Fractal Audio Coding

by

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Abstract

We explore the performance of applying fractal coding on audio data. Some conventional fractal coding problems have been studied with audio data to provide an overview on this subject.

A review of fractal coding is presented. We implement a fractal audio coding scheme to carry out the experiments. The performance of the scheme can be controlled by a number of different parameters, including mapping tolerance, scaling factor, partition range, domain specification, and bit allocation. Empirical results have been obtained from experimenting on various audio data in our testing set under different parameter combinations. Some conclusions and suggestions have been made by analyzing and comparing experimental results. The study leads us to conclude that fractal coding is not an appropriate model to be applied alone to complex audio data. A major barrier is the inability to represent smooth continuous functions.

With the new trend of integrating some other methods in fractal coding research, we discuss some future aspects of fractal audio coding, and some possible improvements by combining it with other techniques such as wavelet transforms.

Acknowledgments

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Chapter 1

Introduction

Fractals were first introduced in the field of geometry. The birth of fractal geometry is usually traced back to the IBM mathematician Benoit B. Mandelbrot and the 1977 publication of his book "The Fractal Geometry of Nature". Later, Michael Barnsley, a leading researcher from Georgia Tech, found a way of applying this idea to image representation and compression with the mathematics of *Iterated Functions Systems* (IFS). Fractal compression algorithms based on IFS are not practical because of their high computational complexity. It is Arnode Jacquin, who finally set a practical fractal coding algorithm using *Partitioned Iterated Function Systems* (PIFS). Since the development of PIFS, fractal image coding has been widely studied and various schemes have been derived and implemented.

Because of fractal image coding's various shortcomings, researchers still can not deliver a practical fractal image coding scheme. For this reason, fractal coding is rarely studied on other types of data except on images. Recently, fractal coding has been extended to audio data as its natural next step. Our interest in this thesis is to explore this possibility. We will experiment on various audio data with conventional

fractal coding schemes to gather performance results. Then, through analyzing and comparing the resulting data, we will attempt to understand fractal coding behaviour on audio data, which may contribute to future research in this aspect.

The thesis is organized starting from some formal fractal theory in Chapter 2. A brief fractal audio model is provided based on the conventional fractal image model. The encoding and decoding algorithms are explained through examples. A review of fractal coding researches is presented in Chapter 3. We mainly address fractal coding and related problems from fractal image compression studies. Our implementation of fractal audio coding and experimental results are described in Chapter 4. We provide details of our implementation by comparing it with conventional fractal image coding implementations. Experiments and empirical results are presented from different perspectives. Some conclusions and suggestions are made by studying the resulting data. Some discussion and future aspects regarding fractal audio coding and fractal systems are proposed in the final chapter.

Chapter 2

Fractal Compression

2.1 Introduction

In this chapter, we shall first present fractal compression as it is usually presented, without any reference to other conventional compression methods. A short mathematical background of fractal compression is provided. The reader unfamiliar with fractal compression is referred to [5] and [11] for a more detailed treatment. We further relate fractal compression with audio data[1] to present the compression and decompression algorithms. Fractal compression advantages and weaknesses from the previous studies on fractal image compression are presented. It is noticeable that the compression scheme is identical on both kinds of data, thus share the same advantages and suffer the same weaknesses. However, we believe the effects may vary since our human perceptions of audio and image are very different.

[1] In this thesis, we use WAVE PCM format, see Appendix B on page 76

2.1.1 Fractal Development

Fractals were not developed for data compression in the first place, but as a different kind of geometry by the IBM mathematician Benoit B. Mandelbrot. In 1981, mathematician John Hutchinson used the theory of iterated function system to model collections of contractive transformations in a metric space as dynamical systems, which later provides the theoretical support of recognizing fractals in metric space. It was Michael Barnsley, eventually, who generated the fractal model using *Iterated Function Systems* (IFS), and led to encoding of images to achieve significant compression.

However, Barnsley's image compression algorithm based on fractal mathematics was inefficient and unpractical suffering a space searching problem that was too large to be practical. In 1988, one of Barnsley's Ph.D. students, Arnaud Jacquin, arrived a modified scheme for representing images called *Partitioned Iterated Function Systems* (PIFS), and implemented the algorithm in his Ph.D. thesis. The basic idea of the algorithm is to convert the whole image into PIFS. It immediately made the fractal image compression algorithm more practical, however, by sacrificing the compression ratio. After Jacquin's PIFS, there were many other modified schemes [5](ch.11 and ch.13), but none of them made any significant progress. Most of the later publications on the subject of fractals follow the PIFS, but focus on some possible improvements. The two big problems of Jacquin's algorithm are the partition selection scheme for encoding, and the speed of decoding.

Fractal compression was lately applied to audio data in [25]. The general belief is that a purely fractal coding scheme is not suitable for audio compression. The reason originates from the fact that fractal compression is mostly based on a fractal

system's ability to approximate discontinuous functions, but audio signals usually exhibit greater smoothness than images. Regarding the remarks above, we think that more experiments should be done with different types of audio data based on conventional fractal coding schemes to demonstrate fractal behavior on audio data, or simply for understanding the nature of fractals.

2.1.2 Fractal Properties

A definition of the term fractal is difficult. People usually regard fractals as a set of properties. Fisher has given a definition of the property set in his book [5](pg.26) as follows:

> If we consider a set F to be a fractal, we think of it as having (some) of the following properties:
>
> 1. F has detail at every scale.
> 2. F is (exactly, approximately, or statistically) self-similar.
> 3. The "*fractal dimension*" of F is greater than its *topological dimension*[2].
> 4. There is a simple algorithm description of F.

We feel this definition of the property set is somehow hard to measure, and not very useful for understanding fractal theory. A more useful and rigorous comparison can be made with *Vector Quantization* (for details, see [6]) for the reader familiar with conventional compression methods.

In general, the principle of mapping an n-dimensional vector to a unique symbol from the codebook is called vector quantization. The quantization process can be viewed as identifying patterns, and storing only a few of the common patterns to

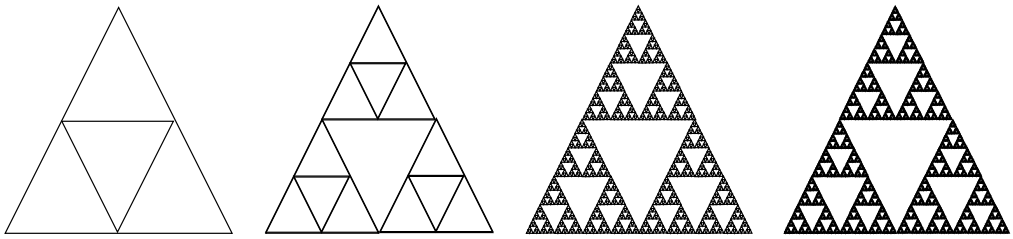
[2]For details on fractal dimension and topological dimension, see [27].

form the codebook such that the compression is achieved. The similarity to fractal compression is apparent. The noticeable difference is that vector quantization uses straight copy which can be lossless, but fractal encoding needs to find out the mapping functions from the domain blocks to the range blocks[3], which is a lossy scheme. More important, fractal compression's codebook, which is referred as a "virtual codebook" in some contexts, is implicitly specified by a set of iteration functions. So fractal compression in theory allows higher compression ratio, but more unpredictable than vector quantization on the other hand.

2.1.3 Fractal Examples

We provide two fractal examples in this section for demonstration purposes.

The first example is generating Sierpinski's Triangle using an IFS. We can see from Figure 2.1 that at each iteration, we shrink the original triangle by a factor of 2, make three copies, and place them to form a new triangle one iteration further.



(a) Initial Image (b) First Iteration (c) Fifth Iteration (d) Sixth Iteration

Figure 2.1: Sierpinski Triangle example of generating sequence from iterating.

This process can be viewed as three transformations mapping the original triangle to the new triangle as demonstrated in Figure 2.2 on the next page.

[3]Domain block and range block are to be defined later in this chapter

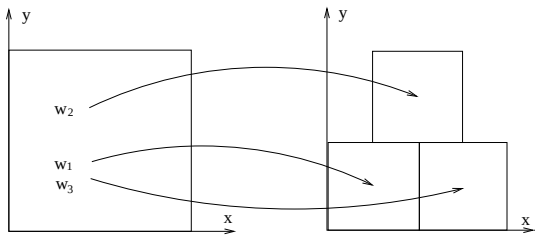


Figure 2.2: Sierpinski Triangle mapping from three transformations.

The mathematical expressions of the transformations are given below:

$$w_1 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.1}$$

$$w_2 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1/4 \\ 1/2 \end{pmatrix} \tag{2.2}$$

$$w_3 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} \tag{2.3}$$

It is important to observe that because of the resolution in Figure 2.1, there is hardly any visible difference between the fifth and the sixth iterations. The Sierpinski's triangle sequence thus visually converges to one triangle within a certain threshold. Reversely, if we take the sequence backward, for the Sierpinski triangle sequence, we only need to know the mapping functions (i.e., w_1, w_2, and w_3) to get the whole picture. This is in essence of how fractal compression works.

The second example is in Figure 2.3 on the next page. It demonstrates one domain-range match identified from the Institution of Eurecom's java applet implementation[4] on conventional fractal image compression algorithm [5].

[4]http://www.eurecom.fr/ image/DEMOS/FRACTAL/english/

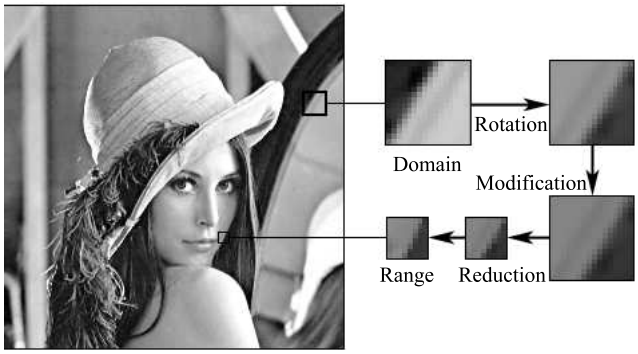


Figure 2.3: Fractal encoding from a domain block maps to a range block on Lena image.

The match is calculated through the rotation and modification. And since the domain block is twice the size of the range block in this case, the reduction is needed afterward. Essentially, in image sense, the rotation and modification are the process to get a proper transform function, which adjust the orientation and luminance of the domain block. The Lena image is a widely used test figure in image compression. It illustrates the idea that natural images have self-similarities or patterns embedded.

2.2 Fractal Mathematical Background

This section is devoted to making the above demonstrations mathematically precise. The mathematics of fractals is related to metric spaces. However, we do not give a concrete treatment on that subject. The interested reader is referred to [5], [11], or any textbook on metric spaces. Fractal notions are better explained here. The image fractal model is based on a conventional fractal image compression scheme, and the audio fractal model is constructed based on the image fractal model.

2.2.1 Complete Metric Spaces

We give the following definitions and theorems on the metric space related to our subject.

Definition 1. *A metric space is a set X on which a real-valued distance function $d : X \times X \to R$ is defined, satisfying the following properties:*

1. *$d(a, b) \geq 0$ for all $a, b \in X$.*
2. *$d(a, b) = 0$ if and only if $a = b$, for all $a, b \in X$.*
3. *$d(a, b) = d(b, a)$ for all $a, b \in X$.*
4. *$d(a, c) \leq d(a, b) + d(b, c)$ for all $a, b, c \in X$ (triangle inequality).*

Such a function d is called a **metric**.

Definition 2. *A map $f : X \to X$ is* **contractive** *over the metric space (X, d) if:*

$$d(f(x), f(y)) \leq s \cdot d(x, y) \qquad \forall x, y \in X$$

where $0 \leq s < 1$ is called the **contractivity** *of f.*

Definition 3. *A sequence $\{x_n\}_{n=1}^{\infty}$ in X is said to* **converge** *to some $x \in X$ where (X, d) is a metric space if $\forall \epsilon > 0$ $\exists N > 0$ such that:*

$$d(x, x_n) < \epsilon \qquad \forall n \geq N.$$

Definition 4. *A sequence $\{x_n\}_{n=1}^{\infty}$ in X is a* **Cauchy** *sequence if $\forall \epsilon > 0$ $\exists N > 0$ such that:*

$$d(x_m, x_n) < \epsilon \qquad \forall n, m \geq N.$$

Definition 5. *A metric space (X, d) is* **complete** *if every Cauchy sequence in X converges to some $x \in X$.*

Definition 6. *$\alpha \in X$ is a* **fixed point** *of the function $f : X \to X$ if $f(\alpha) = \alpha$.*

A well known result is that a Cauchy sequence converges to a fixed point [5] [11]. However, whether a sequence is Cauchy depends on the definition of the distance function d.

By Definition 2, a map is contractive if it brings points closer together by the defined metric d. The contractivity s measures how much closer two points are brought together. All contractive maps have unique fixed points, but not all maps with unique fixed points are contractive. Recall the Sierpinski's triangle mapping example in Figure 2.2 on page 7. The map function is contractive as $x \mapsto x/2$. It is not hard to see that this mapping has a unique fixed point 0. In other words, starting from any initial stage, we will eventually go to a unique fixed picture with such a mapping. It is critical to recognize the contractive mapping in fractal compression, because our goal is to identify the fixed point in fractal coding to achieve compression.

2.2.2 Contractive Mapping Fixed-Point Theorem

There are two main theorems supporting fractal theory. One is the *contractive mapping fixed-point theorem*, the other is recognized as a corollary of the first, the *collage theorem*.

Theorem 1 (Contractive Mapping Fixed-Point Theorem)**.** *Let (X, d) be a complete metric space and $f : X \to X$ be a contractive mapping. Then there exists a unique*

fixed point $x_f \in X$ such that:

$$x_f = f(x_f) = \lim_{n\to\infty} f^{on}(x) \qquad \forall x \in X. \tag{2.4}$$

where $f^{on}(x)$ denotes f composed with itself n times. The value x_f is also called the **attractor** *of the mapping f.*

Proof: See [2] and [5] (ch.2). □

Theorem 2 (Collage Theorem)**.** *If (X, d) is a complete metric space and $f : X \to X$ is a contractive map with fixed point $\alpha \in X$, then:*

$$d(x, \alpha) \leq \frac{1}{1-s} d(x, f(x)) \qquad \forall x \in X. \tag{2.5}$$

where s is the contractivity of f.

Proof: This is a consequence of applying Theorem 1. □

Definition 7. *Let (X, d) be a metric space. A map $w : X \to X$ is* **Lipschitz** *with Lipschitz factor s if $\exists s > 0$, such that:*

$$d(w(x), w(y)) \leq s \cdot d(x, y) \qquad \forall x, y \in X$$

If $s < 1$, then w is contractive with contractivity s.

Using the Lipschitz definition, we can define the *eventually contractive* for a function.

Definition 8. *Let f be Lipschitz function. If there is a number n such that f^{on} is contractive, then f is* **eventually contractive**. *And n is the exponent of eventual contractivity.*

Now, we can get a more generalized collage theorem by using eventual contractivity. This is more interesting for fractal coding theory.

Corollary 1 (Generalized Collage Theorem)**.** *Let (X, d) be a complete metric space, and f be eventually contractive with exponent n, then there exists a unique fixed point $x_f \in X$ such that*

$$x_f = f(x_f) = \lim_{k\to\infty} f^{ok}(x) \qquad \forall x \in X.$$

In this case,

$$d(x, x_f) \leq \frac{1}{1-s}\frac{1-\sigma^n}{1-\sigma} d(x, f(x)),$$

where s is the contractivity of f^{on} and σ is the Lipschitz factor of f.

Remarks: The collage theorem gives us a hope of finding a contractive mapping and identifying the attractor (fixed point), but does not explicitly tell us how to find such a contractive mapping to achieve compression in general.

For example, an image is our space X. Suppose we define a metric d in X that makes (X, d) a complete metric space. Then we need to find a contractive mapping f such that the attractor of f is close to the target image. The collage theorem proves the existence of f. And by the contractive mapping theorem we can use the f to approximate the original image. The reconstruction process from an attractor is by repeatedly applying f in reverse from a random image. Barnsley called this as the ***inverse problem***. It is worth noticing that for audio data, the general idea is the same as for images.

Corollary 1 shows that it is not necessary for f to be contractive for all n that are sufficiently large. It is sufficient if f is contractive for some n. In other words, f is good enough to be eventually contractive to ensure a contractive mapping.

2.2.3 Affine Transformations

Most publications in fractal compression define the mapping function f to be affine to simplify the computation. An *affine transformation* $w : \mathbb{R}^n \to \mathbb{R}^n$ can always be written as $w = Ax + b$, where $A \in \mathbb{R}^{n \times n}$ is an $n \times n$ matrix and $b \in \mathbb{R}^n$ is an offset vector. The transformation is contractive when its linear part is contractive. We know from the previous section that contractiveness depends on the metric used to measure distance. Because affine transformations are easy to compute in Euclidean space, we can use a norm $\|\cdot\|$ in $\mathbb{R}^n$ to define the metric. Then $x \mapsto Ax$ is contractive when

$$\|A\| = \sup_{\overrightarrow{x} \in \mathbb{R}^n} \|A\overrightarrow{x}\| / \|\overrightarrow{x}\| < 1.$$

The contractiveness under the sup norm of a complete metric space is guaranteed if we satisfy the above equation that the left hand side is always less than one. However, Wohlberg and Jager in their review [26] pointed out that this restriction is sufficient but not necessary for convergence; empirical evidence indicates that convergence is often achieved even $\|A\|$ is greater than one, although smaller values provide more rapid convergence in decoding as reported in [5](pg.62). This problem can be related to eventually contractive defined in the previous section. As we have remarked, the eventually contractive mapping function is sufficient to ensure contractiveness or convergence of the mapping. Setting the equation less than one guarantees absolute contractiveness, which is a stronger argument than eventual contractiveness. Unfortunately, the affine transformation and sup norm metric can not explicitly give us a bound to ensure eventual contractiveness.

2.2.4 Partitioned Iterated Function Systems (PIFS)

The Sierpinski triangle in Figure 2.1 on page 6 demonstrates the way of using IFS. However, unlike the example, our real spaces are very irregular. In most cases, it would be rather impossible to find such a perfect mapping for the whole space. Thus Jacquin introduced *Partitioned Iterated Function Systems* (PIFS) in his work [9]. A PIFS is a generalization of an IFS, and attempts to ease the IFS computation by partitioning the whole space into subspaces. In other words, the PIFS is a restricted version of the IFS.

Definition 9. *Let (X, d) be a complete metric space, and let $D_i \subset X$ for $i = 1, \ldots, n$, such that $\bigcup_i D_i = X$. A PIFS is a collection of contractive maps $w_i : D_i \to X$, for $i = 1, \ldots, n$.*

One problem brought up from PIFS is the partition. The space has to be partitioned into subspaces. It is necessary to ensure that the addition of the subspaces covers the original space. Also, the partition scheme dominates the final map set, which is essential to the coding process. Moreover, finding an optimal fractal encoding has been shown as NP-hard [21], and collage based fractal coding may produce a solution of arbitrary distance from the optimal solution. One partition scheme thus can yield very unpredictable results on different spaces.

Jacquin first introduced PIFS on fractal image compression. The image space is naturally recognized as a 2D space. The partition scheme simply partitions the whole space twice into the range set and the domain set. Both sets cover the whole image space, with the domain set allowing overlaps. A shortcoming of using an affine transformation, the partition schemes for the domain set and the range set have to give the same geometric shaped domain and range blocks, which are usually squares or

rectangles. The domain block is set to be twice as big as the range block in Jacquin's original scheme [9], which is widely accepted in fractal image compression field. The reason for allowing domain overlapping is to smooth artifacts between blocks in the decoding process. The mappings between the domain and the range blocks are as demonstrated in Figure 2.4. For each range block, we find a proper domain block to map to. The final map set is composed of mappings for each range block from the range set.

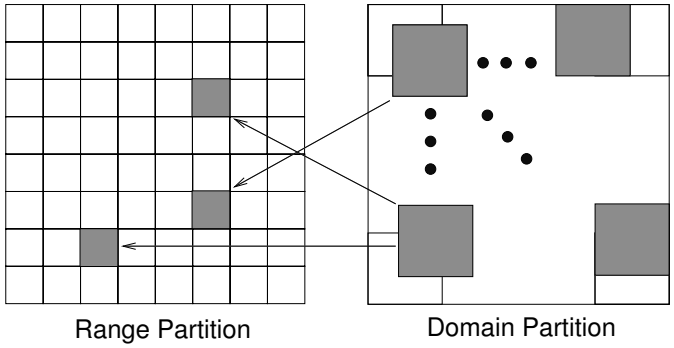
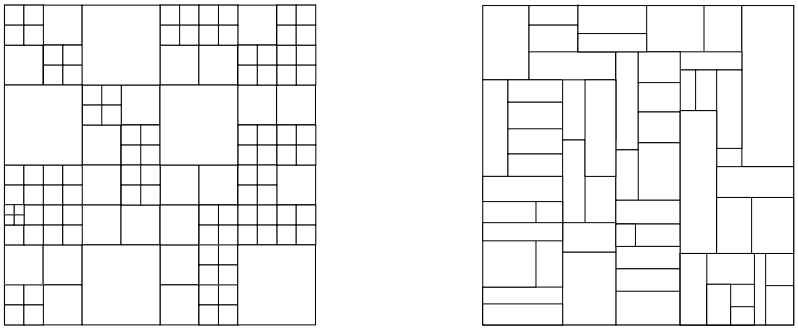


Figure 2.4: Mapping from the domain set to the range set.

Among most of fractal image compression range partition schemes appearing in the literature, *Quadtree* partition and *Horizontal-Vertical* (HV) partition are two of the most popular schemes being used. We show two examples in Figure 2.5 for both partition schemes. Review [26] classifies range partition schemes into right-angled partition schemes, and triangular and polygonal range partition schemes. Both quadtree and HV schemes belong to the first category. The interested reader may refer to [26] for more information.



(a) Quadtree range partition

(b) HV range partition

Figure 2.5: Examples of quadtree and HV range partition schemes

PIFS is recognized as a significant improvement over IFS. It reduces large amount of searching time both theoretically and practically. Furthermore, there are some potentials to improve fractal encoding like applying different partition schemes or taking different mapping methods. Comparing with some other advanced method of generating fractals such as *Weighted Finite Automata*, PIFS also have the beauty of simplicity. For the above reasons, our research on fractal audio coding uses PIFS in the same way as many conventional fractal image compression schemes.

2.2.5 Image and Audio Models

Fractal image and audio models can be naturally generated from fractal theory. In general, for a given space, we need to find a proper metric of the distance measure to define a complete metric space. Then we need to define a PIFS in the metric space, and the mapping method between the domain block and the range block. We provide a fractal image model and a fractal audio model in the following contents in this section.

Fractal Image Model

Image space is a 2D space consisting of pixels. The location of each pixel is given by two co-ordinates. Here, we take monochrome images. The pixel values range from 0 to 255 representing grey levels. An image containing $M \times N$ pixels can be thought of as a vector in an $n = M \cdot N$-dimensional space. Then the space is $\{0, 1, \ldots, 255\}^n \subset \mathbb{R}^n$. Common norms in $\mathbb{R}^n$ are the p-norms, defined by:

$$\|\mathbf{x}\|_p = (|x_1|^p + |x_2|^p + \cdots + |x_n|^p)^{\frac{1}{p}},$$

with the metric defined by:

$$d_p(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_p.$$

The 2-norm is the most widely used metric in fractal image compression, which is referred as the ℓ^2 norm or the rms metric in main literature. Thus, the difference of two images $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$ on the ℓ^2 norm or rms metric is given by

$$d_{rms}(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|_2 = \sqrt{\textstyle\sum_{i=1}^{n}(x_i - y_i)^2}.$$

It is shown that the rms metric is more convenient to use than other metrics because it can be calculated from the standard inner product $\langle \cdot, \cdot \rangle$ given by

$$d_{rms}(\mathbf{x}, \mathbf{y}) = \sqrt{\langle \mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{y} \rangle}.$$

This provides an easy way of solving distance minimization problem, which is critical, because our mapping quality is normally measured by the distance between two blocks in the space. We can find α, β which minimize $d_{rms}(\alpha\mathbf{x} + \beta\mathbf{y}, \mathbf{z})$ by minimizing:

$$\begin{aligned} d^2_{rms}(\alpha\mathbf{x}+\beta\mathbf{y},\mathbf{z}) &= \langle \alpha\mathbf{x}+\beta\mathbf{y}-\mathbf{z}, \alpha\mathbf{x}+\beta\mathbf{y},\mathbf{z}\rangle \\ &= \alpha^2\langle\mathbf{x},\mathbf{x}\rangle + 2\alpha\beta\langle\mathbf{x},\mathbf{y}\rangle + \\ &\quad \beta^2\langle\mathbf{y},\mathbf{y}\rangle - 2\alpha\langle\mathbf{x},\mathbf{z}\rangle - 2\beta\langle\mathbf{y},\mathbf{z}\rangle + \langle\mathbf{z},\mathbf{z}\rangle. \end{aligned}$$

Differentiating with respect to α and β to find the minimum given by:

$$\alpha = \frac{\langle\mathbf{y},\mathbf{z}\rangle\langle\mathbf{x},\mathbf{y}\rangle - \langle\mathbf{y},\mathbf{y}\rangle\langle\mathbf{x},\mathbf{z}\rangle}{\langle\mathbf{x},\mathbf{y}\rangle^2 - \langle\mathbf{x},\mathbf{x}\rangle\langle\mathbf{y},\mathbf{y}\rangle}, \tag{2.6}$$

$$\beta = \frac{\langle\mathbf{x},\mathbf{y}\rangle\langle\mathbf{x},\mathbf{z}\rangle - \langle\mathbf{x},\mathbf{x}\rangle\langle\mathbf{y},\mathbf{z}\rangle}{\langle\mathbf{x},\mathbf{y}\rangle^2 - \langle\mathbf{x},\mathbf{x}\rangle\langle\mathbf{y},\mathbf{y}\rangle}. \tag{2.7}$$

Equations 2.6 and 2.7 can be treated as two coefficients of an affine transformation from $\mathbf{x}$ to $\mathbf{z}$ taking $\mathbf{y}$ as a constant vector. So we actually show that it is easy to calculate the best affine transformation with minimum rms distance between two blocks. This is the primary reason of defining this image model on ℓ^2 space with rms metric. Some other fractal image models are available in [5](ch.13), but are rarely used.

Based on the above development, under ℓ^2, using rms to measure the likelihood between two images seems very intuitive. However, in practise, the *Peak Signal-to-Noise Ratio* (PSNR) is experimentally proved to be more accurate as a distance measure than rms or SNR. We give the equation of calculation below:

$$PSNR = 20\log_{10}(\tfrac{b}{rms}).$$

where b is the largest possible value of the signal. Notice PSNR is a measurement of the similarity between two images, but not a metric in the image space.

Fractal Audio Model

Our fractal audio model starts with recognizing the audio file as a collection of audio samples. We work with uncompressed WAVE format audio files (see Appendix B for

WAVE format). And further restrict the sample type to be *unsigned* such that each sample can be treated as an integer from 0 to 255. Audio data is then a sequentially stored vector of many samples with respect to time. The audio sequence is thus treated in 1D space. It is easy to apply the above fractal image model in ℓ^2 with rms to audio taking the advantage of the easy minimum distance calculation. Theoretically, the two models are the same. Technically, the difference is the way of representing the blocks. An image block is represented as a matrix of pixel values, and an audio block is represented as a vector of sample values.

We have attempted to generate an audio sequence in 2D space, which puts the samples into a matrix based on certain order like music sections, paragraphs, etc. But unfortunately, we realize that a specific order can not be universally applied, and there has not been any identified universal order or pattern that can be used. So, in our implementation, we use 1D representation of audio as a sequential sample vector.

Psychoacoustics is a field that studies human perception of sounds. There is ongoing research to determine models of audio sequences that are perceived by humans to be similar [8]. We take the straight rms distance as the measurement in our experiments since no other proper measurement has been developed so far. Note that it is possible to have two audio sequences with a high rms difference, but sound very similar to our hearing, and vice versa. However, because our focus is on fractal coding here, we think it is still appropriate to use rms as the measurement of the similarity. Regarding more accurate audio measure, human acoustic testing may need to be performed.

2.3 Compression and Decompression Algorithm

We explain the compression and decompression algorithm based on our fractal audio model. Compression is achieved from fractal encoding, and decompression is fractal decoding.

2.3.1 Fractal Encoding

Our encoding is based on the PIFS. Our goal of encoding is to find a contractive map set W whose fixed point is close to the audio space F[5] that we wish to compress. W is the union of a set of contractive maps $w_1, \ldots, w_n$ from the PIFS.

Before we compute W, we have to partition our space F twice. Divide F into disjoint range blocks $R_1, \ldots, R_n$, so that the union of R_i $(i = 1, 2, \ldots, n)$ covers F. Divide F again into domain blocks $D_1, \ldots, D_m$. Then, for each R_i, we compare it with all domain blocks to find D_j that can be mapped to R_i with the smallest rms distance. Store D_j and w_i from encoding as output. The following theorem ensures W is contractive from the union of $w_1, \ldots, w_n$.

Theorem 3. *If $w_1, \ldots, w_n$ are contractive, then*

$$W = \bigcup_{i=1}^{n} w_i$$

is contractive in F with the sup metric.

Proof: Let $s = \max_i s_i$, where s_i are the contractivities of w_i.

[5]Space F with metric d_{sup} (d_{rms} in this case) is complete.

$$
\begin{aligned}
d_{sup}(W(f), W(g)) &= \sup\{|W(f)(x) - W(g)(x)| \ : x \textit{ is a coordinate in the space}\} \\
&= \sup\{\textit{sample value of } |w_i(x, f(x)) - w_i(x, g(x))| : \\
&\qquad\qquad x \textit{ is a coordinate in } D_i, i = 1, \ldots, n\} \\
&\leq \sup\{s_i |f(x) - g(x)| : i = 1, \ldots, n\} \\
&\leq \sup\{s|f(x) - g(x)|\} \\
&\leq s \sup\{|f(x) - g(x)|\} \\
&\leq s d_{sup}(f, g)
\end{aligned}
$$

s_i is from each w_i, so that $s_i < 1$, then $s < 1$. W is thus contractive under d_{sup}. □

The contractiveness of W in F determines a unique fixed point in F by the contractive mapping fixed-point theorem. Note in our audio model, an audio sequence is regarded in 1D space, so that only one coordinate x is needed to locate a sample. We assume each w_i is contractive, and $s_i < 1$. But in fact, from previous developments, we know that it is sufficient for W to be eventually contractive, which may loose the requirements and allow some $s_i \geq 1$.

Our audio encoding under rms and ℓ^2 is now clear. To encode F, we find contractive mapping W by the union of w_i. For each w_i, the method for computing the affine transformation's coefficients has already been specified in the previous section. Our encoding is thus complete.

Encoding Example

We give the following example in Figure 2.6 to demonstrate the encoding process. The given audio sequence has been read in as an array of unsigned integers sample by sample. The range and the domain partitions have been performed. We get a current range block R_i with size of 2, and domain blocks D_j and D_k with size of 4.

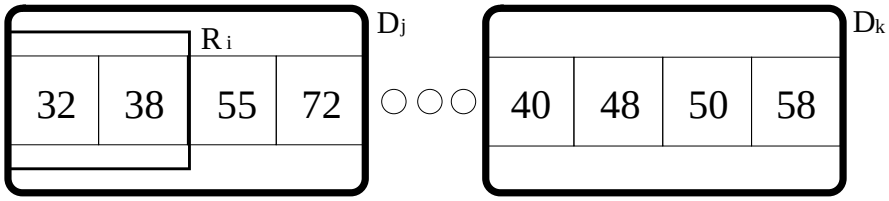


Figure 2.6: Encoding mapping from a domain to a range.

So $F = (\ldots, 32, 38, 55, 72, \ldots, 40, 48, 50, 58, \ldots)$, and

$R_i = (32, 38)$, $D_j = (32, 38, 55, 72)$, $D_k = (40, 48, 50, 58)$.

By affine transformation, our mapping w_i for R_i is:

$$w_i(D) = \alpha_i \phi(D) + \beta_i(1, 1).$$

where we take $\phi(D)(j) = \frac{1}{2}(D(2j) + D(2j+1))$, which averages the adjacent two samples from the domain D and shrinks the domain vector to half so that the output vector is the same size as the range vector's.

Using the rms metric, we can find α and β by applying Equation 2.6 and 2.7 on page 18. Table 2.1 shows the best matchings from D_j and D_k to R_i in this example.

We can see that both of the domain blocks D_j and D_k can be mapped to the range block R_i closely under rms. D_k gives us a perfect mapping with 0 distance. In real audio sequences, we find that it is generally impossible to find such perfect mappings, or even the ones with small difference. So, normally, we set up a tolerance value for the difference. The first mapping calculated within ($\leq$) the tolerance is stored and the map search for that range block is terminated. In our example case, if the rms difference of $D_j \mapsto R_i$ is within the tolerance, the mapping function for R_i is stored, and D_k won't be further tested.

The encoding operation will be performed on all range blocks. The compressed

Range Block (R)	Domain Block (D)	Scale (α)	Offset (β)	Difference (rms)
R_i(32 38)	D_j(32 38 55 72)	0.2105	24.6316	0.0019
R_i(32 38)	D_k(40 48 50 58)	0.6000	5.6000	0.0000

Table 2.1: Range-Domain mappings from Figure 2.6

sequence is represented as an output file storing all scaling and offset coefficients (i.e., α and β) as well as the domain block positions (i.e., index j).

2.3.2 Fractal Decoding

Fractal decoding is a straight iterative process. Begin with any audio sequence f_0[6], we successively compute $W(f_0), W(W(f_0)), \ldots$ until the sequence converges to the attractor f_W. Recall $W = \bigcup_{i=1}^{n} w_i$, at each iteration, we apply all w_i on f and take the union, so that $W(f) = \bigcup_{i=1}^{n} w_i(f)$.

The fractal decoding process is very time-consuming because of the iteration steps. The speed is related with the contractivity of w_i, which decides how fast the sequence converges to the attractor f_W. There has been some research done on fast fractal image decoding by avoiding directly applying W iteratively, see [5](ch.5 and ch.8). We do not provide any fast decoding method on audio data in this thesis. So decoding time is not reported.

2.4 Advantages and Weaknesses

Fractal compression has been mainly used on images. The advantages and weaknesses are apparently addressed in image compression. People generally realize that fractal compression works quite well at a high compression ratio, usually around 40:1 on

[6]Normally, the audio sequence is required to have the same number of samples as the output.

images. Walle gives a very detailed analysis on fractal image encoding performance compared with other conventional image compression methods in [24]. We do not carry out experiments to compare fractal audio compression with other popular audio compression methods such as $MPEG$ and $MP3$ because audio coding is much more complicated than image coding in general. The research here has been focused on the behaviors of fractal coding on various types of audio data. Thus, we present the advantages and weaknesses more from a general fractal model point of view.

Fractal Advantages

Fractal encoding is essentially a process to find close mappings, or transformations if affine is required, for each range block from the domain blocks. We only need to store the domain location and the coefficients of each transform after encoding. Quite a lot of bits can then be saved over the original data. So the most valuable advantage of fractal coding is the ability to achieve high compression ratios. However, the compression ratio is highly dependent on identifiable patterns and self-similarities. And thus, fractal coding with high compression ratio can not be universally applied.

In audio compression, we can see that fractal audio coding is a much simpler scheme at this stage compared with the most popular $MP3$ encoding. The simplicity may be considered as one potential advantage that fractal coding can be used in the audio world. And despite the fact that audio is a very continuous sequence, it still embeds patterns and self-similarities, especially those created by us, like music and instrumental sounds, which gives us a hope of applying fractal coding.

Fractal Weaknesses

Fractal compression has not been put into practical use for its numerous weaknesses. The success of the scheme seems to rely exclusively on exhibiting some self-similarities among part of the space. And there is no guarantee that the probability of matching domain and range blocks is sufficiently high to achieve good compression.

Our restriction of using affine mappings does not guarantee scaling α_i and offset β_i forming a set of independent random variables. This is to say that each w_i may not be able to independent from others. In other words, different orders of applying w_i may result different decoding sequences.

Furthermore, fractal encoding uses a large amount of time because of the extensive search for matching blocks, and fractal decoding can also be a time-consuming process as addressed in the previous section.

2.5 Conclusion

Throughout this chapter, we have given the theoretical background on how fractal compression works. We prove the possibility of applying fractal coding by defining the complete metric space on image and audio. Some details of the fractal encoding and decoding based on ℓ^2 space and rms metric are also discussed. Partition and mapping of the domain and range blocks have been particularly addressed.

The theory behind fractal compression and the motivation of applying fractal coding to audio data have been presented. However, the theoretical performance of fractal audio coding is still far from clear. We thus consider experiments, which may help us to gain more knowledge.

Chapter 3

A Review of Relevant Literature

3.1 Introduction

This chapter is a review of the main literature on the subject of fractals. We take a look at fractal coding from its various perspectives. Some related questions are addressed. We do not give concrete treatments on most of the aspects, which would be out of the scope here. A basic knowledge of fractal coding is assumed.

The fundamental principle of fractal coding consists of the representation of a space by a contractive transform of which the fixed point is close to the space. Most current fractal studies focus on images, because 2D image space is naturally represented as a complete metric space via the norm distance measure. However, fractal encoding is not as simple with no known algorithm for constructing the transform with the smallest possible distance between the corresponding fixed point and the image to be encoded. A common suboptimal approach taken by most researchers in this area is to construct the transform as a "collage" or union of mappings from the image to itself, and a sufficiently small "collage error" (the distance between the

collage and the image) guarantees that the fixed point of that transform is sufficient close to the original image.

Taking the collage approach leads us to the problem of identifying the mappings quickly. This problem was settled by applying Partitioned Iterated Function Systems (PIFS) [9]. The idea is to localize the search to small subsets of the whole space. However, the fractal scheme based on the collage and PIFS clearly leaves considerable latitude in the design of a particular implementation. Wohlberf and Jager classified the majority of existing fractal image coding schemes into five categories [26]:

- The partition imposed on the image determined by the range blocks.
- The composition of the pool of domain blocks.
- The class of transforms applied to the domain blocks.
- The type of search used in locating suitable domain blocks.
- The representation and quantization of the transform parameters.

There are few theoretical results on which design decisions in any of these aspects may be based. So fractal image coding has remained in the research area until now. There have been few attempts to extend fractal coding to other types of data. Fractal coding on video has been investigated in [3] and [29]. Fractal audio coding has been discussed in [24], which is the focus of this thesis.

3.2 Fractal Coding Problems

Fractal coding is essentially a way of identifying self-similarities or patterns in a space through transforms and hopefully to achieve compression by only storing the

transform coefficients. However, to find such self-similarities is not a trivial problem as we have described above. Even with the collage and the PIFS, there are still many choices when designing a fractal coding scheme for a type of data. We review some common problems addressed within the large amount of fractal literature below. Most of them are in fractal image coding, since image compression is the subject where fractal coding was introduced and mainly studied.

3.2.1 Partition Scheme

Based on the PIFS, we have to partition our space into subspaces. The range and the domain partitions determine the process of finding the mappings from the domain blocks to the range blocks. The partition schemes are most critical for the range partition. The domain partition is based on the range partition since the domain shape and size are restricted by the range's, because we use an affine transformation from domain to range. A wide variety of partition schemes have been investigated, with the majority being composed of rectangular blocks. We only discuss some popular schemes here, and for the reader interested in this aspect, refer to [26] for more detailed treatment.

Quadtree

Quadtree partitions were used in the first implementation of the PIFS based fractal image coding [9]. It employs the well-known image processing technique based on a top-down recursive splitting of selected image quadrants. The resulting partition can be represented by a tree structure in which each non-terminal node has four descendants. This partition scheme is easy to implement because of its recursive

structure, which allows us to automatically discard the larger block prior to splitting it into four subblocks if an error threshold was exceeded. Various sized range blocks can be set up by restricting the recursive depth. We can also first partition the space into uniform sized "smallest" blocks. We then proceed using the bottom-up approach to merge those neighboring blocks to get a larger block one level up the quadtree if the error is below the threshold. The first top-down approach has been discussed in [2], [5] (ch.3), and [11] (pg.93-105), and the latter bottom-up approach was introduced in [11] (pg.93-105).

Horizontal-Vertical

The Horizontal-Vertical (HV) partition scheme can be recognized as a generalized quadtree scheme with the splitting done by a horizontal or vertical line. It was discussed in [5] (ch.6). The HV scheme gives more freedom for finding similarities, but more work on defining the domain blocks since the range blocks from the HV partition are more variable in size and shape. Nevertheless, HV partitions still gives a tree structure result and the partitioned blocks are still rectangular.

Non-Rectangular

There are some partition schemes not based on rectangular blocks. In the article by Wohlberg and Jager [26], they give the basic idea of how it works. The motivation behind the non-rectangular partition scheme is to better preserve self-similarities in subspaces after the partition. And in most cases, it is obvious that using only rectangular partitions can not give us the optimal result in terms of maximally preserving self-similarities.

Overlapped Range Blocks

Some out-of-box partition schemes allow overlapping among the range blocks, which is different than the original idea of the PIFS from Jacquin [9]. Reusens used an overlapped quadtree scheme with multiple domain transforms [20] to reduce block artifacts. Walle also claimed to further overcome the block artifact by allowing the range blocks to overlap [24]. Those techniques, while promising some improvements, do increase the complexity of the encoding process.

Comparison

Most of the fractal coding implementations use the quadtree partition scheme. Very few researchers have reported comparisons by applying different schemes. Review [26] gives some comments from different papers. But unfortunately, the results do not agree with each other. We have not found any implementation based on a non-rectangular partition scheme. A difficulty in partitioning with more complicated shapes, is that determining a distance measure becomes exceedingly complicated. Partition schemes of fractal coding on other types of data is basically untouched because fractal coding theory has not been proven efficient on any new type of data except for images. However, it is expected that the partition scheme may be very different when dealing with different types of data.

3.2.2 Domain Pool

The domain pool used in fractal compression is often referred to as a "virtual codebook". The domain pool is a collection of the domain blocks, which are used to

compare the range blocks to find mappings. So the size of the domain pool, determined by how many domain blocks are in the pool, is crucial to the efficiency of the encoding. The general sense is that the larger the domain pool is, the better fidelity (i.e., smaller distance) of the mappings between the domain blocks and the range blocks. On the other hand, larger pools lead to more comparisons, which slows down the encoding. Some research has been done to reduce the size of the domain pool by applying some restrictions when choosing the blocks while not sacrificing mapping fidelity too much.

Global Domain Pool

The naïve domain pool design is to have a fixed domain pool for all range blocks. This design is identified by empirical evidence on image compression where the best domain block for a particular range block is not expected to be spatially close to that range block to any significant degree [5] (pg.69-71) [26]. However, this simple approach results in a domain pool with an enormous size. Further restrictions have to be applied to reduce the size of the pool to bring the encoding process into a manageable time frame.

The common approach started by Jacquin [9] is to restrict the domain block to twice the length [1] of the range block. This is supported by the argument that the larger sized domain block of the two corresponding domain pools usually gives a better compression ratio and fidelity of recovery [30]. Under this restriction, one domain pool only applies to certain sized range blocks. General larger sized domain pools which contain the domain blocks larger than the given range block cannot be

[1] Note that the length is measured by dimensions. So for a 2D image range block, the domain block is twice the length on both the width and the height.

applied here because the distance metric would be very difficult to define.

In most of the designs, the size of the range block is further restricted to guarantee compression. Many fractal image compression schemes require the range block to be no smaller than 4 pixels by 4 pixels, which also implies a size restriction on the domain block from previous descriptions. The restrictions on both the range and the domain are considered when taking fractal coding to other types of data [25] [29].

Classification Domain Pool

The domain blocks in the domain pool are usually classified before the map search. In the encoding process, one range block can be compared to only one class of the domain blocks from the pool that belongs to the same category to reduce the total amount of comparisons. The domain and range blocks are commonly classified into a fixed number of classes according to a certain classification scheme [5] (ch.3, 4) [9]. Other ways of doing classification without a restriction on the number of classes are possible [26], however, difficult to grasp.

There are a variety of classification schemes. It is not hard to see that different schemes are needed for different types of data. We regard classification as a potential way of getting an efficient fractal domain pool on audio data in our research.

Local Domain Pool

In the article by Wohlberg and Jager [26], they describe the effort of localizing the domain pool for each range block to a region about the range block, or a spiral search path that may be followed outwards from the range block position. The idea comes from the experimental tendency for a range block to be spatially close to the matching

domain block on images. The localized domain pool thus gives a smaller search space for each range block. Some evidence from [11] (pg.122) shows that the local pools outperform the global ones. Some other ways of forming local domain pools are also discussed in [26].

However, it is not yet clear how the local domain pool can be applied to fractal coding on other types of data. No experiments have been done to show such a tendency on other types of data except for images. Different ways of forming local domain pools may need to be developed to improve encoding efficiency.

Hybrid Codebooks

It is agreed that Vector Quantization (VQ) may perform slightly better than fractal coding in some cases of image compression with more predictable outcomes. A hybrid domain pool design with VQ was discussed in [17], which also improves coding speed.

The idea of applying adaptive techniques to fractal domain pool design was pointed out in [5] (ch.9), and [25]. The fractal domain pool design with adaptive technique seems attractive to us in applying fractal coding on audio data, because audio is usually represented as a sequential stream.

Discussion

Most of the techniques of designing an efficient domain pool we have discussed here are essentially attempts to localize the domain pool for the range blocks; in other words, to reduce the size of the domain pool to cut the number of comparisons for searching the mappings. Classification is probably a more promising technique than others, while the process is highly dependent on the classification scheme. Hybrid

codebook design is complicated, and hard to be implemented in general.

The differences of the domain pool design are significant and depend on different types of data in most cases, which make the comparison of those techniques difficult.

3.2.3 Decoding

The decoding process of fractal coding is a reconstruction through iteratively applying the mappings (transforms) from the encoding in reverse. Theoretically, by the fixed point theorem, starting from an arbitrary sequence, the reconstruction leads to a fixed sequence. And the collage and complete metric space requirements provide that the final fixed sequence shall be close to the original sequence with a sufficiently small error.

Standard Decoding

Standard decoding is a straightforward process. In most cases, we start from an initial zero vector, and specify the number of iterations. Then apply reversed mappings iteratively. Experiments in [2], [9], and [30] on fractal image decoding have shown that fractal decoded images generally converge to a fixed image after 10 iterations.

The standard decoding suffers a big problem of slow speed. One way to improve the decoding speed is to find mappings that converge faster in the encoding stage, so that fewer iterations are needed in the decoding process. Some studies have been done [5] (C.13) relating the problem with the threshold of the allowable distance between the domain block and the range block when identifying the mappings.

Fast decoding

There are other methods to decode. It is possible to find the exact fixed point directly by inverting a large but sparse matrix [5] (ch.11). Then the further development of the fractal hierarchical model [5] (ch.5) gives a fast decoding algorithm by approximating the fixed point in a lower-dimensional space, where the range block dimensions are doubled at each step, until the desired size is reached. A considerable computational saving is obtained by applying the standard iterative method to full-sized blocks [26].

Postprocessing

Postprocessing is mainly addressed with the problem of blocking artifacts in the decoded image [5] (pg.59) [11] (pg.222-224). Based on PIFS, our range partition results in blocks that are disjoint, which may introduce blocking artifacts after decoding. This problem has not been considered to be serious in fractal coding theory, because allowing the domain blocks to overlap already reduces strong blocking artifacts. The idea of further allowing the range blocks to have certain degree of overlap, which may give a even smoother edge between blocks, has been reviewed in the previous section. However, postprocessing may be very useful when applying fractal coding on other types of data. The processing methods can be more advanced with some sophisticated transform techniques.

Multiple Resolution

One advantage of fractal coding being cited is resolution independence [5] (pg.59) [26]. The fractal model uses functions of infinite resolution. Therefore, we may pick any resolution from the decoding. Taking images for example, we may decode an encoded

image to a larger sized one that still encodes the original information. However, studies [5] (ch.5) have shown that the enlarged image contains artificial data created by the transformations. The resolution independence feature does not show any interest to us for applying fractal coding to other types of data.

Multiple resolution has also been recognized in another way related to wavelet transforms [22] [24] [26]. The wavelet representation of fractal coding gives the potential of joining the two to improve the reconstruction fidelity while maintaining high compression ratio from fractal coding. This theory may also be more generally applied to other types of data.

3.2.4 Efficient Storage

In order to achieve a better compression ratio, the bit allocation schemes for storing the transform coefficients and the partition parameters are important. The simplest scheme is to quantize them uniformly [5] (ch.3). The bit allocation scheme of 5 bits for the scaling coefficient, and 7 bits for the offset coefficient was reported to provide the best performance on fractal image coding [5] (pg.61-65). The partition parameter is based on the partition scheme. In the top-down quadtree partition scheme, only one bit is needed to indicate whether to continue at each recursive partition step for a range block. The domain blocks are usually indexed and referenced by the indices. We can further save bits when the scaling value is zero, and the domain is irrelevant to the transformation and need not be stored.

Some investigation has been done to optimize the bit allocation through non-uniform quantization [26]. However, no significant improvement over uniform quantization was found.

In other work, statistical analysis was used to correlate scaling and offset coefficients between neighboring blocks [10]. This correlation is then used to obtain improved quantization [11] (pg.140-144).

The bit allocation optimizing methods for fractal coding have not been widely implemented because of the complexity added to both encoding and decoding processes. The difficulty of applying some optimizing method suggests that it may be better to stay with the simple uniform quantization with an experimentally optimized setting.

3.3 Hybrid Fractal Coding

In data compression, many conventional methods have been established on different types of data, and often hybrid methods are also investigated by joining two or more compression methods. Walle [24] reviewed fractal coding with some conventional transform methods such as *Discrete Cosine transform* (DCT) or *wavelet transforms*, and demonstrated the relationship between the two. This relationship fits well with fractal coding because it too uses transformations. The conventional transform methods were also introduced to image compression in the first place with a solid statistical foundation. Researchers have found that the transform methods work better at low compression ratios while fractal coding works better at high compression ratios in general.

Different transform methods have been applied with fractal coding to form some hybrid fractal coding schemes. With some transform methods being more widely used on other types of data to achieve good compression, hybrid fractal coding has gained significant interest recently. We address two popular hybrid fractal coding methods that are still under research in this section. The DCT is described in [16] or any data

compression book; for the wavelet treatment related to fractal coding, see [7] [22] [23].

3.3.1 DCT and Fractal Coding

The DCT is a widely used transform technique in image compression, which is part of the JPEG standard. The DCT has been recognized with the ability of removing inter-pixel redundancies with an efficient implementation. Hybrid DCT fractal coding generally tries to combine the advantage of the DCT with the ability of capitalizing on long-range correlations within the image from fractal coding [15]. Fisher, Rogovin, and Shen compared fractal coding with the DCT based JPEG coding [28], and showed their different advantages. Melnikov and Katsaggelos recent developed a jointly optimal fractal/DCT compression scheme [15]. Their hybrid scheme takes the DCT on selected blocks. It then uses fractal coding on the quantized DCT coefficients. The operational optimality comes from applying the Lagrangian multiplier approach on the hybrid transform parameters [15].

The complementing nature of fractal and DCT suggests their joint use to maximally remove the redundancies in an image. However, the DCT has its weaknesses. The most cited one is the block artifact [9]. The hybrid scheme from [15] basically allows the range block to be joined to resolve this problem. The question of whether self-similarities among an image are preserved after the DCT has not been addressed. Furthermore, not enough experiments have been done to fully understand the performance of the hybrid scheme.

We also realize that fractal/DCT hybrid scheme has limited potential since the DCT is primarily developed on the image compression field only [24] [28]. So, generally speaking, the DCT is not very helpful for applying fractal coding on other types

of data.

3.3.2 Wavelet and Fractal Coding

Wavelet analysis was developed in the late 1980s [13] [14], and extensively explained in [18] and [19]. A significant development in fractal coding theory is recognizing fractal coding through wavelet analysis as discovered by a number of researchers independently [7] [22] [23]. Hybrid wavelet and fractal coding schemes use the iterated function system to generate wavelet coefficients from the wavelet transform.

This discovery also gives a better understanding of the mechanism underlying standard fractal coding. If the domain increment is equal to the domain block size, and subject to a few additional restrictions [5] (pg.95), there is a direct correspondence between the domain and range blocks in fractal coding. Under this case, the domain and range mapping is equivalent as the mapping between the subtrees rooted at consecutive resolutions in the Haar wavelet transform [26]. This relationship demonstrates the natural expression of fractal coding from the wavelet transform point of view. And the PIFS based fractal image coding scheme is comparable to a Haar wavelet subtree quantization scheme [4].

A number of hybrid coders have been implemented, combining the subtree mapping of fractal coding with some scalar quantization techniques of various complexities [4] [12] [23] [24]. The high frequency wavelet coefficients are encoded using a collage approach from fractal coding. Walle [23] further allowed the wavelet transform coefficients to be directly stored if no mapping could be identified within certain error threshold in the hybrid image compression scheme. Their study showed that at a low compression ratio (i.e., high picture quality), few mappings can be found

and most wavelet coefficients have to be stored. Fractal compression only starts to play a significant role at high compression ratios. Experimental results [4] [23] also suggest that applying a smoother wavelet with additional vanishing moments leads to a better compression of the signal.

One main advantage of this hybrid scheme is significantly reducing the block artifacts [23]. Through the ability of generating multi-resolutions, artifacts can be overcome by combining different resolutions from the wavelet transform. The rms norm is preserved under the orthogonal wavelet transform, which means minimizing the rms metric between the wavelet transform coefficients is equivalent to minimizing the rms metric between the actual signal blocks. The wavelet framework can also be applied to develop an unconditionally convergent hybrid scheme [4], which admits a fast decoding algorithm.

Patel, Tonkelowitz, and Vernal directly applied the wavelet transform to audio compression to form a lossless scheme [1]. Their experiments suggested that the wavelet transform alone might not be the right paradigm for lossless sound compression. Wannamaker and Vrscay applied their fractal wavelet hybrid scheme on audio data [25], and showed that the compression ratios above 6:1 might ultimately be attainable with a good fidelity signal reconstruction. Applying the wavelet transform on audio data also has the potential advantage to overcome fractal coding's inability to simulate continuous wave signals, which may be a way of helping fractal coding on audio data.

3.4 Conclusion

We have given a short review of the current state of fractal coding. The use of fractal coding is primarily on image data. Unfortunately, to date, we know of no widespread use of fractal image coding.

Compressing audio data with fractal coding is a natural next step, however, there has been very little work done in this area. Therefore, preliminary and exploratory examination of fractal audio coding is required.

It should be noted that the majority of fractal coding algorithms that have been recently developed are not classical fractal coding algorithms relying purely on finding the self-similarity, but incorporate other techniques to identify redundancy. It is still not clear whether fractal coding can capture statistical properties effectively, and compete with other types of compression methods. The potential of fractal coding on image and audio data remains to be seen.

Chapter 4

Implementation and Empirical Results

This chapter describes the implementations of a binary partition fractal coding scheme for audio data. This fractal audio scheme is comparable with the conventional fractal image scheme based on the quadtree partition that has appeared in many previous papers on fractals.

The empirical results from applying the scheme to our test audio data are presented. Various tests concentrating on different aspects of fractal audio coding are performed. We make some conclusions and suggestions from our empirical studies on fractal audio coding at the end.

It is noticeable that our fractal coding scheme is heuristic based. We make no comparison to the state-of-art audio compression methods. Our focus here is to explore the possibility of applying fractal coding on different audio data.

4.1 Fractal Audio with Binary Partition

4.1.1 Encoding

The encoding process follows Fisher's conventional fractal encoding algorithm [5] (pg.19). Binary partition has been used based on the representation of audio data as a sample sequence in 1D space. Simple classification has been applied to improve encoding efficiency.

The Ranges and Domains

The range and domain blocks in fractal audio coding are based on audio samples. The size of the block is the number of audio samples in the block. We restrict the domain block size to be twice of the range block's, and each range block to contain at least four samples. The range blocks are selected based on the binary partition and the affine mapping. The initial partition is to partition an audio sequence into four subsequences. Then the four resulting range blocks are compared with all potential domain blocks from the domain pool to find the optimal domain blocks achieving the minimum rms distance through affine transforms. If the minimum rms distance is above the permitted error threshold, we recursively divide the range block into two subblocks applying the binary partition. This process is repeated until a range block can be mapped to a potential domain block with a minimum rms distance less than the error threshold, or the smallest range block size has been reached. In the latter situation, we store the transform with the minimum rms distance at the deepest recursive level (i.e., with the smallest range block size). Once a mapping w_i has been identified, we store the transform coefficients (i.e., scaling and offset) and

the range block and domain block locations to the output file. The range block which has been mapped is discarded after. The final output file stores parameters of the map $W = \bigcup w_i$ from the encoding process.

We allow three kinds of domain pools $(\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3)$. $\mathbf{D}_1$ has a lattice with a fixed spacing l, which means the domain blocks from $\mathbf{D}_1$ are directly obtained from a fixed size window sliding l samples each time through the whole sequence. Setting l to 1 gives us all possible domain blocks for a range, which is the default in our experiment. $\mathbf{D}_2$ is formed with a spacing given by the domain size divide by l, which gives more small domain blocks and less large ones. The $\mathbf{D}_2$ domain pool thus concentrates on the small size range blocks to ensure mapping quality. $\mathbf{D}_3$ has a lattice as $\mathbf{D}_2$ does but with the opposite spacing-size relationship, which means the largest domain blocks have a spacing corresponding to the smallest domain size divided by l, and vice versa. The $\mathbf{D}_3$ domain pool thus has more large domain blocks, and less small ones. The idea is that it is more important to find a good domain-range fit for larger range blocks, because the encoding will require a fewer number of transforms [5] (pg.57), which also means a higher compression ratio.

Based on the general restrictions on the range size, the range blocks can be sized from the second recursive partition to the minimum size (i.e., 4 samples). For example, an audio sequence that contains 2^n samples can have a maximum range block size of 2^{n-2}, and minimum size of 4. The theoretical largest size range block (i.e., size of 2^{n-1}) is almost impossible to be mapped by the whole sequence with a sufficiently small error, and a range size less than 4 may cause compression failure because too many mappings may occur at the small size range blocks. For the above reasons, we consider the size restriction in our implementation necessary.

The Classification

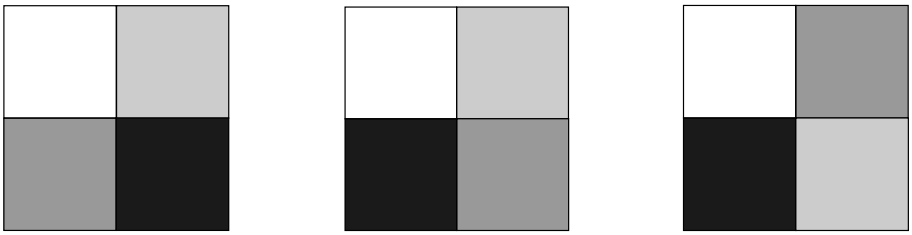
The domain-range comparison step of fractal encoding is very computationally intensive. Different classification schemes have been invented to minimize the number of domain-range comparisons starting from Jacquin's original work on fractal image compression [9]. The basic idea is to categorize the domain blocks under certain criteria before the encoding actually takes place. During the encoding, one range block is classified using the same scheme, and only needs to be compared with the domain blocks in the same category. By reducing the number of domain-range comparisons, the classification improves fractal encoding efficiency. Jacquin classified an image block into flat, edge, and texture regions. Many later classification approaches can be referred to, see [11] and [28].

Our classification scheme for audio fractal encoding is very intuitive, and can be treated as a simplified version of the one presented in [28]. Fisher used a scheme that divided an image block into upper left, upper right, lower left, and lower right quadrants, which are numbered sequentially. For each quadrant, the sum and the variance of the pixel values are computed. So, if the pixel values in quadrant i are r_1, r_2, ..., r_n, for $i = 1, 2, 3, 4$, they compute:

$$\text{Sum: } A_i = \sum_{j=1}^{n} r_j^i \qquad \text{Variance: } V_i = \sum_{j=1}^{n} (r_j^i)^2 - A_i^2.$$

It is shown that it was always possible to orient one block into one of three canonical positions with the brightest (i.e., largest pixel sum) block located at upper left as shown in Figure 4.1. The classification scheme has 3 major classes based on canonical positions, and 24 subclasses consisting of the 24 orderings of the V_i for each major

class. The transformation of orienting one domain into one canonical position is recorded during the classification process.



(a) Major Class 1: $A_1 \geq A_2 \geq A_3 \geq A_4$

(b) Major Class 2: $A_1 \geq A_2 \geq A_4 \geq A_3$

(c) Major Class 3: $A_1 \geq A_4 \geq A_2 \geq A_3$

Figure 4.1: Four quadrants of an image block can always be oriented so that the sum values of the quadrants fall into one of these three canonical positions.

The above classification scheme works well on fractal image encoding with quadtree partition. It is very general as not based on any specific image block attribute. We adopted this scheme into fractal audio coding. However, audio sequence is treated as 1-dimensional data in our case. Binary partition is used for this encoding. We thus simplify the classification scheme to divide an audio block into left half and right half. Organize each block, either range or domain, into the canonical position with the bigger sum of the sample values [1] half left. So, one block is either rotated or not in our scheme. We have 2 classes from the classification by the orderings of the variance V_i, either $V_1 \leq V_2$, or $V_1 > V_2$ as Figure 4.2 on the next page shows.

[1] Each sample is recognized as an unsigned integer in our case, see Appendix B for detail.

(a) Class 1: $V_1 > V_2$ (b) Class 2: $V_1 \leq V_2$

Figure 4.2: After orienting the larger sum half at the left, there are two classes according to the ordering of the variances.

This classification scheme can be further extended based on some higher moment to get a larger number of classes. Generally speaking, if we can partition the domain pool into more classes, the map searching process is more efficient. We do not employ specific audio characteristics into the classification scheme as identifying the audio characteristics from various sounds is a very hard problem.

The Encoding Parameters

For our experimental needs, we allow some user definable encoding parameters in the algorithm. The parameters are based on the previous studies on fractal image encoding [5] (ch.3), which explore some main properties of fractal encoding. We list the parameters below:

- e_{rms}: the maximum rms mean error for a domain-range mapping.
- d_{max}: the maximum depth of the binary partition, which gives the smallest range block size.
- d_{min}: the minimum depth of the binary partition, which gives the largest range block size.
- d_{type}: the type of domain pool to be used.

- l: the lattice spacing used in the domain pool.
- s_{max}: the maximum allowable scaling factor of an affine transform.

In our algorithm, d_{max} is $n-2$ and d_{min} is 2, where n is the power of 2 of the audio sequence size in samples. d_{type} can be set to 1, 2, or 3 to use domain pool $\mathbf{D}_1$, $\mathbf{D}_2$, or $\mathbf{D}_3$. s_{max} is usually 1 to guarantee absolute contractiveness[2].

We have some of other encoding parameters not related with the algorithm. The parameters s_i and o_i specify the numbers of bits to store scaling and offset coefficients of a transform. The full search flag can be set to true to bypass the classification scheme allowing all possible domain blocks to be compared with a range block. The audio header parameter specifies the length of the audio format information (see Appendix B for details). Different versions of the WAVE format may be applied with different header lengths. We allow the maximum audio sequence to contain 2^{20} samples in our experiments. Audio sequences which contains more than 2^{20} samples will be divided into frames with a length of 2^{20} samples each, then encoded frame by frame as in [25].

4.1.2 Decoding

We take the natural fractal decoding process by iterating map W from the encoding in reverse. Fast decoding methods, as mentioned in Section 3.2.3 can not be applied here since the underlying model is based on images. The coefficients of the transforms are read in from the encoded file. A map w_i can be reconstructed based on the affine transform definition that $R = \alpha D + \beta$, where R is a range block, and D is a domain block. W then can be reconstructed by the union of w_i. The decoding algorithm here

[2] see Section 2.2.2 for details.

is not efficient and likely to be time-consuming. However, decoding efficiency is not our interest here.

The Iteration

We start decoding from a zero sequence that has the same number of samples as the original audio sequence. At each iteration, we apply the map W in reverse to the sequence, and replace the sequence with the new one.

Base on the previous studies on fractal image decoding, we use 10 iterations by default. We calculate the difference between consecutive iterations to compare the contractivity. It is also possible to set up a threshold, which can be used to compare the difference between two consecutive iterations. And if the difference is below the threshold value, the decoding process can be halted. However, at this point, we are not sure about which value of the threshold is appropriate. Fractal theory seems to give no support on this issue either.

The Decoding Parameters

Most decoding parameters are straight forward coming from the encoding in order to get the best approximation of the original audio sequence.

- n_i: the number of iterations in the decoding process.
- d_{max}: the maximum depth of the binary partition, which gives the smallest range block size.
- d_{min}: the minimum depth of the binary partition, which gives the largest range block size.

- d_{type}: the type of domain pool to be used.
- l: the lattice spacing used in the domain pool.
- s_{max}: the maximum allowable scaling factor of an affine transform.

$d_{max}, d_{min}, d_{type}$, and l are read in from the encoded file. s_{max} and other parameters s_i and o_i have to be consistent with the encoding. The different setup may cause a bad decoding sequence. The postprocessing flag allows the averaging process between the decoded blocks to take effect after each iteration or after number of iterations. This is somehow a very general approach to see whether smoothing block edges may give us better recovery of an audio sequence from the encoded file.

4.2 Testing Setups

4.2.1 Testing Audio Data

In order to test our compression scheme and parameter settings, we use five groups of audio data in WAVE format from different references. The WAVE files are listed in the table of Appendix A on page 74.

The five categories are intuitive representations of different audio data. Note that we do not formally classify audio data here, but use some common sense. The speech category contains the WAVE files recording different people talking. The instrumental category has the WAVE files with instrumental sounds. The singing songs with background music are in the music song category. The sound clips are various short sounds from different actions. Finally, we make some special WAVE files for our testing purposes in the others category.

Based on the binary partition scheme, we modified every WAVE file to contain a power of 2 audio samples. In general, the audio sequence size restriction may be overcome by appending 0 valued samples to the end of the sequence, or by applying a different partition scheme allowing various block sizes. All WAVE files are also modified to have a single channel. The multiple-channel audio sequence can be divided into single channels with each channel to be encoded separately, and thus are not considered in our tests.

The custom WAVE files are created to test fractal audio compression theoretical limitations. The silent WAVE file is just a copy sequence of a sample, and thus has no variance on its frequency. In other words, the silent sequence contains the maximum pattern in which any subsequence can be mapped by the whole sequence. In this case, fractal coding shall provide a maximum compression ratio. The white WAVE, which is generated by a Gaussian distribution, is recognized as a noise sound sequence. Gaussian distribution provides a high degree of randomness. It is expected that fractal coding performs badly and produces a minimum compression ratio on this sequence. The sine WAVE is designed to show fractal system's inability of simulating continuous functions. The sine audio sequence after the fractal coding process (i.e., encoding and decoding) shall differ from the original sequence in terms of continuity.

4.2.2 Test Measures

We use the rms mean[3] to measure our final compression results. It is important to notice that the rms mean is not a true measure of good performance, but the one that makes sense to be a measure in our fractal scheme. It is possible that we

[3]Note that we do not normalize rms differences, because the absolute rms difference should be applied to measure frequency changes in audio sequences as for example done in [25].

obtain a small rms mean, but the result sequence sounds unlike the original sequence. The only true measure may be through human psychoacoustic testing as mentioned before. Fractal coding in this model is done through the transformations in ℓ^2 space with rms metric. The rms mean is thus a proper measure mathematically.

Another important measure is the compression ratio. We take the general compression ratio definition common in the data compression field as expressed in Equation 4.1.

$$compression\_ratio = raw\_size/compressed\_size. \tag{4.1}$$

where raw_size is the size of the original file, and $compressed\_size$ is the size of the file after encoding. The file size is measured in bytes.

We also keep another measure as the number of transforms stored from the encoding process. This measure generally tells us the performance of our fractal schemes on certain audio sequences. Less number of transforms means a higher compression ratio, or more patterns are identified in some big size blocks. Sometimes the number of transforms gives us more accurate information of the performance than the compression ratio does, because the latter has limited precision as a floating point number. Finally, it is obvious that the comparison of this measure only makes sense to be applied to the same size audio sequences (i.e., containing the same number of samples).

4.3 Sample Testing Results

We present the test results of applying our fractal audio coding method on our test sets. Test data sets were chosen to illustrate the behaviour of our method under a

variety of conditions.

4.3.1 Tolerance

The domain-range mapping error tolerance e_{rms} is a very critical parameter dominating the compression ratio as well as the encoding quality in our fractal scheme. Intuitively, the bigger the e_{rms} value, the higher the compression ratio that can be achieved, because more mappings can be accepted for the large size range blocks, and thus fewer transforms need to be stored. On the other hand, a smaller e_{rms} value gives a better encoding quality. Note that we use the rms mean to measure the quality of encoding.

The experiment on e_{rms} is built upon other fractal coding experimental results with images. We fix the bit allocation using Fisher's optimal results [5](ch.3) on fractal image coding (i.e., $s_i = 5$ and $o_i = 7$), and allow e_{rms} to be varied in the set: $\{4, 5, 6, 7, 8, 9\}$. Throughout the study, we try to find a good tolerance value which balances the compression ratio and encoding quality. Also, the study shall give us some idea of the compression ratio range on applying fractal coding to audio data in general.

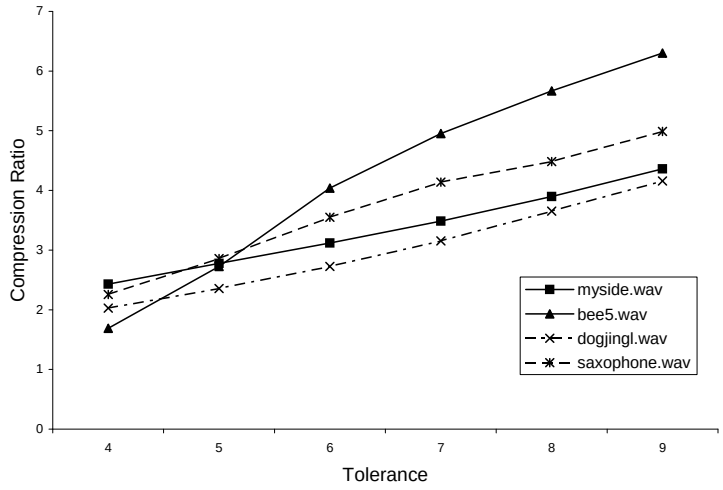


Figure 4.3: The compression ratio versus the e_{rms} with different WAVE files.

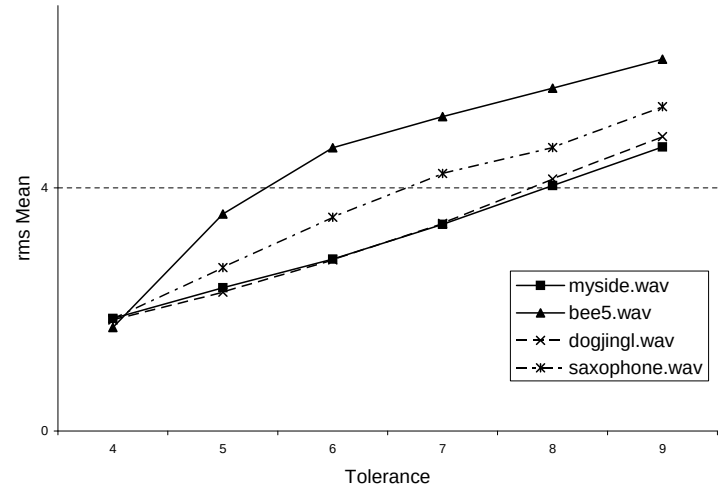


Figure 4.4: The rms mean versus the e_{rms} with different WAVE files.

As is to be expected, Figure 4.3 and Figure 4.4[4] show that a bigger e_{rms} value leads

[4]We demonstrate four WAVE files results with one from each of the four test categories in both figures.

to a higher compression ratio but a larger rms mean error. From the experiments, it seems reasonable to expect the compression ratio in a range from 3 to 6 from fractal audio coding. Compared to fractal image coding, the tolerance (i.e., e_{rms}) value may have to be smaller to achieve a reasonable rms mean error. We consider a value between 5 and 7 for fractal audio coding achieving 4 as the rms mean error, where the value in fractal image coding is 8, see [5] (ch.3).

4.3.2 Scaling and Offset Bit Allocation

Questions related to the bit allocation scheme for the scaling and offset coefficients have been discussed in Section 3.2.4. The issue we want to address here is to find a reasonable assignment of the number of bits to store the scaling and offset with a uniform quantization scheme. In other words, we want to test different bit allocation assignments, and compare the results to get a proper one. It is also interesting to see whether the "optimal" assignment with $s_i = 5$ and $o_i = 7$ in fractal image coding is still a good choice for fractal audio coding, or whether some adjustment needs to be made. Note that we are only interested with the scaling coefficient here (i.e., s_i). The offset (i.e., o_i) is set to 7, because each sample is regarded as an unsigned integer, which implies a value from 0 to 255 (2^8 values). In this case, $o_i = 7$ gives a reasonable precision to represent offset value in any affine transform, although $o_i = 8$ provides a perfect resolution.

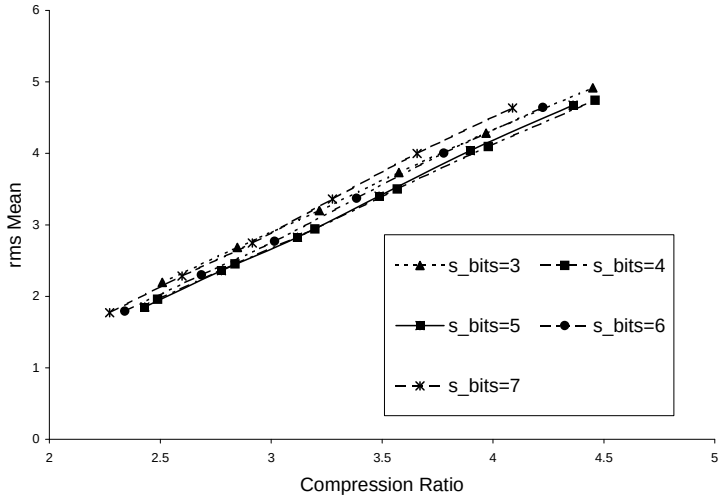


Figure 4.5: The rms mean versus the compression ratio for myside.wav with $o_i = 7$ and $s_i = 3, 4, 5, 6, 7$.

Figure 4.5 shows distributions of the scaling and offset coefficients from a typical encoding. The curves are results of using different numbers of bits for s_i to uniformly quantize the scaling and offset. Different compression ratio and rms mean value are calculated by ranging the tolerance e_{rms} value from 4 to 9. The curves show that $s_i = 5$ and $o_i = 7$ is still a quite reasonable compromise, while $s_i = 4$ may also be a good value. The results are quite similar to fractal image coding findings. Giving more or less bits for scaling coefficient rather than 5 may decrease the performance, because the casting error introduced by saving floating numbers to bytes can increase in both sides.

We also notice that the audio sample values change slowly between consecutive ones like pixel values in some natural images. This leads to a trend that most scaling and offset coefficients are correlated. We thus consider that both coefficients may benefit from some adaptive coding. However, this trend still needs to be justified

through further experiments on more audio data.

4.3.3 Audio Type

Our testing audio set consists of different types of audio data (see Appendix A for descriptions), which has been stated in Section 4.2.1. Intuitively, the fractal audio scheme may have different performances on different types of audio data. We have done some experiments to explore whether the performance is related with the type of audio data, or whether the fractal audio scheme is suitable on one type more than others.

Category	Audio file	Compression ratio	rms mean
Speech			
	bond.wav	2.062	5.254
	giveadamn.wav	3.300	2.858
	kickbutt.wav	4.435	3.658
	myside.wav	3.119	2.827
	messin.wav	6.465	2.947
Instrumental			
	bee5.wav	4.039	4.661
	bee9.wav	3.874	2.891
	lfl.wav	2.902	2.983
	panther1.wav	4.217	3.173
Music Song			
	bbmine.wav	3.760	3.108
	dogjingl.wav	2.726	2.817
	eido.wav	2.097	2.825
	shtheart.wav	2.040	2.863
	umelody.wav	5.442	3.341
Sound Clip			
	applauselong.wav	1.215	4.191
	cameraclick.wav	7.946	2.915
	fastttalk.wav	6.164	3.199
	saxophone.wav	3.549	3.517
	whistleshort.wav	2.570	2.213

Table 4.1: Various test audio encoding results.

Table 4.1 shows the results of fractal coding on all testing WAVE files from four test categories. The coding parameters are chosen from previous experiments as $e_{rms} = 6.0$, $s_i = 5$, and $o_i = 7$.

From the results, it is generally hard to conclude which type of audio is more suitable to be applied with fractal coding. Each category has some WAVE showing favorable result, and some showing relatively low compression ratio. We originally thought the instrumental category would be the most likely one in favor of fractal coding. However, our testing results do not provide such evidence. WAVE files in

the sound clips category are small in sequence size. We find the recovered sequences in that category are somehow worse than other categories. The music song category WAVE files can be treated as a mix of a person singing and background instrumental sound. The result shows some support that this category may be harder to be compressed than speech and instrumental categories.

We thus give no suggestion on which type of audio is more suitable for fractal coding, but rather, advise that a lower level categorization of the audio sequences may be needed in fractal audio coding research later. We consider the reason stemming from the fractal scheme that works on hunting patterns in sample sequence levels.

4.3.4 Scaling Factor

Choosing a scaling factor s_{max} is the subject of ongoing mathematical research. The arguments have been presented in Section 2.2.2. Under the affine transform requirement, s_{max} is always set to 1 to guarantee a contractive (absolute contractive) mapping. However, this is proved to be a stronger requirement than needed. Thus assigning s_{max} a value bigger than 1 may still give us a contractive (eventually contractive) mapping, and may even deliver a higher compression ratio with a faster coding. Unfortunately, the fractal model does not explicitly give calculations for such a value of s_{max}. We try to explore this interesting point here through some experiments with various s_{max} values.

Scaling factor	Test files			
	myside.wav	bee5.wav	dogjingl.wav	saxophone.wav
0.5	2.750	3.733	2.405	2.774
1	3.119	4.039	2.726	3.549
1.5	3.228	4.026	2.829	3.599
2	3.247	4.005	2.838	3.596
4	3.252	3.914	2.782	3.530

Table 4.2: The compression ratios from applying different scaling factors with selected WAVE files.

Scaling factor	Test files			
	myside.wav	bee5.wav	dogjingl.wav	saxophone.wav
0.5	4.308	4.715	5.480	3.580
1	2.827	4.661	2.817	3.517
1.5	2.726	4.627	2.826	3.522
2	2.731	4.621	2.834	3.532
4	2.806	4.620	2.802	3.527

Table 4.3: The rms mean from applying different scaling factor values with selected Wave files.

As demonstrated in Table 4.2, it is not always the case that relaxing the scaling factor will increase the compression ratio. Setting $s_{max} > 1$ does not provide any significant benefit in most cases. The rms mean error does not necessarily decrease either in Table 4.3. The results are more unpredictable with $s_{max} > 1$ with respect to eventual contractiveness. On the other hand, further restricting s_{max} to a value less than 1 (i.e., 0.5 in Table 4.2 and Table 4.3) has significantly decreased the compression ratio and increased the rms mean error in most cases. Thus, further restriction to a lower value less than 1 for s_{max} is not a good choice. Taking $s_{max} = 1$ to guarantee absolute contractiveness seems necessary from a practical point of view.

It has been shown through some experiments in fractal image coding that using $s_{max} > 1$ may provide better encoding for the scaling coefficients [5] (pg.64). However, this trend has not been observed in our experiments with audio data. The encoding time with $s_{max} > 1$ is shorter in general compared with $s_{max} \leq 1$, but not significant.

4.3.5 Fractal Limitation

The compression results of applying our fractal compression scheme on the custom WAVE files are discussed here. Some fractal limitations are shown from the results.

The compression results with different tolerance e_{rms} settings for the silent audio sequence are listed in Table 4.4. Theoretically, the encoding shall not change with different e_{rms} values because a perfect matching can always be found for each range block.

e_{rms} value	Compression ratio	rms mean	♯ of transforms
4	4096.000	0.465	2
5	4096.000	0.465	2
6	4096.000	0.465	2
7	4096.000	0.465	2
8	4096.000	0.465	2
9	4096.000	0.465	2

Table 4.4: The silent audio sequence encoding with different e_{rms} settings. ($s_i = 5$ and $o_i = 7$)

The compression ratio and rms mean are invariant from e_{rms} for the silent audio sequence in Table 4.4. The fractal encoding process ends in the first partition[5] (i.e., two range blocks) with two identical transforms where $\alpha = 0$ and β is the integer value of a sample in the sequence. The compression ratio is irrelevant here, since

[5]We allow the minimum partition $d_{min} = 1$ in this case.

we can make the silent sequence longer (i.e., more samples) to get an even bigger compression ratio. The mapping in this case is perfect. The rms mean is caused by casting the unsigned integer value of the sample into o_i bits. This can be adjusted by giving one more bit to the offset (i.e., $o_i = 8$), and we get 0 on rms mean.

The performance of fractal encoding on the white noise audio sequence is generally the worst case in our experiment. The results with different e_{rms} are listed in Table 4.5.

e_{rms} value	Compression ratio	rms mean	♯ of transforms
4	1.231	7.574	8192
5	1.231	7.574	8192
6	1.231	7.573	8191
7	1.231	7.582	8186
8	1.232	7.586	8183
9	1.234	7.602	8171

Table 4.5: The white noise sequence encoding with different e_{rms} settings. ($s_i = 5$ and $o_i = 7$)

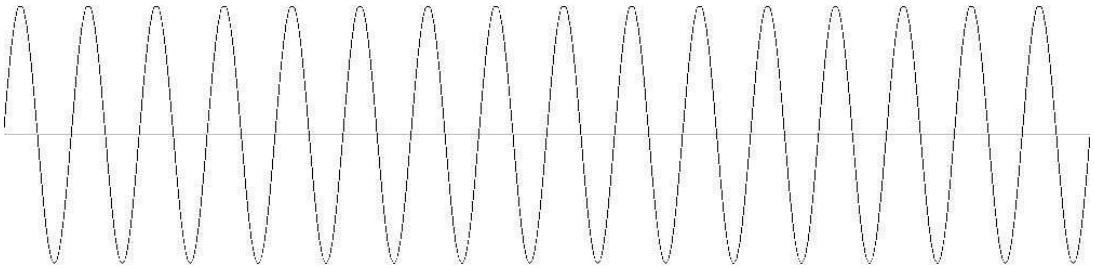
The compression ratio and the rms mean are not very good. The low compression ratio and the high rms mean show that the scheme can not identify enough patterns from the sequence to achieve a good compression. The white noise is a worst case scenario for fractal coding.

The sine audio sequence coding results are listed in Table 4.6 on the next page. Figure 4.6 shows the comparison between the original sine sequence and the fractal coded (i.e., encode and decode) sequence in a graphic representation.

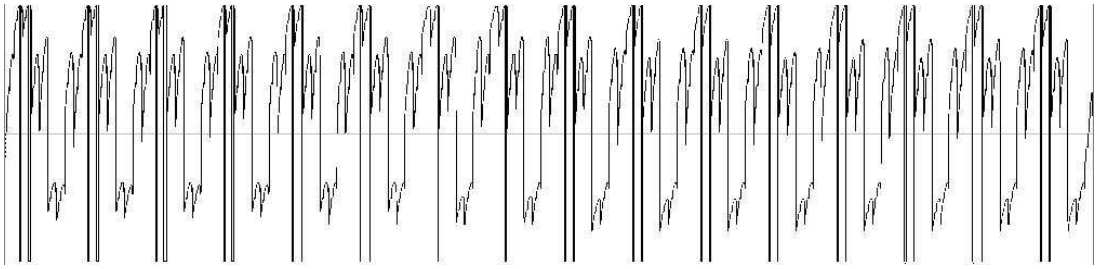
e_{rms} value	Compression ratio	rms mean	♯ of transforms
4	60.680	1.069	197
5	91.273	2.910	134
6	91.273	2.910	134
7	91.273	2.910	134
8	96.374	2.850	128
9	96.374	2.850	128

Table 4.6: The sine sequence encoding with different e_{rms} settings. ($s_i = 5$ and $o_i = 7$)

Fractal coding in terms of compression is supposed to work well on this custom sine sequence which contains 16 full sine cycles. The compression ratio is very high from Table 4.6 which proves our scheme does find the patterns in the sequence. However, Figure 4.6 shows the inability of simulating the continuous sine wave with the fractal system. The resulting sequence after fractal coding shows a lot of local cliques, and much less smoothness.



(a) Original sine audio sequence.



(b) Result sine audio sequence ($s_i = 5, o_i = 7$ and $e_{rms} = 5$).

Figure 4.6: Original and result sine audio sequences in the wave form.

The wave representations demonstrated above in Figure 4.6 also unveils the accuracy problem of the rms measure. The encoding rms mean for $e_{rms} = 5$ is only 2.910 in Table 4.6. However, we can see through Figure 4.6 that the two audio sequences actually differ a lot visually.

From the above studies on some special audio sequences, we explore some limitations of fractal coding based on our scheme. However, neither the silent audio sequence nor the white noise audio sequence gives us any practical insight on fractal audio compression. The experiments on these two audio sequences only give us some information about how fractal coding may act in its best and worst cases with different audio sequences. We can draw no conclusion from these two results, but show that the fractal scheme performance is highly unpredictable on different audio

sequences. The result on the sine audio sequence on the other hand supports the theoretical hypothesis that fractals are not able to simulate continuous functions. This disadvantage is a major challenge when trying to apply fractal coding on audio data.

4.3.6 Other Issues

Our implementation allows us to experiment on some other issues related to fractal coding, such as domain pool selection, domain classification, and postprocessing. Again, many issues have been studied in fractal image coding. However, we do not present the experimental results on those issues here, because most of them belong to a later stage of development, and fractal audio coding research is still in its early period.

Among those issues, postprocessing is an interesting topic which may be further studied for the reason that some constant background noise is observed from the audio sequences after fractal coding. We suspect a strong block artifact problem from our fractal audio coding scheme. Unfortunately, our simple postprocessing method of averaging the edges between two blocks does not seem to work well to solve this problem. We suspect that some extra work needs to be done with our scheme to solve this problem.

Preprocessing is an issue that we do not address in our experiment. It has been shown to be very useful in this field by taking some transform technique such as a wavelet transform, as reviewed in Section 3.3. Preprocessing may also be a method to detect whether fractal coding can be applied with the given audio sequences. A trend that has been observed from our empirical results is that it does not seem appropriate to apply fractal coding on small audio sequences. In most of those cases, the loss from

applying fractal coding is significant and the quality of recovery is very bad.

4.4 Conclusion

In this chapter, we provided our implementation of a fractal audio coding scheme and some empirical results from our experiment with various audio data. We shall conclude based on our results, and give suggestion for future studies on this subject.

- The compression ratio from applying fractal audio coding may be in a range from 3 to 6 in general, with a tolerance value between 5 and 7.
- Comparing with fractal image coding, fractal audio coding requires more precision on mapping (i.e., smaller e_{rms} value). The compression ratio is much lower than in the image case, because the weakness of fractal system's ability of simulating continuous functions. Fractal coding may thus not be appropriate to be directly applied on audio.
- Unlike with images, it is in general hard to conclude which type of audio is more suitable to be a fractal subject. Our results show no strong support for any type of audio. Furthermore, fractal coding's highly unpredictability has been demonstrated through our experiment with some custom audio sequences.
- Under the affine transformation assumption, the scaling factor s_{max} value is still better to be 1 to guarantee the absolute contractiveness. Values that are greater than 1 give more unpredictable results, and are thus unreliable.
- Fractal coding does not seem to be a good choice for small audio sequences (in terms of number of samples). The loss from applying fractal coding on those

sequences is usually significant.

- Preprocessing can be considered to improve fractal audio coding. Good preprocessing shall help to break the audio sequence's continuity in order to make the fractal system work well later. Preprocessing can also be used to decide whether fractal coding is applicable for a particular sequence.

- It seems that the block artifact produces some background noise through our fractal audio coding scheme. The same problem is solved in fractal image coding allowing range block to overlap, or taking wavelet transform as the preprocessing step. The latter approach has been taken by [25]. But no discussion has been posted regarding the block artifact problem on fractal audio coding yet.

Chapter 5

Discussion

5.1 Summary and Conclusion

This thesis presents a quick overview on fractal coding. We review fractal coding development through the main literature on this subject. Fractal theory is provided in Chapter 2 with some focus on the fractal audio model. We implement a fractal audio coding scheme to carry out our experiment described in Chapter 4. Some aspects of fractal coding have been practised through empirical studies on various audio sequences.

Ultimately, our experience suggests that fractal coding may not be good enough to stand alone in audio compression, while fractal systems are still of a great interest in the compression field. As stated at the beginning of this thesis, our purpose to carry out this research on fractal coding is an attempt to extend fractal coding to audio data, and explore how the fractal system performs under its audio model. Our study tries to contribute to future fractal audio coding research, or in general fractal system applications with experimental results and evidence.

5.2 Recommendations for Future Work

Through a large amount of studies on fractal image coding, it has been demonstrated that fractal coding can be nicely integrated into the framework of conventional compression techniques such as vector quantization and transform methods. The future aspect on fractals seems to focus on this kind of integration. Built upon the success of applying wavelets in image compression, hybrid fractal wavelet image coding schemes are introduced in [4] [7] [23]. This idea has also been practised in fractal audio coding too [25]. This direction may also lead to a potential success on applying fractal audio coding in practise.

In terms of improvement, preprocessing may be promising. A preprocessing step can be used to break the continuity in audio sequences, which may provide a way of compensating for the shortcomings of fractal systems. In fractal image coding, we have seen the applications with wavelets and DCT to supply this function. We expect that the same concept may be applicable in fractal audio coding too.

Specific to audio data, a proper classification shall help the compression scheme greatly. Fractal audio coding can clearly take the advantage of the classification to improve the mapping quality as well as the searching efficiency. However, the classification of audio data is probably a more general topic to be concerned, which may be addressed from other different perspectives.

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Appendix A

Testing Audio File

Category	Audio File	Size∗	Resource	Description
Speech				
	bond.wav	2^{14}	Michael's Home Turf	Roger Moore:"My Name's James Bond."
	giveadamn.wav	2^{16}	Movie Wavs Page	'Gone With The Wind' conversation
	kickbutt.wav	2^{15}	Movie Wavs Page	Charleton Heston:"Put some pants on kid so I can kick your butt."
	myside.wav	2^{16}	Movie Wavs Page	Homer:"I'm drawing a line down the center of the house around 'I Love Lucy'. You stay on your side and I'll stay on my side!"
	messin.wav	2^{16}	Movie Wavs Page	Detective James Carter (Chris Tucker) talking

Category	Audio File	Size∗	Resource	Description
Instrumental				
	bee5.wav	2^{15}	Joe's Original Wave Files	The Beethoven's Fifth
	bee9.wav	2^{17}	Joe's Original Wave Files	The Beethoven's Ninth
	lfl.wav	2^{17}	Michael's Home Turf	Yanni - 'A Love for Life'
	panther1.wav	2^{20}	Movie Wavs Page	Panther commercial music
Music Song				
	bbmine.wav	2^{17}	Michael's Home Turf	Michael Jackson - 'Baby be Mine'
	dogjingl.wav	2^{17}	Michael's Home Turf	dog Jingle Bell song
	eido.wav	2^{16}	Michael's Home Turf	Bryan Adams - 'Everyting I do, I do it for you'
	shtheart.wav	2^{16}	Michael's Home Turf	Bon Jovi - 'Shot Through The Heart'
	umelody.wav	2^{16}	Michael's Home Turf	Righteous Brothers - 'Unchained Melody'
Sound Clip				
	applauselong.wav	2^{16}	A1 Free Sound Effects	Applause sound clip
	cameraclick.wav	2^{13}	A1 Free Sound Effects	Camera shutter click
	fastttalk.wav	2^{14}	A1 Free Sound Effects	Fast play tape sound
	saxophone.wav	2^{14}	A1 Free Sound Effects	A saxophone note sound
	whistleshort.wav	2^{14}	A1 Free Sound Effects	A short whistle sound
Others				
	silence.wav	2^{15}	custom	Silent sound sequence
	sine.wav	2^{15}	custom	16 sine cycle sequence
	white.wav	2^{15}	custom	Gaussian distribution random sequence

∗ The size is measured by the number of samples in the WAVE files.

Appendix B

WAVE PCM Format

There are various formats for storing digital sound files. In computing, the sound is normally digitized through sampling, on which we take a sample of the wave frequencies that occur in the analog sound. So the more samples we take when we digitize, the higher fidelity of the digitized sound will be to the analog original. One sample is a certain fixed number of bits, normally 8-bit or 16-bit, with each bit setting to "on"(1) or "off"(0). The number of the bits per sample is the bit resolution. The sample rate is specified in kilohertz (kHz), that is, in thousands of samples per second. The highest possible pitch in the sound is equal to one half of the sample rate. So on a music CD, where sound is sampled at 44.1 kHz, the highest pitch possible is 22.050 kHz, which is also the top of the human hearing range. Audio sampling is also called *Pulse Code Modulation* (PCM).

The digital sound format we are interested in this thesis is the WAVE file format, which was created by Microsoft and IBM. The WAVE file format directly stores the samples from ADPCM (*analog-to-digital PCM*) supporting a variety of bit resolution, sample rates, and channels of audio. It also allows some different compression methods. However, we are only looking for the uncompressed WAVE files as our raw data to apply our compression scheme here.

Data Organization

The WAVE file format uses the Microsoft version of the *Electronic Arts Interchange File Format* method for storing data in"chunks". All data is stored in 8-bit bytes, arranged in Intel 80x86 (i.e.

little endian) format. The bytes of multiple-byte values are stored with the low-order (i.e., least significant) bytes first. Two types of the bit resolutions are demonstrated in Figure B.1 (i.e., LSB: least significant bit; MSB: most significant bit).

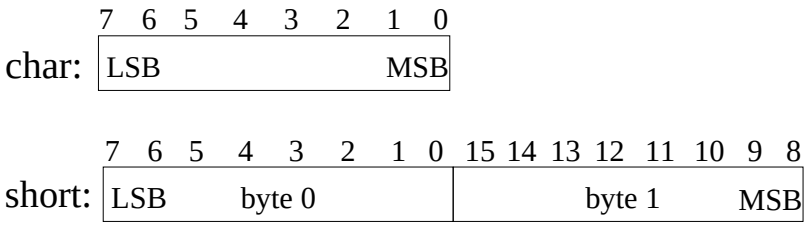

Figure B.1: 8-bit and 16-bit resolution data organizations.

File Structure

A WAVE file is a collection of a number of different types of chunks. The basic chunks are the descriptor chunk, the format chunk, and the data chunk. The descriptor chunk is the WAVE header. The format chunk contains some important parameters describing the waveform, such as the sample rate, byte rate, and bits per sample. The data chunk indicates the size of the sound data and contains the raw data. The chunk structure of the WAVE file format is described in Figure B.2.

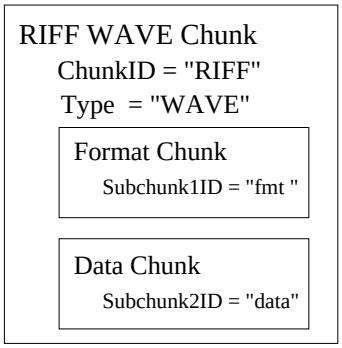

Figure B.2: WAVE format chunk structure diagram.

The Descriptor and Format Chunks

The descriptor chunk contains chunk ID, which is always "RIFF" (stands for *Resource Interchange File Format*), chunk size, and chunk format, which is "WAVE" under the WAVE file format. The format chunk consists of different subchunks. Notice that there are quite some different format chunk specifications, and we only give the one used in our program. We show this chunk specifications in Figure B.3.

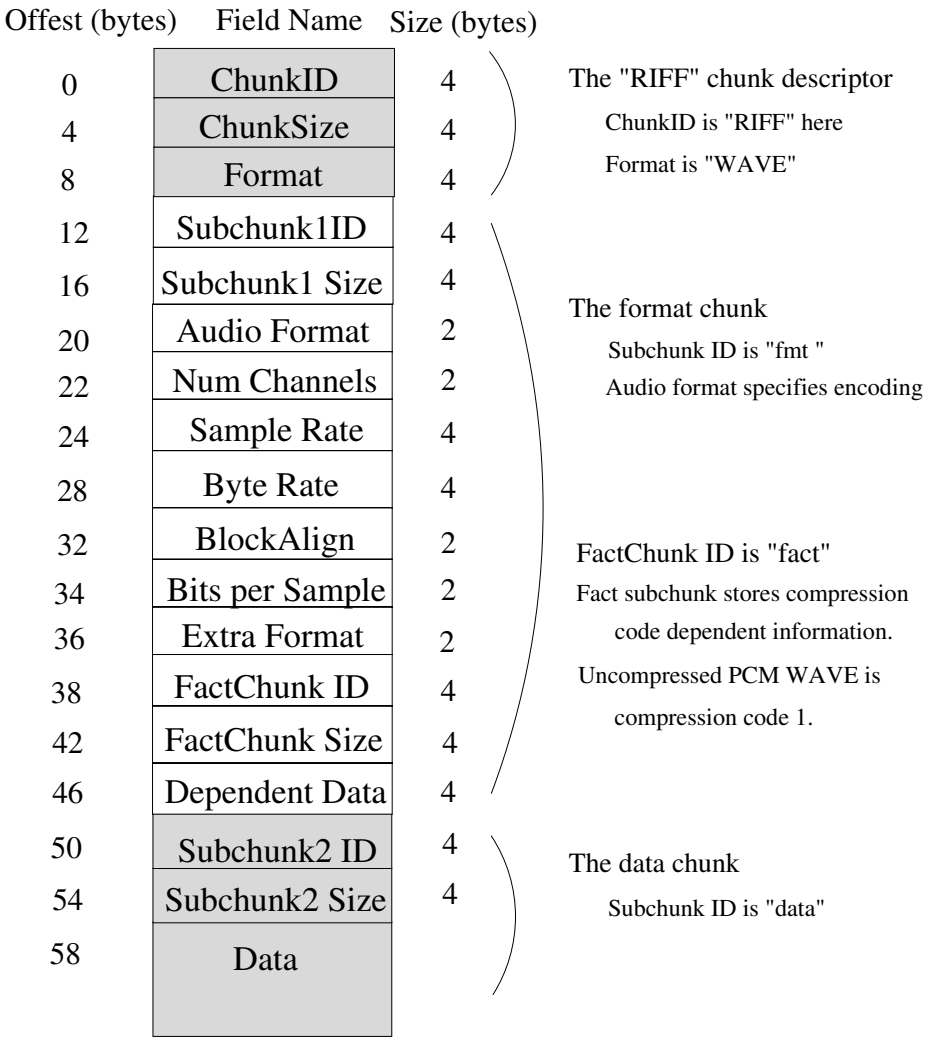

Figure B.3: WAVE chunk specifications.

Data Chunk

The data chunk contains the actual sample frames. The ID is always "data". Chunk size is the number of bytes in the chunk. The data chunk is required, and only one data chunk in a WAVE file. Raw data are stored by samples sequentially.

Example

Figure B.4 gives an example of the interpretation of the bytes shown as hexadecimal numbers from a WAVE file.

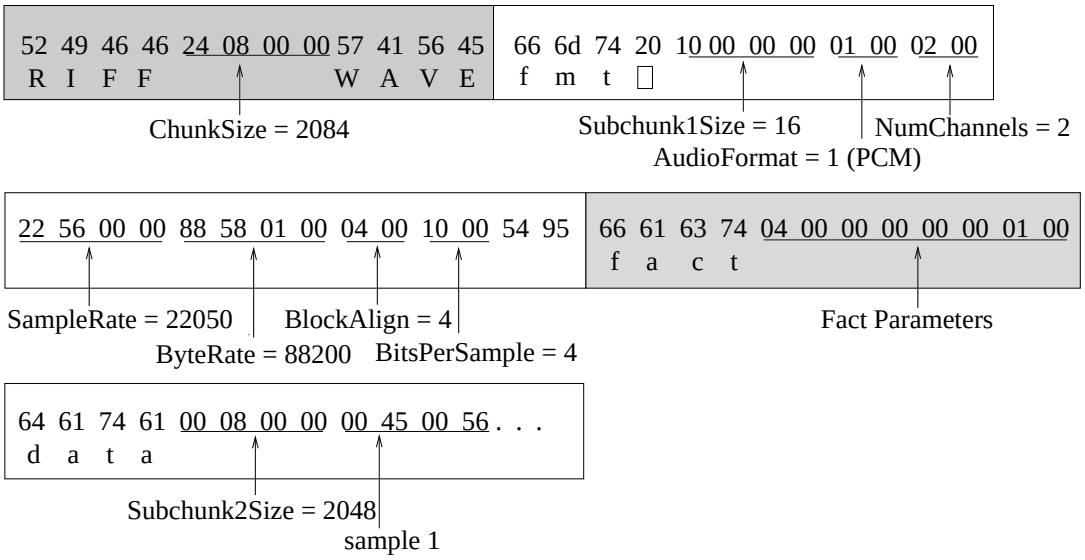

Figure B.4: WAVE audio file example interpretation.