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Research

Fractal Architecture Could Be Good for You

Abstract. The deployment of fractal principles in art and architecture seems to be a phenomenon of all times, and is in no way restricted to the period after the systematic mathematical understanding and description of fractals from the1970s onwards. Nowadays, computer-generated fractal art, and the software to generate it, are widely available on the Internet. Fractal principles are also at work in more "traditional" arts or crafts, such as some Dalì paintings, mandalas, mosaics, floor decorations, and so on. This paper presents some of the architectural appropriations of fractal geometry. The concluding sections argue that fractal architecture is in a sense "good" for us.

The architectural utility of fractal geometry

Before I embark on a review of fractal forms in architecture, you should know that this type of geometry is used in architecture for two main reasons. First, some scholars have promoted fractal geometry as a creative tool. For example, Carl Bovill [1996] uses fractal rhythms, created by midpoint displacement, to generate a wide range of architectural organizations, such as planning grids, strip windows, noise abatements, and so on. Nikos Salingaros [1998] has emphasized three-dimensional applications of fractals in architecture and fervently argues for the recurrence of self-similar architectural elements on different scales of the built form. From a study of natural entities, he concludes that the scaling relationship between these elements should obey the ratio 2.7 to be aesthetically pleasing. Finally, note that an intuitive understanding of the creative value of fractal geometry was already present in classical architectural composition, which has lead to some remarkable similarities with well-known fractals (fig. 1). Andrew Crompton [2002] has tentatively argued that this is because some classical composition rules favour fractal forms.

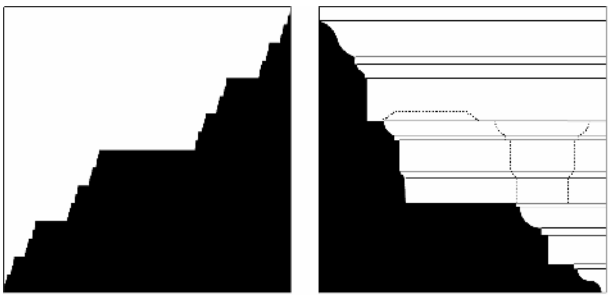

Fig. 1. A classical Doric entablature (left) has a remarkable similarity with the Devil's Staircase[1] (right). Picture credits: Andrew Crompton

A second way in which fractal geometry can be related to architecture is to use some of its typical measurement techniques to analyse the structure of buildings. Many readers are perhaps familiar with the box counting dimension, which is a measure for the recursiveness

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of detail on ever smaller scales.[2] Carl Bovill [1996] has applied this method to different building styles. He found that Wright's organic architecture shows a 'cascade of detail' on different scales, while in Le Corbusier's modernist architecture, the box counting dimension quickly drops to 1 for smaller scales. This finding is consistent with the fact that 'Wright's organic architecture called for materials to be used in a way that captured nature's complexity and order ... [while] Le Corbusier's purism called for materials to be used in a more industrial way, always looking for efficiency and purity of use' [Bovill 1996, 143]. Similar to Bovill, Daniele Capo [2004] applied the box counting method to the classical orders and found that there is detail up to 1/256th of the height of the entire order. Burkle-Elizondo and Valdéz-Cepeda [2006] also used fractal measurement techniques to establish the complexity of thirty-five Mesoamerican pyramids, and found that the monuments had a fractal dimension of around 1.3.

Two-dimensional fractals in architecture

Let me now show how fractal forms are, and have been, integrated in architecture. On first sight there does not seem to be an all-encompassing factor that binds the following buildings together. Sometimes, the fractal form is an expression of a worldview or a social idea, while on other occasions the architect just found it an attractive shape. Nevertheless, in the final sections I tentatively propose that there is perhaps a deeper-lying reason why such patterns are integrated in architecture, throughout all ages and cultures.

I start off with an overview of two-dimensional fractal forms in architecture, which are mostly present in the ground plans of buildings. You can find this application in a wide range of architectural structures, ranging from the plans of fortifications, to the organization of traditional Ba-ilia villages (Zambia). The global form of the latter settlements reoccurs in the family ring, which consists of individual houses, which are, again, similar to the overall shape of the village.

Interestingly, the scaling hierarchies governing this whole are a reflection of the social hierarchy in these communities [Eglash and Odumosu 2005]. As is noted by George Hersey [1993], a fractal organization is also characteristic of the plan of Bramante's design for St. Peter's in Rome:

> Symmetrically clustered within the inside corners formed by the cross's arms are four miniature Greek crosses, that, together, make up the basic cube of the church's body. The arms of these smaller crosses consist of further miniatures. And their corners, in turn, are filled in with smaller chapels and niches. In other words, Bramante's plan ... may be called fractal: it repeats like units at different scales [Hersey 1993].

The fractal ground plan that has perhaps received most theoretical attention is Wright's Palmer House (Ann Arbor, Michigan). In order to understand its fractal character, it is important to note that architects sometimes use a 'module' as the main organizational element. In a sense, such an element can be understood as the conceptual 'building block' of the house (e.g., a circle). Wright often applied this procedure to his work. Initially, the geometry governing his architecture created with the aid of such modules remained Euclidean. In later works, however, these elements were sometimes so organized that they gave the building a remarkable fractal organization. The Palmer House seems to be the culmination point of this evolution. Here, one geometric module – an equilateral triangle –

is repeated in the ground plan on no less than 7 different scales [Eaton 1998] (fig. 2). Another Wright building, whose fractal nature is visible – but in elevation – is the Town Hall in Marin County (San Francisco). In this structure, above each arch a window or arch is placed that is somewhat smaller than the previous one. This gives the structure self-similarity up to five scales [Portoghesi 2000].

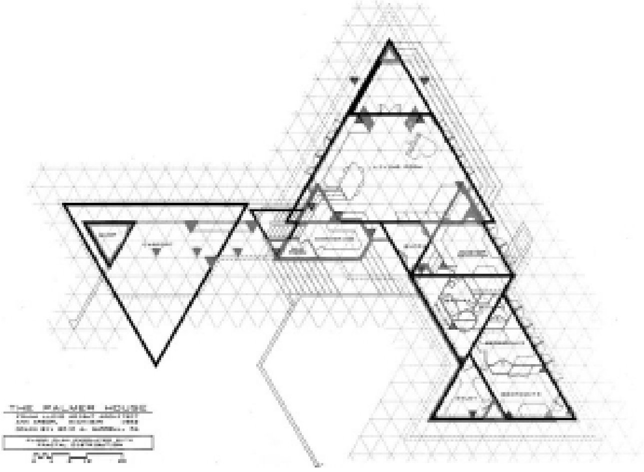

Fig. 2. Ground plan of Wright's Palmer House. Drawing by Eric Murrell, from [Eaton 1998]. Reproduced with permission, Kim Williams Books

Three-dimensional fractals in architecture

An obvious disadvantage of fractal ground plans is that the fractal component is barely visible for the viewer in a normal architectural experience. In this sense, it could be claimed that it loses some of its significance, and that three-dimensional applications are more convincing. A three-dimensional method, which some have linked to fractals, is to tessellate the architectural façade [Jencks 2002]. On first sight, the link with fractal geometry could seem obvious: such patterns are rich in detail, which is an intrinsic characteristic of fractals.

The contemporary architectural group Ashton Raggatt McDougall was perhaps one of the first to apply fractal tiling to architecture [Jencks 2002]. They covered the façade and the interior of Storey Hall (Melbourne) with polygon tiles that are inspired on Penrose tiling. Penrose tilings were discovered by the British mathematical physicist Roger Penrose and consist of a small number of simple tiles that can cover the plane in a non-repetitive manner. Such tessellations can be related to fractal geometry because they can be generated by Iterated Function Systems and L-systems. A 'tiling-approach' has also been adopted by the Lab Architecture Studio for Federation Square in Melbourne and its adjacent buildings. The main units of this 'fractally incremental system' are triangles, which are organized by five into panels, while five panels form the main constructional module (fig. 3).

I find the relation of tiled façades with fractal geometry difficult to judge. While some scholars, such as Charles Jencks, have unambiguously related such creations with fractals, it can also be observed that such constructs have no detail within detail, no tiles within tiles.

In a sense, the patterns are no more fractal than a grid of squares is fractal. They contain many details, but zooming in on the structure does not reveal new detail. On the other hand, in the case of Storey Hall and Federation Square, the tiles are organized into 'higher-order' wholes, with the aid of texture, colour and lines. This gives them a profounder sense of self-similarity. Finally, some might argue that such fractal interventions are merely surface treatment: essentially, these are not architectural but decorative interventions. Indeed, leaving out the patterns would probably wipe out the fractal character of the building altogether.

Fig. 3. Federation Square, Melbourne (Lab Architecture Studio). Picture credits: Steven Connor

But do there exist instances of modern architecture where the fractal component is eminently three-dimensional – where it pertains to the architectural form and/or structure? Such appropriations seem rare. On the Internet I came along the website "Fractal Architecture" (http://www.fractalarchitect.com), which shows building designs that are the result of marrying fractal principles and modernist forms. In the twentieth century Russian artist Malevich has created a series of architectural designs (*Arkhitektoiniki*) of which some have a remarkable fractal component. In one example, the main architectural form is surrounded by smaller versions of the whole building, which are again surrounded by even smaller fragments. The relation between their number and size is claimed to obey a $1/f$ relation. More recently, Steven Holl Architects' Simmons Hall has been related to fractal geometry, because it is inspired by a sponge whose openings have a fractal distribution.

For other eminent examples of three-dimensional fractal architecture, you have to go back in time. Sometimes it is noted that some of Leonardo Da Vinci's cathedral designs are fractal, because the domes are repeated for different sizes. However, this example (and the previous ones) cannot meet up to the profound fractal character of certain Hindu temples

[Trivedi 1989; Portoghesi 2000]. The fractal character of Hindu temples is strongly intertwined with Hindu cosmology (fig. 4).

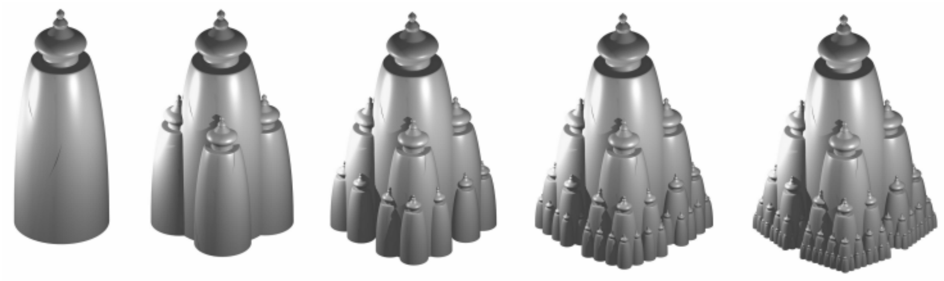

Fig. 4. Fractal generation of central dome of a Hindu Temple

In fact, these edifices should be understood as models of the Hindu cosmos. In Hindu philosophy the cosmos is (more or less) conceived as a hologram, where each part of the whole ís the whole itself, and contains all the information about the whole. Some schools of Hindu thought adhere to the (related) view that the macrocosm is 'enclosed' in the microcosm:

> The entire cosmos can be visualized to be contained in a microcosmic capsule, with the help of the concept of subtle elements called 'tanmatras'. The whole cosmic principle replicates itself again and again in ever smaller scales. The human being is said to contain within itself the entire cosmos [Trivedi 1989, 245-246].

Interestingly, both cosmological conceptions can be straightforwardly related to fractal self-similarity. Here also, the global structure recurs – over and over again – in the microstructure.

In order to maintain a harmonious worldview, man-made objects and artistic expressions were made in accordance with the central principles governing the Hindu cosmos – the result being a profound three-dimensional fractal architecture. The fractality of Hindu temples can be traced back to a set of typical architectural interventions. A survey of these methods is not only theoretically interesting, but also offers a set of concrete guidelines for enhancing the fractal character of architecture.

1. Splitting or breaking up a form, and repeating it horizontally, vertically or radially.
2. In the ground plan, iteratively replacing plain sides by sides that contain interior and exterior projections, or more detail. This method can also be applied in three dimensions.
3. Three-dimensional self-similar iteration of the central spire of a Hindu temple.
4. Repeating similar shapes horizontally and/or vertically.
5. Three-dimensional superimposition of architectural elements ('... motifs are inscribed within different kinds of motifs and several different kinds of themes and motifs are condensed and juxtaposed together into one complex and new entity' [Trivedi 1989, 257].)

Fig. 5. Notre Dame Cathedral, Paris

Fig. 6. Gothic architecture (City Hall, Bruges). The arched form reoccurs throughout the building façade on different hierarchical scales

In the West, the fractal Hindu temple seems to have its counterpart in Gothic architecture (figs. 5 and 6). From the illustrations you can easily see which specific methods are at the root of the fractality of the Gothic. In fact, these methods are quite similar to the ones used in Hindu architecture. For example, the shape of the main portal in fig. 5 recurs on a smaller scale in the two side portals. Further and smaller versions of this form can be found in the different arched windows or openings. In some Gothic buildings the windows are divided in constituent parts, where the stained glass, with its vibrant and colourful patterns, adds even more detail to the façade. The complexity of Gothic façades is further increased by the mere repetition of forms. For example, in fig. 5 the contours of the main and side portals are repeated inwardly, and circumscribed with a wealth of sculpted figures. The fractal component of Gothic cathedrals is also evident in the rose windows. These often contain flower-like patterns of varying sizes, and the stained glass often enhances the fractality even further.

Why fractal architecture could be good for you

Perhaps the last examples of fractal architecture are the most appealing to mathematicians, because the fractal component is clearly visible. However, as a philosopher it strikes me that these buildings have a strong aesthetic 'pull'. I can testify to this first-hand because I have the privilege of living in the medieval town of Bruges (fig. 8). Loads of tourists are fascinated by the Gothic buildings, and are ever so keen to take some snapshots, at the risk of being run over by the typical horse carriages that go around here. Of course there is an interplay of different factors here: people also go to see these monuments because the tourist industry thinks that they are worth seeing. Yet, I am certain that the mathematical – i.e., the fractal – structure of the buildings plays a role too. Some years ago I had the chance to embark on a research project that tried to address this issue. In essence, the research has been strongly interdisciplinary, and brings together findings from the fields of art, mathematics and psychology.

As strange as it may sound, I found that there is reason to believe that we are in a sense attuned to fractals, and hence, to fractal architecture. One line of evidence shows that the human nervous system is governed by time fractals. More specifically, analyses of brain functioning shows that it displays typical noise signals, commonly referred to as '1/f noise' or 'pink' noise [Anderson and Mandell 1996]. The fractal character of such noise can be easily appreciated, because like spatial fractals, it shows self-similar detail when you zoom in on it. But what is the function of these time fractals? A common answer is that the natural world – and the way it changes over time – is also characterized by pink noise, which suggests that our fractal minds are optimized to process the fractal characteristics of natural scenes (see, for example, [Knill et al. 1990]). This hypothesis is supported by the finding that discriminating fractal contours is best for those that have the same (fractal) properties as natural scenes [Gilden et al. 1993]. Interestingly, these findings can (tentatively) explain the creation of fractal artwork, and fractal architecture in particular. Such art should be understood as an exteriorization of the fractal aspects of brain functioning [Goldberger 1996]. Or as Goldberger [1996] puts it: '... the artwork externalizes and maps the internal brain-work ... Conversely, the interaction of the viewer with the artform may be taken as an act of self-recognition'.

However, as plausible as this explanation might sound, I don't find it satisfactory. The main reason is that it cannot account for why fractal patterns have a strong aesthetic component. A factual description of the human perceptual system in terms of time fractals does not explain why we find fractal structure beautiful. For a more plausible explanation for the aesthetic appeal of fractal architecture we have to delve into some findings from the field of environmental psychology (for a related account, see [Salingaros 2006]). Here, the empirical literature has shown, over and over again, that humans display a positive emotional affiliation with a specific set of natural elements and settings, namely vegetation (trees, plants and flowers) and savannas (see [Ulrich 1993] for an excellent review). More specifically, researchers found that these elements (a) lead to positive aesthetic responses, and (b) reduce psychological and physiological stress in humans [Ulrich et al. 1991]. These emotional affiliations are claimed to be 'hardwired' in the human brain and are sometimes framed in terms of "biophilia" (literally, "love for life"; see e.g., [Kellert and Wilson, 1993]). They are remnants of our shared human evolution in East-African savannas. Having these inborn "biofilic" responses was advantageous because they motivated our ancestors to explore and approach the natural settings and the elements it contained. This increased survival chances: vegetative elements were eminent sources of food, and could offer protection, while savannas are known to be high-quality habitats. People will also feel more relaxed in places that are good for living, hence the stress-reducing effect of such settings and of their constituent (vegetative) elements [Ulrich 1993].

At this point, the reader interested in mathematics might ask what this all has to do with fractals. Let's make my point. While the fractality of nature has been amply demonstrated, there is now reason to believe that the presence of fractal geometry (in a sense) underlies these biophilic responses. To put it very crudely, it is not the tree that the causes these emotional responses, but the fractal mathematics of the tree. Preliminary research that supports this hypothesis has been conducted by Caroline Hägerhäll and colleagues [2004]. They found that the emotional states towards vegetated/natural landscapes can be predicted by typical fractal characteristics (i.e., the fractal dimension). This study, and others, also indicate that the aesthetic reactions peak when the natural settings, or the fractal pattern underlying it, have an intermediate fractal dimension (around 1.3 – 1.5) [Aks and Sprott 1996; Abraham et al. 2003; Spehar et al. 2003]. What is even more surprising is that images with this range of fractal dimension also dampen stress in humans [Wise and Taylor 2002]. I, and others, have speculated that this reflects our inborn emotional affiliation with savanna-type landscapes, which are intermediately complex environments [Joye 2007]. In fact, I believe that the brain constantly evaluates settings with regard to their habitability. It would be adaptive if it could calculate the fractal dimension of a setting: this quickly conveys basic information about the complexity of a particular setting, which is a strong indicator whether it is a good place for living. Settings with a high fractal dimension could contain hidden dangers, such as ambushing predators, while those with a low fractal dimension probably do not contain enough elements, in order to offer protection and sources of food. Hence, we will be more aesthetically attracted and more relaxed in environments with an intermediate fractal dimension.

Now what is the link with architecture? In essence, the previous argument points out that, as a result of evolution, the brain has a preference for fractal structures, and feels relaxed when surrounded by these. This means that one of the reasons why we like the fractals in Gothic and Hindu architecture is that they remind us of our ancient, natural habitats. Because our brains have not fundamentally changed since prehistory, these biofilic responses are still at work. However, in the modern world, the fractal forms (e.g. trees) which we crave for are increasingly pushed back from our current habitats, and often replaced by simple Euclidean forms. Some scholars argue that this discrepancy can lead to an increased release of stress-related hormones, which, in the long run, can have deleterious effects on human health [Parsons 1991]. Others think that this discrepancy could be one of the underlying causes for psychopathologies in westernized societies [Gullone 1999]. With all this I do not claim that Euclidean buildings are inherently wrong or unhealthy; in fact, we are as much cultural as biological beings. However, the dominance of Euclidean shapes could prove harmful, and seems at variance with some of the mathematical preferences of our brains. As we all intend to live good lives, it wouldn't be a bad idea to replace some of them by architectural work that implements some essential fractal characteristics. Our preference for a specific fractal dimension further indicates that the effects of such buildings can be maximized for intermediate levels of complexity. So, the next time you feel stressed from a hard day's work, stimulate your brain with some fractals – albeit natural or architectural ones.

Notes

1. You construct this 'borderline' fractal as follows. Consider a square whose side length is 1. In the first step, make a column on the middle third part of the square with length 1/3 and height 1/2. In the second step, erect a column of height 1/4 over the interval 1/9 - 2/9 and one of height 3/4 over the interval 7/9 - 8/9. In the third step make four columns of heights 1/8, 3/8, 5/8, 7/8. For k steps 2 k-1 columns are drawn of heights 1/2k, 3/2k,... , (2k–1)/2k.
2. You obtain the box counting dimension as follows. First, place a rectangular grid over a (two-dimensional) representation of the architectural object. Count the number of boxes across the bottom of the grid (B1), and the number of boxes that contain a portion of the representation (N1). Next, decrease the size of the boxes of the grid, and again count the number of them at the bottom of the grid (B2), and the number that contains a fragment of the object (N2). Finally, plot a log (B) versus log (1/N) on a log-log diagram. The slope of this (straight) line approximates the box counting dimension.

References

ABRAHAM, F. D., et al. 2003. Judgements of time, aesthetics, and complexity as a function of the fractal dimension of images formed by chaotic attractors. http://www.blueberry-brain.org/silliman/JEM%20ms2.htm.

AKS, D.J. and J. C. SPROTT. 1996. Quantifying aesthtic preference for chaotic patterns. *Empirical studies of the arts* **14**: 1-6.

ANDERSON, C.M. and A.J. MANDELL. 1996. Fractal time and the foundations of consciousness: vertical convergence of 1/f phenomena from ion channels to behavioural states Pp. 75-126 in *Fractals of brain, fractals of mind*, E. MacCormac and M.I. Stamenov eds. Amsterdam/Philadelphia: John Benjamins.

BOVILL, C. 1996. *Fractal Geometry in Architecture and Design*. Boston: Birkhäuser.

BURKLE-ELIZONDO, G. and VALDÉZ-CEPEDA, R.D. 2006. Fractal Analysis of Mesoamerican Pyramids. *Nonlinear Dynamics, Psychology, and Life Sciences* **10**: 105-122.

CAPO, D. 2004. The Fractal Nature of the Architectural Orders. *Nexus Network Journal* 6, 1: 30-40.

CROMPTON, A. 2002. Fractals and picturesque composition *Environment and Planning B: Planning and Design* **29**: 451-459.

EATON, L. 1998. Fractal Geometry in the Late Works of Frank Lloyd Wright. Pp. 23-38 in *Nexus II: Architecture and Mathematics*, K. Williams, ed. Fucecchio (Florence): Edizioni Dell'Erba.

EGLASH, P. and T.B. ODUMOSU. 2005. Pp. 101- 109 in *What Mathematics from Africa?*, G. Sica (ed.) Monza: Polimetrica International Scientific Publisher.

GOLDBERGER, A.L. 1996. Fractals and the birth of Gothic: reflections on the biological basis of creativity. *Molecular Psychiatry* 1: 99-104.

GILDEN, D.L., et al. 1993. *Psychological Review* 100: 460-78.

GULLONE, E. 2000. The biophilia hypothesis and life in the 21st century: increasing mental health or increasing pathology. *Journal of Happiness Studies* 1: 293-321.
HÄGERHÄLL, C., et al. 2004. Fractal dimension of landscape silhousette outlines as a predictor of landscape preference. *Journal of Environmental Psychology* 24: 247-255.
HERSEY, G. 1993. *High Renaissance Art in St. Peter's and the Vatican: An Interpretive Guide*. Chicago: University of Chicago Press.
JENCKS, C. 2002. *The New Paradigm in Architecture*. New Haven & London: Yale University Press.
JOYE, Y. In preparation. Towards Nature-Based Architecture: Drawing Lessons from Psychology. *Review of General Psychology* (2007).
KNILL, D.C., et al. 1990. Human discrimination of fractal images. *Journal of the Optical Society of America A* 7: 1113-1123.
PARSONS, R. 1991. The potential influences of environmental perception on human health. *Journal of Environmental Psychology* 11: 1-23.
PORTOGHESI, P. 2000. *Nature and Architecture*. Milan: Skira.
SALINGAROS, N. 1998. A scientific basis for creating architectural forms. *Journal of Architectural and Planning Research* 15: 283-293.
SPEHAR, B., et al. 2003. Universal aesthetic of fractals. *Computers & Graphics* 27: 813-820.
TRIVEDI, K. 1988. Hindu temples: models of a fractal universe. *Space design* 290: 234-258.
ULRICH, R.S., et al. 1991. Stress recovery during exposure to natural and urban environments. *Journal of Environmental Psychology* 11: 201-230.
ULRICH, R.S. 1993. Biophilia, biophobia, and natural landscapes. Pp. 73-137 in *The Biophilia Hypothesis*, S.R. Kellert and E.O. Wilson, eds. Washington: Island Press.
WISE, J.A. and R. TAYLOR. 2002. Fractal design strategies for enhancement of knowledge work environments. Pp. 854-859 in *Proceedings of the 46th meeting of the Human Factors and Ergonomics Society*. Santa Monica, CA: Human Factors and Ergonomics Society.

About the author

Yannick Joye obtained his Ph.D. in philosophy at the University of Ghent, Belgium. His research focuses on the question which architectural interventions can be deduced from the fact that humans have evolved in natural, fractal-like habitats. He is thereby among those that try to provide a theoretical framework for the upcoming field of biophilic architecture.