

# Foundations of Mechanics

Second Edition

Revised, enlarged, and reset

*A mathematical exposition of classical mechanics  
with an introduction to  
the qualitative theory of dynamical systems  
and applications to the three-body problem*

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