

## FOURIER COSINE–LAPLACE GENERALIZED CONVOLUTION INEQUALITIES AND APPLICATIONS

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*Abstract.* We introduce several weighted  $L_p(\mathbb{R}_+)$ -norm inequalities and integral transform related to the generalized convolution with a weight function for the Fourier cosine and Laplace transforms. Some applications of these inequalities to estimate the solutions of some partial differential equations are considered. We also obtained solutions of a class of the Toeplitz plus Hankel integro-differential equations in closed form.

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