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VOLUME IX

**FOUNDATIONS OF THE LOGICAL THEORY
OF SCIENTIFIC KNOWLEDGE (COMPLEX LOGIC)**

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BOSTON STUDIES IN THE PHILOSOPHY OF SCIENCE

VOLUME IX

EDITED BY ROBERT S. COHEN AND MARX W. WARTOFSKY

A. A. ZINOV'EV

FOUNDATIONS
OF THE LOGICAL THEORY
OF SCIENTIFIC KNOWLEDGE
(COMPLEX LOGIC)

Revised and Enlarged English Edition

with an Appendix by

G. A. Smirnov, E. A. Sidorenko, A. M. Fedina, and L. A. Bobrova



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EDITORIAL INTRODUCTION

Boston Studies in the Philosophy of Science are devoted to symposia, congresses, colloquia, monographs and collected papers on the philosophical foundations of the sciences. It is now our pleasure to include A. A. Zinov'ev's treatise on complex logic among these volumes. Zinov'ev is one of the most creative of modern Soviet logicians, and at the same time an innovative worker on the methodological foundations of science. Moreover, Zinov'ev, although still a developing scholar, has exerted a substantial and stimulating influence upon his colleagues and students in Moscow and within other philosophical and logical circles of the Soviet Union. Hence it may be helpful, in bringing this present work to an English-reading audience, to review briefly some contemporary Soviet investigations into scientific methodology.

During the 1950's, a vigorous new research program in logic was undertaken, and the initial published work – characteristic of most Soviet publications in the logic and methodology of the sciences – was a collection of essays, *Logical Investigations* (Moscow, 1959). Among the authors, in addition to Zinov'ev himself, were the philosophers A. Kol'man and P. V. Tavanec, and the mathematicians and linguists, S. A. Janovskaja, A. S. Esenin-Vol'pin, S. K. Šaumjan, G. N. Povarov. Two principal themes dominate this work: first, that the *results* of mathematical logic have practical importance for the sciences (and that modern logic may be understood as of general philosophical significance); and second, that it is impossible to provide a successful and profound methodology of science without using the formal apparatus and methods of contemporary logic. This relationship, between research into mathematical and formal logic on the one hand and the methodology of the sciences on the other, became a principal focus of logical research in the Soviet Union during the subsequent period, and it remains so at present.

Perhaps the pre-eminent group of Soviet logicians and methodologists is gathered together in Moscow at the Institute of Philosophy of the Academy of Sciences of the U.S.S.R. Among its number are P. V. Tava-

nec, D. P. Gorskij, G. I. Ruzavin, V. A. Smirnov, A. A. Zinov'ev. These scholars, together with other colleagues in the section on logic of the Institute of Philosophy have been the most productive of Soviet logicians and methodologists; they have published a series of volumes which, in effect, has fulfilled the function of a periodical of logical and methodological studies.

Among the most interesting of these volumes, in addition to the *Logical Investigations* of 1959 have been: *Philosophical Questions of Contemporary Formal Logic*, 1962; *Problems of the Logic of Scientific Knowledge*, 1964 (English translation, 1970, D. Reidel); *Logical Structure of Scientific Knowledge*, 1966; *Logical Semantics and Modal Logics*, 1967; *Investigation of Logical Systems*, 1970; *Non-Classical Logics*, 1970.

Further, a number of individual monographs have been published, including: N. I. Stjažkin, *History of Mathematical Logic from Leibniz to Peano*, 1964 (English translation, 1969, MIT Press); A. I. Uemov, *Things, Properties and Relations*, 1963; D. P. Gorskij, *Questions of Abstraction and Notions of Construction*, 1961; and a number of monographs by Zinov'ev himself (see below).

Logical research has also been undertaken in the sections on logic of the departments of philosophy of the Moscow State University and the University of Leningrad by E. K. Vojšvillo, A. A. Ivin, O. F. Serebrjannikov among others, and also at other Soviet academic institutions: for example, A. I. Uemov at the University of Odessa, V. N. Sadovskij at the Institute of the History and Theory of the Natural Sciences and Technology of the Soviet Academy in Moscow, M. V. Popovič at the Institute of Philosophy of the Ukrainian Academy of Sciences in Kiev, and many others. The literature has been voluminous but two works from Zinov'ev's own colleagues particularly should be noted: O. F. Serebrjannikov, *Heuristic Principles and Logical Calculi*, 1970, and A. A. Ivin, *Foundations of the Logic of Value*, 1970.

Despite the immense effort devoted to the elaboration of the technical apparatus of formal logic, particularly by Zinov'ev, Smirnov, and Serebrjannikov, the primary aim of this trend in Soviet logic has been the application of formal logic to the decisive solution of a range of problems in the methodology of the sciences. Among Soviet scholars, this topic is called the 'logic of science'. Zinov'ev's group of colleagues has been formed during the past few years. It consists of former research students of

Zinov'ev who either continue to collaborate with him or to work independently on the further development of his ideas. They include A. A. Ivina at the Department of Philosophy of the Moscow State University, G. A. Smirnov at the Institute of the History and Theory of the Natural Sciences in Moscow, H. Vessel of the Department of Philosophy at the Humboldt University in Berlin, E. A. Sidorenko and A. M. Fedina at the Institute of Philosophy in Moscow, L. A. Bobrova, Dept. of Logic, Moscow State University.

The distinctive nature of the logical investigations of Zinov'ev and his colleagues are quite fully expounded in the present book. It is, briefly, the attempt to construct a particular logical conception of contemporary logic, broad enough in scope to encompass the whole range of issues beginning with a general theory of signs and concluding with logical analyses of such scientific problems as motion, causality, space and time.

In Zinov'ev's conception, the theory of deduction has a central place. But this theory is carefully to be distinguished from generally accepted theoretical interpretations in the systems of classical and intuitionist mathematical logic. As he writes in an earlier essay, which may be taken as a technical introduction to this monograph ('Logical and Physical Implication', p. 91);

Under the influence of the mathematization of sciences and the successes of mathematical logic in the last few decades, a special branch of logical-philosophical research has developed. Its essence is the use of the ideas, the apparatus (*calculi*) and methods of mathematical logic and mathematics (exact methods) in the solution of a series of traditional problems of formal logic and philosophy as well as of new problems of the methodology of science specifically connected with the development of contemporary science.

In this branch one considers the epistemological interpretation of formal systems of logic, constructs formal systems for the express purpose of describing various aspects of human cognitive activity, solves certain problems of philosophy by means of logical-mathematical constructions, and uses the accomplishments of logic to overcome philosophical difficulties in the natural sciences.

Among the problems thus researched we find causal and nomological statements, scientific laws, operational and inductive definitions, models, reasons for and means of limiting the rules of judgment in various domains of science, construction and inter-relating of theory, etc. There are numerous works on these subjects, the study of each of which requires definite specialisation.

There are philosophers, logicians, and mathematicians who, for a variety of reasons, are inclined to exclude this branch of logical-philosophical investigation from the sphere of philosophy. On the other hand, there are others who hold that the application of exact methods in philosophy does not fall outside the realm of philosophy if the results thereof are strictly compared with the previous methods and results of philoso-

phy. Regardless of the outcome of this dispute, the fact remains that exact methods are applied to problems which have always been considered philosophical.

Efforts to use exact methods in philosophy proper are found in the works of the positivist philosophers who reduced the problems of philosophy to problems of formal logic. Whence the impression that the application of such methods in philosophy is a mark of positivism. This view is incorrect. The use of exact methods in philosophy (as the refusal to use them) in itself says nothing [about] the philosophical position of the user. And if there are erroneous philosophical views attached to such use, they can be successfully opposed not by ignoring exact methods in solving philosophical problems but rather by carefully and expertly using them and by developing new methods of this type.

The use of exact methods as a possible mode of philosophical investigation can, if the object and tasks of philosophy are properly understood, lead to great progress in bringing it into consonance with the thought-structure of contemporary science. The application of these methods marks a transition to the theoretical level in the solution of philosophical problems. At this level, new knowledge [of] the objects of investigation comes not through observation and experiment (as happens on the empirical level) but through logical judgments in the framework of a given or newly developed theory (i.e., special groups of concepts and statements united by rules of logic). The value of the theoretical level is well known and we need not discuss it here. The same is true in philosophy (not as an object about which one can talk but as a means of investigation).

As regards the question [of] the non-reducibility of philosophy to formal logic, the application of these methods makes it possible not just to declare this as a preconceived notion but strictly to demonstrate it for any philosophical problem.

and a few pages later,

In the wide sense of the term the problem of logical implication can be formulated as follows: is a given logical construction suited to the description of the properties of logical implication? Do the formulae of a given formal construction of logic correspond to the intuitive understanding of logical implication? By intuitive understanding of logical implication we here mean the understanding which grows up in people perforce of habitually judging (reasoning, drawing conclusions) and observing such activity in others. The habit of judging according to the rules of logical implication comes as the result of personal experience, education and acquired science. What form must a logical system have in order to satisfy the intuitive understanding of logical implication?

One here talks about intuitive understanding because logic presents the results of its investigations in the form of an apparatus for the practical use of those who judge, infer, reason, prove, etc. And they, of course, compare these results precisely with their usually clear understanding of the rules of these operations. Of course, intuitive understanding is not something once and for all given and absolutely universal. But there are, all the same, some stable and general aspects and they suffice for mutual understanding. What is more, the task of logical constructions is not limited to following intuition passively. Their basic task consists in clarifying and standardizing intuitive understanding, in systematizing the rules of logical implication, in providing the means of establishing their reliability and the means of predicting such reliable rules which have not yet been met in the experience of judgment or which have not been actively realized. What form must a logical system have so that it – without thereby being limited – will be as close as possible to the intuitive bases?

After the appearance in 1967 of the first edition of the present work, Zinov'ev published several further studies. Based upon discussion (by R.S.C.) with him during Summer 1971, we can briefly mention these new results. The most recent, *The Logic of Science*, 1971, mainly coincides with the present work. However, in it the author provides a considerably more detailed and thereby more easily grasped explanation of the basic principles and fundamental notions of his conception of logic; moreover he expounds this conception in the 1971 work with a minimal formal apparatus. It should be accessible to a much wider circle of readers. In addition, the new book has an extensive section devoted to the methodology of physical science with elaborate studies of space, time, causality, motion, etc. There, Zinov'ev criticizes the point of view which claims that a special logic [quantum logic] is necessary for micro-physics, different from the logical and methodological formalism of macro-physics. In a related section, Zinov'ev expounds his conception of the universality of logic, by which he understands the independence of logical rules from the specific domains of application of these rules in the sphere of objects.

In the 1971 book, Zinov'ev proceeds from an analysis of 'ontological terminology' (his phrase), perhaps more easily identified as 'physical terminology', and from his exposition of logical rules of operation with these ontological terms, to his major conclusions. Many problems which are discussed in the philosophy of physics, and which are particularly connected with modern discoveries, are shown to be only terminological, independent of the success or inadequacy of physics proper. Such, for example, is the central problem of the reversibility of time. Indeed, in Zinov'ev's analysis, many assertions which traditionally have been construed to be empirical or physical, turn out to be the implicit consequences of definitions of terms; or at any rate they may be conceived thus without contradiction or empirical refutation. An example is the assertion that a body cannot be in different places at the same time.

In another work, his newer *Complex Logic* published in 1970, Zinov'ev offers a systematic account of the formal apparatus of logical implication. Here the most interesting part is perhaps his theory of quantifiers. Zinov'ev formulates the entire range, the totality, of different logical systems of the theory of quantifiers which satisfy differing corresponding intuitive premises, and he investigates their properties. In particular, the 1970 monograph provides a fuller investigation of the strict theory of quantifiers.

Evidently, Zinov'ev's conception of logic is opposed to certain intellectual trends of contemporary logic and methodology of science. He ruefully contrasts the insignificance of many problems treated in the standard methodology of science with the grandiosity of methodological claims, and with the tendency to apply methodological formulations beyond their domain. And he is critical of the misuse of deductive logic and the accepted formulations of the methodology of natural science in social-scientific investigations.

A. A. Zinov'ev foresees fruitful applications of his conception of 'complex logic' throughout methodological investigations into the natural and the social sciences, and in the theory of values as well, but he also sees the need for extensive further work in pure logic. We warmly anticipate his ongoing investigations.

*Boston University Center for the
Philosophy and History of Science*
Spring 1972

R. S. COHEN
M. W. WARTOFSKY

NOTE

Some of Zinov'ev's studies have appeared in English:

[*SSP: Soviet Studies in Philosophy* (International Arts and Sciences Press, White Plains, New York)]

1. *Philosophical Problems of Many-Valued Logic* (ed. and transl. by G. Kung and D. D. Comey), Humanities Press, New York; and D. Reidel, Dordrecht, Holland; 1963.
2. 'Logical and Physical Implication', in: *Problems of the Logic of Scientific Knowledge*, (ed. by P. V. Tavanec), pp. 91-159 (transl. by T. J. Blakeley) (Humanities Press, New York; and D. Reidel, Dordrecht, Holland; 1970). Original: *Problemy logiki naučnogo poznanija*, Moscow, 1964.
3. 'Two-Valued and Many-Valued Logic', *SSP* 2, 69-84 (Summer-Fall 1963); from: *Filosofskie voprosy sovremennoj formal'noj logiki* (ed. by P. V. Tavanec), Institute of Philosophy, U.S.S.R. Academy of Sciences, Moscow, 1962.
(Note: 7 of the 12 papers in the original volume appeared in this issue of *SSP*.)
4. 'On the Application of Modal Logic in the Methodology of Science', *SSP* 3, 20-26 (Winter 1964-65); from *Voprosy filosofii*, 1964, No. 8.
5. 'On Classical and Non-Classical Situations in Science', *SSP* 7, 24-33 (Spring 1969); from: *Voprosy filosofii*, 1968, No. 9.

6. 'On the Logic of Microphysics', *SSP* 9, 222–236 (Winter 1970–71); from: *Voprosy filosofii* 1970, No. 2 (Part of a symposium on Logic and Quantum Mechanics, with other contributions by B. G. Kuznecov: 'On Quantum-Relativistic Logic'; R. A. Aronov: 'Toward a Logic of the Microworld'; I. P. Staxanov: 'The Logic of "Possibility"').

PREFACE

Logical theory of scientific knowledge is the investigation of scientific knowledge within the framework of the concepts and methods of logic. The bases for such investigations in contemporary logic were provided by Frege, Russell, Lewis, Łukasiewicz, Carnap, Reichenbach, Tarski, Ajdukiewicz, and many other scientists, whose works are generally quoted in logical-philosophical writings.

The present work offers a somewhat systematic construction of that conception of the logical theory of scientific knowledge which was to be found in incomplete form in the author's earlier works. This construction has to do only with the fundamentals of the theory of scientific knowledge. Therefore, the book is to be taken neither as a textbook nor as a presentation of what is generally done in corresponding branches of logic.

Some details of this way of looking at things are truisms to be found in most other works on the same subject. But in its general character and on the most essential points it is essentially different from such other works, as the reader can see by carrying out the comparison.

The basic object of this book is to present as simply and systematically as possible the ideas and principles which seem to us to be the most promising for the theory of scientific knowledge. Therefore, the formal logical apparatus which could be developed on this basis has been held to an absolute minimum.

Mathematical logic has carried the day in the theory of scientific knowledge. But one finds in logical circles a prejudice that mathematical logic as it is found in textbooks (propositional calculus and predicate calculus, with some expansions) is the only possible logical apparatus for the solution of all problems of the theory of scientific knowledge. The fact of the matter is that mathematical logic in its normal form is only a fragment of the theory of scientific knowledge and the other sections cannot be reduced to it. This is particularly true of the theory of terms, of the forms of logical entailment, syllogistics, physical entailment, and other sections of logic which are stressed in the present volume.

This could be called “complex logic” for the following reasons. Contemporary logic has developed into an extensive and sophisticated science. It has need of systematization. This is not simply a question of a suitable presentation of its results in teaching. It is more an effort to find a notion of logic itself such that the various calculi, theories, trends, etc., appear as natural fragments of a single system. Our present effort is in this direction. In particular, classical logic, intuitionist logic, the system of strong implication, and other logical systems, which are usually taken as different solutions of one and the same problem of the definition of the rules of logical inference, are here viewed as solutions of different problems, i.e., as different fragments of a single logical system. For this purpose we need a unified logical structure – a type of logical “base” – which must contain the various logical calculi and which must itself have the form of a deductive system. Complex logic is intended for this purpose. Further, the method of construction we have chosen – i.e., the construction of the different branches of logic through appropriate additions to the general theory of deduction (to propositional logic) – means that whole groups of logical laws fall outside the purview of logic. We mean the laws which join propositions with different structures and do not occur among the formulae of certain calculi. Such are the laws joining modal signs and quantifiers, implication signs and relation signs, signs of predication and of class-inclusion, etc. What is more, we find in logic implicit assumptions which can be explicated only by formulating a system of “residual” assertions in order to unify the various sections of logic into a unified, complex logical system. Our presentation tries to take such “residual” laws of logic into account. We would note, in conclusion, that this conception of logic makes possible a more differentiated analysis of logical forms than is usually the case. This happens most particularly in the treatment of the various forms of entailment.

To make our formulations as compact and intuitive as possible, we will use the symbols

$$\cdot, \vee, \therefore, \sim, \rightarrow, \leftrightarrow$$

in the following sense:

1) $X \cdot Y$ for “ X and Y ”, “Each of X, Y ”; $X^1 \cdot X^2 \cdot \dots \cdot X^n$ – “ X^1 and X^2 and ... and X^n ”, “Each of X^1, \dots, X^n ”; here and below X, Y, X^1, \dots, X^n are any sentences;

2) $X \vee Y$ for “at least one of X and Y ”; $X^1 \vee X^2 \vee \dots \vee X^n$ for “at least one of X^1, X^2, \dots, X^n ”.

3) $X:Y$ for “Either X or Y ”, “ X or Y ”, “One and only one of X, Y ”; $X^1:X^2:\dots:X^n$ – “Either X^1 , or X^2 , ..., or X^n ”, “One and only one of X^1, X^2, \dots, X^n ”;

4) $\sim X$ for “Non- X ”, “It is not as affirmed in X ”;

5) $X \rightarrow Y$ for “If X then Y ”;

6) $X \leftrightarrow Y$ for “ X if and only if Y ”; an abbreviation for $(X \rightarrow Y) \cdot (Y \rightarrow X)$.

In the sequel we will be more precise about the signs “and”, “or” and “not”. For the moment, however, we will assume that their meaning is known to the reader at least to the extent necessary for explaining the matter at hand. The same is the case for the signs, “if... then” and “if and only if”. In other words, we assume that the reader already has some skill in handling logical tools, i.e., that he has the logical minimum.

Definitions and assertions will be numbered with the help of Di , Ai and Ti , where i is the ordinative numeral of the definition or assertion in a given paragraph; A indicates that the sentence is taken as an axiom; T indicates that the sentence can be obtained as an inference from axioms. In cross-referencing, the chapter and paragraph numbers will be written after the i . For example, $T3V7$ will designate the third theorem of the seventh paragraph in the fifth chapter.

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