

**Mathematical
Surveys
and
Monographs**
Volume 116

Fourier Analysis in Convex Geometry

Alexander Koldobsky



American Mathematical Society

Contents

Chapter 1. Introduction	1
Chapter 2. Basic Concepts	13
2.1. Star bodies	13
2.2. Convex bodies	16
2.3. Radon transforms	27
2.4. The Gamma-function	30
2.5. The Fourier transform of distributions	33
2.6. Fractional derivatives	39
2.7. Positive definite distributions	40
2.8. Stable random variables and the function γ_q	44
Chapter 3. Volume and the Fourier Transform	49
3.1. The first examples: hyperplane sections of ℓ_q -balls	49
3.2. A general formula for the volume of hyperplane sections	53
3.3. The parallel section function and the Fourier transform	55
3.4. Parseval's formula on the sphere	62
3.5. Remarks and further results	69
Chapter 4. Intersection Bodies	71
4.1. A Fourier analytic characterization	71
4.2. k -intersection bodies	75
4.3. L_p -balls as k -intersection bodies	80
4.4. The second derivative test	85
4.5. Remarks and further results	91
Chapter 5. The Busemann-Petty Problem	95
5.1. A Fourier analytic solution	95
5.2. How can one make the answer affirmative?	98
5.3. The affirmative part via spherical harmonics	100
5.4. Zvavitch's generalization to arbitrary measures	105
5.5. Remarks and further results	110
Chapter 6. Intersection Bodies and L_p -Spaces	115
6.1. L_p -spaces and positive definite functions	115
6.2. Schoenberg's problems on positive definite functions	121
6.3. Intersection bodies and embeddings in L_p , $p < 0$	126
6.4. Remarks and further results	139
Chapter 7. Extremal Sections of ℓ_q -Balls	143
7.1. The case of the cube, K. Ball's theorem	143

7.2. The case $0 < q \leq 2$	147
7.3. Remarks and further results	149
Chapter 8. Projections and the Fourier Transform	151
8.1. A formula for the volume of hyperplane projections	151
8.2. Extremal hyperplane projections of ℓ_q -balls	152
8.3. Projection bodies	155
8.4. The Shephard problem	157
8.5. Remarks and further results	161
Bibliography	163
Index	169