



Fractal dimension analysis for seismicity spatial and temporal distribution in the circum-Pacific seismic belt

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MS received 25 October 2016; revised 16 July 2017; accepted 17 April 2018; published online 4 January 2019

In this study, we present the fractal characteristics of the spatio-temporal sequence for seismic activity in the circum-Pacific seismic belt and vicinity regions, which is one of the most active seismic zones worldwide. We select the seismic dataset with magnitude $M \geq 4.4$ in the circum-Pacific seismic belt region and its vicinity from 1900–2015 as the objects. Based on the methods of capacity dimension and information dimension, using $\ln(1/\delta) - \ln N(\delta)$ of the relationship to evaluate and explain, the results show that (1) in the circum-Pacific seismic belt and the surrounding areas, for the seismic activity with magnitude $M \geq 4.4$, the time series dimension is 0.63, the spatial distribution dimension is 0.52 and they have fractal structure. (2) For the earthquakes with $M \geq 7.0$, the time series dimension increases greatly, which indicates that the cluster characteristics in time is greatly reduced. And the earthquakes with magnitude $7.0 \geq M \geq 4.4$ have significant impact on the characterized by clustering in time in the study region. (3) There is significant fractal structure at spatio-temporal distribution of earthquakes in the circum-Pacific seismic belt. It reveals the tectonic movements keep continuous, obvious anisotropism characteristic of geological structure and the distribution of surface stress field is spatio-temporal heterogeneity in the study area.

Keywords. Fractal dimension analysis; circum-Pacific seismic belt; spatio-temporal distribution.

1. Introduction

As an important branch of non-linear science, fractal theory can be used to reveal the characteristics of non-stability and discontinuity in the complex system (Higuchi 1988; Goldberger 1996). At present, the fractal theory has been applied to the fields of chemistry, geochemistry, biophysics,

geology, geophysics (Birdi 1993; Turcotte 1997; Dimri 2000) and the application in geography is still in exploring stage. The research can further promote the application of fractal theory in the study of geography (Padhy *et al.* 2015). As its greatest advantage in studying the spatial-temporal propagation characteristics of seismic activities, fractal theory is not limited on the

cognition of the earth's interior (Ogata and Abe 1991). We can study on earth plate movement and seismic activity in a new perspective by the fractal theory.

The circum-Pacific seismic belt is the high incidence area of earthquakes and about 80% of the earthquakes in the world occur in this area. The high frequency and high intensity seismic activity has caused serious damage at the area and vicinity regions (Zheng *et al.* 2017). In recent years, many researchers have paid closed attention to the area.

Most of the researches on seismicity spatio-temporal sequences (Zhuang *et al.* 2002) are based on the former seismic models to analyze or predict, and are deficient in seismic theory and complexity of geographical structure. It is difficult to obtain the spatio-temporal characteristic of seismic activity completely. From the aspects of magnitude and intensity, fractal theory presents a new avenue for studying the spatio-temporal distribution of earthquakes. In the 1980's, Kagan and Kuopoff (Kagan 1991) applied the fractal theory to the study of the spatial distribution of the epicenter. The results show that there are scale invariance and fractal structure with the spatio-temporal sequence. In the early 1990's, the Soviet Union Geilikman (Geilikman *et al.* 1990), selected seismic data set of the Pamir-Tianshan, Caucasus and California regions, and found that there are variability among areas in the discontinuity, sparsely and densely sectors characteristic of singular spectrum and fractal dimensions of seismic activities, which could be used as indicators for prediction. In sum, fractal theory can be used to study the spatial-temporal distribution characteristics of earthquakes.

As its research object, this paper selects the original seismic dataset from 1900 to 2015 in the circum-Pacific seismic belt. From the aspects of time series and space distribution sequences, the spatial-temporal distribution of seismic activities in the circum-Pacific seismic belt are analysed by fractal theory to provide a theoretical basis for the prevention and exploration of earthquakes.

2. Fractal dimension

Fractal dimension is an important index to quantitatively describe the fractal. It is a measurement in self-similar characteristic with the complexity degree of system. It is a key factor to distinguish different shape complexity (Li and Wu

2002). Although the fractal theory is still in the starting stage about the study of geological disasters, it is an inevitable trend to combine the two. Fractal theory will be an important starting point for the analysis of spatio-temporal distribution characteristics in seismic activity (Dimri 2005). In the study of seismic activity, the capacity dimension and the information dimension in the fractal dimension can reflect the fractal characteristics of the spatio-temporal distribution sequence by the spatial correlation scale and the pattern intensity (Sunmonu *et al.* 2001). The fractal feature of seismic activity is the random self-similar structure of the earthquake in the time series and the spatial distribution sequence, and the larger capacity dimension value indicates that the seismic activity is more active in the time series, and the seismic activities are more concentrated in the spatial distribution and *vice versa* (Li and Wu 2002). Based on fractal theory, in this study, therefore, it aims to analyze the spatio-temporal distribution characteristics of the seismic activity in the seismic belt from 1900 to 2015 by the method of capacity dimensions and information dimensions.

Using the box counting dimension, the study researches fractal characteristic of the seismic spatio-temporal space in the circum-Pacific seismic belt by calculating capacity dimension and information dimension of the time series and the spatial distribution in the seismic activity. Each seismic activities are set as a data point in the seismic dataset; δ as minimum spatio-temporal interval. With the minimum spatio-temporal scale δ , $N(\delta)$ is number of non-empty subsets under the whole seismic dataset. As the δ value head towards zero, the fractal dimension is calculated. The capacity dimension D_c set as (Sunmonu *et al.* 2001):

$$D_c = -\lim_{\delta \rightarrow 0} \frac{\ln N(\delta)}{\ln(\delta)}. \quad (1)$$

The capacity dimension values present the degree of activity of the earthquake in time.

When the i^{th} component contains the number of data points as $N_i(\delta)$, the probability for a point data belongs to the i^{th} subset with the spatio-temporal scale δ , is defined as (Sunmonu *et al.* 2001):

$$p_i(\delta) = \frac{N_i(\delta)}{N(\delta)}. \quad (2)$$

The information dimension D_i is (Sunmonu *et al.* 2001):

$$D_i = - \lim_{\delta \rightarrow 0} \frac{1}{\ln \delta} \sum_i^N P_i(\delta) \ln \frac{1}{P_i(\delta)}. \quad (3)$$

The information dimension is closely related to the density level of the earthquake in the spatial sequence.

3. Study area

As one of the largest and most seismically active regions in the world, around 80% earthquake activities in the world occur in the circum-Pacific seismic belt region and in its vicinity (Duda 1965). Most of the seismic activities in the area are shallow and high-frequency. Thereby, the seismic activities is a serious threat for the area. The research selects the circum-Pacific seismic belt as the study area. It is analysed in this study that the spatio-temporal distribution of the seismic activities in this area for further understanding the occurrence mechanism and attenuation law of seismic activities. There is notable significance in studying the seismic activities in the study area.

All original seismic data used in this research were collected from the earthquakes hypocentre file of the United States Geological Survey (USGS). These dataset covered 70°N–50°S latitude and 100°E–50°W longitude from 1900 to 2015 (figure 1).

For all the original seismic data from 1900 to 2015 in the circum-Pacific seismic belt and its vicinity region, according to the Gutenberg–Richter’s law (Zheng *et al.* 2017), the minimum magnitude is 4.4 after the integrity analysis. And the number of magnitude $M \geq 4.4$ seismic activities in this area is 65,535, which is far beyond the minimum number requirements of data points (50 data points) (Wang and Lee 1997). In other words, our study data, the magnitude $M \geq 4.4$ earthquakes in the study area collected from USGS database, are sufficient for the fractal dimension analysis. Therefore, in this study we select earthquakes from 1900 to 2015, with $M \geq 4.4$ level as the sample for subsequent analysis.

4. Fractal analysis of seismic temporal sequence

As there is continuous movement and collision of plates, the longer the intervals between two adjacent earthquakes, the more energy builds up and unleash. Therefore, the time interval between two adjacent seismic activities reflects the temporal characteristics of the earthquakes in a certain extent. The seismic activity in the study area can be regarded as a discontinuous point set with seismic attributes. Each time point corresponds to an earthquake. Accordingly, the time interval between

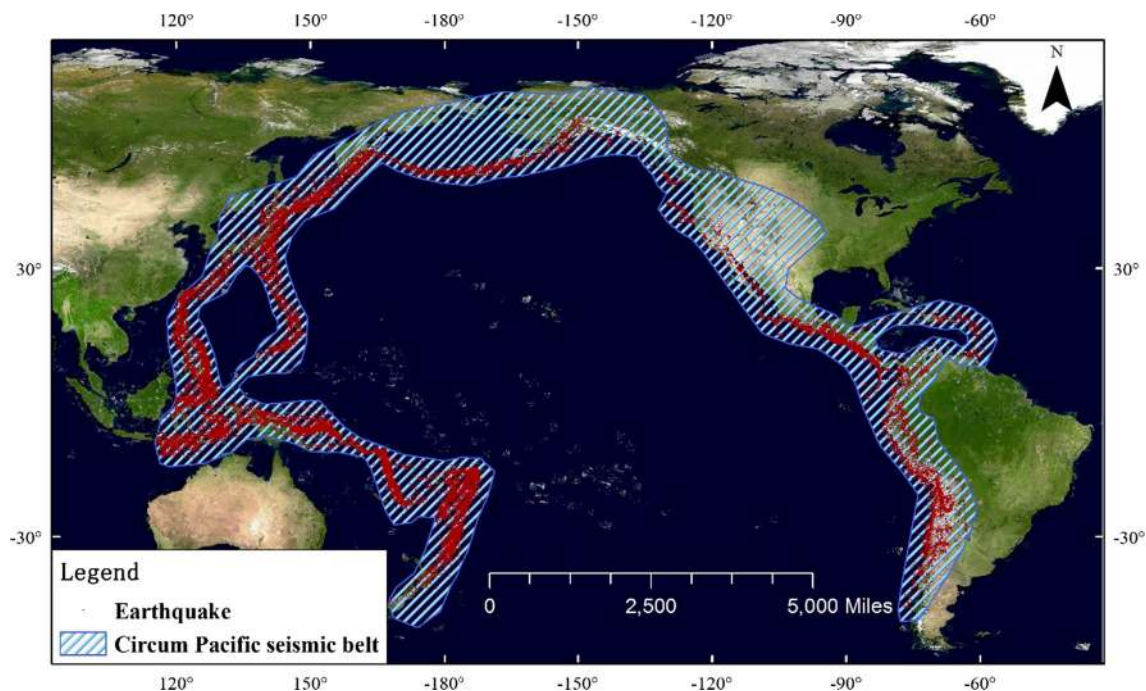


Figure 1. Earthquake distribution in the study area between 1900 and 2015.

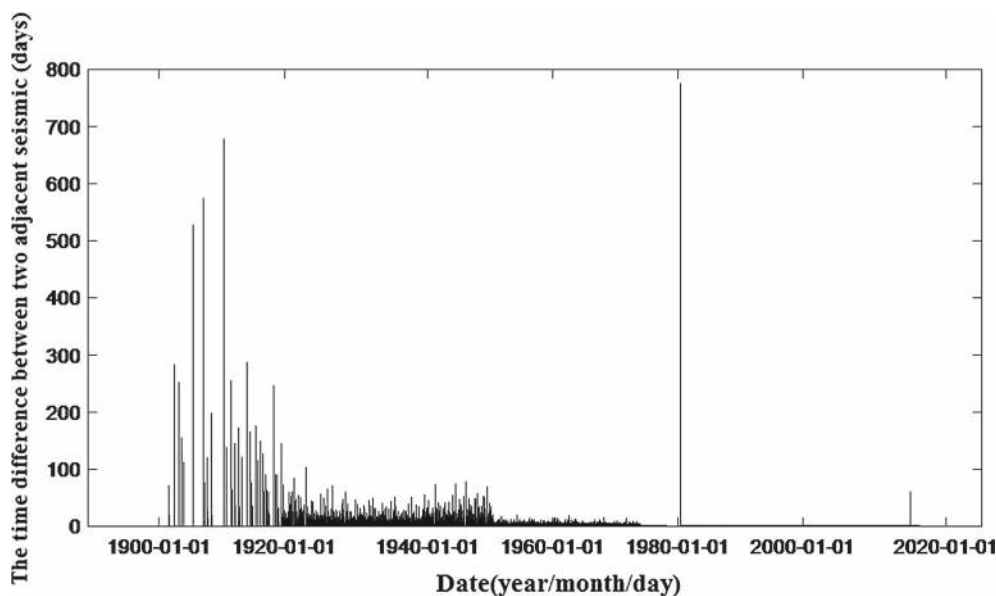


Figure 2. Time interval map of adjacent earthquakes in the study area.

the seismic activities in the research area was counted (figure 2). After the seismic data of the study area are pre-treated, the changes of the time interval are fluctuation over time. The trend of time interval is drastically reduced, and finally tends to be stable. As shown in figure 2, the maximum time interval for adjacent earthquakes is around 1980, while the longer interval is more concentrated from 1900 to 1920. Between 1900 and 1975, the frequency of earthquakes was decreased, and after 1975, the frequency of seismic activity increased gradually. Seismic activity is heterogeneous in time dimension.

In this study, each seismic activity is regarded as a data point in the study area, and δ represents the minimum seismicity time interval that is not zero, called the minimum time scale. At the minimum time scale δ , $N(\delta)$ is the minimum number of covering required to cover all seismic activity time series. The capacity dimension D_c of the seismic activity time series of the study area is calculated by the formula (1), shown as $\ln(1/\delta) - \ln N(\delta)$ in figure 3.

The $\ln(1/\delta) - \ln N(\delta)$ curve of the seismic activities has a turning point, which acts as the demarcation point of the non-scale region. It indicates the presence of fractal structures in the seismic time series of the study area. The capacity dimension D_c is a quantitative parameter that measures the fractal structure of fractal objects. The slope of the non-scale region has a capacity dimension of 0.63, thereby, indicating the seismicity appears to be active in time.

From the $\ln(1/\delta) - \ln N(\delta)$ curve, with the increase of $\ln(1/\delta)$ value, $\ln N(\delta)$ is increased primarily and latterly in a steady state. It suggests that the seismic frequency increases rapidly, then increase slowly, and then almost without an increase.

Many small earthquakes occur before and after a strong earthquake, thereby indicating a certain relationship between these earthquakes. To analyse such relationships, seismic data with the magnitude $M \geq 7.0$ were extracted and analysed via fractal analysis. The interval time of earthquakes with high-magnitude ($M \geq 7.0$) shows the uneven distribution of seismic activities in time in this area.

The time points of the large earthquakes in the study area were used as interval points to partition the seismic activities in the entire seismic zone. The information dimension (equation 2) of a small earthquake swarm in different segments were calculated and analysed to determine the relationship between strong and small earthquake swarms (figure 4). The fractal dimension value of seismic activities greatly fluctuated along with time, especially around 1930, 1940, and 1990. This indicates that the seismic activity in the study area is more regular in these time periods. It could be noticed that prior to each peak value, the fractal dimension reaches the minimum. The change dramatically from the early minimum to later peak value, it reveals the seismic alternatively phenomenon of the active and quiet period. And also shown in figure 4, the fractal dimension attenuate sharply

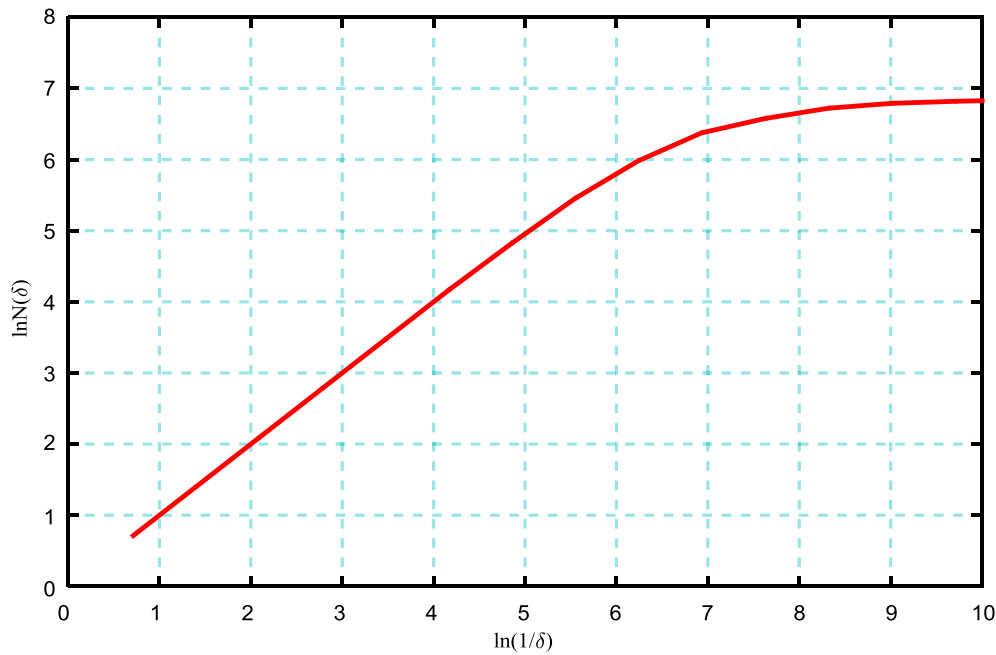


Figure 3. $\ln(1/\delta)$ - $\ln N(\delta)$ curve of the seismic temporal distribution for the capacity dimension.

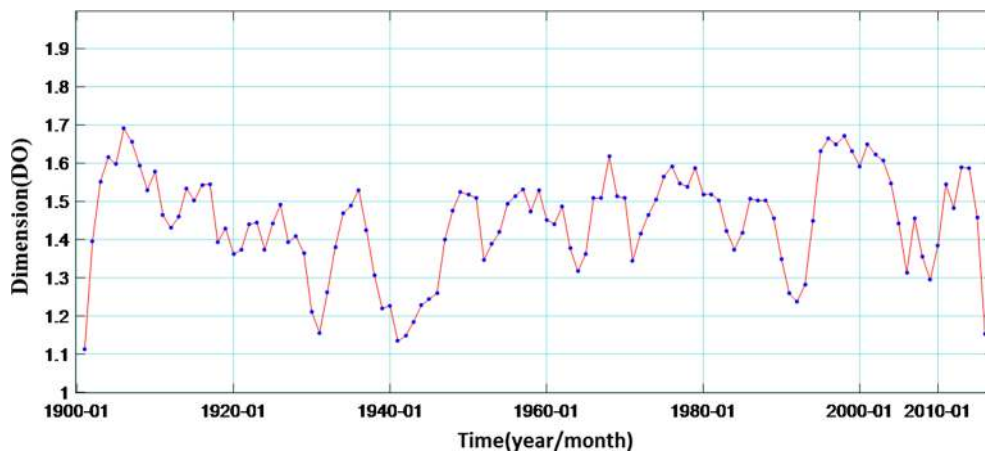


Figure 4. Plot of the information fractal dimension of the independent earthquake swarm in the study area.

after reaching its peak. It could be explained by the release of energy after seismic activities in active periods.

5. Fractal analysis of the spatial distribution sequence in the study area

In the fractal analysis of the seismic spatial distribution in the study area, our research use grid transformation to process the seismic dataset of which magnitude is greater than 4.4 and spatial distribution sequence in the area is from 1900 to 2015 as the dataset pre-processing. By means of grid transformation, all seismic point data in the study area are divided into different grid areas

according to the spatial distribution. That means all the data are divided into sub-datasets. Therefore, in the study of fractal analysis of spatial distribution, the minimum grid scale that seismic activity area is not empty is chosen to be the value of δ , that is the smallest grid scale. The value of the smallest grid scale that we choose in this research is 0.4° . $N(\delta)$ means the minimum coverage number, which is required to cover the spatial distribution of all seismic activity at the minimum grid scale δ . The capacity dimension of the spatial distribution of seismic activity in the study area which is called D_c is calculated by equation (1) shown in figure 5.

Compared with the result of fractal dimension analysis, the plot of $\ln(1/\delta)$ - $\ln N(\delta)$ between the

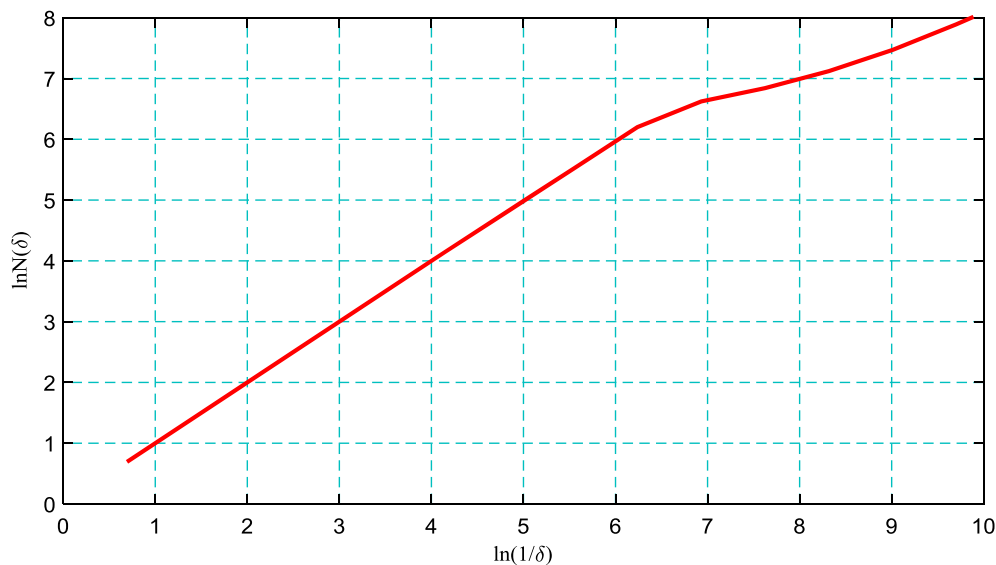


Figure 5. $\ln(1/\delta)$ – $\ln N(\delta)$ curve of the seismic spatial distribution for the capacity dimension.

temporal sequence and spatial distribution for the seismic dataset (figures 3 and 5) shows that

- (1) The trends of the two curves are similar and there are inflection points. The seismic activities in the study area is divided into scale region and non-scale region by the points.
- (2) There is a second inflection point in the curve of the spatial distribution sequence of $\ln(1/\delta)$ – $\ln N(\delta)$. Non-scale region exists between the two turning points. The double logarithm graph shows a complex change, which indicates that the fractal structure of the seismic activity is highly complex and variable in the spatial distribution sequence.
- (3) The slope of the non-scale region of the seismic activity in the spatial distribution sequence is obtained via one element linear regression. The spatial distribution sequence has a capacity dimension of 0.52, and the time dimension of the time series is 0.63. It shows that the self-similarity in spatial distribution is more obvious than the time series, and the seismicity shows the characteristics of temporal and spatial agglomeration in the study area.

6. Discussion and conclusion

As one of the largest earthquake belt in the world, the circum-Pacific seismic belt, it has a profound influence on the rim of the Pacific Ocean and surrounding areas. In the study, using the fractal theory, it analyses the spatio-temporal distribution

characteristics of the seismic activities from 1900 to 2015, to explore the spatio-temporal variation mechanism and law of seismic activities in the area.

- (1) Some self-similarity and non-scale region exit in the time and space series of the seismic activities. In this research, the fractal method is used to analyse the temporal and spatial distribution characteristics of the earthquake in the study area, and there are significant inflection points in the $\ln(1/\delta)$ – $\ln N(\delta)$ graph, which is the demarcation point of the non-scale region. The earthquakes in the study area are self-similar in the space and time. The slope of the curve of $\ln(1/\delta)$ – $\ln N(\delta)$ represents the capacity dimension. The capacity dimension on the spatio-temporal distribution sequence is greater than 0.5, which suggests that the seismic activity in the time series is active and clustered on the spatial distribution sequence.
- (2) It shows that there is a close spatial and temporal clustering relationship between the great earthquake and the before-and-after small earthquakes. After fractal analysis of the $M \geq 7.0$ earthquake in the area, it can be seen that the fractal dimension of the earthquake group is from 1.1 to 1.7. Compared with the fractal dimension of the earthquakes with $M \geq 4.4$ (the value is 0.63), there is a significant increase. The clustering characteristics of the seismic activity in the study area are mostly caused by the cluster characteristics of weak earthquakes ($4.4 \leq M < 7.0$).

(3) The fractal structure of the seismic activity in the spatial distribution of the study area is more complicated than that in the time series. According to the spatio-temporal distribution of the earthquake activity in figure $\ln(1/\delta) - \ln N(\delta)$, the fractal structure in the time series linearly increases and then reaches a steady state, that means there is a clear turning point. In the spatial distribution, there is also a turning point, but the difference is that after the turning point there is an approximately rising stage. This indicates that the fractal structure and heterogeneity feature in space is more prominent than in time. It also shows that the anisotropy of the geological structure of the study area is more obvious, and the distribution of surface stress field is with a high degree of heterogeneity in the study area.

Acknowledgements

Research supported by Fundamental Research Funds for the Central Universities (Nos. SWU-117063, SWU116183 and XDJK2011C086); Funds for International S&T Cooperation and Exchange R&D Project of Sichuan Province (Grant No. 2017HH0054); Opening Fund of State Key Laboratory of Virtual Reality Technology and Systems (Beihang University) (Grant No. BUAA-VR-16KF-11).

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Corresponding editor: N V CHALAPATHI RAO