Pure Appl. Opt. 3 (1994) 249–253. Printed in the UK

LETTER TO THE EDITOR

Fourier transform evaluation of digital interferograms for diffusivity measurement

G Schirripa Spagnolo†, D Paoletti†, D Ambrosini‡, V Bagini‡ and M Santarsiero‡

† Dipartimento di Energetica, Università Dell'Aquila, Località Monteluco di Roio, 67040 Roio Poggio (AQ), Italy

‡ Dipartimento di Fisica, Universita' 'La Sapienza', 00185 Roma, Italy

Received 1 February 1994

Abstract. A Fourier transform algorithm for practical processing in terms of phase distribution of TV speckle interferograms in quantitative measurements of isothermal diffusion coefficient of transparent mixtures is discussed. Some applications illustrating the fringe analysis for diffusivity measurements of a binary liquid mixture are presented.

In separate papers [1, 2] digital speckle pattern interferometry has been considered as an interesting valid alternative to holographic methods for measurements of the diffusion coefficient of a liquid mixture. In an interferogram the quantitative measurement of the refractive index variation is based on the determination of the phase distribution from the recorded and stored fringe patterns. Conventional fringe finding and tracking methods have been used [2, 3]. In this letter a Fourier transform algorithm [4–8] is developed for accurate phase evaluation of video correlograms obtained by means of a TV-speckle interferometer. The method allows to calculate the interference phase pointwise, even between fringe extremes and thus has some advantages on other computation procedures. This phase distribution is then combined by the computer with geometric data relative to experimental set-up to yield refractive index distributions.

The digital speckle interference patterns produced with a conventional out-of-plane displacement sensitive TV speckle interferometer [2, 9] are recorded by a CCD camera. The interferogram is converted into corresponding video signal and sampled to yield a digital picture, which can be stored in a digital frame memory. Linear fringes are introduced by rotating the object beam of the interferometer of an angle $\Delta\alpha$. The details of the experimental apparatus are described in [2].

The correlation fringes, representing the loci of constant variation of the refractive index, are obtained by subtracting a reference frame (that depicts an initial state, for example a fringe pattern captured at the time t_1) from a second frame (captured at the time $t_2 = t_1 + \Delta t > t_1$) and squaring the difference. In practice the intensity of this video correlogram with carrier fringes can be written [2, 9]

$$b_{sq} = [I(x, y, t_1) - I(x, y, t_2)]^2 = a(x, y) - a(x, y)\cos[2\pi u_0 x + \varphi(x, y, \Delta t)]. \qquad (1)$$

The function $a(x, y)$ represents the speckle pattern averaging over speckles [2], u_0 is the spatial frequency of the carrier fringes due to the rotation of the illuminating beam ($u_0 = \Delta\alpha/\lambda$), and the phase $\varphi(x, y, \Delta t)$ is related to the refractive index variation Δn by

$$\varphi(x, y, \Delta t) = kl\Delta n(x, y, \Delta t) \qquad (2)$$

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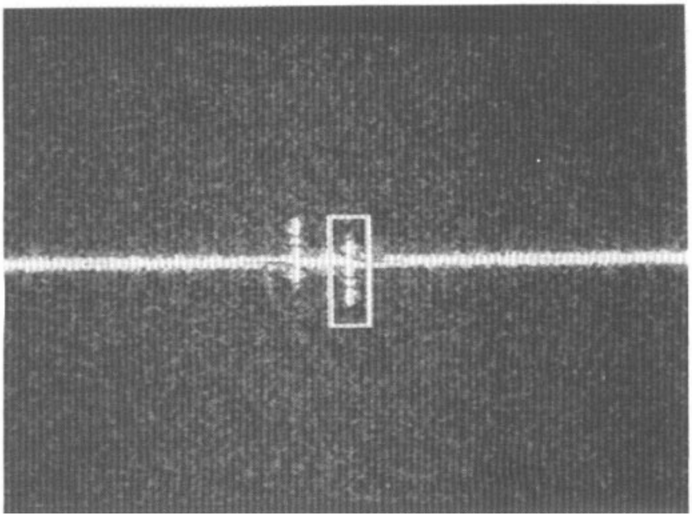

Figure 1. Typical Fourier spectrum of a specklegram with filtering mask.

where k is the wavenumber and l is the thickness of the diffusion cell.

Equation (1) may be reformulated as

$$b_{sq}(x, y, \Delta t) = a(x, y) - q(x, y, \Delta t)\exp(j2\pi u_0 x) - q^*(x, y, \Delta t)\exp(-j2\pi u_0 x) \tag{3}$$

where

$$q(x, y, \Delta t) = \tfrac{1}{2}a(x, y)\exp[j\varphi(x, y, \Delta t)]. \tag{4}$$

The Fourier transform of $b_{sq}(x, y, \Delta t)$ came out to be

$$B_{sq}(u, v, \Delta t) = A(u, v) - Q(u - u_0, v, \Delta t) - Q^*(-u - u_0, -v, \Delta t) \tag{5}$$

where capital letters are used for Fourier transforms.

If we assume that the spatial variations along the x-axis of $a(x, y)$ and $\varphi(x, y, \Delta t)$ are slow compared with the spatial frequency u_0, the function $B_{sq}(u, v, \Delta t)$ has a zero-order peak and two first-order peaks around $u = \pm u_0$. There are also higher order peaks at $u = \pm 2u_0, \pm 3u_0, \ldots$, but these values result from non-linearities in the recording and are small enough to be neglected. Only one of the first-order peaks is used to calculate the spatial phase distribution. The function $Q(u - u_0, v, \Delta t)$ can be isolated with a filtering mask centred at u_0 (see figure 1). Incidentally, it can be noted that the speckle averaging question already assumed in equation (1) can also be performed in the Fourier domain. The carrier frequency can be removed by shifting $Q(u - u_0, v, \Delta t)$ by u_0 towards the origin of the Fourier plane to obtain $Q(u, v, \Delta t)$. The inverse Fourier transform of $Q(u, v, \Delta t)$ with respect to u and v yields equation (4). From equation (4) the wrapped phase can be calculated pointwise by

$$\varphi_w(x, y, \Delta t) = \tan^{-1}\left(\frac{\Im[q(x, y, \Delta t)]}{\Re[q(x, y, \Delta t)]}\right) \tag{6}$$

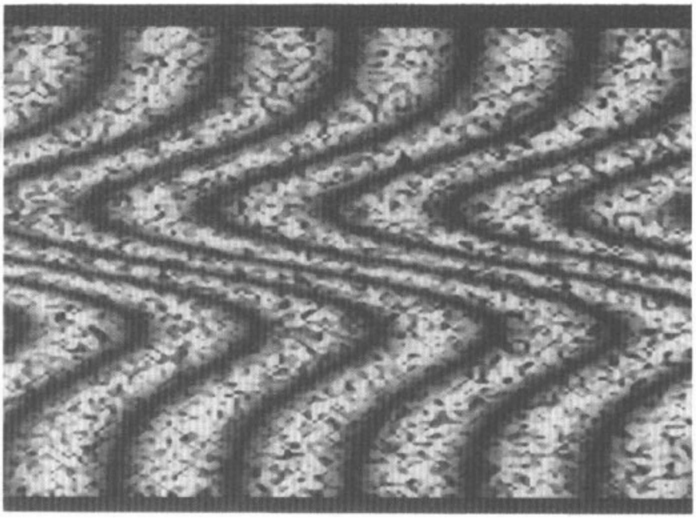

Figure 2. Synthetic fringe pattern.

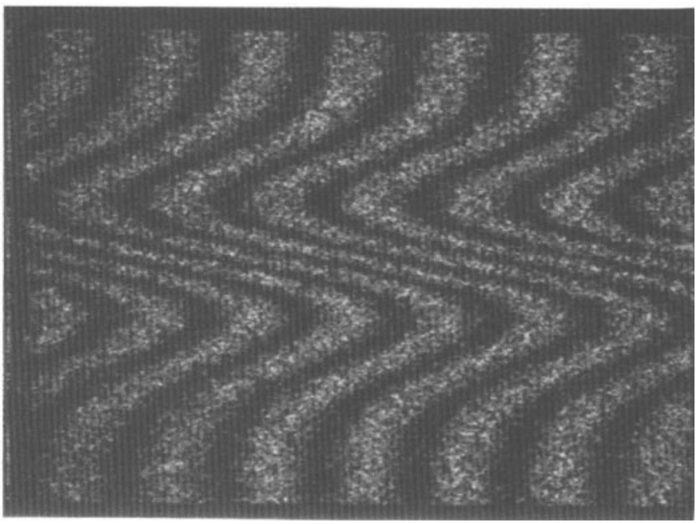

Figure 3. Typical DSPI correlogram ($t_1 = 890$ s, $t_2 = 990$ s).

where $\Re[q(x, y, \Delta t)]$ and $\Im[q(x, y, \Delta t)]$ denote the real and imaginary parts of $q(x, y, \Delta t)$, respectively. The phase distribution is wrapped into this range and 2π jumps occur for variations of more than 2π. Using equation (6) and taking into account the signs of $\Im$ and $\Re$, then phase principal values ranging from $-\pi$ to π are obtained. These discontinuities can be corrected by adding or subtracting 2π according to the phase jump ranging from π to $-\pi$ or vice versa. A practical phase unwrapping algorithm developed by Macy [5] is used for processing module 2π phase maps of equations (6) in reasonable times. Successively the refractive index variation calculated by the unwrapped phase φ_u according to equation (2) can be expressed as

$$\Delta n(x, y, \Delta t) = \frac{\lambda}{2\pi} \frac{\varphi_u(x, y, \Delta t)}{l} + \text{const.} \tag{7}$$

The constant of equation (7) is undetermined. Indeed with the application of Fourier method for the phase retrieval there is no way to calculate the overall constant additive phase term ($\varphi = \varphi_u + \text{const}$).

Table 1. Observed values of the diffusion of BrLi (1 M).

Picture	t_1 (s)	t_2 (s)	W (cm)	D (10^{-5} cm^2 s^{-1})
1	890	990	0.32	1.406
2	1780	2000	0.45	1.400
3	2400	2700	0.53	1.398
4	4200	4800	0.70	1.399

Now an ideal free diffusion process in which two mixtures of concentrations c_1 and c_2 are brought into contact at time $t = 0$ so that their boundary coincides with the plane $y = 0$, is considered. The variation of the refractive index is assumed to occur only along the y direction (one-dimensional diffusion). For this reason in the following expressions we omit, for the sake of brevity, the dependence on x. For small ranges of concentration, the refractive index (n) of a dilute liquid solution varies linearly with the concentration (c), and this relationship allows direct translation of optical path changes into concentration changes [10]. To a first approximation, we can write [11]

$$n(y, t) = \left(\frac{\mathrm{d}n}{\mathrm{d}c}\right)_0 c(y, t) + n_0 \tag{8}$$

where n_0 = constant and $(\mathrm{d}n/\mathrm{d}c)_0$ is the mean value of the derivative $\mathrm{d}n/\mathrm{d}c$ for the applied concentration range. The solution of Fick's second law, which rules a free diffusion process is [12]

$$c(y, t) = \frac{(c_1 + c_2)}{2} + \frac{(c_2 - c_1)}{\sqrt{\pi}} \int_0^{y/\sqrt{4Dt}} \mathrm{e}^{-\eta^2}\,\mathrm{d}\eta. \tag{9}$$

Using equations (8) and (9), the change of the refraction index of the mixture between the two recording times takes the form

$$\Delta n(y, \Delta t) = \left(\frac{\mathrm{d}n}{\mathrm{d}c}\right)_0 \frac{(c_2 - c_1)}{\sqrt{\pi}} \left(\int_0^{y/\sqrt{4Dt_2}} \mathrm{e}^{-\eta^2}\,\mathrm{d}\eta - \int_0^{y/\sqrt{4Dt_1}} \mathrm{e}^{-\eta^2}\,\mathrm{d}\eta \right). \tag{10}$$

The shape of this curve as a function of y presents two characteristic extremes, whose positions are related to the diffusion coefficient D by

$$D = \frac{w^2[(1/t_1) - (1/t_2)]}{8 \ln(t_2/t_1)} \tag{11}$$

where w is the distance between the two extremes (for more details see references [2, 12, 13]).

Owing to the linear relationship between Δn and φ_u (see equation (7)), measurements of D may be made automatically from the unwrapped phase of the interferogram [14].

In order to test the efficacy of the above discussed method and to evaluate the effects of the related errors, we made a computer simulation. Synthetic fringes may be calculated with speckle noise generated according to a multiplicative noise model [15]. Figure 2 shows the grey-scale monitor display of the synthetic fringe pattern which could be obtained for a coefficient of diffusion of 1.404×10^5 cm^2 s^{-1}, tilt of object beam of $\Delta\alpha = 0.1$ rad, recording times $t_1 = 890$ s and $t_2 = 990$ s. The estimated accuracy of this method is better than 0.5%. The method has also been tested experimentally on a set of interferograms, captured during the diffusion process relative to 1 M aqueous solution of LiBr at 25 °C. Figure 3 shows a typical interferogram. From each interferogram a value of the diffusion constant D was deduced from the parameter t_1, t_2 and w. The results are shown in table 1.

The average value was 1.401×10^5 cm^2 s^{-1}. This compares favourably with the handbook value of 1.404×10^5 cm^2 s^{-1} [9].

The authors thank F Gori for his keen interest and encouragement. We acknowledge G Pasqualoni for everything concerning image processing and A Colagrande for technical assistance.

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