FOUNDATIONS OF PERTURBATIVE QCD

The most non-trivial of the established microscopic theories of physics is QCD: the theory of the strong interaction. A critical link between theory and experiment is provided by the methods of perturbative QCD, notably the well-known factorization theorems. Giving an accurate account of the concepts, theorems, and their justification, this book is a systematic treatment of perturbative QCD.

As well as giving a mathematical treatment, the book relates the concepts to experimental data, giving strong motivations for the methods. It also examines in detail transverse-momentum-dependent parton densities, an increasingly important subject not normally treated in other books. Ideal for graduate students starting their work in high-energy physics, it will also interest experienced researchers wanting a clear account of the subject.

JOHN COLLINS is Distinguished Professor of Physics at Penn State University. He has a long experience in perturbative QCD. He has proved a number of the fundamental theorems that form the main content of this book, and has a record of formulating and deriving novel results in QCD. During his career he has received several awards, including a Guggenheim fellowship, a Humboldt Research Award, a Mercator professorship, and the JJ Sakurai prize.

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Contents

	Acknowledgments	<i>page</i> xii
1	Introduction	1
1.1	Factorization and high-energy collisions	3
1.2	Why we trust QCD is correct	6
1.3	Notation	6
1.4	Problems and exercises	7
2	Why QCD?	8
2.1	QCD: statement of the theory	9
2.2	Development of QCD	10
2.3	Deeply inelastic scattering	16
2.4	Parton model	23
2.5	Asymptotic freedom	28
2.6	Justification of QCD	29
2.7	QCD in the full Standard Model	31
2.8	Beyond the Standard Model	32
2.9	Relation between fields and particles	33
Exerc	ises	34
3	Basics of QCD	36
3.1	Quantization	36
3.2	Renormalization	39
3.3	Renormalization counterterms of QCD	45
3.4	Meaning of unit of mass, renormalization scale	48
3.5	Renormalization group	51
3.6	Solution of RG equations	54
3.7	Values of RG coefficients	58
3.8	Symmetries and approximate symmetries of QCD	60
3.9	Dealing with quark masses	62

 $\textcircled{\sc c}$ in this web service Cambridge University Press

vi	Contents	
3.10	CWZ (ACOT) method for heavy quarks	64
3.11	Relating CWZ subschemes with different numbers of active quarks	66
Exerci	ses	67
4	Infra-red safety and non-safety	69
4.1	e^+e^- total cross section	69
4.2	Explicit calculations	75
4.3	Evolution of state	81
4.4	Dispersion relation and effective virtuality of final-state quarks and gluons	84
4.5	Generalizations	86
Exerci	ses	86
5	Libby-Sterman analysis and power-counting	87
5.1	High-energy asymptotics and mass singularities	88
5.2	Reduced graphs and space-time propagation	91
5.3	Examples of general reduced graphs	92
5.4	One-loop vertex graph	105
5.5	Power-counting for vertex graph	111
5.6	Which reactions have a pinch in the Glauber region?	125
5.7	Coordinates for a PSS	130
5.8	Power-counting	134
5.9	Catalog of leading regions	146
5.10	Power-counting with multiple regions	148
5.11	Determination of Glauber-like regions	156
Exerci	ses	158
6	Parton model to parton theory: simple model theories	161
6.1	Field theory formulation of parton model	161
6.2	When is the parton model valid?	170
6.3	Parton densities as operator matrix elements	174
6.4	Consequences of rotation and parity invariance: polarization dependence	177
6.5	Polarization and polarized parton densities in spin- $\frac{1}{2}$ target	179
6.6	Light-front quantization	180
6.7	Parton densities as number densities	185
6.8	Unintegrated parton densities	190
6.9	Properties of parton densities	190
6.10	Feynman rules for pdfs	201
6.11 E	Calculational examples	202
Exerci	ses	210
7	Parton theory: further developments	213
7.1	DIS with weak interactions, neutrino scattering, etc.	213
7.2	Light-front perturbation theory	217

	Contents	vii
7.3	Light-front wave functions	225
7.4	Light-front quantization in gauge theories	227
7.5	Parton densities in gauge theories	229
7.6	Feynman rules for gauge-invariant parton densities	235
7.7	Interpretation of Wilson lines within parton model	237
Exerci	ses	241
8	Factorization for DIS, mostly in simple field theories	243
8.1	Factorization: overall view	243
8.2	Elementary treatment of factorization	245
8.3	Renormalization of parton densities	251
8.4	Renormalization group, and DGLAP equation	261
8.5	Moments and Mellin transform	262
8.6	Sum rules for parton densities and DGLAP kernels, including in QCD	263
8.7	Renormalization calculations: model theory	264
8.8	Successive approximation method	269
8.9	Derivation of factorization by ladder method	271
8.10	Factorization formula for structure functions	276
8.11	Transverse-spin dependence at leading power?	280
Exerci	ses	282
9	Corrections to the parton model in QCD	284
9.1	Lowest order	284
9.2	Projections onto structure functions	284
9.3	Complications in QCD	285
9.4	One-loop renormalization calculations in QCD	286
9.5	One-loop renormalization by subtraction of asymptote	293
9.6	DIS on partonic target	295
9.7	Computation of NLO gluon coefficient function	296
9.8	Choice of renormalization scale μ	300
9.9	NLO quark coefficient	301
9.10	Hard scattering with quark masses	306
9.11	Critique of conventional treatments	307
9.12	Summary of known higher-order corrections	309
9.13	Phenomenology	310
Exerci	ses	312
10	Factorization and subtractions	313
10.1	Subtraction method	314
10.2	Simple example of subtraction method	320
10.3	Sudakov form factor	321
10.4	Region approximator T_R for Sudakov form factor	323
10.5	One-loop Sudakov form factor	330

viii	Contents	
10.6	Rationale for definition of T_R	345
10.7	General derivation of region decomposition	351
10.8	Sudakov form factor factorization: first version	359
10.9	Factorization in terms of unsubtracted factors	374
10.10	Evolution	374
10.11	Sudakov: redefinition of factors	378
10.12	Calculations for Sudakov problem	388
10.13	Deduction of some non-leading logarithms	393
10.14	Comparisons with other work	394
Exercis	bes	395
11	DIS and related processes in QCD	398
11.1	General principles	398
11.2	Regions and PSSs, with uncut hadronic amplitude	399
11.3	Factorization for DIS	404
11.4	Renormalization of parton densities, DGLAP evolution	411
11.5	DIS with weak interactions	412
11.6	Polarized DIS, especially transverse polarization	412
11.7	Quark masses	413
11.8	DVCS and DDVCS	415
11.9	Ward identities to convert K gluons to Wilson line	416
Exercis	ses	425
12	Fragmentation functions: e^+e^- annihilation to	
	hadrons, and SIDIS	426
12.1	Structure-function analysis of one-particle inclusive	
	cross section	427
12.2	Statement of factorization etc. for $e^+e^- \rightarrow h(p) + X$	429
12.3	LO calculation	432
12.4	Introduction to fragmentation functions	433
12.5	Leading regions and issues in a gauge theory	439
12.6	Which gauge to use in a proof?	444
12.7	Unitarity sum over jets/sum over cuts	447
12.8	Factorization for $e^+e^- \rightarrow h(p) + X$ in gauge theory	448
12.9	Use of perturbative calculations	461
12.10	One-loop renormalization of fragmentation function	461
12.11	One-loop coefficient functions	464
12.12	Non-perturbative effects and factorization	465
12.13	Generalizations	466
12.14	Semi-inclusive deeply inelastic scattering	470
12.15	Target fragmentation region: fracture functions	475
Exercis	es	477

	Contents	ix
13	TMD factorization	479
13.1	Overview of two-particle-inclusive e^+e^- annihilation	479
13.2	Kinematics, coordinate frames, and structure functions	481
13.3	Region analysis	485
13.4	Collinear factors	491
13.5	Initial version of factorization with TMD fragmentation	494
13.6	Factorization and transverse coordinate space	495
13.7	Final version of factorization for e^+e^- annihilation	496
13.8	Evolution equations for TMD fragmentation functions	501
13.9	Flavor dependence of CS and RG evolution	502
13.10	Analysis of CS kernel K: perturbative and non-perturbative	503
13.11	Relation of TMD to integrated fragmentation function	507
13.12	Correction term for large $qh_{\rm T}$	513
13.13	Using TMD factorization	514
13.14	NLO calculation of TMD fragmentation function at small $b_{\rm T}$ and	
	at large $k_{\rm T}$	519
13.15	SIDIS and TMD parton densities	526
13.16	Polarization issues	532
13.17	Implications of time-reversal invariance	533
Exercis	es	537
14	Inclusive processes in hadron-hadron collisions	540
14.1	Overview	540
14.2	Drell-Yan process: kinematics etc.	542
14.3	Glauber region example	545
14.4	Factorization for Drell-Yan	553
14.5	TMD pdfs and Drell-Yan process	562
14.6	Calculations with initial-state partons	569
14.7	Production of hadrons	570
Exercis	es	571
15	Introduction to more advanced topics	573
15.1	Light-front wave functions and exclusive scattering at large	
	momentum transfer	574
15.2	Exclusive diffraction: generalized parton densities	574
15.3	Small-x, BFKL, perturbative Regge physics	575
15.4	Resummation, etc.	576
15.5	Methods for efficient high-order calculations	577
15.6	Monte-Carlo event generators	577
15.7	Heavy quarks	578
15.8	Large x	579
15.9	Soft-collinear effective theory (SCET)	579
15.10	Higher twist: power corrections	580

х	Contents	
Apper	ndix A: Notations, conventions, standard mathematical results	582
A.1	General notations	582
A.2	Units, and conversion factors	582
A.3	Acronyms and abbreviations	583
A.4	Vectors, metric, etc.	584
A.5	Renormalization group (RG)	584
A.6	Lorentz, vector, color etc. sub- and superscripts	585
A.7	Polarization and spin	585
A.8	Structure functions	586
A.9	States, cross sections, integrals over particle momentum	587
A.10	Dirac, or gamma, matrices	587
A.11	Group theory	588
A.12	Dimensional regularization and $\overline{\text{MS}}$: basics	589
A.13	Dimensional regularization: standard integrals	590
A.14	Properties of Γ function	591
A.15	Plus distributions, etc.	591
A.16	Feynman parameters	592
A.17	Orders of magnitude, estimation, etc.	592
Apper	ndix B: Light-front coordinates, rapidity, etc.	595
B.1	Definition	595
B.2	Boosts	596
B.3	Rapidity	596
B.4	Pseudo-rapidity	598
B.5	Rapidity distributions in high-energy collisions	598
Appendix C: Summary of primary results		600
	References	603
	Index	617

To Mary, Dave, and George

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