Formulation, Interpretation and Application of Non Commutative Quantum Mechanics

F G Scholtz

National Institute for Theoretical Physics (NITheP)
Stellenbosch University

Non-commutative Structures and Non-relativistic (super)symmetries, LMPT Tours
22 June 2010

Outline

Formulation of non commutative quantum mechanics

- Non commutative configuration space is defined by the commutation relations $[\hat{x}, \hat{y}] = i\theta$

Formulation of non commutative quantum mechanics

- Non commutative configuration space is defined by the commutation relations $[\hat{x}, \hat{y}] = i\theta$
- Defining annihilation and creation operators $b = \frac{1}{\sqrt{2\theta}}(\hat{x} + i\hat{y}) \quad , b^\dagger = \frac{1}{\sqrt{2\theta}}(\hat{x} - i\hat{y})$ non commutative configuration space, $\mathcal{H}_c$, is isomorphic to boson Fock space

Formulation of non commutative quantum mechanics

- Non commutative configuration space is defined by the commutation relations $[\hat{x}, \hat{y}] = i\theta$
- Defining annihilation and creation operators $b = \frac{1}{\sqrt{2\theta}}(\hat{x} + i\hat{y}) \quad , b^\dagger = \frac{1}{\sqrt{2\theta}}(\hat{x} - i\hat{y})$ non commutative configuration space, $\mathcal{H}_c$, is isomorphic to boson Fock space
- Hilbert space of the non commutative quantum system

$$\mathcal{H}_q = \left\{ \psi(\hat{x}, \hat{y}) : \psi(\hat{x}, \hat{y}) \in \mathcal{B}\left(\mathcal{H}_c\right), \ \mathrm{tr_c}(\psi^\dagger(\hat{x}, \hat{y})\psi(\hat{x}, \hat{y})) < \infty \right\}$$

Formulation of non commutative quantum mechanics

- Non commutative configuration space is defined by the commutation relations $[\hat{x}, \hat{y}] = i\theta$
- Defining annihilation and creation operators $b = \frac{1}{\sqrt{2\theta}}(\hat{x} + i\hat{y}) \quad , b^\dagger = \frac{1}{\sqrt{2\theta}}(\hat{x} - i\hat{y})$ non commutative configuration space, $\mathcal{H}_c$, is isomorphic to boson Fock space
- Hilbert space of the non commutative quantum system

$$\mathcal{H}_q = \left\{\psi(\hat{x}, \hat{y}) : \psi(\hat{x}, \hat{y}) \in \mathcal{B}\left(\mathcal{H}_c\right),\ \mathrm{tr_c}(\psi^\dagger(\hat{x}, \hat{y})\psi(\hat{x}, \hat{y})) < \infty\right\}$$

- This space has a natural inner product and norm

$$(\phi(\hat{x}_1, \hat{x}_2), \psi(\hat{x}_1, \hat{x}_2)) = \mathrm{tr_c}(\phi(\hat{x}_1, \hat{x}_2)^\dagger \psi(\hat{x}_1, \hat{x}_2)).$$

Formulation of non commutative quantum mechanics

- Non commutative configuration space is defined by the commutation relations $[\hat{x},\hat{y}] = i\theta$
- Defining annihilation and creation operators $b = \frac{1}{\sqrt{2\theta}}(\hat{x} + i\hat{y}) \quad , b^\dagger = \frac{1}{\sqrt{2\theta}}(\hat{x} - i\hat{y})$ non commutative configuration space, $\mathcal{H}_c$, is isomorphic to boson Fock space
- Hilbert space of the non commutative quantum system

$$\mathcal{H}_q = \left\{\psi(\hat{x},\hat{y}) : \psi(\hat{x},\hat{y}) \in \mathcal{B}\left(\mathcal{H}_c\right), \ \mathrm{tr_c}(\psi^\dagger(\hat{x},\hat{y})\psi(\hat{x},\hat{y})) < \infty\right\}$$

- This space has a natural inner product and norm

$$(\phi(\hat{x}_1,\hat{x}_2),\psi(\hat{x}_1,\hat{x}_2)) = \mathrm{tr_c}(\phi(\hat{x}_1,\hat{x}_2)^\dagger\psi(\hat{x}_1,\hat{x}_2)).$$

- We denote states in this space by $|\psi)$

Formulation of non commutative quantum mechanics

- The next step in building the quantum system is to find a representation for the non-commutative Heisenberg algebra on $\mathcal{H}_q$. In two dimensions this reads

$$\left[x_i, p_j\right] = i\hbar\delta_{i,j}, \quad \left[x_i, x_j\right] = i\theta\epsilon_{i,j} \quad \left[p_i, p_j\right] = 0.$$

Formulation of non commutative quantum mechanics

- The next step in building the quantum system is to find a representation for the non-commutative Heisenberg algebra on $\mathcal{H}_q$. In two dimensions this reads

$$\left[x_i, p_j\right] = i\hbar\delta_{i,j}, \quad \left[x_i, x_j\right] = i\theta\epsilon_{i,j} \quad \left[p_i, p_j\right] = 0.$$

- A unitary representation of this algebra in terms of operators $\hat{X}_i$ and $\hat{P}_i$ acting on $\mathcal{H}_q$ is easily found to be

$$\hat{X}_i\psi(\hat{x}_1, \hat{x}_2) = \hat{x}_i\psi(\hat{x}_1, \hat{x}_2), \quad \hat{P}_i\psi(\hat{x}_1, \hat{x}_2) = \frac{\hbar}{\theta}\epsilon_{i,j}[\hat{x}_j, \psi(\hat{x}_1, \hat{x}_2)],$$

i.e., the position acts by left multiplication and the momentum adjointly.

Formulation of non commutative quantum mechanics

- It is also useful to introduce the following operators on $\mathcal{H}_q$

$$B = \frac{1}{\sqrt{2\theta}}\left(\hat{X}_1 + i\hat{X}_2\right), \quad B^\ddagger = \frac{1}{\sqrt{2\theta}}\left(\hat{X}_1 - i\hat{X}_2\right),$$
$$\hat{P} = \hat{P}_1 + i\hat{P}_2, \quad \hat{P}^\ddagger = \hat{P}_1 - i\hat{P}_2.$$

Formulation of non commutative quantum mechanics

- It is also useful to introduce the following operators on $\mathcal{H}_q$

$$B = \frac{1}{\sqrt{2\theta}}\left(\hat{X}_1 + i\hat{X}_2\right), \quad B^\ddagger = \frac{1}{\sqrt{2\theta}}\left(\hat{X}_1 - i\hat{X}_2\right),$$
$$\hat{P} = \hat{P}_1 + i\hat{P}_2, \quad \hat{P}^\ddagger = \hat{P}_1 - i\hat{P}_2.$$

- These operators act as follow

$$\begin{aligned} B\psi(\hat{x}_1,\hat{x}_2) &= b\psi(\hat{x}_1,\hat{x}_2), \\ B^\ddagger\psi(\hat{x}_1,\hat{x}_2) &= b^\dagger\psi(\hat{x}_1,\hat{x}_2), \\ P\psi(\hat{x}_1,\hat{x}_2) &= -i\hbar\sqrt{\frac{2}{\theta}}[b,\psi(\hat{x}_1,\hat{x}_2)], \\ P^\ddagger\psi(\hat{x}_1,\hat{x}_2) &= i\hbar\sqrt{\frac{2}{\theta}}[b^\dagger,\psi(\hat{x}_1,\hat{x}_2)]. \end{aligned}$$

Interpretation of non commutative quantum mechanics

- The interpretation is as in usually quantum mechanics with $\mathcal{H}_q$ representing the state space, i.e., physical observables are represented by hermitian operators on $\mathcal{H}_q$, a measurement yields an eigenvalue, a, with probability $\mathrm{tr}\,(\rho\pi_a)$ with ρ the density matrix and $\pi_a = |a)(a|$ the projection on the eigenstate $|a)$.

Interpretation of non commutative quantum mechanics

- The interpretation is as in usually quantum mechanics with $\mathcal{H}_q$ representing the state space, i.e., physical observables are represented by hermitian operators on $\mathcal{H}_q$, a measurement yields an eigenvalue, a, with probability $\mathrm{tr}\,(\rho\pi_a)$ with ρ the density matrix and $\pi_a = |a)(a|$ the projection on the eigenstate $|a)$.
- Position measurement is, however, different as we cannot construct simultaneous eigenstates of $\hat{X}_1$ and $\hat{X}_2$. However, we can give meaning to this in the sense of a weak measurement.

Position measurement in non commutative quantum mechanics

- First note that since position acts from the left it is natural to introduce the following states in $\mathcal{H}_q$: $|z,n) \equiv |z\rangle\langle n|$, with n labeling an arbitrary basis in $\mathcal{H}_c$ and $|z\rangle = e^{-\bar{z}z/2}e^{zb^\dagger}|0\rangle$ a coherent state.

Position measurement in non commutative quantum mechanics

- First note that since position acts from the left it is natural to introduce the following states in $\mathcal{H}_q$: $|z,n) \equiv |z\rangle\langle n|$, with n labeling an arbitrary basis in $\mathcal{H}_c$ and $|z\rangle = e^{-\bar{z}z/2}e^{zb^\dagger}|0\rangle$ a coherent state.
- These states satisfy

$$B|z,n) = z|z,n)$$

and are minimal uncertainty states.

Position measurement in non commutative quantum mechanics

- First note that since position acts from the left it is natural to introduce the following states in $\mathcal{H}_q$: $|z,n) \equiv |z\rangle\langle n|$, with n labeling an arbitrary basis in $\mathcal{H}_c$ and $|z\rangle = e^{-\bar{z}z/2}e^{zb^\dagger}|0\rangle$ a coherent state.
- These states satisfy

$$B|z,n) = z|z,n)$$

and are minimal uncertainty states.
- Since $B = \frac{1}{\sqrt{2\theta}}\left(\hat{X}_1 + i\hat{X}_2\right)$, we naturally identify $z = x_1 + ix_2$ with x_1, x_2 being the average x_1, x_2 positions.

Position measurement in non commutative quantum mechanics

- These states also have the property

$$1 = \int \frac{d^2z}{\pi} \sum_n |z,n)(z,n|$$

Position measurement in non commutative quantum mechanics

- These states also have the property

$$1 = \int \frac{d^2 z}{\pi} \sum_n |z, n)(z, n|$$

- We can thus use them to construct a POVM as

$$\pi_{z,n} = |z, n)(z, n|$$

Note that these operators are not orthogonal, which is why they constitute a POVM and not a PVM. In the language of POVM's the probability of finding the system in state $|z, n)$ is then $\mathrm{tr}\,(\rho \pi_{z,n})$ and we have to relax the von Neumann projection axiom.

Position measurement in non commutative quantum mechanics

- If we want to measure position, we are not interested in n, but rather the total probability of finding the particle and position z, irrespective of n. The POVM for this is

$$\pi_z = \sum_n |z,n)(z,n| = |z,\bar{z})e^{\overleftarrow{\partial_{\bar{z}}}\overrightarrow{\partial_z}}(z,\bar{z}| = |z,\bar{z})\star(z,\bar{z}|.$$

with $|z,\bar{z}) = \frac{1}{\sqrt{2\pi\theta}}|z\rangle\langle z|$.

Position measurement in non commutative quantum mechanics

- If we want to measure position, we are not interested in n, but rather the total probability of finding the particle and position z, irrespective of n. The POVM for this is

$$\pi_z = \sum_n |z,n)(z,n| = |z,\bar{z})e^{\overleftarrow{\partial_{\bar{z}}}\overrightarrow{\partial_z}}(z,\bar{z}| = |z,\bar{z})\star(z,\bar{z}|.$$

  with $|z,\bar{z}) = \frac{1}{\sqrt{2\pi\theta}}|z\rangle\langle z|$.
- Thus the probability of finding the particle at position z for a pure state $\rho = |\psi)(\psi|$ is then

$$P(x_1,x_2) = (\psi|\pi_z|\psi) = \langle z|\psi^\dagger|z\rangle\star\langle z|\psi|z\rangle$$

Applications: Harmonic Oscillator

- The Hamiltonian is

$$\hat{H} = \frac{1}{2m}\hat{P}_1^2 + \frac{1}{2m}\hat{P}_2^2 + \frac{1}{2}m\omega^2\hat{X}_1^2 + \frac{1}{2}m\omega^2\hat{X}_2^2,$$

Applications: Harmonic Oscillator

- The Hamiltonian is

$$\hat{H} = \frac{1}{2m}\hat{P}_1^2 + \frac{1}{2m}\hat{P}_2^2 + \frac{1}{2}m\omega^2\hat{X}_1^2 + \frac{1}{2}m\omega^2\hat{X}_2^2,$$

- Introducing creation and annihilation operators the Hamiltonian can be rewritten as

$$\hat{H} = \frac{\lambda_1}{2m}(2\hat{A}_1^\ddagger\hat{A}_1 + 1) + \frac{\lambda_2}{2m}(2\hat{A}_2^\ddagger\hat{A}_2 + 1),$$

with

$$\begin{aligned} \lambda_1 &= \frac{1}{2}\left(m^2\omega^2\theta + m\omega\sqrt{4\hbar^2 + m^2\omega^2\theta^2}\right), \\ \lambda_2 &= \frac{1}{2}\left(-m^2\omega^2\theta + m\omega\sqrt{4\hbar^2 + m^2\omega^2\theta^2}\right) \end{aligned} \quad (1)$$

Applications: Harmonic Oscillator

- The ground state wave functions is found to be

$$\psi_0 = e^{\frac{\alpha}{2\theta}(\hat{x}_1^2+\hat{x}_2^2)},$$

with

$$\alpha = \ln\left(1 - \frac{\theta}{\hbar^2}\lambda_2\right) = -\ln\left(1 + \frac{\theta}{\hbar^2}\lambda_1\right).$$

Applications: Harmonic Oscillator

- The ground state wave functions is found to be

$$\psi_0 = e^{\frac{\alpha}{2\theta}(\hat{x}_1^2+\hat{x}_2^2)},$$

with

$$\alpha = \ln\left(1 - \frac{\theta}{\hbar^2}\lambda_2\right) = -\ln\left(1 + \frac{\theta}{\hbar^2}\lambda_1\right).$$

- Note that the two frequencies are not identically so that the spectrum is less degenerate than in the commutative oscillator. Indeed, one can show that $A_1^\ddagger$ creates 1 unit of angular momentum, while $A_2^\ddagger$ creates -1 unit of angular momentum and that the ground state has zero angular momentum. Thus one observes a breaking of time reversal.

Applications: spherical Well

- The Hamiltonian for the spherical well reads

$$\hat{H} = \frac{P^2\psi}{2\mu} + (V_1 P + V_2 Q).$$

with

$$P = \sum_{n=0}^{M} |n\rangle\langle n|, \quad Q = \sum_{n=M+1}^{\infty} |n\rangle\langle n|.$$

The radius of the disc is given by $R^2 = \theta(2M+1)$.

Applications: Spectrum of the infinite spherical Well

- The energies of the infinite well for positive angular momentum is obtained as

$$L_{M+1}^{m}\left(\frac{\theta k^2}{2}\right)=0,\ m\geq 0,\quad k^2=\frac{2\mu E}{\hbar^2},$$

and for negative angular momentum as

$$L_{M+m+1}^{m}\left(\frac{\theta k^2}{2}\right)=0,\ -M\leq m<0,\quad k^2=\frac{2\mu E}{\hbar^2},$$

Note that the spectrum truncates at angular momentum $-M$.

Applications: Spectrum of the infinite spherical Well

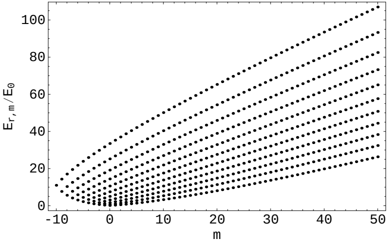

Figure: Spectrum of the infinite non commutative well

Applications: Thermodynamics of a non commutative Fermi gas

- From the spectrum we may expect strong differences in the thermodynamics of a Fermi gas at high enough densities:

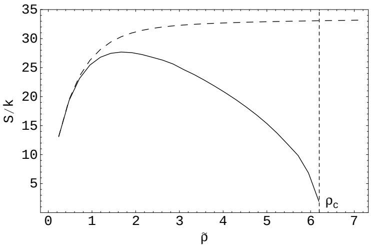

Figure: Entropy of fermi gas as a function of density

Applications: Bound states of a finite well

- For a finite well one can study the bound states:

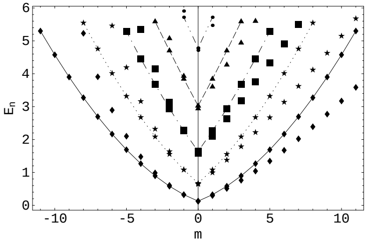

Figure: Commutative and non-commutative bound state energies for a finite well. Connected symbols are the commutative energies and unconnected ones indicate the non-commutative energies.

Applications: Scattering from a finite well

- One can also study scattering and compute phase shifts:

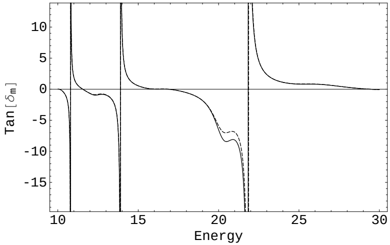

Figure: Tangent of the phase shift in the m=4 channel of a finite well

Path integral representation of transition amplitude

- Use the completeness relations

$$\int d^2p\,|p)(p| = 1_Q\,, \quad \int 2\theta dz d\bar{z}\,|z,\bar{z})\star(z,\bar{z}| = 1_Q,$$

with

$$|p) = \sqrt{\frac{\theta}{2\pi\hbar^2}}e^{i\sqrt{\frac{\theta}{2\hbar^2}}(\bar{p}b+pb^\dagger)}, \quad |z,\bar{z}) = \frac{1}{\sqrt{2\pi\theta}}|z\rangle\langle z|.$$

Path integral representation of transition amplitude

- The overlaps

$$(z,\bar{z}|p) = \frac{1}{\sqrt{2\pi\hbar^2}} e^{-\frac{\theta}{4\hbar^2}\bar{p}p} e^{i\sqrt{\frac{\theta}{2\hbar^2}}(p\bar{z}+\bar{p}z)}.$$

Path integral representation of transition amplitude

- The overlaps

$$(z,\bar{z}|p) = \frac{1}{\sqrt{2\pi\hbar^2}} e^{-\frac{\theta}{4\hbar^2}\bar{p}p} e^{i\sqrt{\frac{\theta}{2\hbar^2}}(p\bar{z}+\bar{p}z)}.$$

- The transition amplitude is

$$(z_f, t_f|z_0, t_0) = N \exp\left(-\vec{\partial}_{z_f}\vec{\partial}_{\bar{z}_0}\right) \int \mathcal{D}z\mathcal{D}\bar{z} \exp(\frac{i}{\hbar}S)$$

Path integral representation of transition amplitude

- The overlaps

$$(z,\bar{z}|p)=\frac{1}{\sqrt{2\pi\hbar^2}}e^{-\frac{\theta}{4\hbar^2}\bar{p}p}e^{i\sqrt{\frac{\theta}{2\hbar^2}}(p\bar{z}+\bar{p}z)}.$$

- The transition amplitude is

$$(z_f,t_f|z_0,t_0) \quad = \quad N\exp\left(-\vec{\partial}_{z_f}\vec{\partial}_{\bar{z}_0}\right)\int \mathcal{D}z\mathcal{D}\bar{z}\exp(\frac{i}{\hbar}S)$$

- The action S is given by

$$S=\int_{t_0}^{t_f}dt\ \left[\frac{\theta}{2}\dot{\bar{z}}(t)(\frac{1}{2m}-\frac{i\theta}{2\hbar}\partial_t)^{-1}\dot{z}(t)-V(\bar{z}(t),z(t))\right].$$

Path Integral: Free particle

- Equation of motion for the free particle is $K\ddot{z}_c(t) = 0 \quad ; \quad K = (\frac{1}{2m} - \frac{i\theta}{2\hbar}\partial_t)^{-1}$. This operator has trivial kernel so that the equation of motion is equivalent to $\ddot{z} = 0$ with solution $z_c(t) = z_0 + \frac{z_f - z_0}{T}(t - t_0)$

Path Integral: Free particle

- Equation of motion for the free particle is $K\ddot{z}_c(t) = 0 \quad ; \quad K = (\frac{1}{2m} - \frac{i\theta}{2\hbar}\partial_t)^{-1}$. This operator has trivial kernel so that the equation of motion is equivalent to $\ddot{z} = 0$ with solution $z_c(t) = z_0 + \frac{z_f - z_0}{T}(t - t_0)$
- Substituting the above solution in the path integral and acting with the boundary operator yields

$$(z_f, t_f | z_0, t_0) = N \exp\left[-\frac{m}{2(i\hbar T + m\theta)}(\vec{x}_f - \vec{x}_0)^2\right]$$

Path Integral: Free particle

- Equation of motion for the free particle is $K\ddot{z}_c(t) = 0 \quad ; \quad K = (\frac{1}{2m} - \frac{i\theta}{2\hbar}\partial_t)^{-1}$. This operator has trivial kernel so that the equation of motion is equivalent to $\ddot{z} = 0$ with solution $z_c(t) = z_0 + \frac{z_f - z_0}{T}(t - t_0)$
- Substituting the above solution in the path integral and acting with the boundary operator yields

$$(z_f, t_f|z_0, t_0) = N \exp\left[-\frac{m}{2(i\hbar T + m\theta)}(\vec{x}_f - \vec{x}_0)^2\right]$$

- The constant can be determined as $N = \frac{m}{2\pi(\theta m + i\hbar T)}$

Path Integral: Harmonic oscillator

- The equation of motion for the harmonic oscillator is $K\ddot{z}_c(t) + 2m\omega^2 z_c(t) = 0$

Path Integral: Harmonic oscillator

- The equation of motion for the harmonic oscillator is $K\ddot{z}_c(t) + 2m\omega^2 z_c(t) = 0$
- Making the ansatz $z_c(t) = e^{i\gamma t}$ yields the frequencies

$$\gamma_\pm = \frac{1}{2\hbar}(m\omega^2\theta \pm \omega\sqrt{m^2\omega^2\theta^2 + 4\hbar^2}).$$

Path Integral: Harmonic oscillator

- The equation of motion for the harmonic oscillator is $K\ddot{z}_c(t) + 2m\omega^2 z_c(t) = 0$
- Making the ansatz $z_c(t) = e^{i\gamma t}$ yields the frequencies

$$\gamma_\pm = \frac{1}{2\hbar}(m\omega^2\theta \pm \omega\sqrt{m^2\omega^2\theta^2 + 4\hbar^2}).$$

- Noting that the one frequency is positive and the other negative, we can write the general classical solution as $z_c(t) = a_+ e^{i\omega_+ t} + a_- e^{-i\omega_- t}$, which reflects time reversal symmetry breaking.

Path Integral: Harmonic oscillator

- Substituting into the action and acting with the boundary operator yields $(z_f, t_f|z_0, t_0) = N \exp\left(Z^\dagger \Lambda Z\right)$ where $Z^\dagger = \left(z_0^*, z_f^*\right)$ and Λ is the 2×2 matrix

$$\Lambda = \frac{m\theta}{m\theta Q_{12} - \hbar} \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} + \frac{m\theta\omega_+\omega_-}{\hbar} & Q_{22} \end{pmatrix}.$$

Path Integral: Harmonic oscillator

- Substituting into the action and acting with the boundary operator yields $(z_f, t_f|z_0, t_0) = N \exp\left(Z^\dagger \Lambda Z\right)$ where $Z^\dagger = \left(z_0^*, z_f^*\right)$ and Λ is the 2×2 matrix

$$\Lambda = \frac{m\theta}{m\theta Q_{12} - \hbar}\begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} + \frac{m\theta\omega_+\omega_-}{\hbar} & Q_{22} \end{pmatrix}.$$

- Q is the matrix

$$Q = \begin{pmatrix} -\frac{\omega_- + \omega_+ e^{iT(\omega_- + \omega_+)}}{1 - e^{iT(\omega_- + \omega_+)}} & \frac{\omega_- + \omega_+}{e^{-iT\omega_+} - e^{iT\omega_-}} \\ \frac{\omega_- + \omega_+}{e^{-iT\omega_-} - e^{iT\omega_+}} & -\frac{\omega_+ + \omega_- e^{iT(\omega_- + \omega_+)}}{1 - e^{iT(\omega_- + \omega_+)}} \end{pmatrix}.$$

Path Integral: Harmonic oscillator

- The normalization can be fixed by studying the time evolution of the harmonic oscillator ground state

$$N = \frac{1}{2\pi}\left[\frac{m\omega_-}{\hbar} - \left(1 + \frac{m\omega_-\theta}{\hbar}\right)\frac{mQ_{11}}{m\theta Q_{12} - \hbar}\right] e^{-\frac{iT}{2}(\omega_- + \omega_+)}$$

Conclusions

- Non commutative quantum mechanics can be formulated and interpreted within the normal axioms of quantum mechanics. The only generalization required is the notion of weak measurements.

Conclusions

- Non commutative quantum mechanics can be formulated and interpreted within the normal axioms of quantum mechanics. The only generalization required is the notion of weak measurements.
- Within this framework simple systems such as the harmonic oscillator and well can readily be solved and the energies of bound states and phase shifts can be computed.

Conclusions

- Non commutative quantum mechanics can be formulated and interpreted within the normal axioms of quantum mechanics. The only generalization required is the notion of weak measurements.
- Within this framework simple systems such as the harmonic oscillator and well can readily be solved and the energies of bound states and phase shifts can be computed.
- The thermodynamics of a non commutative Fermi gas deviates strongly from that of the commutative gas at high densities. In particular the entropy is non extensive.

Conclusions

- Non commutative quantum mechanics can be formulated and interpreted within the normal axioms of quantum mechanics. The only generalization required is the notion of weak measurements.
- Within this framework simple systems such as the harmonic oscillator and well can readily be solved and the energies of bound states and phase shifts can be computed.
- The thermodynamics of a non commutative Fermi gas deviates strongly from that of the commutative gas at high densities. In particular the entropy is non extensive.
- We have found that the non commutative path integral is non local in time and time reversal is broken.

Conclusions

- The propagator for the free particle and harmonic oscillator can be computed explicitly.

Conclusions

- The propagator for the free particle and harmonic oscillator can be computed explicitly.
- Collaborators: B Chakraborty (S N Bose), S Vaidya (Indian Institute of Science), J Govaerts (Univ Louvain), N Thom (Stellenbosch Univ), C Rohwer (Stell Univ) and A Hafver (Stell Univ).