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FORMULATION OF MATHEMATICAL MODEL FOR STRESS-STRAIN RELATIONSHIP OF NORMAL AND HIGH STRENGTH CONCRETE UNDER COMPRESSION

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Abstract

The research includes a new model proposed for the stress-strain relationship of unconfined concrete in compression valid for normal and high strength concrete. A wide range of experimental data with varied lab circumstances has been used for fitting and other data for verifying the model. It is noted that the current model has a good agreement with the experimental data for both its ascending and descending branches in normal and high strength concrete. Depending on the mean of average values of experimental to calculated stresses, coefficient of variation, and difference ratio. Where values of the average experimental to calculated stresses ranged from 0.723 to 1.354 for 38 samples with a mean of 0.994, while the coefficient of variation values ranged from 16.099 to 48.562 with a mean of 27.704 % for these specimens. Also, difference ratio values ranged from 0.86 % to 31.804 % with a mean of 9.009 % for these specimens. The model gives the best results in comparison with other models.

Keywords:

Stress-strain relationship; Unconfined concrete; Normal strength concrete; High strength concrete; Coefficient of variation.

1 Introduction

In the past decades, valuable attempts have been conducted to evaluate and define the stress-strain relationship of concrete in compression for its vital role in the determination of sectional non-linear behavior under compression stresses and analysis of reinforced concrete structures and then the design of their different members to resist the applied loads. Many factors affect this relation including testing method, loading rate, curing technique, mix design, compressive strength, and etc. These factors are varied from one research to others, and this makes the distribution of compressive stresses fairly difficult to be predicted. As always, the compressive strength, strain at peak, and initial modulus of elasticity are considered the main demands for designers to simplify their calculations and that make the researchers directed toward stress-strain expressions as a function of these [1].

Firstly a second-degree parabola expression was proposed for stress-strain relation by Hognestad, (1951) [2]. A simple model as a function of compressive strength, strain at peak, and a material parameter depending on stress-strain shape β was suggested by Popovics (1973) [3] and then it has been followed by some researchers after modification of the parameter β to be more adequate [4-7]. Later, a more complicated formula was used by Sargin et al. (1971) [8] that became a base for other researchers to propose their modified models [9, 10].

Unusually, Wee et al. in 1996 [6] conducted a fitting of four existing expressions, (Hodnestad, 1951), (Wang et al., 1978), (CEB Model Code 90, 1990) and (Carreira and Chu, 1985) [2, 4, 9, 10] on a wide range of experimental data to consider the effect of mix design. It is found that Wang et al. (1978) [9] model gives the best fit and because of its complexity, they proposed a new model based on Carreira and Chu (1985) [4] equation. Also an assessment of these four expressions was viewed by Wee et al. (1996) [6]. Hodnestad (1951) [2] expression is limited for compressive strength to 60 MPa and adequate for ascending part only and a computational effort has been required for Wang et al. (1978) model [9]. While in the CEB model (1990) [10] a too much steep drop of the descending branch is considered, and Carreira and Chu (1985) [4] model has no longer adequate prediction of post-peak portion for a wide range of compressive strength.

In the same way, Lu and Zhao (2010) [11] investigated some empirical stress-strain models with published experimental data and a note of these models viewed. Limited applicability in the modeling of descending part was revealed in Wee et al. (1996) model [6]. Furthermore, the residual stresses in Van Gysel and Taerwe (1996) model [12] tend to zero at high strains while a discontinuity as recognized in Hsu and Hsu (1994) model [5] at 0.3 f'_c of descending part.

Some remarks are noted on the previously proposed equations for stress-strain curve such as:

1) The limited range of compressive strength values for adopted experimental data in some research.

2) Most literature attempted to evaluate and calibrate stress-strain relation in compression based on their experimental results rather than the others. Therefore, they were limited and may not be applicable for other experimental data.

3) Some models depended on parameters taken from experimental results which makes it cannot be applied as an independent model used in the analysis.

4) Some models were very complicated in their mathematical expressions that complicate using them.

5) Some models do not satisfy the boundary conditions for the true stress–strain curve or continuity requirements at common points between the two branches of its equation.

Based on these remarks, the current study is directed toward trying to propose a simple new model as a function of compressive strength and parameters concerned with the shape of the curve to simulate the complete behavior of the stress-strain curve in compression so that it is valid for normal and high strength concrete and overcome the previous shortcomings. Unlike before, to make the model more applicable for any data as possible, a list of wide experimental data of different concrete mixes and different test conditions provided in literature for many previous researches was used to fit the model, find shape parameters and calibrate the proposed model in comparison with other exists models.

2 Present study technique

The present study is based on collecting a wide range of experimental data where compressive strength ranges from 16 MPa to 122 MPa with different test conditions and mix designs for twelve researchers [2, 5-7, 9, 13-19]. These data are divided into two groups, the first one contains 76 samples as shown in Table 1 used for fitting the proposed model and the second contains 38 samples used for verifying the adequacy of the model equation. This technique is to confirm that the proposed equation is suitable for any other experimental data. Also, a comparison has been conducted between the proposed model with some others that exist in the literature [3-6, 11].

The fitting process to find the parameters of the proposed equation is based on the comparison between experimental and theoretical curves for 76 samples using convergence criteria. This comparison reflects the agreement between experimental and theoretical results. The best convergence is at minimum differences between stresses taken from experimental and theoretical curves at points along them. The percentage ratio between the summation of absolute differences between experimental stresses and theoretical stresses to the summation of experimental stresses is adopted as statistical criteria (DR) to evaluate the fitting process as follows:

$$DR = \frac{\sum_{i=1}^{n}\left|\sigma_{exp.} - \sigma_{theo.}\right|}{\sum_{i=1}^{n}\sigma_{exp.}} x100\% \quad , \tag{1}$$

where σ_{exp} is the experimental stress, σ_{theo} is the theoretical stress, n is the number of stress points on the curve.

Table 1: Details of 76 samples used for regression analysis.

Specimen	ε_o	f'_c	Specimen	ε_o	f'_c	Specimen	ε_o	f'_c
Tasnimi-1	0.002269	24.3082	Wee-6	0.002748	122.392	Ayub-10	0.002299	84.3722
Tasnimi-10	0.002884	35.6212	Hsu-1	0.003015	66.1109	Ayub-11	0.00233	86.8395
Tasnimi-12	0.002971	37.2238	Hsu-2	0.002356	33.1257	Ayub-2	0.002415	72.164
Tasnimi-13	0.003128	36.4697	Hsu-4	0.003003	73.9417	Ayub-3	0.002318	75.4842
Tasnimi-15	0.002953	38.4494	Hsu-6	0.003253	83.3093	Ayub-6	0.002299	77.4083
Tasnimi-16	0.002988	39.4864	Dahl-2	0.002735	31.9078	Ayub-8	0.002431	80.4897
Tasnimi-17	0.003175	55.4444	Dahl-4	0.002587	65.1311	Ayub-9	0.002415	85.7172
Tasnimi-19	0.002545	47.5755	Dahl-5	0.002647	93.5914	shah-3	0.003166	88.7999
Tasnimi-20	0.002971	47.4325	Dahl-6	0.002804	105.789	shah-4	0.003656	91.0966
Tasnimi-21	0.003107	46.431	Hognstad-3	0.002223	35.2372	shah-5	0.003251	89.1976
Tasnimi-22	0.002579	47.5755	Hognstad-5	0.001981	51.8546	Shah -7	0.003606	82.5045
Tasnimi-23	0.002971	44.2849	Wang-1	0.002914	20.737	shah-8	0.003437	83.4562
Tasnimi-25	0.003073	42.2819	Wang-3	0.002942	50.3707	chen-1	0.004238	81.8122
Tasnimi-27	0.002409	39.9928	Wang-4	0.003598	74.1335	chen-10	0.003222	76.6435
Tasnimi-28	0.002988	40.1358	Wang-6	0.003102	30.3469	chen-12	0.003232	80.7945
Tasnimi-3	0.002221	26.4394	Wang-7	0.003186	38.9509	chen-13	0.003868	86.4889
Tasnimi-4	0.0024	27.9	Slate-1	0.002004	25.0706	chen-15	0.003929	91.1736
Tasnimi-5	0.002509	27.5051	Slate-3	0.002793	71.4588	chen-18	0.002804	77.9537
Tasnimi-6	0.002726	28.5707	Slate-5	0.003291	36.4046	chen-19	0.002906	82.7594
Tasnimi-7	0.002622	31.1904	Slate-6	0.003791	56.9402	chen-2	0.003824	88.6471
Tasnimi-8	0.002604	32.1332	Almusallam-2	0.002305	34.7023	chen-20	0.002846	75.9648
Tasnimi-9	0.002849	32.3217	Almusallam-3	0.002495	42.723	chen-3	0.003658	90.2412
Wee-2	0.001993	46.7827	Almusallam-6	0.002593	82.5587	chen-4	0.003207	86.8374
Wee-3	0.002345	66.7706	Ali-2	0.001944	25.676	chen-6	0.003245	88.2026
Wee-4	0.002329	86.7585	Ali-5	0.002243	43.6782	chen-7	0.003131	85.0963
						chen-9	0.003673	89.1244

3 Formulation of proposed equation

The present proposed model is based on a mathematical function represented by a power function. This type of function can represent complicated curves as stress-strain curves without needing to use complex mathematical functions. The derivation of the new expression will be shown as follows:

Firstly, we put the stress–strain relationship in a normalized way, $y = \sigma/f'_c$ and $x = \epsilon/\epsilon_0$, where f'_c and ϵ_0 are stress and strain at peak point while σ and ε are stress and strain at any point on the relationship curve.

If a linear relation between x and y is adopted for ascending part of the relationship and its inverted form for descending part we get a basic relationship curve as shown in Fig. 1 where:

$y = x$ for ascending, (2)

$y = x^{-1}$ for descending**.** (3)

The shape of this function is similar to the general shape of stress–strain curve and coincides with it at the initial, peak, and final points. To make this function appropriate to the representation of stress–strain curve and satisfies all its boundary conditions, the following formula can be used as an improved expression for the basic equations (2), (3) as follows:

$y = x^{a(1-x)}$. (4)

This equation is more capable to represent the stress–strain relationship as shown in Fig. 1 where the curve ascends when the power of the equation is positive (at $x < 1$) and it descends when the power of the equation is negative (at $x > 1$). This equation is considered a good single expression for two branches of the curve and satisfies all required boundary conditions as follows:

at $x = 0$, $y = 0$
at $x = 1$, $y = 1$ and $dy/dx = 0$
at $x = \infty$, $y = 1$ and $dy/dx = 0$

Based on comparison with shapes and properties of experimental curves of previously tested samples for different values of concrete strength, some improvements to the formula of equation (4) can be made to ensure the closest agreement for the proposed equation with the actual response as shown in Fig. 1. Therefore, the formula will be as follows:

$$y = x^{a\left(\frac{1-x^c}{1+bx^c}\right)}, \tag{5}$$

where a, b, and c are parameters that depend on values of concrete compressive strength. The values of these parameters are responsible for the degree of nonlinearity of the relationship and the curving of its two branches including the turning point that lies in descending part at which the slope of the curve turns from increasing to decreasing. The parameters are defined as functions of f'_c and their expressions will be analytically determined depending on the fitting with experimental data.

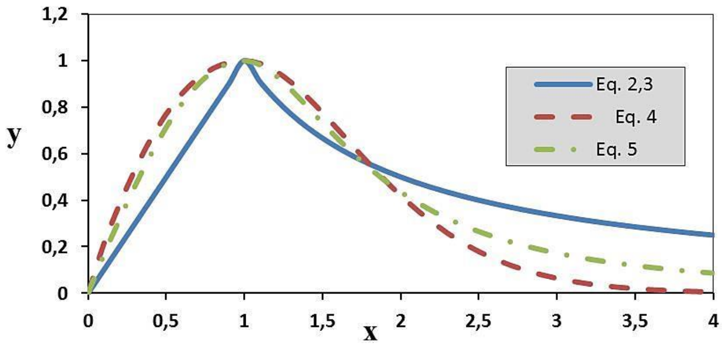

Fig. 1: General shape of constructed mathematical model for stress-strain relationship.

4 Proposed stress-strain model

To obtain the full equation for the model of stress–strain relationship, the experimental data of 76 specimens listed in Table 1 are used to determining the parameters of the equation (5) using the regression analysis. The analysis showed that parameter a has a value smaller than one for ascending part and larger than one for descending part and it must be raised to the power ϵ/ϵ_0 to avoid the sharp descent in this part for high values of ε . The parameter b is a negative value and does not exceed - 0.95 for ascending part but it equals one for descending part. Also, the parameter c equals one for ascending part while for descending part, the analysis showed that the parameter c is equal to the value of a parameter a, therefore the descending part can be written in terms of the parameter c only.

Powered expression as functions of f'_c are proposed to predict the values of parameters a, b, and c, and their coefficients were determined using the regression analysis. Fig. 2, 3, and 4 show the data fitting of these parameters where their equations are confirmed with points of experimental results with acceptable values of correlation factor. It is noted that the values obtained from experimental data are dispersed due to the variety in test conditions and sample properties. Therefore, the fitting process gives the equation that has the best correlation with test data. Based on this regression analysis, the final formulas for the proposed model and its parameters will be as follows:

$$\sigma = f'_c\left(\frac{\varepsilon}{\varepsilon_o}\right)^{\beta}, \tag{6}$$

$$\beta = \frac{a\left(1-\frac{\varepsilon}{\varepsilon_\circ}\right)}{\left(1+b\frac{\varepsilon}{\varepsilon_\circ}\right)} \; for\, \varepsilon \le \varepsilon_\circ \; (ascending\ branch)\,, \tag{7}$$

$$a=0.7f_c^{'\frac{1}{15}} \le 1\,, \tag{8}$$

$$b=-0.02f_c^{'0.8} \ge -0.95\,, \tag{9}$$

$$\beta = \frac{c^{\frac{\varepsilon_\circ}{\varepsilon}}\left[1-\left(\frac{\varepsilon}{\varepsilon_\circ}\right)^c\right]}{\left[1+\left(\frac{\varepsilon}{\varepsilon_\circ}\right)^c\right]} \; for\ \varepsilon > \varepsilon_\circ \; (descending\ branch)\,, \tag{10}$$

$$c=0.02f_c^{'}\,. \tag{11}$$

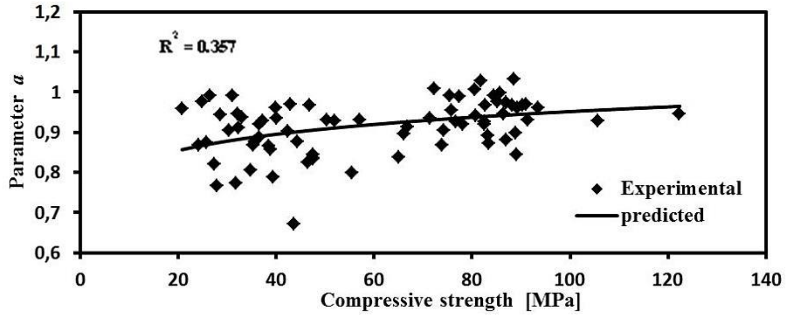

Fig. 2: Relation between the parameter *a* and concrete compressive strength.

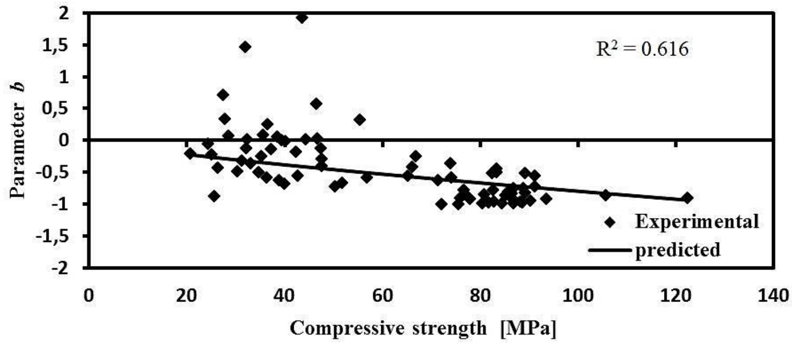

Fig. 3: Relation between the parameter *b* and concrete compressive strength.

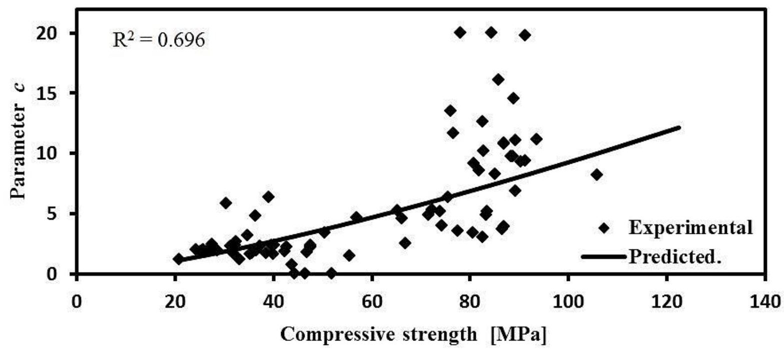

Fig. 4: Relation between the parameter *c* and concrete compressive strength.

5 Verification of the proposed equation

As previously mentioned, results of stress–strain curves for 38 test samples were used for checking the adequacy of the proposed equation. Also, the proposed equation will be compared with equations of five models by previous researchers [3-6, 11].

Three statistical criteria were adopted to represent the adequacy of the proposed equation and the comparison with models in the literature:

1) The average value (AV) that represents an average of the ratios of experimental to calculated stress for all points on stress–strain curve for one sample.

2) The coefficient of variation (COV) that represents the standard deviation of experimental to calculated stress ratios divided by their average for one sample, where this criteria measures the dispersion between the experimental and calculated stresses on stress–strain curve.

3) The difference ratio DR that measures the summation of differences between experimental and calculated stresses on stress–strain curve as a percentage from the summation of experimental stresses for all points on the curve for one sample as shown in equation (1).

The model is more accurate as the AV value converges from unity and as COV and DR values are smaller.

Table 2 reviews the details of the previous model's equations that were selected for comparison purposes. Tables 3, 4, and 5 listed the statistical results for agreement between the experimental and predicted results of 38 stress–strain curves.

From these tables, it can be noted that the proposed equation has good convergence with the experimental results where AV values ranged from 0.723 to 1.354 with a mean of 0.994 for all specimens, while COV values ranged from 16.099 to 48.562 with a mean of 27.704 % for all specimens and DR values ranged from 0.86 % to 31.804 % with mean 9.009 % for all specimens.

The adequacy of the proposed equation is more obvious by comparison with the other model. Based on AV values, Table 3, the proposed model and Hsu and Hsu (1994) model [5] are the best where AV values for these models are very close to unity 0.9944, 1.0057 respectively with very slight priority for the proposed equation. While AV values for other models are 0.866, 0.9, 0.928, and 0.973 respectively. This means that these models give larger stresses compared with the experimental results and that makes these models overestimate the stresses of this relationship.

Based on COV values, Table 4, the proposed equation gives the smaller values for numerous samples. Also the proposed equation and Wee et al. (1996) model [6] give the smaller mean values for COV for all specimens 27.704 % and 27.441 % respectively with slight priority for Wee et al. (1996) model [6] while mean COV values for the other equations were 28.696 %, 29.073 %, 29.931 %, and 29.939 %. This means that these equations have less agreement with the experimental results by comparison with the proposed and Wee et al. (1996) equations [6].

Based on DR values, Table 5, the proposed equation gives the smaller values for a larger number of samples if it is compared with the other equations. Also, the proposed equation gives the smaller mean values of DR for all 38 specimens 9.009 % while mean DR values for the other equations were 9.946 %, 11.001 %, 11.956 %, 12.093 %, and 12.562 %. Lu and Zhao (2010) model equation [11] gives closer results to the proposed equation by comparison with the others where the DR value for it ranged from 1.035 % to 34.918 % with the mean value of 9.946 %. This means that these equations have less agreement with the experimental results by comparison with the proposed equation.

Table 2: List of equations of stress-strain models for previous researchers.

Authors	Model	Parameter description
Popovics (1973) [2]	$\frac{f_c}{f'_c}=\frac{\beta\frac{\varepsilon_c}{\varepsilon_o}}{\beta\text{-}1+\left(\frac{\varepsilon_c}{\varepsilon_o}\right)^{\beta}}$	$\beta=0.058f'_c+1$
Carreira and Chu (1985) [3]	$\frac{f_c}{f'_c}=\frac{\beta\frac{\varepsilon_c}{\varepsilon_o}}{\beta\text{-}1+\left(\frac{\varepsilon_c}{\varepsilon_o}\right)^{\beta}}$	$\beta=\frac{1}{1\text{-}\left(\frac{f'_c}{\varepsilon_o E_{it}}\right)}$ $\frac{\varepsilon_o E_{it}}{f'_c}=\frac{24.82}{f'_c}+0.92$ $\varepsilon_o=\left(168+0.71f'_c\right)x10^{-5}$
Hsu and Hsu (1994) [4]	$0\le\varepsilon_c\le\varepsilon_d....\frac{f_c}{f'_c}=\frac{n\beta\frac{\varepsilon_c}{\varepsilon_o}}{n\beta\text{-}1+\left(\frac{\varepsilon_c}{\varepsilon_o}\right)^{n\beta}}$ $\varepsilon_c\ge\varepsilon_d....\frac{f_c}{f'_c}=0.3\,exp.\left[\text{-}0.8\left(\frac{\varepsilon_c}{\varepsilon_o}\text{-}\frac{\varepsilon_d}{\varepsilon_o}\right)^{0.5}\right]$	$\beta=\left(\frac{f'_c}{65.23}\right)^3+2.59$ $n=1...for...0<f'_c<62$ $n=2...for...62\le f'_c\le 76$ $n=3...for...76\le f'_c\le 90$ $n=5...for...f'_c\ge 90$
Wee et al. (1996) [5]	$0\le\varepsilon_c\le\varepsilon_o....\frac{f_c}{f'_c}=\frac{\beta\frac{\varepsilon_c}{\varepsilon_o}}{\beta\text{-}1+\left(\frac{\varepsilon_c}{\varepsilon_o}\right)^{\beta}}$ $\varepsilon_c>\varepsilon_o....\frac{f_c}{f'_c}=\frac{k_1\beta\frac{\varepsilon_c}{\varepsilon_o}}{k_1\beta\text{-}1+\left(\frac{\varepsilon_c}{\varepsilon_o}\right)^{k_2\beta}}$	$\beta=\frac{1}{1\text{-}\left(\frac{f'_c}{\varepsilon_o E_{it}}\right)}$ $E_{it}=10200f'^{\frac{1}{3}}_c$ $\varepsilon_o=0.00078f'^{\frac{1}{4}}_c$ $k_1=\left(\frac{50}{f'_c}\right)^3$ $k_2=\left(\frac{50}{f'_c}\right)^{1.3}$
Lu and Zhao (2010) [10]	$0\le\varepsilon_c\le\varepsilon_1....\frac{f_c}{f'_c}=\frac{\left(\frac{E_{it}}{E_c}\right)\left(\frac{\varepsilon_c}{\varepsilon_o}\right)-\left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2}{1+\left(\frac{E_{it}}{E_c}-2\right)\left(\frac{\varepsilon_c}{\varepsilon_o}\right)}$ $\varepsilon_c>\varepsilon_1....\frac{f_c}{f'_c}=\frac{1}{1+\lambda\left[\frac{\left(\frac{\varepsilon_c}{\varepsilon_o}\text{-}1\right)}{\left(\frac{\varepsilon_1}{\varepsilon_o}\text{-}1\right)}\right]^{2(1\text{-}\lambda)}}$	$\varepsilon_1=\varepsilon_o\left[\frac{1}{10}\left(\frac{E_{it}}{E_c}\right)+\frac{4}{5}+\sqrt{\left(\frac{1}{10}\left(\frac{E_{it}}{E_c}\right)+\frac{4}{5}\right)^2\text{-}\frac{4}{5}}\right]$

Table 3: Values of (AV) for all specimens using the proposed and previous models equations.

Specimens No.		*fc*' [MPa]	Proposed model	Popovics 1973 model	Carreira and Chu 1985 model	Hsu and Hsu 1994 model	Wee et al. 1996 model	Lu and Zhao 2010 model
Tasnimi, 2004	23	25.507	2.008	3.679	6.150	2.758	7.644	2.445
	24	36.658	6.565	5.323	2.662	3.247	2.887	3.710
	25	39.298	3.656	3.151	2.043	2.051	4.372	1.721
	26	48.720	7.466	7.599	5.418	3.596	1.897	2.429
	27	42.425	5.042	4.117	2.141	1.689	2.235	1.035
	28	42.282	3.393	3.181	1.308	1.442	4.646	3.242
Wee et al., 1996	5	30.521	10.983	5.975	1.524	4.991	4.585	19.367
	6	104.886	12.441	15.273	15.294	20.772	4.933	8.028
Hsu and Hsu, 1994	5	65.912	8.860	14.180	16.561	8.710	16.182	9.858
	6	79.741	5.299	16.567	17.744	8.843	14.063	7.844
	7	90.000	7.481	25.872	26.380	15.239	18.994	14.657
Dahl, 1992	5	21.610	7.338	5.211	5.216	6.059	5.521	16.101
	6	50.745	6.286	15.654	18.230	20.185	21.296	11.985
Hognstad, 1951	3	20.156	9.536	5.162	3.185	8.648	1.523	1.889
	4	46.688	2.524	2.053	4.510	6.105	8.351	2.701
Wang et al., 1978	6	38.350	4.640	10.846	15.496	15.178	18.475	9.524
	7	23.554	31.804	35.282	39.785	32.059	41.940	9.811
	8	54.499	13.635	24.793	28.092	31.652	31.621	21.406
Slate et al., 1986	5	48.357	3.724	1.303	2.167	3.106	4.181	2.350
	6	19.176	7.484	9.940	12.265	7.252	12.927	4.602
Almusallam and Alsayed, 1995	4	28.097	7.098	6.597	6.314	6.640	7.164	19.529
	5	48.594	8.291	5.134	7.240	8.484	9.877	2.534
	6	68.973	11.283	2.555	3.818	4.420	5.556	7.963
Ali et al., 1990	3	28.094	5.781	2.538	1.334	2.871	3.681	6.490
	4	32.483	10.410	7.740	5.292	6.826	3.907	2.120
Ayub et al., 2014	8	71.499	18.726	15.433	17.161	14.325	17.635	23.873
	9	72.160	13.656	31.026	33.682	17.719	33.228	10.825
	10	82.486	12.542	17.952	19.341	8.231	17.742	6.261
	11	80.469	19.521	13.914	13.430	20.354	5.238	10.760
	12	86.157	0.860	18.784	19.915	11.272	17.868	10.726
Shah et al., 1981	6	93.729	12.245	25.672	25.722	17.165	20.406	34.197
	7	91.833	11.354	23.222	23.387	15.883	19.860	35.952
	8	87.299	8.649	18.078	19.012	11.687	14.353	4.879
	9	88.892	4.750	5.426	5.247	10.604	4.029	4.462
Chen, 1995	15	94.034	2.911	2.844	2.819	10.226	2.068	6.193
	16	87.763	4.711	4.988	4.780	10.498	3.397	5.469
	17	90.245	26.648	39.149	39.634	28.566	38.116	28.912
	18	86.109	2.743	3.341	3.074	8.692	1.918	2.112
		Mean	9.009	12.093	12.562	11.001	11.956	9.946

Table 4: Values of COV % for all specimens using the proposed and previous models.

Specimens No.		*fc*' [MPa]	Proposed model	Popovics 1973 model	Carreira and Chu 1985 model	Hsu and Hsu 1994 model	Wee et al. 1996 model	Lu and Zhao 2010 model
Tasnimi, 2004	23	25.507	0.936	0.907	0.878	0.920	0.861	0.967
	24	36.658	1.040	1.027	0.978	0.989	0.925	0.982
	25	39.298	1.010	1.005	0.961	0.961	0.909	0.973
	26	48.720	1.080	1.092	1.053	1.022	0.991	0.925
	27	42.425	1.036	1.035	0.995	0.985	0.944	0.953
	28	42.282	0.959	0.964	0.923	0.913	0.871	0.956
Wee et al., 1996	5	30.521	1.031	0.984	0.915	0.971	0.865	1.153
	6	104.886	1.302	1.063	1.109	1.417	1.015	1.096
Hsu and Hsu, 1994	5	65.912	1.061	0.877	0.846	1.063	0.825	0.860
	6	79.741	1.010	0.823	0.811	1.098	0.827	0.874
	7	90.000	0.940	0.762	0.759	1.155	0.786	0.815
Dahl, 1992	5	21.610	1.035	0.990	0.937	1.047	0.917	1.172
	6	50.745	1.027	0.925	0.886	0.855	0.836	0.872
Hognstad, 1951	3	20.156	1.082	1.007	0.947	1.082	0.928	0.999
	4	46.688	0.922	0.930	0.901	0.882	0.857	0.990
Wang et al., 1978	6	38.350	0.915	0.855	0.816	0.819	0.821	1.149
	7	23.554	0.723	0.699	0.674	0.716	0.662	1.023
	8	54.499	0.831	0.791	0.771	0.748	0.743	0.781
Slate et al., 1986	5	48.357	0.921	0.899	0.882	0.870	0.857	0.918
	6	19.176	0.821	0.786	0.764	0.816	0.759	0.875
Almusallam and Alsayed, 1995	4	28.097	0.907	0.877	0.849	0.880	0.833	1.391
	5	48.594	1.030	0.923	0.888	0.867	0.848	0.914
	6	68.973	1.031	0.913	0.900	0.990	0.903	1.031
Ali et al., 1990	3	28.094	1.023	0.985	0.934	0.990	0.899	1.008
	4	32.483	1.175	1.148	1.088	1.125	1.035	0.954
Ayub et al., 2014	8	71.499	1.033	0.844	0.818	1.033	0.800	1.564
	9	72.160	0.841	0.727	0.712	0.833	0.706	0.872
	10	82.486	0.993	0.812	0.800	1.076	0.803	0.961
	11	80.469	1.354	1.560	1.381	1.502	0.918	0.880
	12	86.157	1.044	0.811	0.797	1.112	0.788	1.075
Shah et al., 1981	6	93.729	0.983	0.861	0.860	1.112	0.863	0.738
	7	91.833	0.951	0.867	0.864	1.074	0.859	0.720
	8	87.299	1.041	0.834	0.826	1.123	0.836	0.953
	9	88.892	1.024	1.024	1.021	1.113	1.013	1.002
Chen, 1995	15	94.034	0.994	0.994	0.993	1.105	0.986	0.886
	16	87.763	0.958	0.963	0.959	1.043	0.947	0.962
	17	90.245	0.762	0.732	0.730	0.858	0.720	0.804
	18	86.109	0.956	0.965	0.960	1.053	0.942	0.936
		Mean	0.9944	0.928	0.900	1.0057	0.866	0.973

Table 5: Values of DR % for all specimens using the proposed and previous models equations.

Specimens No.		*fc'* [MPa]	Proposed model	Popovics 1973 model	Carreira and Chu 1985 model	Hsu and Hsu 1994 model	Wee et al. 1996 model	Lu and Zhao 2010 model
Tasnimi, 2004	23	25.507	0.936	0.907	0.878	0.920	0.861	0.967
	24	36.658	1.040	1.027	0.978	0.989	0.925	0.982
	25	39.298	1.010	1.005	0.961	0.961	0.909	0.973
	26	48.720	1.080	1.092	1.053	1.022	0.991	0.925
	27	42.425	1.036	1.035	0.995	0.985	0.944	0.953
	28	42.282	0.959	0.964	0.923	0.913	0.871	0.956
Wee et al., 1996	5	30.521	1.031	0.984	0.915	0.971	0.865	1.153
	6	104.886	1.302	1.063	1.109	1.417	1.015	1.096
Hsu and Hsu, 1994	5	65.912	1.061	0.877	0.846	1.063	0.825	0.860
	6	79.741	1.010	0.823	0.811	1.098	0.827	0.874
	7	90.000	0.940	0.762	0.759	1.155	0.786	0.815
Dahl, 1992	5	21.610	1.035	0.990	0.937	1.047	0.917	1.172
	6	50.745	1.027	0.925	0.886	0.855	0.836	0.872
Hognstad, 1951	3	20.156	1.082	1.007	0.947	1.082	0.928	0.999
	4	46.688	0.922	0.930	0.901	0.882	0.857	0.990
Wang et al., 1978	6	38.350	0.915	0.855	0.816	0.819	0.821	1.149
	7	23.554	0.723	0.699	0.674	0.716	0.662	1.023
	8	54.499	0.831	0.791	0.771	0.748	0.743	0.781
Slate et al., 1986	5	48.357	0.921	0.899	0.882	0.870	0.857	0.918
	6	19.176	0.821	0.786	0.764	0.816	0.759	0.875
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	6	68.973	1.031	0.913	0.900	0.990	0.903	1.031
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	4	32.483	1.175	1.148	1.088	1.125	1.035	0.954
Ayub et al., 2014	8	71.499	1.033	0.844	0.818	1.033	0.800	1.564
	9	72.160	0.841	0.727	0.712	0.833	0.706	0.872
	10	82.486	0.993	0.812	0.800	1.076	0.803	0.961
	11	80.469	1.354	1.560	1.381	1.502	0.918	0.880
	12	86.157	1.044	0.811	0.797	1.112	0.788	1.075
Shah et al., 1981	6	93.729	0.983	0.861	0.860	1.112	0.863	0.738
	7	91.833	0.951	0.867	0.864	1.074	0.859	0.720
	8	87.299	1.041	0.834	0.826	1.123	0.836	0.953
	9	88.892	1.024	1.024	1.021	1.113	1.013	1.002
Chen, 1995	15	94.034	0.994	0.994	0.993	1.105	0.986	0.886
	16	87.763	0.958	0.963	0.959	1.043	0.947	0.962
	17	90.245	0.762	0.732	0.730	0.858	0.720	0.804
	18	86.109	0.956	0.965	0.960	1.053	0.942	0.936
		Mean	0.9944	0.928	0.900	1.0057	0.866	0.973

Generally and based on these three criteria, the proposed equation can be considered as the best equation and it gives results closer to the experimental results than the other models. The slight priority for Wee et al. (1996) model [6] is ruled out because this model has the smaller value of AV 0.866, this means that equation of this model significantly overestimate the stresses values on stress-strain relationship. Also it gives high value for DR 11.956 % if compared with value that given by the proposed equation 9.009 % that makes the priority for the proposed equation rather than Wee et al. (1996) model [6].

Although that the Lu and Zhao (2010) model [11] has good agreement with experimental results, but there is an problem in using its equation, where calculation of concrete stresses depends on the practical value of initial modulus of elasticity that is taken from tests. This results in impossibility of application this model theoretically and there is a need to construct a theoretical expression for initial modulus of elasticity so that this model is able to theoretical use. Certainly, using theoretical expression for initial tangent modulus of elasticity will reduce the agreement of Lu and Zhao (2010) model [11] with the experimental results.

Parts of Fig. 5 show the comparison between the analytical stress-strain curve results from the proposed equation with the experimental stress-strain curve for eight different samples. From this Figure one can note the good agreement between the proposed equation curve with the experimental curve for all stages of loading and for different values of concrete compressive strength.

Parts of Fig. 6 show the comparison between the analytical stress-strain curves result from the proposed and other previous equations with the experimental stress-strain curve for other eight samples. This Figure confirms that the conclusions obtained from previous tables of statistical results that show the good convergence of proposed model with the experimental results by comparison with the other models that generally give curves lie above the experimental curve especially in descending part of these curves.

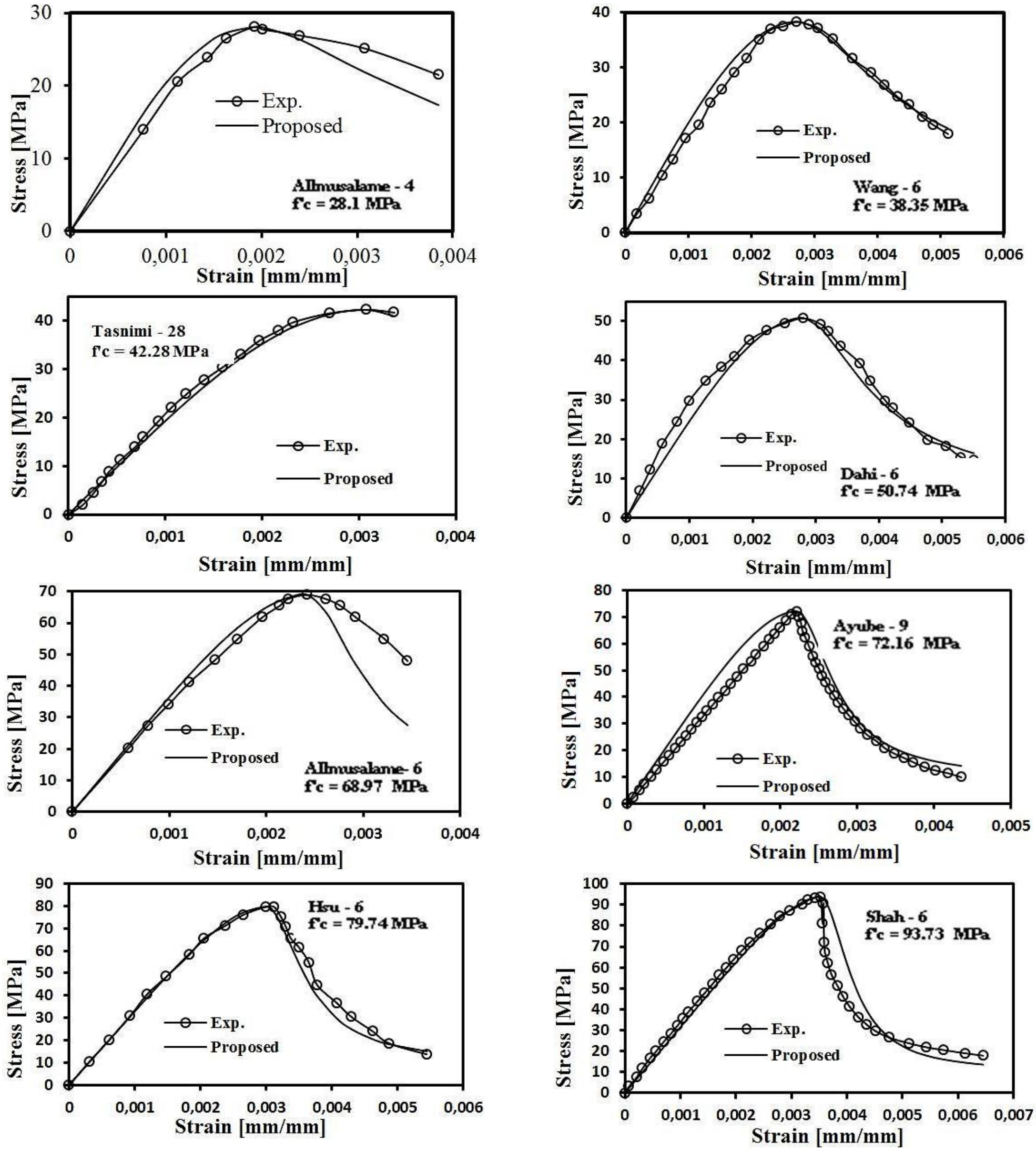

Fig. 5: Comparison between experimental and proposed analytical stress-strain curves.

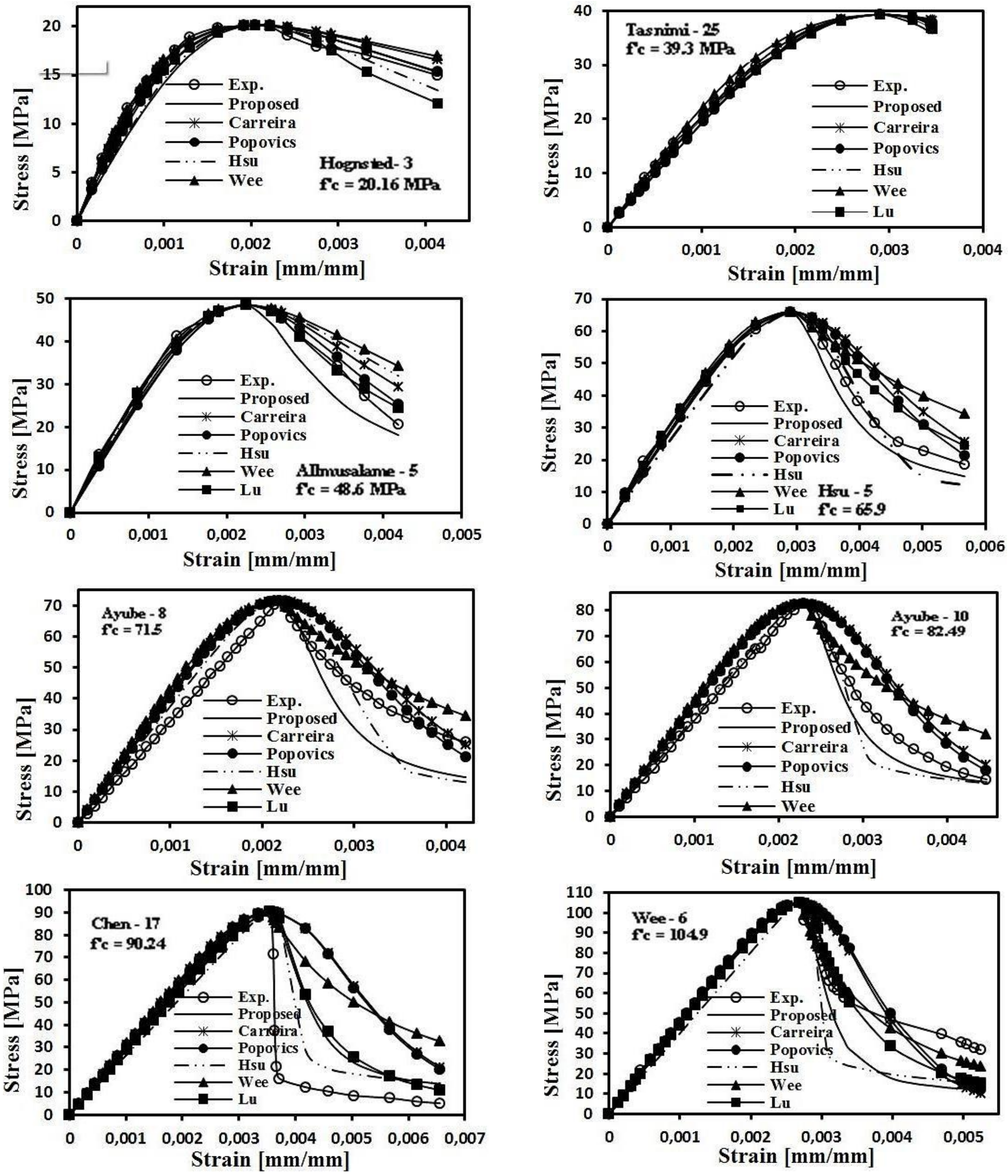

Fig. 6: Comparison between experimental and analytical stress-strain curves by using proposed and other previous equations.

6 Strain at peak stress

For application of the proposed equation theoretically without need any information from test results, mathematical expression for concrete strain at peak stress must be derived depending on test data of 76 samples. An empirical powered expression was proposed here for the strain in terms of concrete strength f'_c for normal and high strength concrete as follows.

$$\varepsilon_o = c_1 f_c^{'c_2} \ . \tag{12}$$

By reliance on collected data of 76 previously tested specimens, the regression analysis has been performed on above equation to evaluate coefficients c_1 and c_2 depending on the best affinity with experimental results. The results of regression analysis give c_1 = 0.00125 and c_2 = 0.2 therefore, the expression of ε_o as will be follows:

$$\varepsilon_o = 0.00125 f_c^{'0.2} \ . \tag{13}$$

An assessment has been conducted to verify this equation by comparing with the other formulas found in previous works for some researchers [4, 6, 19]. This assessment is performed on test data of group 2 (verification group of 38 samples). Table 6 gives the statistical values for agreement between the experimental results and analytical expressions results for this strain using proposed and the other equations.

From this table, one can note that the proposed equation was the more agreement with the experimental results than the other equations where average value of experimental to calculated strains for all samples was (1) with smaller values of COV 16.919 % and DR 14.697 %. Tasnimi (2004) formula [19] underestimates value of the strain while Carrira and Chu (1985) [4] and Wee et al. (1996) [6] formulas overestimate this value.

Table 6: Statistical values for (Exp. /Theo.) strains for 38 sample.

Criteria	Method			
	Proposed Eq.	**Tasnimi Eq.**	**Carrira Eq.**	**Wee Eq.**
AV	1.000	0.889	1.320	1.315
COV %	16.919	17.606	16.968	17.034
DR %	14.697	17.731	24.399	24.315

Also the agreement of the proposed equation with the experimental results can be noticed in Fig. 7 while the other equations have less agreement.

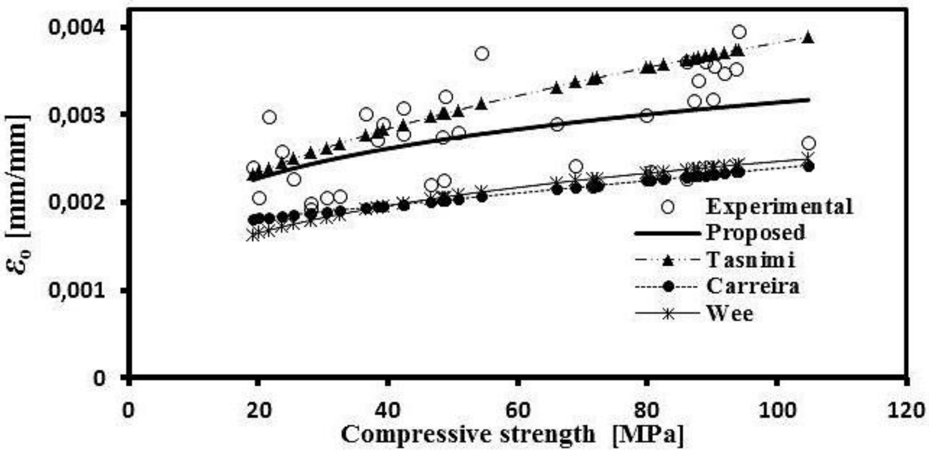

Fig. 7: Comparison between the proposed and previous equations with the experimental results for strain at peak stress of concrete stress strain curves.

7 Compression behaviour of concrete using the proposed equation

From previous paragraphs, it is concluded that the proposed equation has good agreement with wide range of concrete compressive strength. This means that the equation is able to describe the behavior of normal and high strength concrete under compressive stresses. Fig. 8 shows the stress-strain relationship for concrete with varying its strength using the proposed model. From this Figure, one can note that concrete behaves in ascending part as elastic material in the earlier stages of loading. This is evident from the linear part of the curve, then, the relation between the stresses and strains starts to be nonlinear. The nonlinearity of the relation increases arriving to the peak stress where the curve has horizontal tangent. As compressive strength increases, the linear part is larger and the nonlinearity of the curve decreases. In very high strength concrete, the largest part of ascending branch seems as linear, i.e. it fairly behaves as elastic material within this branch.

For descending branch, the curve descends with growing slope until a certain point (turning point) then it starts to descend with slow down slope where the stresses are small at high values of strains and die out at very high strains. With increasing the strength, the slope of descending is larger and more steeper. At high values of strains, the curves are close to each other and the stresses values for all curves are small (i.e. there are residual stresses) and they trend to the zero value at very high strain values.

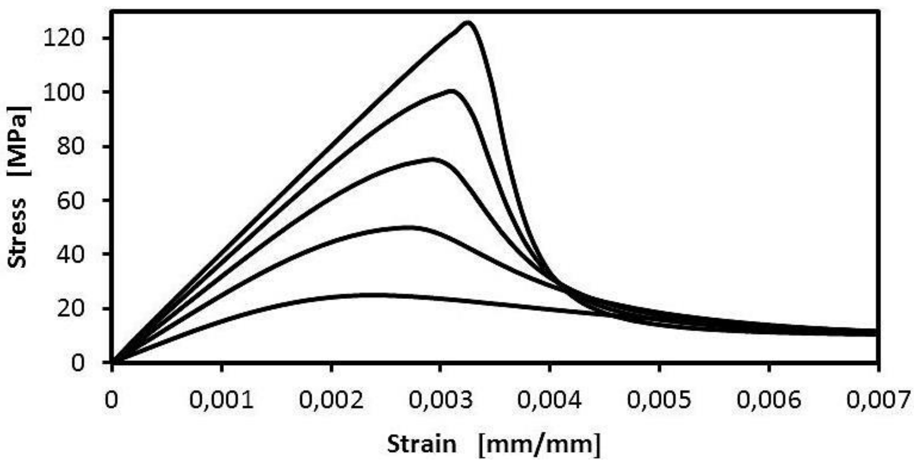

Fig. 8: Stress-strain relationship of concrete using the proposed equation.

8 Conclusion

The following conclusions can be summarized from the study:

1) The proposed equation has good agreement with experimental results and it is more accurate than five equations provided in literature.

2) The curves that drawn using the proposed equations for normal and high strength concretes are close to experimental curves for both ascending and descending branches of the curves.

3) The proposed expression for ε_o was more accurate than some previous expressions.

4) Using the proposed model to understand the behaviour of concrete under compression loading shows that increasing concrete strength results in larger linear portion at ascending part and steeper falling curve at descending part.

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