spherical or non-spherical).

After the long but important digression to establish the fact that the local Lorentz transformation (LLT) and its inverse of SR can take on the forms of systems (100) and (101) or systems (102) and (103) only within or outside a gravitational field, for every pair of frames of reference in relative motion, and the gravitational local Lorentz transformation and its inverse can take on the forms of systems (85) and (86) or systems (88) and (89) or systems (91) and (92) in every gravitational field (spherically symmetric or not), let us return to the subject of this sub-section, which is writing the results of SR and combined SR and TGR on the flat four-dimensional spacetime (Σ, ct) from the corresponding results of ϕSR and combined ϕSR and ϕTGR on flat intrinsic spacetime $(\phi\rho, \phi c\phi t)$.

Either system (100) or (101) or the explicit form in terms of the speed v (102) or (103) leads to local Lorentz invariance (LLI) (of SR) on the flat relativistic spacetime (Σ, ct) of TGR in a gravitational field. That is,

$$c_\gamma^2 \tilde{\tilde{t}}^2 - \tilde{\tilde{x}}^2 - \tilde{\tilde{y}}^2 - \tilde{\tilde{z}}^2 = c_\gamma^2 \tilde{t}^2 - \tilde{x}^2 - \tilde{y}^2 - \tilde{z}^2 \tag{106}$$

This is the outward manifestations on the flat (Σ, ct) in the context of SR of the intrinsic local Lorentz invariance (ϕLLI) (of ϕSR) (42) on flat $(\phi\rho, \phi c\phi t)$.

The Lorentz transformation (LLT) and its inverse of systems (100) and (101) or system (102) and (103) and the LLI (106) they imply, obtain within every local Lorentz frame in every gravitational field (spherically symmetric or not). The LLI has thus been validated on the flat relativistic spacetime (Σ, ct) that evolved in the context of TGR in every gravitational field, as shall also be re-done purely analytically in the second part of this paper. It may be recalled that LLI remains an assumption (without theoretical validation) but with abundant experimental support in the general theory of relativity (GR) [10, 11, etc].

The intrinsic special-relativistic length contraction and intrinsic special-relativistic time dilation formulae in the alternative forms of Eqs. (43a) and (43b) and Eqs. (44a) and (44b) in the context of ϕSR on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ of ϕTGR in a gravitational field, are made manifest outwardly (or physically) in special-relativistic length contraction and special-relativistic time dilation formulae in the context of SR on the flat relativistic spacetime (Σ, ct) of TGR in every gravitational field respectively as follows

$$\tilde{\tilde{x}} = \tilde{x}\cos\psi_d;\ \tilde{\tilde{y}} = \tilde{y};\ \text{and}\ \tilde{\tilde{z}} = \tilde{z} \tag{107a}$$
$$\tilde{\tilde{t}} = \tilde{t}\sec\psi_d \tag{107b}$$

or

$$\tilde{\tilde{x}} = \gamma_d(v)^{-1}\tilde{x} = (1 - v^2/c_\gamma^2)^{1/2}\tilde{x};\tilde{\tilde{y}} = \tilde{y};\ \text{and}\ \tilde{\tilde{z}} = \tilde{z} \tag{108a}$$
$$\tilde{\tilde{t}} = \gamma_d(v)\tilde{t} = (1 - v^2/c_\gamma^2)^{-1/2}\tilde{t} \tag{108b}$$

The intrinsic gravitational-relativistic cum special-relativistic length contraction and intrinsic gravitational-relativistic cum special-relativistic time dilation formulae on the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in the context of combined ϕTGR and ϕSR, in the alternative forms of Eqs. (48a-b) and (50a-b), are likewise made manifest outwardly on the flat four-dimensional spacetime (Σ, ct) of TGR as gravitational-relativistic cum special-relativistic length contraction and gravitational-relativistic cum special-relativistic time dilation formulae in the context of combined TGR and SR respectively as follows

$$\tilde{\tilde{x}} = \tilde{x}' \cos\psi_g(r') \cos\psi_d; \;\; \tilde{\tilde{y}} = \tilde{y}'; \;\; \tilde{\tilde{z}} = \tilde{z}' \quad (109a)$$

$$\tilde{\tilde{t}} = \tilde{t}' \sec\psi_g(r') \sec\psi_d \quad (109b)$$

$$\tilde{\tilde{x}} = \gamma_g(r')^{-1}\gamma_d(v)^{-1}\tilde{x}' = (1 - \frac{V_g'(r')^2}{c_g^2})^{1/2}(1 - \frac{v^2}{c_\gamma^2})^{1/2}\tilde{x}';$$

$$\tilde{\tilde{y}} = \tilde{y}'; \;\; \tilde{\tilde{z}} = \tilde{z}' \quad (110a)$$

$$\tilde{\tilde{t}} = \gamma_g(r')\gamma_d(v)\tilde{t}' = (1 - \frac{V_g'(r')^2}{c_g^2})^{-1/2}(1 - \frac{v^2}{c_\gamma^2})^{-1/2}\tilde{t}' \quad (110b)$$

or

$$\tilde{\tilde{x}} = \gamma_g(r')^{-1}\gamma_d(v)^{-1}\tilde{x}' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2}(1 - \frac{v^2}{c_\gamma^2})^{1/2}\tilde{x}';$$

$$\tilde{\tilde{y}} = \tilde{y}'; \;\; \tilde{\tilde{z}} = \tilde{z}' \quad (111a)$$

$$\tilde{\tilde{t}} = \gamma_g(r')\gamma_d(v)\tilde{t}' = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{-1/2}(1 - \frac{v^2}{c_\gamma^2})^{-1/2}\tilde{t}' \quad (111b)$$

The affine spacetime coordinates with prime label $c_\gamma\tilde{t}'$, $\tilde{x}'$, $\tilde{y}'$ and $\tilde{z}'$ are those of the particle's frame in the context of the primed special theory of relativity (SR′), involving the motion of the rest mass m_0 of the particle relative to the observer, within a local Lorentz frame at radial distance r' from the center of the rest mass M_0 of the gravitational field source, in the proper Euclidean 3-space Σ' of the flat proper (or classical) spacetime (Σ', ct') (in Fig. 11 of [6]), which evolved in the context of absolute intrinsic gravity/absolute gravity (ϕAG/AG) – assuming relative gravity was still absent – at the first stage of evolutions of spacetime/intrinsic spactime and parameters/intrinsic parameters in a gravitational field. The coordinates $c_\gamma\tilde{\tilde{t}}$, $\tilde{\tilde{x}}$, $\tilde{\tilde{y}}$ and $\tilde{\tilde{z}}$ are those of the observer's frame in the context of the unprimed special theory of relativity (SR), involving the motion of the gravitational-relativistic mass m of the particle relative to the observer, within a local Lorentz frame at radial distance r from the center of the gravitational-relativistic mass M of the gravitational field source, in the relativistic Euclidean 3-space Σ of the flat relativistic spacetime (Σ, ct) of TGR in Fig. 1, which evolved at the second stage of evolutions of spacetime/intrinsic spactime and parameters/intrinsic parameters in a gravitational field.

However while the resultant time dilation formula in the context of combined TGR and SR of Eq. (109b), (110b) or (111b) is valid for an arbitrary orientation of the coordinates of 3-space $\tilde{\tilde{x}}$, $\tilde{\tilde{y}}$ and $\tilde{\tilde{z}}$ of the observer's frame relative to a radial direction from the center of the mass M of the gravitational field source in Σ, within a local Lorentz frame, the resultant length contraction formula (109a), (110a) or (111a) is valid for the particular orientation of the spatial coordinates in which $\tilde{\tilde{x}}$ along which motion of the particle relative to the observer occurs, lies along a radial direction from the center of M.

In a situation where the coordinate $\tilde{\tilde{x}}$ along which the motion of the particle relative to the observer occurs does not lie along a radial direction from the center of the gravitational field source M in Σ, on the other hand, the length contraction formula (109a), (110a) or (111a) must be modified appropriately. If, for instance, the coordinates $\tilde{\tilde{x}}$ and $\tilde{\tilde{y}}$ are orientated perpendicular to a radial direction from the center of M, while the coordinate $\tilde{\tilde{z}}$ lies along a radial direction from the center of M within a local Lorentz frame for a give moment, then the proper (or classical) coordinate $\tilde{x}'$ will suffer special-relativistic contraction solely, the proper (or classical) coordinate $\tilde{z}'$ will suffer gravitational-relativistic contraction solely, while the proper (or classical) coordinate $\tilde{y}'$ will suffer no contraction for that moment. Then system (109a), (110a) or (111a) must be modified accordingly for this situation. For instance system (111a) must be modified as follows

$$\tilde{\tilde{x}} = (1 - \frac{v^2}{c_\gamma^2})^{1/2}\tilde{x}'; \;\; \tilde{\tilde{y}} = \tilde{y}'; \;\; \tilde{\tilde{z}} = (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2}\tilde{z}' \tag{112}$$

while Eq. (111b) remains unchanged.

The outward manifestations in the context of SR on the flat four-dimensional gravitational-relativistic spacetime (Σ, ct), of the intrinsic local Lorentz transformation and its inverse in terms of the little gravitational-relativistic intrinsic metric spacetime interval $(d\phi\rho, \phi cd\phi t)$ contained within the gravitational-relativistic intrinsic mass $(\phi m, \phi\varepsilon/\phi c^2)$ of the particle and the little gravitational-relativistic cum special-relativistic intrinsic metric spacetime interval $(d\phi\overline{\rho}, \phi cd\phi\overline{t})$ contained within the gravitational-relativistic cum special-relativistic mass intrinsic $(\phi\overline{m}, \phi\overline{\varepsilon}/\phi c^2)$ of the particle, in the alternative forms of systems (51) and (52) and systems (53) and

(54) in the context of ϕSR are the following

$$\left.\begin{aligned} dx &= d\bar{x}\sec\psi_d - cd\bar{t}\tan\psi_d;\ dy = d\bar{y};\ dz = d\bar{z}; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \\ cdt &= cd\bar{t}\sec\psi_d - d\bar{x}\tan\psi_d; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \tag{113}$$

and

$$\left.\begin{aligned} d\bar{x} &= dx\sec\psi_d + cdt\tan\psi_d;\ d\bar{y} = dy;\ d\bar{z} = dz; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \\ cd\bar{t} &= cdt\sec\psi_d + dx\tan\psi_d; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \end{aligned}\right\} \tag{114}$$

or

$$\left.\begin{aligned} dx &= \gamma_d(v)(d\bar{x} - vd\bar{t});\ d\bar{y} = dy;\ d\bar{z} = dz; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \\ dt &= \gamma_d(v)(d\bar{t} - \frac{v}{c_\gamma^2}d\bar{x}); \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \tag{115}$$

and

$$\left.\begin{aligned} d\bar{x} &= \gamma_d(v)(dx + vdt);\ d\bar{y} = dy;\ d\bar{z} = dz; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \\ d\bar{t} &= \gamma_d(v)(dt + \frac{v}{c_\gamma^2}dx); \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \end{aligned}\right\} \tag{116}$$

The metric spacetime coordinate intervals dx, dy, dz and cdt in systems (113) – (116) are the dimensions of the gravitational-relativistic mass (m, ε/c^2) of the particle that evolved on the flat relativistic spacetime (Σ, ct) in the context of TGR, while $d\bar{x}$, $d\bar{y}$, $d\bar{x}$ and $cd\bar{t}$ are the dimensions of the gravitational-relativistic cum special-relativistic mass ($\overline{m}$, $\overline{\varepsilon}/c^2$) that evolved on (Σ, ct) in the context of combined TGR and SR.

The special-relativistic length contraction and special-relativistic time dilation formulae implied by systems (113) and (114) and systems (115) and (116), are the outward manifestations of Eqs. (56a-b) and Eqs. (57a-b) in the context of ϕSR, given as follows

$$d\bar{x} = dx\cos\psi_d;\ d\bar{y} = dy;\ d\bar{z} = dz; \tag{117a}$$

$$d\bar{t} = dt\sec\psi_d \tag{117b}$$

and

$$d\bar{x} = \gamma_d(v)^{-1}dx; = (1 - \frac{v^2}{c_\gamma^2})^{1/2}dx; \;\; d\bar{y} = dy; \;\; d\bar{z} = dz; \qquad (118a)$$

$$d\bar{t} = \gamma_d(v)dt; = (1 - \frac{v^2}{c_\gamma^2})^{-1/2}dt \qquad (118b)$$

Only the dimension dx of the particle (or object), a box, say, along which its motion relative to the observer occurs, suffers special-relativistic contraction relative to the observer according to system (117a) or (118a).

The intrinsic special-relativistic cum gravitational-relativistic length contraction and special-relativistic cum gravitational-relativistic time dilation of the little proper intrinsic metric spacetime interval ($d\phi\bar{\rho}$, $\phi c d\phi\bar{t}$) contained within the special-relativistic cum gravitational-relativistic intrinsic mass ($\phi\overline{m}$, $\phi\bar{\varepsilon}/\phi c^2$) of the particle or object on the flat relativistic intrinsic spacetime ($\phi\rho$, $\phi c\phi t$), in the context of combined ϕTGR and ϕSR, given in the alternative forms of Eqs. (58a-b), (59a-b) and (60a-b), are likewise made manifest on the flat four-dimensional relativistic spacetime (Σ, ct) in the context of combined TGR and SR respectively as follows

$$d\bar{x} = dx' \cos\psi_g(r')\cos\psi_d; \;\; d\bar{y} = dy'; \;\; d\bar{z} = dz'; \qquad (119a)$$

$$d\bar{t} = dt' \sec\psi_g(r')\sec\psi_d \qquad (119b)$$

or

$$\begin{aligned} d\bar{x} &= \gamma_g(r')^{-1}\gamma_d(v)^{-1}dx'; \\ &= (1 - \frac{V_g'(r')^2}{c_g^2})^{1/2}(1 - \frac{v^2}{c_\gamma^2})^{1/2}dx'; \;\; d\bar{y} = dy'; \;\; d\bar{z} = dz'; \qquad (120a) \\ d\bar{t} &= \gamma_g(r')\gamma_d(v)dt'; \\ &= (1 - \frac{V_g'(r')^2}{c_g^2})^{-1/2}(1 - \frac{v^2}{c_\gamma^2})^{-1/2}dt' \qquad (120b) \end{aligned}$$

or

$$\begin{aligned} d\bar{x} &= \gamma_g(r')^{-1}\gamma_d(v)^{-1}dx'; \\ &= (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2}(1 - \frac{v^2}{c_\gamma^2})^{1/2}dx'; \;\; d\bar{y} = dy'; \;\; d\bar{z} = dz'; \qquad (121a) \\ d\bar{t} &= \gamma_g(r')\gamma_d(v)dt'; \\ &= (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})^{-1/2}(1 - \frac{v^2}{c_\gamma^2})^{-1/2}dt' \qquad (121b) \end{aligned}$$

Again the length contraction formulae of the dimensions of the particle or object of system (119a), (120a) or (121a) is valid in a situation where its dimension $d\overline{x}$, along which its motion relative to the observer occurs within a local Lorentz frame, is orientated along a radial direction from the center of the mass M of the gravitational field source in Σ. Otherwise systems (119a), (120a) and (121a) must be modified appropriately.

The intrinsic mass relation (74) or (75), the intrinsic total energy expression (76) and the intrinsic kinetic energy relation (77), derived graphically in the context of ϕSR on the flat relativistic intrinsic spacetime ($\phi\rho$, $\phi c\phi t$) in a gravitational field earlier, are made manifest in mass relation, total energy expression and kinetic energy relation on the flat relativistic spacetime (Σ, ct) in the context of SR in a gravitational field respectively as follows

$$\overline{m} = \gamma_d(v)m = m \sec\psi_d = m(1 - \frac{v^2}{c_\gamma^2})^{-1/2}, \tag{122}$$

$$\overline{m}^2 c_\gamma^4 - \overline{m}^2 c_\gamma^2 v^2 = m^2 c_\gamma^4 \tag{123}$$

and

$$\begin{aligned} \overline{T} &= mc_\gamma^2(\gamma_d(v) - 1) \\ &= mc_\gamma^2\Big((1 - v^2/c_\gamma^2)^{-1/2} - 1\Big) \end{aligned} \tag{124}$$

The intrinsic mass relation in the context of combined ϕTGR and ϕSR derived graphically in sub-sections 2.2 and 3.2 and presented in the alternative forms of Eqs. (78), (79) and (80) are made manifest on the flat four-dimensional relativistic spacetime (Σ, ct) in the context of combined TGR and SR in the following alternative forms

$$\begin{aligned} \overline{m} &= m_0\gamma_g(r')^{-2}\gamma_d(v) \\ &= m_0 \cos^2\psi_g(r') \sec\psi_d && (125) \\ &= m_0(1 - \frac{V_g'(r')^2}{c_g^2})(1 - \frac{v^2}{c_\gamma^2})^{-1/2} && (126) \\ &= m_0(1 - \frac{2GM_{0a}}{r' c_g^2})(1 - \frac{v^2}{c_\gamma^2})^{-1/2} && (127) \end{aligned}$$

The mass $\overline{m}$ is the gravitational-relativistic cum special-relativistic mass in the context of combined TGR and SR.

The intrinsic gravitational-relativistic cum special-relativistic kinetic energy in the context of combined ϕTGR and ϕSR presented in the forms of Eqs¿ (80) – (82) are likewise made manifest on flat four-dimensional relativistic spacetime (Σ, ct) in the context of combined TGR and SR respectively as follows

$$\overline{T} = m_0 c_\gamma^2 \cos^2 \psi_g(r')[\sec \psi_d - 1] \quad (128)$$

$$= m_0 c_\gamma^2 (1 - \frac{V_g'(r')^2}{c_g^2})[(1 - \frac{v^2}{c_\gamma^2})^{-1/2} - 1] \quad (129)$$

$$= m_0 c_\gamma^2 (1 - \frac{2GM_{0a}}{r' c_g^2})[(1 - \frac{v^2}{c_\gamma^2})^{-1/2} - 1] \quad (130)$$

The kinetic energy $\overline{T}$ is the gravitational-relativistic cum special-relativistic kinetic energy in the context of combined TGR and SR.

Finally the intrinsic total energy expression (76) on the flat relativistic intrinsic spacetime $(\phi\rho\,, \phi c\phi t)$ in the context of combined ϕTGR and ϕSR is made manifest in total energy expression on the flat relativistic spacetime (Σ, ct) in the context of combined TGR and SR as follows

$$\overline{m}^{\,2} c_\gamma^4 - \overline{m}^{\,2} c_\gamma^2 v^2 = m^2 c_\gamma^4 \quad (131)$$

where $\overline{m}$ is given by Eq. (126) or (127) in the context of TGR+SR and m is given by Eq. (97), (98) or (99) in the context of TGR. By using Eq. (126) or (127) and Eq. (97), (98) or ((99) in Eq. (131) we have

$$\tilde{m}^2 c_\gamma^4 - \tilde{m}^2 c_\gamma^2 v^2 = m_0^2 c_\gamma^4 \quad (132)$$

where $\tilde{m} = \gamma_d(v) m_0$ is the special-relativistic mass expression (usually written as $m = \gamma m_0$) in the context of the primed special theory of relativity (SR′) on flat proper spacetime (Σ', ct') in the absence of relative gravity, with the geometry of Fig. 11 of [6], at the first stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field of arbitrary strength.

The effect of gravitational relativity (that is, the effect of TGR) cancels out in the gravitational-relativistic cum special-relativistic expression (131), thereby making the pure special-relativistic expression (132) to remain unchanged with position in a gravitational field.

Every result of TGR, SR and combined TGR and SR on the flat relativistic spacetime (Σ, ct) of TGR has its corresponding results in the context of ϕTGR, ϕSR and combined ϕTGR and ϕSR on the flat relativistic intrinsic spacetime $(\phi\rho\,, \phi c\phi t)$

of ϕTGR in a gravitational field, where most of the results of TGR, SR and combined TGR and SR can be obtained by simply removing the symbol ϕ from the results of ϕTGR, ϕSR and combined ϕTGR, ϕSR. There is certainly a graphical approach to TGR, SR and combined TGR and SR *via* the graphical approach to ϕTGR, ϕSR and combined ϕTGR and ϕSR, as developed in this first part of this paper.

The fact that TGR, SR and TGR+SR, etc, on the flat four-dimensional relativistic spacetime (Σ, ct) of TGR are outward (or physical) manifestations of ϕTGR, ϕSR, ϕTGR+ϕSR, etc, on the flat relativistic intrinsic spacetime $(\phi\rho\,, \phi c\phi t)$ of ϕTGR in a gravitational field, establishes a notion that non-observable intrinsic physics in intrinsic spacetime determines the observed physics in spacetime. The formal establishment of this notion at this point in the present theory is crucial, because it (the notion) authenticates one of the background philosophical stand-point of the present theory, that the domain of physics transcends the domain of experience.

There are actually two possible approaches to each of TGR, SR and combined TGR and SR on the flat four-dimensional relativistic spacetime (Σ, ct) of TGR in a gravitational field namely,

1. The graphical approach to TGR, SR and combined TGR and SR *via* the graphical approach to ϕTGR, ϕSR and combined ϕTGR and ϕSR, as developed in this first part of this paper, and
2. An analytical approach to TGR, SR and combined TGR and SR on the flat four-dimensional spacetime (Σ, ct) of TGR, to be developed in the second part of this paper, to complement the graphical approach. However the analytical approach to SR, which has been developed by Einstein in 1905, shall not be repeated.

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