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Fractal Antennas
A Novel Miniaturization Technique for wireless networks

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ABSTRACT

In the recent years and with the multiplication and miniaturization of telecommunications systems and their integration in restricted environments, such as Smart-phones, tablets, cars, airplanes, and other embedded systems. The design of compact multi-bands and Ultra Wide Band (UWB) antennas becomes a necessity. One of the interesting techniques to provide this kind of antenna is the use of fractal structures.

Keywords: Fractal antennas, Broad-Band antennas, Ultra Wide Band Antennas, Multi-Band antennas, wireless communications.

1 Introduction

The numerous applications of telecommunication to the advances of technology have necessitated the exploration and utilization of most of the electromagnetic spectrum. Also, the advents of broadband systems have demanded the design of broadband and multi-band antennas. In addition, the use of simple, small, lightweight, and economical antennas, designed to operate over the entire frequency band of a given system, would be most desirable. In recent years, one of the techniques used to design this kind of antenna is the use of the fractal structures.

The term “Fractal” means linguistically “broken” or “fractured” from the Latin “fractus.” This term was created by Benoît MANDELBROT 40 years ago in 1974.

Fractals are geometric shapes, which cannot be defined using Euclidean geometry, are self-similar and repeating themselves on different scales like clouds, mountains, coasts, lightning, etc. [1, 2].

The fractal geometry has been applied to many fields such as:

- Medicine: structure of the lungs, intestines, heartbeat,
- Meteorology: clouds, vortex, ice, rogue waves, turbulence, lightning structure,
- Volcanology: prediction of volcanic eruptions, earthquakes.
- Astronomy: the description of the structures of the universe, craters on the Moon, distribution galaxies. [3].

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Also, fractal geometry has been used in the electromagnetic, and especially in the design of antennas. Several studies have adopted fractal structures and showed that this technique can improve the performances of the antenna and it is one of the techniques to design antennas with multi-band and broad-band behavior [4].

In this paper, we give some generalities of fractal geometries and their dimensions, after that, we describe some linear fractal geometries such as KOCH, SIERPINSKI, DRAGON, TREE, CIRCULAR, CANTOR SET, HILBERT, MINKOWSKI, and finally we discuss the applications of these geometries and their performances in the design of the miniaturized antennas.

2 The Fractal Dimension

Usual dimensions used are integer values. For example, the dimension of the line is 1; the dimension of a cube is 3. For fractal geometries, the dimension used is not necessarily integer value, but we use HAUSDORFF dimension [5]. A fractal consists of smaller replicas of itself. Its HAUSDORFF dimension can be calculated as follows:

$$d = \frac{\ln(n)}{\ln(h)} \tag{1}$$

With: the fractal consists of (n) copies whose size has been reduced by a factor of h.

Here is an example of calculating a HAUSDORFF dimension of KOCH Fractal.

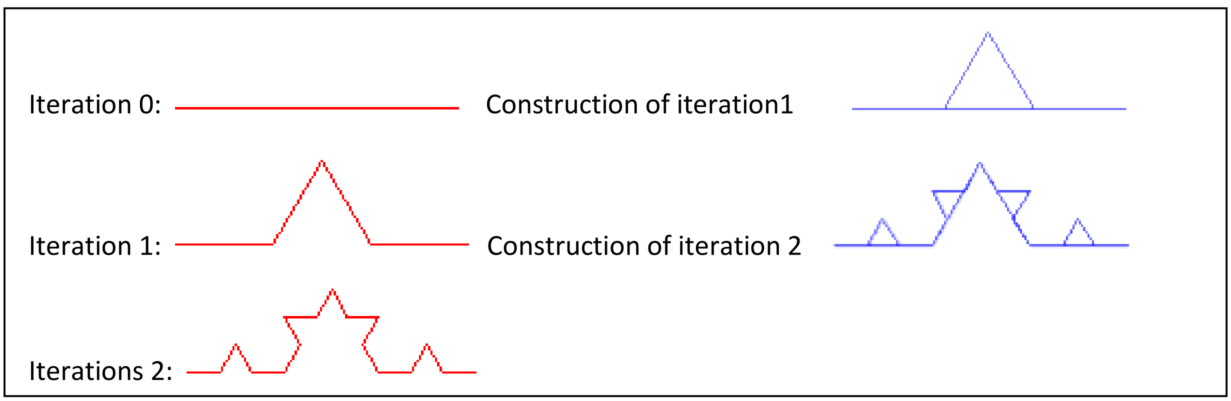


Figure 1: Construction of KOCH fractal iterations

As shown in the figure 1, the geometry of the first iteration is made by four copies of the basic geometry (iteration 0), so n =4. Also, the lengths of the segments making up the geometry of the first iteration, are reduced by a factor of 3, so h = 3.

The HAUSDORFF dimension of KOCH Fractal is:

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(4)}{\ln(3)} = 1.26 \tag{2}$$

3 Types of Fractals

Fractals are classified among three major categories.

- Linear: based on the iteration of linear equations (HILBERT, KOCH, SIERPINSKI, Dragon ...),
- Nonlinear: based on the iteration of complex numbers (MANDELBROT, JULIA ...),
- Random: based on the introduction of a random parameter in the iteration to obtain irregular shapes (such as mountains or clouds)

4 Linear Fractal Geometries

In this section we study the famous fractal geometries, the methods used for the generation of different iterations and calculating their Hausdorff dimensions.

4.1 The KOCH Structure

This structure was invented by the Swedish mathematician HELGE VON KOCH in 1906 before the invention of the term "fractal". There are several variations of this structure:

4.1.1 The KOCH Curve

As shown in Figure 2, the construction of this curve is made from a segment by applying the following steps:

1. The segment is divided into three segments of equal length.
2. An equilateral triangle whose base is the middle segment of the first stage is constructed.
3. Segment which was the base of the triangle of the second step is eliminated.

After these three steps, the resulting object has a shape similar to a cross section of a witch hat.

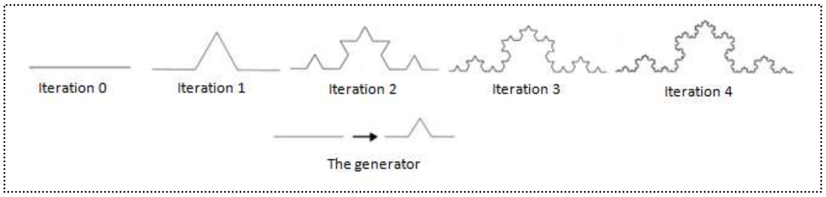


Figure 2: the 4 iterations of the KOCH Curve

4.1.2 The KOCH snowflake

The procedure to build the Koch snowflake is the same as that used for the construction of the KOCH curve except that the base is a triangle, which means that the procedure is repeated three times for each iteration (Figure 3).

The HAUSDORFF dimension of the KOCH structure is given by the equation (2).

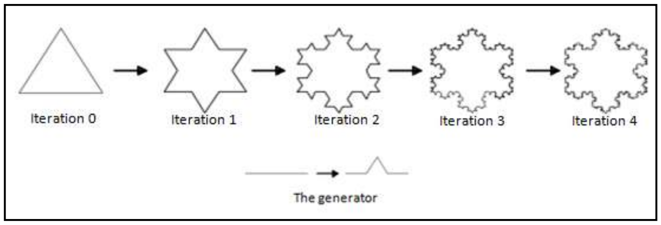


Figure 3: the 4 iterations of the KOCH Snowflake

4.2 The SIERPINSKI Structure

This structure was invented by Polish mathematician SIERPINSKI. There are several variations of this structure:

4.2.1 The SIERPINSKI triangle

The construction of this triangle is made from a solid equilateral triangle and applying the following steps:

a) An equilateral triangle is built and will be taken as a base.
b) Subdivide it into four smaller congruent equilateral triangles and remove the central one.
c) Repeat step 2 with each of the remaining smaller triangles.

The Figure 4 shows the first four iterations of the SIERPINSKI triangle.

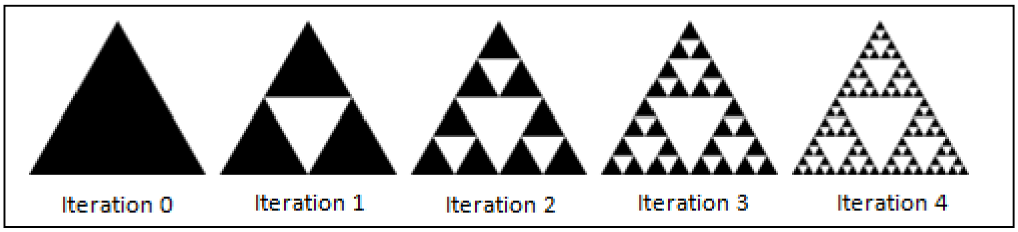


Figure 4: the 4 iterations of the SIERPINSKI Triangle

The HAUSDORFF dimension of the SIERPINSKI Triangle is given by the equation (3).

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(3)}{\ln(2)} = 1.58 \tag{3}$$

4.2.2 The SIERPINSKI carpet

The construction of this structure is made from a solid square and applying the following steps:

a) The square is cut into 9 congruent sub-squares in a 3-by-3 grid
b) The central sub-square is removed.
c) The same procedure (1 and 2) is then applied recursively to the remaining 8 sub-squares.

The Figure 5 shows the first four iterations of the SIERPINSKI Carpet.

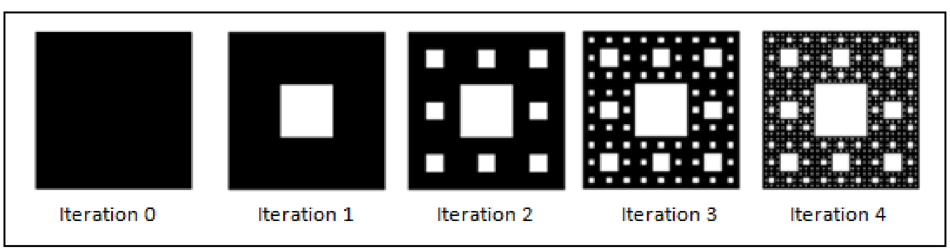


Figure 5 : the 4 iterations of the SIERPINSKI Carpet

The HAUSDORFF dimension of the SIERPINSKI Carpet is given by the equation (4).

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(8)}{\ln(3)} = 1.89 \quad (4)$$

4.3 The Dragon structure

The dragon's name comes from the fact that, for high iterations, the shape of the structure is close to that of the Dragon. The construction of this structure is made from a simple line and applying the following steps:

a) For the first iteration (Figure 6)

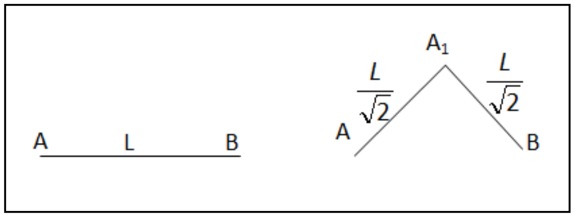


Figure 6: Generation of the first iteration of the Dragon structure

- We move from the segment [AB] to the segment [AA_1] by a rotation with the centre A and the angle $\frac{\pi}{4}$ followed by a scaling whose center is A and ratio $\frac{\sqrt{2}}{2}$;
- We move from the segment [AB] to the segment [A_1B] by a rotation with the center B and the angle $-\frac{\pi}{4}$ followed by a scaling whose center is B and ratio $\frac{\sqrt{2}}{2}$;

b) For the other iterations, we apply the same procedure on each segment. The figure 7 shows the iterations and the 20th iteration of the DRAGON structure.

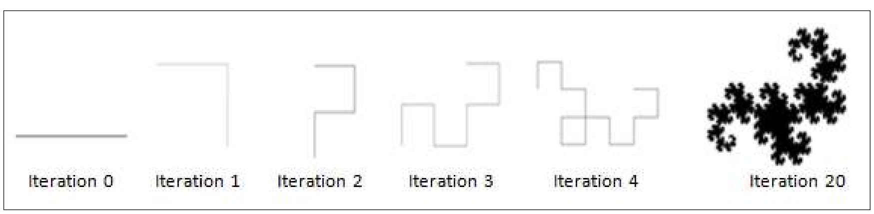


Figure 7: the four first iterations and the 20th iteration of the DRAGON structure

The HAUSDORFF dimension of the DRAGON structure is given by the equation (5).

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(2)}{\ln(\sqrt{2})} = 2 \quad (5)$$

4.4 The Tree Structure

This structure has the same shape of a Tree; there are several kinds of this structure:

4.4.1 The Tree

To generate this kind of fractal structure, we apply the following steps:

1. For the initiator or "iteration 0", the structure has 3 branches, the vertical one is the "parent" the 2 others are the "Childs" with an inclination (θ).
2. At each iteration, the same shape is generated with a reduction factor "h" (Figure8).

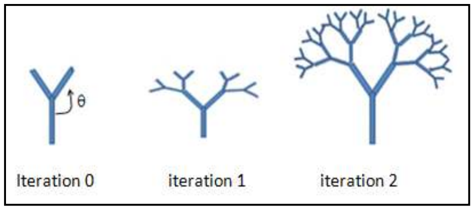


Figure 8: The three first iterations of the TREE Fractal structure

The HAUSDORFF dimension of the Tree structure is given by the equation (6).

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(4)}{\ln(h)} \quad (6)$$

If the reduction factor h=2, the HAUSDORFF dimension is given by the equation (7).

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(4)}{\ln(2)} = 2 \quad (7)$$

4.4.2 The H-Tree

The H-TREE geometry is a modified Tree geometry with the same concept (Figure9). The initiator is a structure like the letter "H". On each iteration, we create 4 copies of the previous iteration with a reduction factor "R".

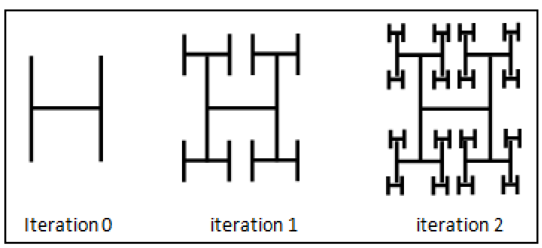


Figure 9: the three first iterations of the H-TREE Fractal structure

For the H-TREE structure, the HAUSDORFF dimension is given by the equation (8).

$$d = \frac{\ln(n)}{\ln(R)} \quad (8)$$

If the reduction factor R =2, the HAUSDORFF dimension is given by the equation (9).

$$d = \frac{\ln(n)}{\ln(R)} = \frac{\ln(4)}{\ln(2)} = 2 \quad (9)$$

4.4.3 PYTHAGORE Tree

This structure is constructed with squares. It is named “PYTHAGORE” because each triple square in touch creates a right triangle. To construct this fractal structure, we apply the following steps:

1. we built a simple square,
2. On this square, we construct two other squares, each one is smaller by a factor of $\frac{\sqrt{2}}{2}$, such as the corners of the squares are in contact.
3. The procedure is applied recursively to each square, to infinity.
4. The figure 10 shows the three iterations of the PYTHAGORE Tree Structure.

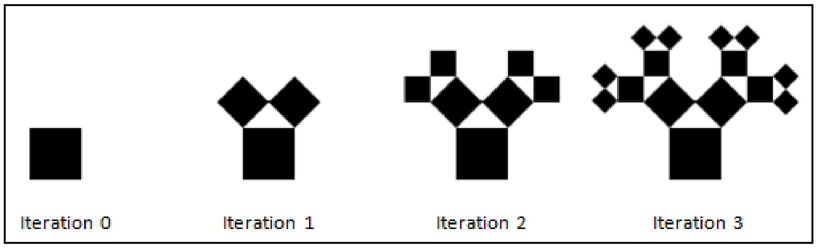


Figure 10: the three first iterations of the PYTHAGORE Tree Structure

The HAUSDORFF dimension for the PYTHAGORE Tree Structure is given by the equation (10).

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(2)}{\ln(\frac{2}{\sqrt{2}})} = 2 \tag{10}$$

4.5 The circular structure “APOLLONIUS circle”

This structure was invented by the Greek mathematician APOLLONIUS of Perga. Apollonius circles are tangent to one over other.

For the construction of this geometry the following steps are followed:

1. We begin with three circles C1, C2 and C3 of any size, each of which is tangent to the others.
2. According to Apollonius, there are two other circles which do not intersect, C4 and C5, which have the property of being tangent with the original three circles - these were called Apollonius circles.
3. To construct the other circles, each time we take 3 tangent circles and we repeat the step 2.

The Figure 11 shows some iteration of the APOLLONIUS Circles.

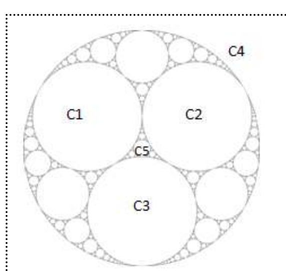


Figure 11: APOLLONIUS Circles

The HAUSDORFF dimension for the Apollonius Circles Structure is 1.3 [6].

4.6 CANTOR Set

CANTOR Set was invented by the German mathematician Georg CANTOR. It is built iteratively from the segment [0, T] by removing a central portion (a third for example); then the operation is repeated on the remaining two segments, and so on. The figure 12 shows the 6 first iterations of the Cantor set structure.

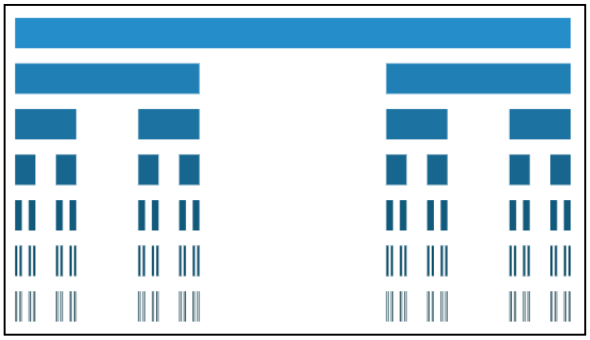


Figure 12: the 6 first iterations of the CANTOR SET structure

The HAUSDORFF dimension of the CANTOR Set fractal is given by the equation (11):

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(2)}{\ln(\alpha)} \tag{11}$$

Where $1/\alpha$ is the remaining part of the initial length of the first iteration.

In the example (figure 12), α =3.

So the HAUSDORFF dimension is given by the equation (10).

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(2)}{\ln(3)} = 0.63 \tag{12}$$

4.7 The HILBERT curve

The Hilbert curve is described for the first time by the German mathematician David Hilbert in 1891 [7]. The Hausdorff dimension is 2 and the method of construction is described in Figure 13 [8].

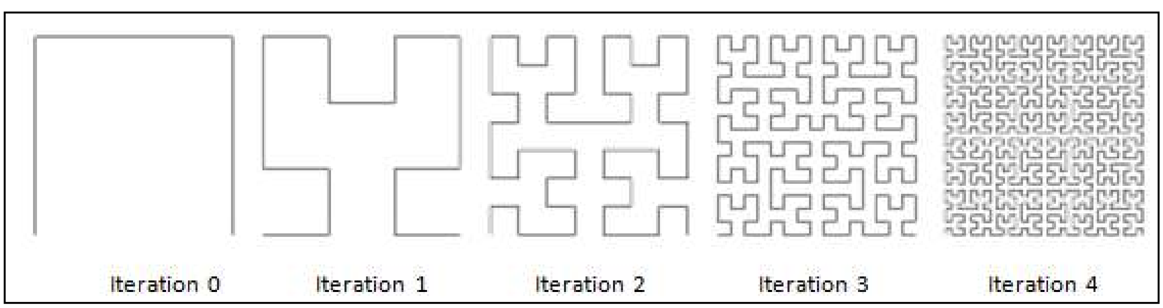


Figure 13: the first four iterations of the HILBERT Curve Structure

4.8 The MINKOWSKI Curve

This curve was invented by the German mathematician Hermann MINKOWSKI. The generation of this fractal is described in Figure 14.

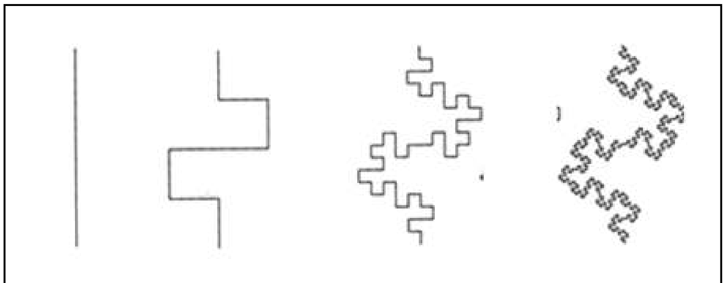


Figure 14: the first three iterations of the MINKOWSKI Curve Structure

The HAUSDORFF dimension of the MINKOWSKI curve fractal is given by the equation 13.

$$d = \frac{\ln(n)}{\ln(h)} = \frac{\ln(8)}{\ln(4)} = 1.5 \qquad (13)$$

5 Fractal Antennas Geometries

5.1 Background of the study and issues

With the proliferation and miniaturization of telecommunications systems and their integration in restricted environments, such as Smart-phones, tablets, cars, airplanes, and other embedded systems. The design of compact multi-bands and Ultra Wide Band (UWB) antennas becomes a necessity.

For designing this kind of antennas, two techniques are used:

a) Designing multi-bands antennas operating in several frequencies bands. Several studies have been made to design this kind of antennas by using fractal geometries or adding slots to the radiating elements [9]-[14].

b) Designing UWB antennas operating in the frequencies bands exceeding 500MHz or having a fractional bandwidth of at least 0.20, UWB wireless communication occupies a bandwidth from 3.1 to 10.6 GHz (based on the FCC "Federal Communication Commission") [14]-[22].

According to our literature searches, the use of fractal antennas allows to have multi-band and Broad-Band behavior.

5.2 The use of the fractal antennas

Several fractal geometries have been adopted to design fractal antennas including: the KOCH curve, the KOCH snowflake, the SIERPINSKI triangle, the SIERPINSKI carpet and other structures.

5.2.1 The KOCH structures

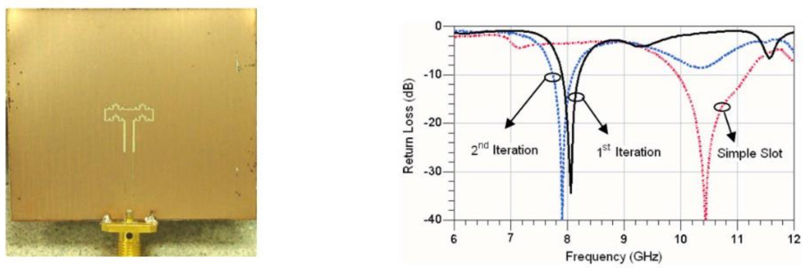
- ***Wire antennas***

The KOCH curve was used for the design of wire antennas with good performances. In 2000, PUENTE demonstrated that we can improve the efficiency of the antennas by increasing the number of iterations [23]. In 2002, BEST confirmed that the resonance frequency and the radiation resistance decrease when we increase the number of iterations while maintaining a fixed length of the structure [24]. In 2003 and 2004, VINOY demonstrated that the dimension of the fractal geometry affects the resonance frequencies. For each dimension, the relationships between the resonance frequencies are unchanged even if the number of iterations increases [25-26].

- ***Patch antennas***

The KOCH curve was used also for the design of patch antennas. In fact, it was used for the construction of the radiating elements, or for the setup of the slots on the radiating elements, or for the design of the ground planes.

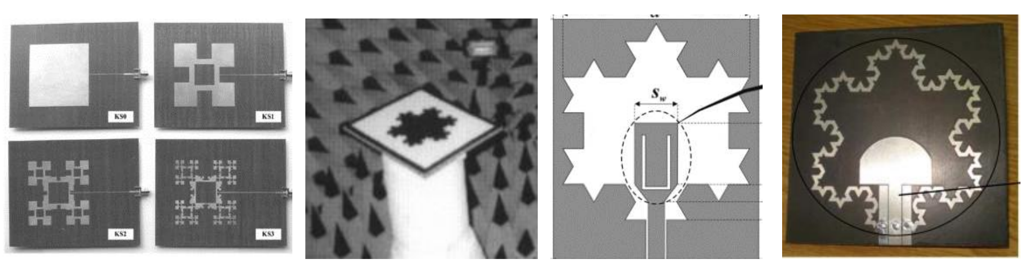
In 2007, SUNDARAM used the KOCH curve to design slots on a patch antenna, increasing the number of iterations; we decrease the resonant frequencies without increasing the size of the patch. This technique is a way to miniaturize the antennas. The figure 15 shows the structure of the proposed antenna and its measured return loss [27].



(a) The structure of the proposed antenna **(b) The measured return loss**

Figure 15: the structure and the measured return loss of the proposed antenna by SUNDRAM [27]

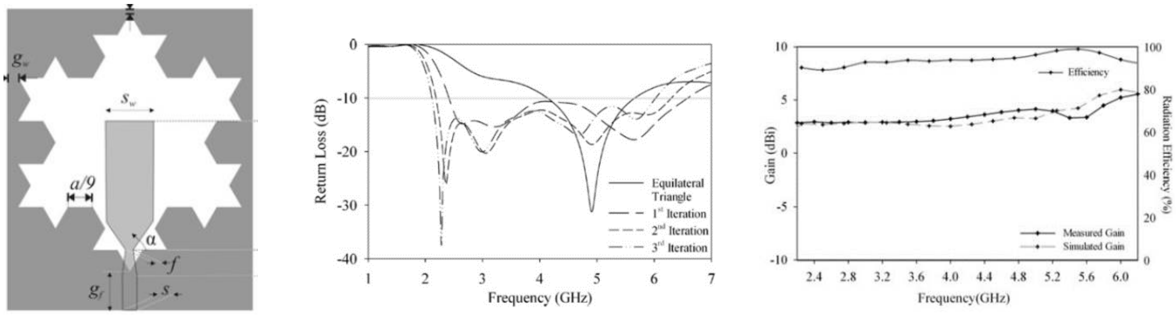
In 2008, CHEN designed a patch antenna using the KOCH and SIERPINSKI structures with a lower resonant frequency and a largest bandwidth compared to the traditional patch antenna [28]. Also, ANGUERA presented the performances of a patch antenna based on a KOCH fractal structure [29]. KRISHNA and PATNAM used the KOCH structures in the CPW-fed patch ground plane antennas and have increased the bandwidth and decreased the resonant frequencies of the antennas [30]-[31]. The figure 16 shows the proposed antenna by CHEN, ANGUERA, KRISHNA and PATNAM.



(a) **(b)** **(c)** **(d)**

Figure 16: the structures proposed by (a) CHEN (b) ANGUERA (c) KRISHNA (d) PATNAM [28]-[32]

In 2009, KRISHNA has designed another patch antenna having a KOCH structure on the ground plane with an Ultra Wide Bandwidth and a high gain. The figure 17 shows the structure of the proposed antenna, the return loss versus the frequency and versus the iterations number, the gain and the efficiency versus the frequency [32].



(a) **(b)** **(c)**

Figure 17: the structures proposed by KRICHNA (a), the return loss versus the iteration number and versus the frequency (b), the gain and the efficiency of the proposed antenna (c) [32]

In 2010, BIN YOUNAS presented a patch antenna with a high directivity. The resonant element and a slot are designed with the KOCH curve [33]. YUSOP used the KOCH structure for the design of an antenna fed by induction and has achieved an important gain [34]. The figure 18 shows the structures proposed by BIN YOUNAS and YUSOP.

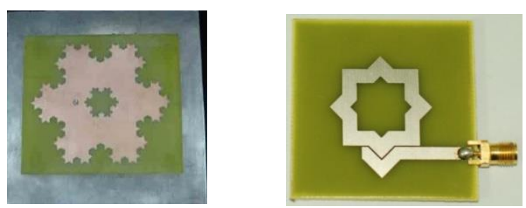
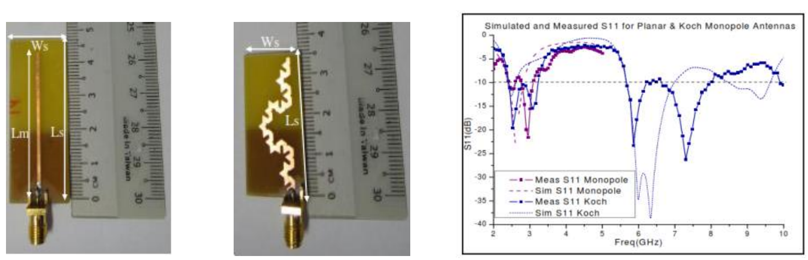


Figure 18: the structures proposed by BIN YOUNAS (a), and by YUSOP (b) [33]-[34]

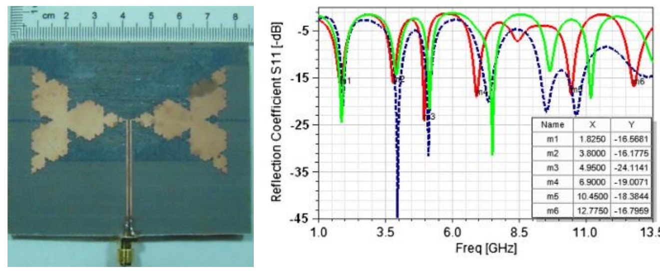
In 2011, ISMAHAYATI presented a comparative study between the performances of a simple monopole antennas and a KOCH monopole structure. The last one presented a multi-band and broad-band behavior [35]. The figure 19 shows the two structures and the comparison of the return Loss versus the frequency for the two structures.



(a) **(b)** **(c)**

Figure 19: the structures proposed by ISMAHAYATI, (a) Simple Monopole (b) KOCH Monopole, and (c) the comparison of the return Loss versus the frequency for the two structures [35]

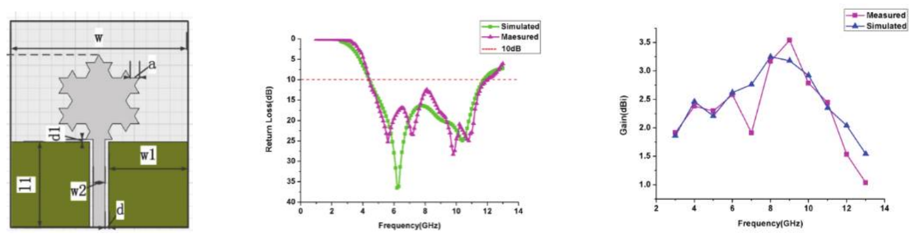
In 2012, DAOTIE LI combined the two structures BOW-TIE and KOCH. This antenna presents a multi-band and wide-band behavior [36]. The figure 20 shows the structure of the proposed antenna and the return loss of the antenna versus the frequency versus the iterations number.



(a) **(b)**

Figure 20: the structure proposed by DAOTIE LI (a) and its return loss versus the frequency and versus the iterations number [36]

Also, DONG LI presented a miniaturized patch antenna as a structure KOCH 2nd iteration with significant gains and a wide bandwidth [37]. The figure 21 shows the structure proposed by DAOTI LI, the simulated and measured return loss, and the simulated and measured gain.

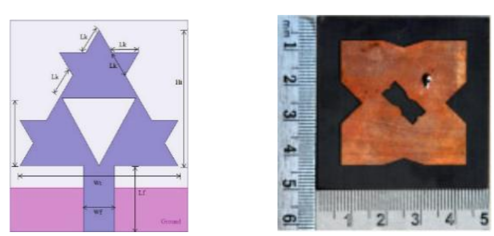


(a) **(b)** **(c)**

Figure 21: the structure proposed by DAOTIE LI (a), the simulated and measured return loss versus the frequency (b), and the simulated and measured gain [37]

In 2013, JEEMON combined the two structures KOCH and SIERPINSKI to design a miniaturized antenna for UWB applications and for WIMAX and WLAN applications [38].

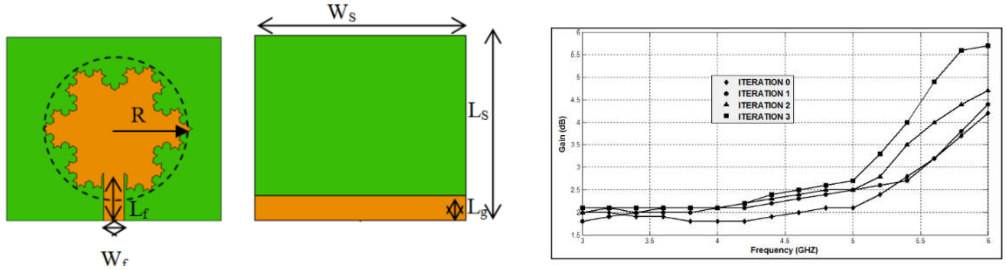
In 2014, REDDY designed a patch antenna with a first iteration KOCH curve borders. This structure performed a high gain and it is operational for WIMAX and WLAN applications [39]. The figure 22 shows the structures proposed by JEMON and REDDY.



(a) **(b)**

Figure 22: the proposed antennas by JEEMON (a), and REDDY (b) [37]-[39]

Also REHA studied the behavior of a CPW-Fed KOCH SNOWFLAKE Fractal Antenna for UWB Wireless Applications and demonstrated that increasing the number of iterations allows obtaining a low profile antenna with a high gain, a multi-band and a broad-band behavior [14]. The figure 23 shows the structure studied and the gain of the antenna versus the frequency and the iterations number.



(a) (b)

Figure 23: the antennas studied by REHA (a), and the simulated gain versus the frequency and the iterations number (b) [14]

5.2.2 The SIERPINSKI structures

- ***SIERPINSKI triangles***

In 1998, BORJA has implemented a patch antenna using SIERPINSKI triangle network and has obtained a multi-band antenna by incorporating more iteration [40]. Also, PUENTE has modeled the behavior of this kind of fractal antennas [41].

In 1999 GONZALEZ studied experimentally the distribution of currents in a SIERPINSKI fractal antenna and demonstrated that this distribution follows the principle of self-similarity [42]. In 2000, he predicted this kind of antenna when the flare angle is modified [43]. The figure 24 shows the structure studied by BORJA, PUNTE, and GONZALEZ.

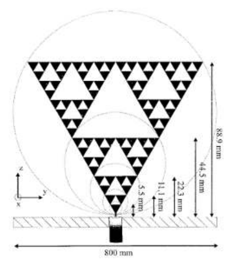
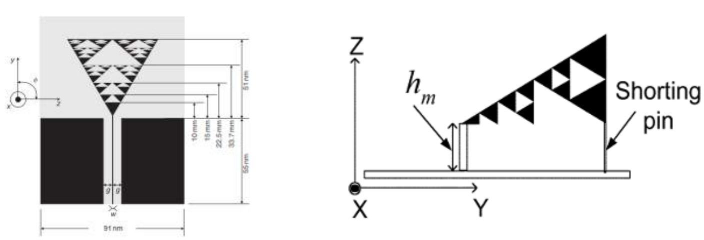


Figure 23: the antenna studied by BORJA, PUNTE, and GONZALEZ [41]

In 2004, KITLINSKI confirmed the results found previously and improved the behavior of the SIERPINSKI fractal antenna by adopting the CPW-feeding technique [44]. SONG studied the behavior of these antennas by leaving only half of the antenna and adding short circuits. This technique allows miniaturization while maintaining the same performance of the complete antennas [45]. The figure 24 shows the antennas studied by KITLINSKI and SONG.



(a) (b)

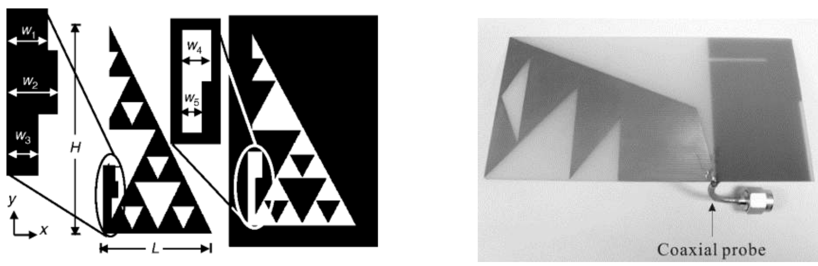
Figure 24: the antennas studied by KITLINSKI (a), and SONG (b) [44]-[45]

In 2006, ANGUERA set up the SIERPINSKI fractal antenna on several layers of substrate and designed a multi-band antenna operating for all mobile networks 2G-4G with good performances [46]. The figure 25 shows the proposed structure and its return loss.

(a) (b)

Figure 25: the structure proposed by ANGUERA (a) and its return loss versus the frequency (b) [46]

In 2007, HWANG and VEMAGRI have developed semi-SIERPINSKI fractal antennas respectively for UHF-RFID (Ultra High Frequency - Radio Frequency Identification) and mobile networks 2G-4G [47-48]. The figure 26 shows the structures proposed by **HWANG and VEMAGRI** .



(a) (b)

Figure 26: the structures proposed by HWANG (a) and VEMAGRI (b) [47]-[48]

In 2009, KRZYSZTOfiK presented a modified and miniaturized SIERPINSKI Fractal structure to cover the two ISM (Industrial, Scientific and Medical) bands (2.4 and 5.2 GHz) with better performance compared to traditional structures [49].

In 2011, SOH has setup the SIRPINSKI structure in a PIFA (Planar Inverted F Antenna) for IEEE 802.15 WPAN LR (Low Rate Wireless Personnal Area Network) applications [50].

In 2013, VIANI has changed the basic structure of the SIERPINSKI geometry and has designed an antenna with high gains for LTE and WiMAX applications [51]. The figure 27 shows the structures proposed by **KRZYSZTOfiK, SOH and VIANI**.

(a) (b) (c)

Figure 27: the structures proposed by KRZYSZTOfiK (a), SOH (b), and VIANI [49]-[51]

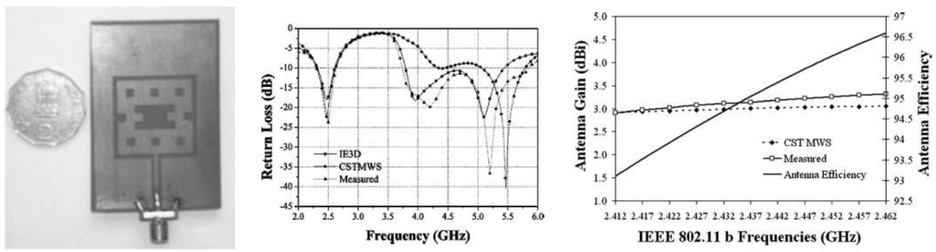
- ***SIERPINSKI Carpet***

In 1999, Werner showed the effect of the iterations number on the behavior of fractal antennas in general and in particular of the SIERPINSKI Carpet fractal structure [52].

In 2004, Ban-Leong OOI proposed a modified SIERPINSKI carpet structure to improve the bandwidth and the gain compared to a standard SIERPINSKI structure [53].

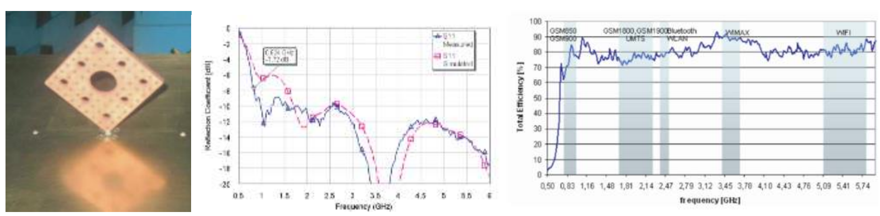
In 2008, GHATAK proposed other changes to the standard SIERPINSKI carpet structure to adapt it to wireless networks IEEE 802.11a / b WLAN and HyperLAN2 applications with a good efficiency [54]. The figure 28 shows the proposed structure, its return loss, its gain and efficiency versus the frequency.

In 2010, ANGUERA has designed a 3D SIERPINSKI carpet antenna for WiFi, 2G, 3G, WIMAX, and Bluetooth applications, with very high gains and good efficiency [55]. The figure 29 shows the proposed structure, its return loss, and its efficiency versus the frequency.



(a) (b) (c)

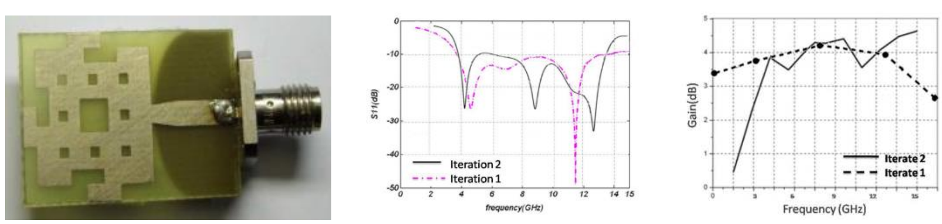
Figure 28: the structure proposed by GHATAK (a), its return loss (b), its gain and efficiency versus the frequency(c) [54]



(a) (b) (c)

Figure 29: the structure proposed by ANGUERA (a), its return loss (b), and its efficiency versus the frequency (c) [55]

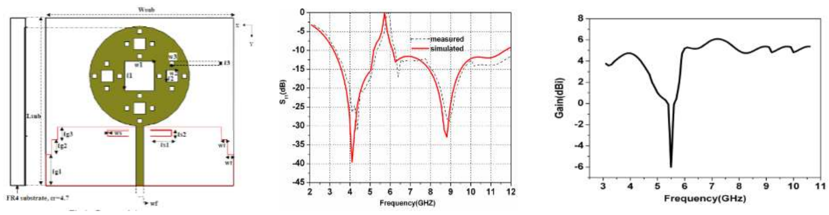
In 2011, ORAIZI combined the SIERPINSKI Carpet structure with the GIUSEPE PEANO structure. The result is a miniaturized Ultra Wide Band antenna, with high efficiency and high gain [56]. The figure 30 shows the proposed structure, its return loss, and its gain versus the frequency.



(a) (b) (c)

Figure 30: the structure proposed by ORAIZI (a), its return loss (b), and its gain versus the frequency (c) [56]

In 2013, BISWAS has set up the SIERPINSKI Carpet structure on the circular patch antenna and fed by a microstrip line and having slots on the ground plane. This new structure allowed having an Ultra Wideband behavior with significant gains [57]. The figure 31 shows the proposed structure, its return loss, and its gain versus the frequency.



(a) (b) (c)

Figure 31: the structure proposed by BISWAS (a), its return loss (b), and its gain versus the frequency (c) [57]

5.2.3 The Tree Structure

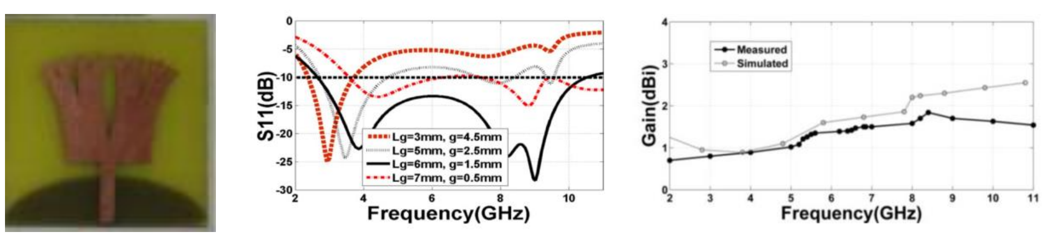
In 1986, according to our literature searches, KIM has published the first paper on the fractal antennas and especially on the random tree fractal antennas [58].

In 1999, WERNER studied the effect of iterations number on the behavior of fractal antennas in general and in particular of the Tree structures [59].

In 2004, PETKO studied the 3D TREE Fractal antennas and found out the relationship between the geometry parameters and the performances of this kind of fractal structures [60].

In 2009, He studied the array of TREE Fractal antennas and he demonstrated that we can have better results with this technique [61].

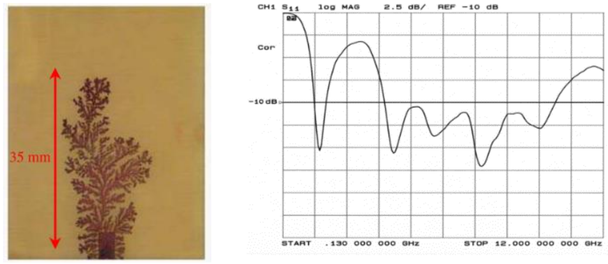
In 2011, POURAHMADAZA created a modified patch antennas fed by microstrip lines and having the shape of "PYTHAGORE TREE". He demonstrated that by increasing the number of iterations we can have miniaturized antennas with more resonance frequencies, an Ultra Wideband behavior, good efficiency and better matching [62]. The figure 32 shows the proposed structure, its return loss, and its gain versus the frequency.



(a) (b) (c)

Figure 32: the structure proposed by POURAHMADAZA (a), its return loss (b), and its gain versus the frequency (c) [62]

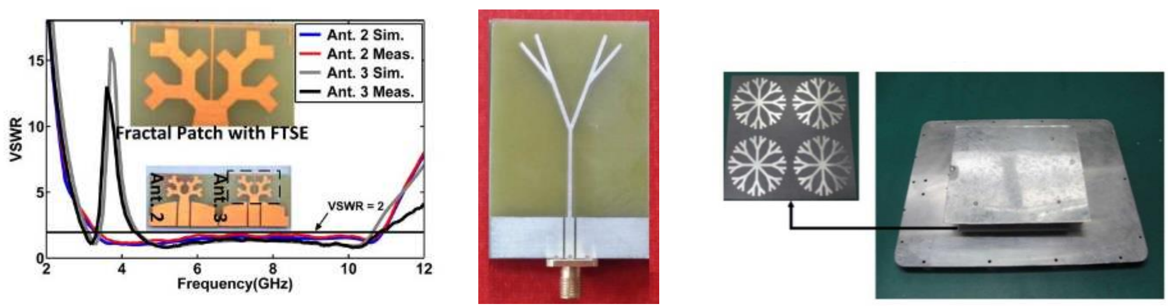
In 2013, DUMOND studied experimentally the Random 2D-TREE fractal antennas and he demonstrated that with this kind of structures, we can have antennas with good performances, multi-band and UWB behavior [63].



(a) (b)

Figure 33: the structure proposed by DUMOND (a) and its return loss (b) [63]

Also, NASER-MOGHADASI, has introduced a miniaturized TREE fractal structure operational for all the UWB applications, he set up also parasite elements in order to eliminate some frequencies bands [64]. VARADHAN proposed a Tri-bands TREE fractal antenna for The RFID applications [65]. LIU proposed another structure with a high gain for the UHF-RFID, this antenna is composed of a traditional patch and another layer composed of a four TREE fractal elements [66]. The figure 34 shows the proposed structures by -MOGHADASI, VARADHAN, and LIU.



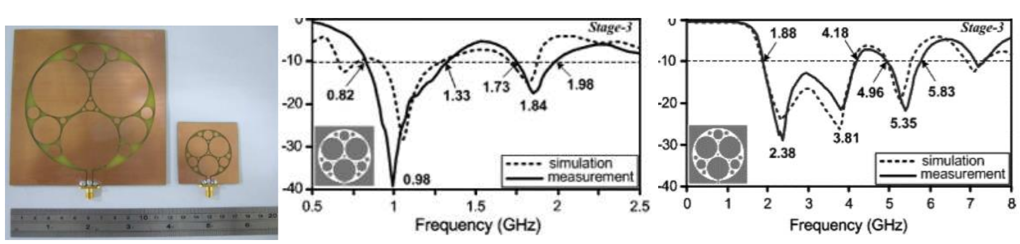
(a) (b) (c)

Figure 34: the proposed structures by NASER-MOGHADASI (a) , VARADHAN (b), and LIU (c) [64]-[65]

5.2.4 The circular Structures

- ***Apollonius circle***

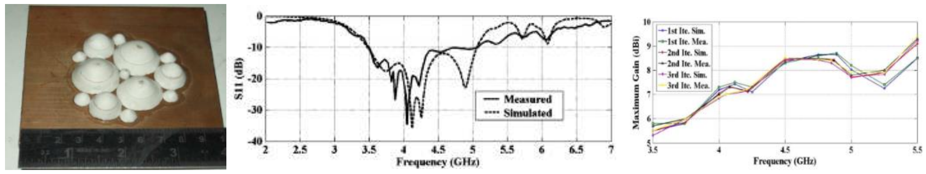
In 2008, CHANG proposed two Apollonius circles antennas with very high gains and a multi-band behavior [67]. The figure 35 shows the proposed structures and its return loss for the $\lambda/2$ and $\lambda/4$ design.



(a) (b) (c)

Figure 35: the proposed structures by CHANG (a), its return loss for the $\lambda/2$ design (b), and $\lambda/4$ design (c) [67]

In 2014, MUKHERJEE has set up a 3D Apollonius circles with high gain and a broad-band behavior [68]. The figure 36 shows the proposed structure, its return loss, and its gain.



(a) (b) (c)

Figure 36: the proposed structure by MUKHERJEE (a), its return loss (b), and its gain (c) [68]

- ***Other circular fractal structures***

Several studies have been made on the circular fractal geometries without being as Apollonius circles. In 2006, DING has designed a circular fractal antenna fed by a microstrip line, combining circles and triangles as shown in Figure 37. The proposed antenna is a small structure, broad-band and can be used in UWB applications [69].

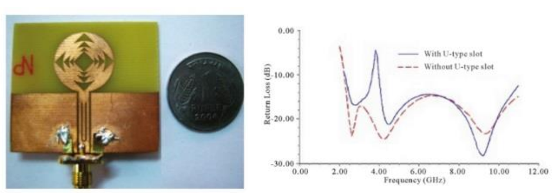
(a) (b)

Figure 37: the circular fractal structures proposed by DING (a) and its the return loss (b) [69]

In 2008, as shown in Figure 38, KUMAR proposed the same modified structure. The proposed structure is a miniaturized antenna with multi-band behavior and a high gain [70].

Figure 38: the circular fractal structure proposed by KUMAR [70]

In 2010, Kumar proposed another circular fractal antenna (Figure 39). The proposed structure provides a broad-band behavior. The setup of the slots on the microstrip line allows the reject of some frequencies bands [71]. The figure 39 shows the proposed structure and its return loss with and without the slot.



(a) (b)

Figure 39: the circular fractal structure proposed by KUMAR [71]

In 2012 MOHAMMADSHAH has proposed the circular fractal structure shown in the Figure 40. The proposed structure is a broad-band antenna, operational from 2 to 21GHz with a good efficiency [72].

Figure 40: the circular fractal structure proposed by MOHAMMADSHAH [72]

In 2014, REDDY has studied the effect of the introduction of the semicircular fractals on the edges of a patch antenna as shown in Figure 41. The proposed structure provides important gains for WiFi, WiMAX and WLAN applications [73].

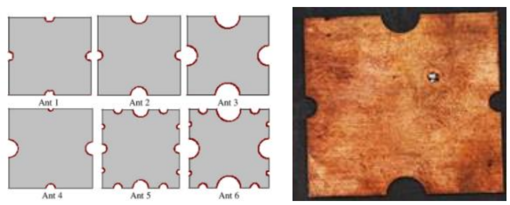
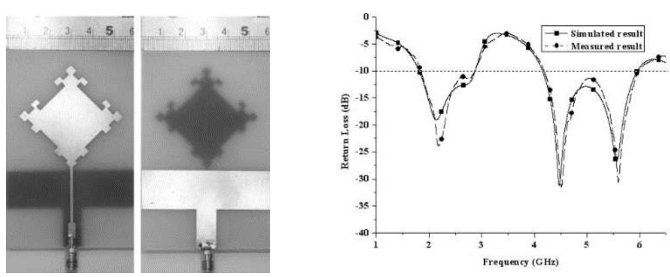


Figure 41: the circular fractal structure proposed by REDDY [73]

5.2.5 The MINKOWSKI curve

In 2002 and 2003, GIANVITTORIO and BEST have studied the effect of iterations number on the resonance frequencies number. They showed that for the MINKOWSKI curve wire antennas, the number of resonant frequencies increases by increasing the number of iterations [74-75].

In 2009, MAHATTHANAJATUPHAT has designed a modified patch antenna with MINKOWSKI Curve on the borders of the radiating element for UMTS, WLAN and WIMAX applications [76]. The figure 42 shows the proposed structure and its return loss.



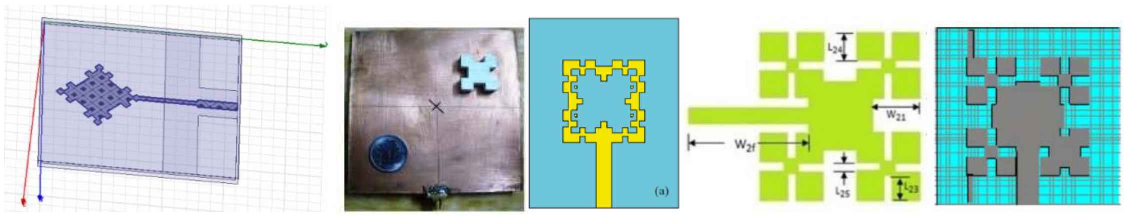
(a) **(b)**

Figure 42: the structure proposed by MAHATTHANAJATUPHAT (a) and its return loss (b) [76]

In 2011, CHEN has designed a modified patch antenna by MINSOWSKI curve to reduce the RCS (Radar Cross Section) [77]. Also, MORAES has studied the difference between a square patch and a triangular patch antenna. The antennas have been optimized to have initially the same resonant frequency. The introduction of MINKOWSKI curve contour allows having lower resonance frequencies compared to the introduction of KOCH curve contour [78].

In 2012, NAJI has studied a patch antenna with Proximity-coupled feed, modified by a lot of MINKOWSKI curve contour configurations. The proposed structures are very miniaturized, with an important gain, and operational for 5.8GHz-RFID applications [79-80].

In 2013 and 2014, LATA, DHAR, AHMAD, SINGH AND ANCY have proposed patch antennas with modified contours with a MINKOWSKI curve. These structures have a Multi-band and a broad-band behavior [81]-[85]. The figure 43 shows the structures proposed.



(a) (b) (c) (d) (e)

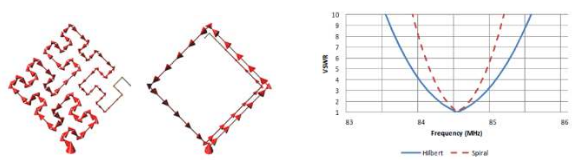
Figure 43: the structures proposed by LATA (a), DHAR (b), AHMAD (c), SINGH (d), and ANCY (e) [81]-[85]

5.2.6 The HILBERT Curve

In 2003, GONZALEZ-ARBESÚ has studied the parameters and the behavior of 2D and 3D HILBERT curve wire antennas [86].

In 2006, MURAD has developed a modified patch antenna using the Hilbert curve. The proposed antenna is a miniaturized structure for the 2.45GHz RFID applications [87].

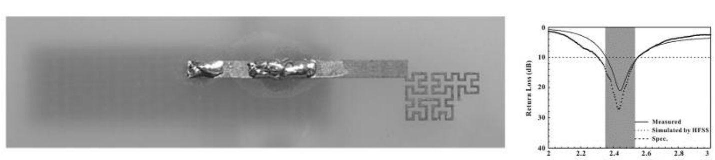
In 2010, SANZ has studied the difference between the spiral wire antennas and the wire antennas based on the Hilbert geometries. He has showed that this second structure provides a higher bandwidth [88]. The figure 44 shows the studied structures and its VSWR (Voltage Standing Wave Ratio).



(a) (b)

Figure 44: the studied structures by SANZ (a) and its VSWR (b) [88]

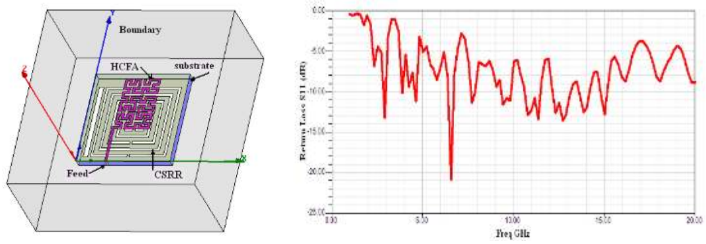
HUANG has set up a PIFA antenna based on a HILBERT structure, operational for the 2.4GHz applications [89]. The figure 45 shows the proposed structure and its return loss.



(a) (b)

Figure 45: the proposed structure by HUANG (a) and its Return Loss (b) [89]

In 2012, SUGANTHI has studied some modified patch antennas based on the HILBERT structure. The proposed antennas are operational for medical applications [90]. The figure 46 shows the proposed structure and its return loss.

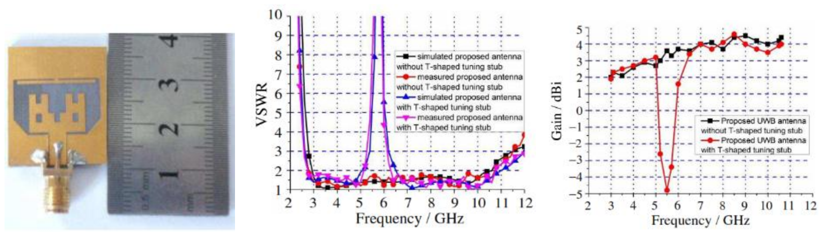


(a) (b)

Figure 46: the proposed structure by SUGANTHI (a) and its Return Loss (b) [90]

5.2.7 CANTOR Set

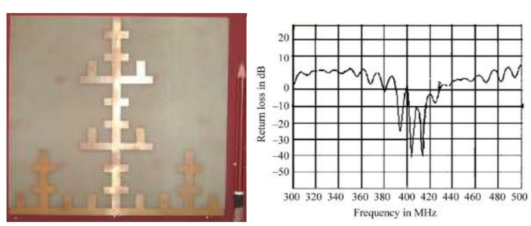
In 2011, LI has designed a miniaturized and modified patch antenna fed by a microstrip line having a radiating element in the form of a CANTOR set. This structure is operational for UWB applications and having an important gain except for the 5 - 6.3GHz.applications [91]. The figure 47 shows the proposed structure, its VSWR, and its gain.



(a) (b) (c)

Figure 47: the proposed structure by LI (a), its VSWR (b), and its gain (c) [91]

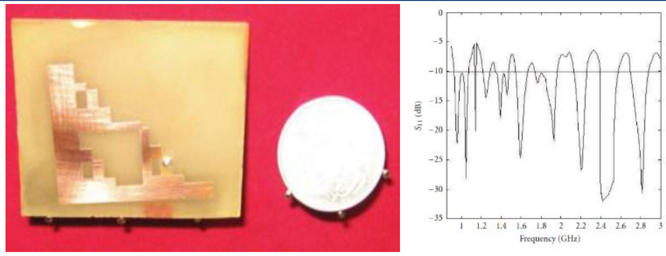
Also, SRIVATSUN has proposed the modified CANTOR set structure with for MICS (Medical Implant Communication Service) applications [92]. The figure 48 shows the proposed structure, and its return loss.



(a) (b)

Figure 48: the proposed structure by SRIVATSUN (a), and its return loss (b) [92]

In 2012, SRIVATSUN has the same structure for the IEEE 802.11b WLAN applications, 802.15, PCS, GSM, DCS, IMT, UMTS, Wi-Fi, and WLAN with good performances [93]. The figure 49 shows the proposed structure and its return loss.



(a) **(b)**

Figure 49: the proposed structure by SRIVATSUN (a), and its return loss (b) [93]

6 Conclusion

In this paper, some generalities of fractal geometries and their dimensions are presented. We have described also some linear fractal geometries such as KOCH, SIERPINSKI, DRAGON, TREE, CIRCULAR, CANTOR SET, HILBERT, MINKOWSKI, and we have discussed the applications of these geometries and their performances in the design of the miniaturized antennas.

The common feature of all the presented works is that the fractal antennas allow a Broad-Band and a Multi-band behavior. Also, the fractal antenna allows having miniaturized antennas with good performances.

In fact, for the KOCH fractal structures, the wire antennas allows a multi-bands behavior, the combination with the patch antennas decreases the resonant frequencies and the introduction of the CPW feeding allows a Broad-Band behavior. For the SIERPINSKI fractal structures, the patch antennas allows having a multi-band behavior, also, the introduction of the CPW feeding allows a broad-Band behavior. For the TREE fractal structures, we can have multi-band antennas. The use of PYTHAGORE TREE structures allows having broad-band antennas. The Circular fractal structures allows having a multi-band behavior, the use of 3D structures allows having a broad-band antennas with a high gains. The use of MINKOWSKI fractal structures, allows having a multi-band and broad-band antennas. For the HILBERT fractal structures, we can have multi-band antennas. For the CANTOR set fractal structures, we can design broad-band antennas.

According to our literature search, it is clear that some have been extensively used such as KOCH, SIERPINSKI and TREE structures, other structures were rarely used such as CIRCULAR structures, CANTOR Set, but some of them weren’t studied such as DRAGON, H-TREE structures. Also, several studies have combined two fractal structures or have modified the original fractal structures.

In the next works, we will try to study the fractal structures that have never been studied such as DRAGON and H-Tree structures, and we will try also to study the performances of the fractal antennas on the networking configurations.

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