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# Foundations of Micropolar Mechanics

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# Preface

Mechanics of Micropolar Continua (also known as Cosserat Continua) is in the focus of scientists since the end of the nineteenth century. A first summary of the theory with independent force and moment (couple) actions was given in 1909 by the Cosserat brothers in their centurial book “Théorie des corps déformables” [1]. Since that time there were published tens of books and thousands of papers in this field, see e.g., the well-known books by Eringen [2, 3], Nowacki [4], and the recent collections edited by Maugin and Metrikine [5], Altenbach, Maugin and Erofeev [6], Markert [7], and Altenbach and Eremeyev [8] where the state of the art in the field of the Cosserat theory is presented. It is worth mentioning the special issue of International Journal of Engineering Science in memory of A. C. Eringen, which is published as Volume 49(2011), No. 12, edited by G. A. Maugin and J. D. Lee. All the contributions on the Cosserat continuum are focussed on the fact that the continuum translations and rotations can be defined independently. In other words, force and moment actions in the continuum can be introduced independently as in dynamics of rigid body or structural mechanics. In a micropolar medium, each material particle has six degrees of freedom, they are three translational and three rotational degrees of freedom. Besides ordinary stresses in the theory of micropolar continuum there are introduced couple stresses. These characteristic features of the Cosserat continuum model give a possibility to describe more complex media, for example, micro-inhomogeneous materials, foams, cellular solids, lattices, masonries, particle assemblies, magnetic rheological fluids, liquid crystals, etc. The aim of this book is as follows:

- the presentation of the basics of the micropolar continuum mechanics including a short but comprehensive introduction of stress and strain measures, derivation of motion equations, and discussion of the differences between Cosserat and classical (Cauchy) continua, and
- the discussion of more specific problems related to the constitutive modeling, i.e., constitutive inequalities, symmetry groups, acceleration waves, etc., which are original contributions in the field.

**Chapter 1** (Introduction) is a brief review of some important publications in the research area under consideration. Such a review cannot be complete—there are too many publications. But it is a starting point for further studies. In **Chap. 2** we recall kinematics of the micropolar continuum and introduce the directors and the microrotation tensor. In **Chap. 3** we consider the theory of stresses and couple stresses. Here, we present the motion equations of micropolar continuum and Lagrangian and Eulerian statements of the boundary value problems. **Chapter 4** is devoted to the theory of constitutive equations. We formulate the principles of determinism, local action, and material frame-indifference. Here, we consider the constitutive equations of the hyperelastic micropolar continuum. We introduce the natural relative Lagrangian strain measures using three different approaches. Then we present the definition of the material symmetry group and discuss the corresponding constitutive equations. We introduce constraints which lead for example to the model of the Cosserat continuum with the constrained rotations. We establish the condition of strong ellipticity of equilibrium equations and formulate other constitutive inequalities. The detailed analysis of acceleration waves in thermoelastic micropolar continuum is the subject of **Chap. 5**. We prove that the conditions of the existence of acceleration waves coincide with the condition of strong ellipticity of equilibrium equations. A few examples are presented.

The book also contains some necessary information about tensor analysis. Using Cosserat's approach we briefly present some elements of rigid body dynamics and the mechanics of rods and shells to demonstrate that Cosserat's description of continua fills up the gap between the Continuum Mechanics and Mechanics of rigid bodies and thin structures.

In what follows, we use the notations: the numeration of formulae, figures, and tables is given as (i,j), where i is the number of the chapter and j is the number inside the chapter; the vectors are denoted by semibold roman font like  $\mathbf{A}$ ,  $\mathbf{a}$ , the second-order tensors are denoted by semibold san serif font like  $\mathbb{A}$ ,  $\mathbb{a}$ , the higher order tensors are denoted by blackboard bold font like  $\mathbb{A}$ , Greece indices take values 1 and 2, whereas Latin indices are arbitrary. We also employ the Einstein summation convention.

This book is written for specialists in continuum mechanics as well as for Masters and PhD students investigating Cosserat and other generalized continua.

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