Fractal dimension versus density of the built-up surfaces in the periphery of Brussels.

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Abstract.- This paper aims at showing the usefulness of the fractal dimension for characterizing the spatial structure of the built-up surfaces within the periurban fringe. We first discuss our methodology and expectations in terms of operationality of the fractal dimension theoretically and geometrically. An empirical analysis is then performed on the southern periphery of Brussels (Brabant Wallon). The empirical analysis is divided into two parts: first, the effect of the size and shape of the windows on the fractal measures is empirically evaluated; this leads to a methodological discussion about the importance of the scale of analysis as well as the real sense of fractality. Second, we show empirically how far fractal dimension and density can look alike, but are also totally different. The relationship between density and fractality of built-up areas is discussed empirically and theoretically. Results are interpreted in an urban sprawl context as well as in a polycentric development of the peripheries. These analyses confirm the usefulness but also the limits of the fractal approach in order to describe the built-up morphology. Fractal analysis is a promising tool for describing the morphology of the city and for simulating its genesis and planning.

Keywords: Fractals – dimension – periurbanisation – Brussels

Note to the ERSA2004 referees:
This is the state of our paper on April 30th 2004. It is not finished nor checked by an English native but results seem quite promising. Please take contact with the corresponding author for the latest version of the paper at the moment of the refereeing process or at the moment of editing the proceedings, if necessary. We thank you for your comments and questions.

1. Introduction and literature review

We know that the shape of the cities results from a compromise between a need for space for human activities and some exogenous constraints fixed for instance by relief, surrounding walls or town-planning prescriptions. Hence, the evolution of each city is tightly bound to its history and to its situation, even some global trends and general evolution cycles have been depicted in the European urbanisation (e.g. Anas*,* Arnott and Small, 1998 ; Champion, 2001-a et b). The recent explosive urban growth has led to be particularly interested in urban peripheries, that is to say mixed areas where urban and rural spaces interfere and compete (see e.g. Johnson, 2001; Longley, Batty and Chin, 2002; Caruso, 2002; Cavailhès et al., 2002 and 2004). This paper aims at analysis the morphology of built-up areas in one **urban periphery** (Brussels) by using one specific tool: fractal analysis.

By definition, **a fractal** is a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a smaller copy of the whole. Fractals are generally self-similar (bits look like the whole) and independent of scale (they look similar, no matter how close you zoom in). This justifies its interest in urban modelling. The use of fractals in urban analysis mainly developed in the 90s. Indeed, former papers have shown that cities can be conceptualized at several scales as fractals. These scales are inter-related. Let us mention some examples. At the **regional scale** rank-order plots of city size follow a fractal distribution (Frankhauser 1994) and population scales with city area as a power law (Batty and Longley 1994). In many examples, each city is individually selected, considered as a unit and fractality measured at one or several periods of time. At this level, cities display a surprising degree of universality (Arlinghaus, 1985; White and Engelen, 1993; Batty and Longley, 1994; Frankhauser 1994; MacLennan et al 1991; Schweitzer and Steinbrick, 1998; Longley and Mesev, 2000 and 2002; Wentz, 2000 or Shen, 2002). The fractal dimension of such urban entities is considered as a global measure of areal coverage. Some authors link the population growth to the fractal nature of transportation networks (Benguigui, 1992 and 1995). However, **detailed measures** of spatial distribution are clearly needed to complement the description of the morphology of an urban area; a few number of papers pertaining to this scale are to be found (Carvalho et al., 2003; Schweitzer and Steinbrick, 1998 ; Batty and Xie, 1996 ; Frankhauser, 1997). At a local scale, it has been suggested that urban space resembles a Sierpinksi carpet (Frankhauser 1994; Batty and Longley 1994). Our paper considers microscale analysis of fractality.

This paper aims at showing (1) how far the fractal dimension can be useful for characterizing periurban realities, (2) how far the way of selecting the basic spatial units (windows) biases the results. Hence, we try to see if there is a best way of defining a window when studying periurbanization by means of fractal tools. Last but not least, we want to see (3) if there is a relationship between the morphology of the built up area, the built-up density and the history of the urban sprawl. As far as we know, these objectives are quite novel in fractal analysis of the urban reality.

Let us here mention two former and related papers of the same authors and hence better position this paper. In a first paper presented in ERSA-2003 Conference and published in Geographical Analysis (De Keersmaecker, Frankhauser and Thomas, 2003), we've analyzed the fractal dimension of fully urban built up areas. This analysis was limited to the center of Brussels. Several methods for estimating the fractal dimension were tested: correlation, dilation, radial; on surfaces and on borders. Only one way of defining the windows was used (a gliding window of fixed size with overlapping). The objective was to compare several fractal-based parameters and to explain the observed spatial variations by means of variables commonly used in geography, urban economics and land use planning. Interesting statistical associations were found with the structure of the housing market, rent, distance to the city center, income of the households as well as some planning rules. In another paper published in French (De Keersmaecker, Frankhauser and Thomas, 2004), we have analysed the southern periurban area of Brussels, once again with only one way of defining the windows (26 windows of fixed size centred on the communes). Several estimation techniques were applied: correlation and dilation on surfaces, correlation after dilation on borders. Each method leaded to different results, but correlation on surfaces seemed to be the most promising technique. The fractal dimensions obtained by correlation were then related to traditional socio-economical and morphological indicators. These former papers help us to better orient the simulations performed in this paper (southern periphery of Brussels; dimension by correlation).

The present paper is concerned with the periurbanisation and more particularly with the measurement of the morphology of the built-up areas in the southern periphery of Brussels. It shows the meaning and usefulness of fractal indices for characterising urbanisation at a micro level compared to that of density. Hence, we first provide a theoretical and geometrical explanation of our expectations (Section 2) and then develop the empirical findings (Section 3). Several ways of defining the window are considered: fixed size versus variable size, gliding versus windows centred on the villages, large versus small windows. Fractal dimensions are compared and analysed in terms of scale, MAUP, as well as data biases. Several micro scale examples illustrate the results. Special attention is put on the interpretation of the fractal realities and in their differences to density. Links are also made between the morphology of the built up area, the built-up density and the history of the urban sprawl. Conclusion and discussion are reported in Section 4.

2. Theoretically and geometrically …

In spatial analysis, most of the currently used measures for describing urban patterns are based on the notion of **density**. Since one the main goals of the present paper is to study the potential link between density and fractal dimension in a periurban context, we will first focus on the fundamental difference existing between these two concepts. Both measures aim at characterizing the spatial distribution of objects (in our case buildings). Density is usually computed by dividing a mass (e.g. the built-up area of a township) by the surface on which this mass is localized and which serves as reference (e.g. the total area of the township). From a mathematical point of view this must however be considered as a rather rough interpretation of the notion of density.

Let us go further into the notion of density. Let us come back to the most general but rather theoretical definition of density. We assume that we characterize a given site in a two-dimensional space by polar coordinates (ε,φ), where φ is the angle with respect to an origin and a given axis, and ε the radial distance to this origin (Figure 1).

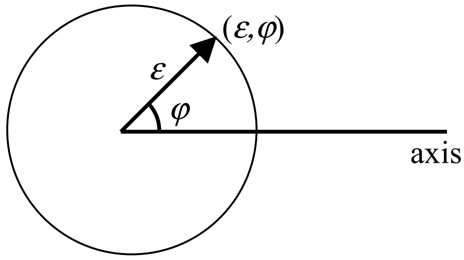


Figure 1: *(ε,φ)* coordinates

In each point (ε,φ), we define a local density $\rho_{loc}(\varepsilon,\varphi)$ characterizing the presence of a mass in (ε,φ). Hence the total mass $M(\varepsilon)$ lying within a circle of radius ε is

$$M(\varepsilon) = \int_{\varepsilon'=0}^{\varepsilon} \int_{\varphi=0}^{2\pi} \rho_{loc}(\varepsilon,\varphi)\,\varepsilon\, d\varphi\, d\varepsilon' \qquad [1]$$

If this mass is homogeneously distributed, the local density is constant: $\rho_{loc}=\rho$. We then integrate over the circular disk of radius ε

$$M(\varepsilon) = \int_{\varepsilon'=0}^{\varepsilon} \int_{\varphi=0}^{2\pi} \rho\,\varepsilon\, d\varphi\, d\varepsilon' = \rho \int_{\varepsilon'=0}^{\varepsilon} \int_{\varphi=0}^{2\pi} \varepsilon\, d\varphi\, d\varepsilon' = \rho\,\pi\,\varepsilon^2 \qquad [2]$$

Since $A_{tot}(\varepsilon) = \pi\,\varepsilon^2$ is the surface of the circle, we find the current definition of the density:

$$\rho = \frac{M(\varepsilon)}{A_{tot}(\varepsilon)} \qquad [3]$$

It is obvious that if local density depends on site (ε,φ), the integration is not trivial. As pointed out, the usual way for computing density for empirical structures is to determine the mass present in a given area, e.g. on a disk of radius ε, and to divide this mass by the total surface of the area. This corresponds mathematically to the computation of the **mean density** of the mass distributed on the disk. Figure 2 geometrically shows that this mean density does not allow for distinguishing different types of spatial distributions within a given area. In Figure 2(a) the black elements are distributed homogeneously, whereas the Figure 2(b) and 2(c) correspond to fractal objects: Figure 2(b) illustrates the second iteration step of a Sierpinski carpet and Figure 2(c) corresponds to a randomised version of this Sierpinski carpet.

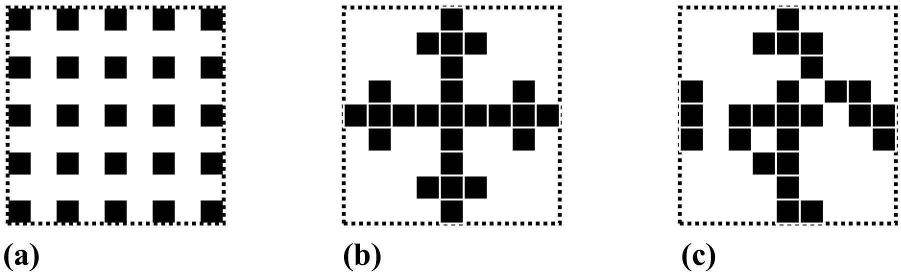


Figure 2: Three different spatial arrangements of the a given number of square-like black elements (25) on a fixed quadratic surface

For regular homogeneous patterns, densities remain constant when changing the size of the reference surface $A_{tot}(\varepsilon)$; this doesn't hold anymore when the local density $\rho_{loc}(\varepsilon,\varphi)$ varies like this the case in a fractal structure. This phenomenon is illustrated in Figure 3, where two regular patterns ((a) and (b)) are compared with the Sierpinski carpet (c) already proposed in Figure 2. For all three patterns densities are computed for two square-like reference surfaces of different size. Results are reported in Table 1**.**

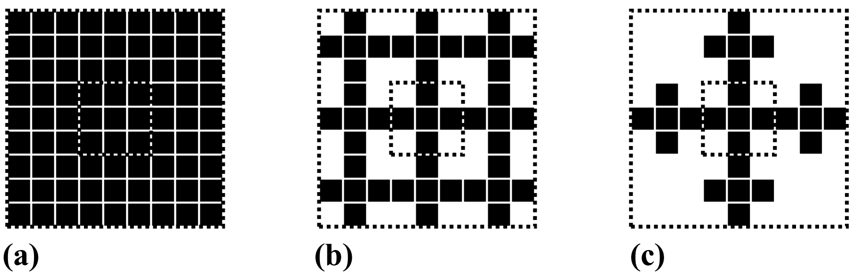


Figure 3: Studied examples of homogeneous ((a) and (b)) and fractal (c) textures.

Reference surfaces Patterns	base length: 3 units surface: 9 units	base length: 9 units surface: 81 units
Figure 3(a)	9/9 = **1**	81/81 = **1**
Figure 3(b)	5/9 = **0.56**	45/81 = 5/9 = **0.56**
Figure 3(c)	5/9 = **0.56**	25/81 = **0.31**

Table 1: Mean densities computed for the three textures suggested in Figure 3 (a to c) and for two sizes of the square-like reference surfaces (see Figure 3).

Mathematically the generalized relation for the mean density $\overline{\rho}(\varepsilon)$, which depends now on the radius ε, is obtained by combining equations [1] and [3]:

$$\overline{\rho}(\varepsilon) = \frac{M(\varepsilon)}{A_{tot}(\varepsilon)} = \frac{1}{A_{tot}(\varepsilon)} \int_{\varepsilon'=0}^{\varepsilon} \int_{\varphi=0}^{2\pi} \rho_{loc}(\varepsilon,\varphi)\, \varepsilon \, d\varphi \; d\varepsilon' \quad [4]$$

where again polar coordinates are used. Hence, the mean density depends on the mass distribution described by the local density $\rho_{loc}(\varepsilon,\varphi)$. For fractals the mass lying within a radius ε around the mass centre varies according to the scaling law:

$$M(\varepsilon) = a\ \varepsilon^{D} \qquad [5]$$

In this relationship, parameter D corresponds to the ***fractal dimension,*** which characterizes the scaling behaviour of fractals, i.e. the fact that the same structure appears on smaller and smaller scales, like in Figure 4 (see also Mandelbrot, 1989; Batty and Longley, 1994; Frankhauser, 1994).

The second parameter is the prefactor a; it includes all the influences that may be at the origin of the deviations from fractal law (Gouyet, 1992). An example is given in Figure 5: by adding two fractals to each other, the mass of the object is simply doubled – without affecting the fractal dimension.

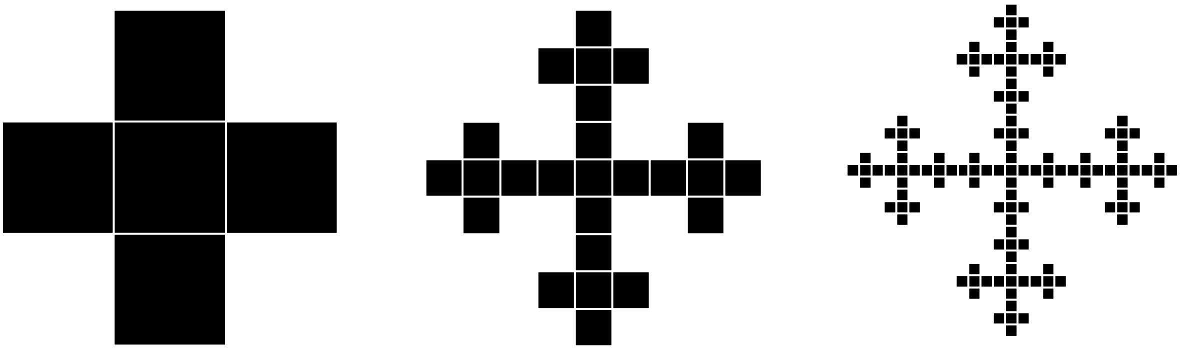


Figure 4: Generating a fractal: in the central figure each square is replaced by a reduced copy of the *LHS* figure, etc.

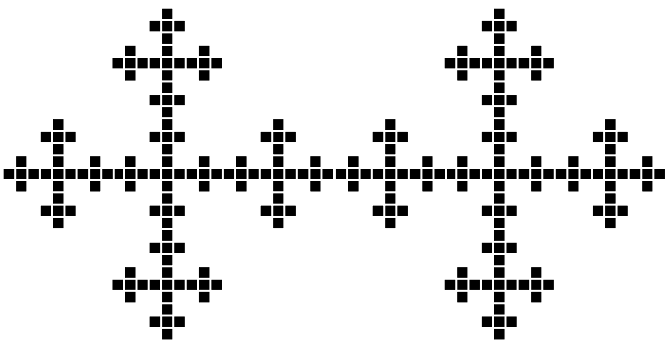


Figure 5: Two fractals (Figure 4-c) attached to each other; the total mass is multiplied by a = 2.

When inserting the fractal law [5] in equation [4], we may compute the mean density

$$\bar{\rho}(\varepsilon) = a\,\frac{\varepsilon^{D}}{\pi\,\varepsilon^{2}} = \frac{a}{\pi}\,\varepsilon^{-(2-D)} \qquad [6]$$

which is no longer constant, but decreases according to a hyperbolic, since $D < 2$. **This shows that changing the reference surface affects the observed density, whereas the fractal dimension remains constant whatever the distance ε may be.**

Let us estimate the variation of the density with respect to changes in radius ε by computing the derivation of the mean density:

$$\frac{d\bar{\rho}(\varepsilon)}{d\varepsilon} = -\frac{a}{\pi}(2-D)\varepsilon^{-(D-3)} \qquad [7]$$

what yields the relation, already discussed Batty and Kim (1992):

$$\frac{d\bar{\rho}(\varepsilon)}{\bar{\rho}(\varepsilon)} = -(2-D)\frac{d\varepsilon}{\varepsilon} \qquad [8]$$

This relationship indicates that the relative changes in mean density and distance are linked by the fractal dimension. The case of a homogeneous pattern is obtained for the limit value $D = 2.0$, for which the mean density remains constant. When fractal dimension D is close to 2.0, density doesn't vary very much, whereas for low values of D the mean density drops down rapidly. The nethermost value of $D = 0$ corresponds to an isolated point in which mass M is concentrated. **Hence, the fractal dimension *D* measures how contrasted a fractal pattern is across the scales.**

$D = 1$ is a limit case: beyond this value, the elements can no longer be connected. Hence such a fractal structure corresponds to a Fournier dust (see Figure 6), whereas values higher than 1 correspond to a Sierpinski carpet, a Fournier dust or a mixed structure.

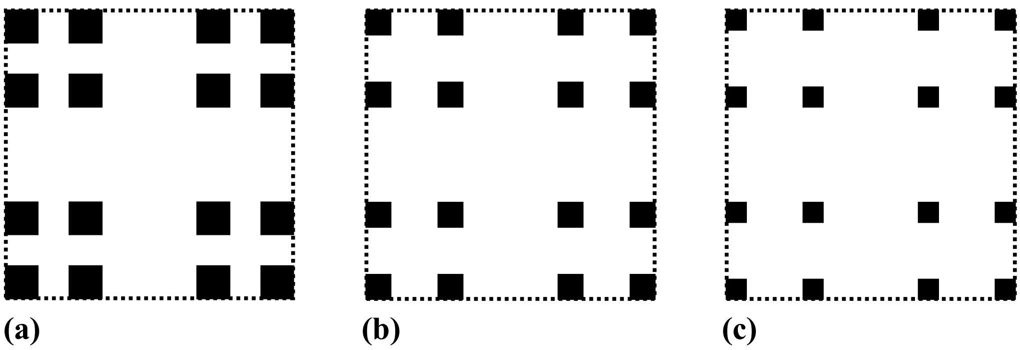


Figure 6: Three Fournier dusts corresponding to $D = 1.26$(a), $D = 1.0$(b) and $D = 0.86$(c).

However, fractal dimension D doesn't provide information about the mean density since this parameter fluctuates across the scales. Hence, from a fractal point of view, arrangements proposed in Figure 3(a) and 3(b) are equivalent, since mass is homogeneously distributed. Hence, mean density is not equal to fractal dimension: both parameters express different aspects of the urban built-up pattern. However, if built-up mass is not distributed in a homogeneous way in urban patterns, density measures become of course ambiguous. Hence when comparing different spatial patterns, the only way for obtaining comparable results is to use strictly the same reference areas for the entire sample. **Changing the size of the reference area affects density values; thus results for density refer strictly to the selected scale of analysis.**

Let us now consider from a theoretical point of view (and later empirically) the relationship between mean density $\bar{\rho}(\varepsilon)$ and fractal dimension D when using a constant reference surface $A_{tot}(\varepsilon)=\pi\varepsilon_{fix}^2$.

We assume that we have a sample of n empirical structures e.g. urban patterns. For each j = 1, 2...n patterns we estimate the mean density $\bar{\rho}_j(\varepsilon)$ as well as the fractal dimension D_j and the prefactor values a_j. According [6] we obtain:

$$\bar{\rho}_j(D_j, a_j) = a_j \frac{\varepsilon_{fix}^{D_j}}{\pi\,\varepsilon_{fix}^{2}} = a_j \alpha\, \varepsilon_{fix}^{D_j} = a_j \alpha\, e^{\beta D_j} \quad with \quad \alpha = \frac{1}{\pi\,\varepsilon_{fix}^{2}}, \quad \beta = \ln \varepsilon_{fix} \qquad [9]$$

where α, β are constant values since we have fixed the size $\varepsilon = \varepsilon_{fix}$ of the surface. If we assume that the prefactor a_j is approximately the same for all patterns j = 1, 2...n, we get aware that **there exists for fixed reference surfaces – and only in this case – an exponential relationship between mean density and fractal dimension:**

$$\bar{\rho}_j \approx const\ e^{\beta D_j}$$

Hence if we measure the values $\bar{\rho}_j, D_j$ for a set of settlements, we can at least approximately obtain an exponential relation for $\bar{\rho}_j(D_j)$. Deviations from this law may be observed when the a_j-values differ considerably from one pattern to the other. Hence, the best the exponential law is fulfilled, the best D_j (fractal dimension) characterizes the pattern. Indeed, in this case, the parameter a_j describing the non-fractal influences on pattern morphology doesn't hardly affect the shape of the curve.

This direct relationship between density and fractal dimension may be seen as quite surprising and perhaps inconsistent with the phenomenon observed in Figure 2, where for a given surface we found different ways of distributing a given mass: the homogeneous distribution in Figure 2(a) corresponds to D = 2, whereas the fractal dimension in the other arrangements amounts according to fractal theory to D = 1.46 (Mandelbrot, 1977, Frankhauser, 1994, Batty and Longley, 1994). The answer can be found by taking into account parameter α_j. Let us assume that we have two structures j = 1, 2 for which we observe $D_1 \neq D_2$ but the same mean density $\bar{\rho}_1 = \bar{\rho}_2$, what yields according to [9]:

$$\begin{aligned} & a_1 e^{\beta D_1} = a_2 e^{\beta D_2} \\ \Rightarrow \quad & a_1 = e^{\beta(D_2 - D_1)} a_2 \end{aligned} \qquad [10]$$

Hence the same mean density can only be observed if the values of the prefactors a_j differ and are linked in a strict relation. This relationship is no longer valid when choosing another value ε_{fix} since $\beta = \log\ \varepsilon_{fix}$. Since the exponential function is monotonous, such an ambiguous situation may occur for a given couple D_1, D_2 only for one unique β-value.

On the one hand, we cannot *a priori* expect in real world situations to observe the same prefactor values a_j for all patterns j. Hence, even for urban patterns having approximately the same fractal dimension, the mean density may vary by means of the a_j-values. On the other hand, the more a fractal law dominates the structure of the pattern, the more the values a_j should be close to 1.0, as non-fractal phenomena become marginal. Then for a sample made of patterns with different D_j values, we should observe an empirical relationship $\bar{\rho}_j(D_j)$, which follows an exponential law, according to relation [9]:

$$\bar{\rho}_j(D_j) \propto e^{\beta D_j} \qquad [11]$$

Let us now test our expectations for the southern periphery of Brussels and interpret them in terms of urban sprawl and town-planning.

3. Empirically …

3.a The studied area

Brussels is the capital city of Belgium, located almost in the center of Belgium and containing globally 1 million inhabitants. As in most urban analysis, defining its limits is an objective on its own (see e.g. GEMACA 1995, Thomas *et al*. 2000, Vanderhaegen et al., 1996; Caruso, 2002): the city sprawls far beyond its original boundaries. In administrative sense, the Brussels Capital-Region is one of the three Regions of the Belgian federal state. Spatially, it corresponds to the enlarged city centre (19 communes) and hence excludes recent peripheral wards that mainly extend in the two other administrative and linguistic regions: Flanders and Wallonia. This makes the problem of data more acute (no integration). Hence, this paper only refers to the southern part of the suburbs which corresponds here to an administrative entity: Brabant Wallon.

Brabant Wallon is a province on its own (approximately 1.090 km² and 358.012 inhabitants). The present aspect of the province results from a historical evolution. In the 15th - 18th centuries, it was indeed mainly rural, with many small villages (less than 250 inhabitants). During the 19th century, industries located in the west (ironworks) and center (papermills); the eastern part remained mainly agricultural. Railways and better roads increased the accessibility. In the first half of the 20th century, industries started closing one after the other. In the sixties, the region is increasingly polarised by Brussels: as in many European cities, people started to move from the centre of Brussels to the countryside, while keeping their activities in the city. Later on (seventies) a university was created in Louvain-la-Neuve and several industrial grounds were planned all over the area. The Southern periphery of Brussels is now characterised by old villages and small towns, a new town and many allotments, old industrial locations as well as new planned ones ("zonings industriels"), highly urbanised communes close to Brussels as well as more residential areas, woods and agriculture as well as employment centres. In other words, a mosaic of landscapes, a polynuclear structure and hence a quite interesting spatial pattern.

In this paper, we restrict ourselves to the analysis of the morphology of the built-up areas. The digitised topological maps IGN 39 and 40 are used. Indeed, at scale 1:10.000, built-up areas correspond to a layer where each building can be isolated. However, no information is provided about the function of the buildings (residence, public service, industry, and farm), or about their occupants (number of inhabitants or number of jobs within a building). Moreover, details about the centers of two towns (Nivelles and Wavre) are lacking: for generalisation reasons, these very dense urbanised zones are represented by a uniform grey pattern. No data base is perfect …

3.b Data processing

Three definitions of the windows are considered.

Type A: Gliding windows were used from left to right, covering the entire studied area. 151 windows represent the Brabant Wallon; they are 850×850 pixels large. No overlapping is tolerated. By using this method, we obtain the complete covering of the studied area but the windows are quite heterogeneous: some are fully built, others are characterised by only two or three farms or by two villages.

Type B: in Belgium, communes were redefined in 1977. On the average, a "new commune" is made of 5 "old communes". These old communes mainly correspond to former villages. Each village center is now considered as a center of a window. The size and shape of the window are adapted to the shape of the commune. Hence, in this case, size and shape of the windows vary but each window is always centred on the village core. Given the shape of some communes, some parts of the studied area were ignored. Overlapping is almost non-existing (less than 3 %). The average size of the windows is 517 (standard deviation: 218) × 526 (standard deviation: 217). Each commune of the Brabant Wallon belonging to map 39 and 40 are considered, that is to say 88 communes. A 89th was added: we indeed created by means of the present statistical wards an entity around Louvain-la-Neuve (new town).

Type C: in this third definition, windows have a fixed size (1900×1600) and centred on the present administrative centre of the new communes. Given the shape and size of the present communes, some overlapping is observed, but once again it is marginal (less that 5 %). 26 windows are defined.

For each window, a measure of **fractal dimension** *D* is computed using the software *Fractalyse 2.12* (Frankhauser and Vuidel, 2002). As mentioned in Section 2, *D* describes how the mass *M* is concentrated in a given surface *A*. A value of *D* close to 2.0 is synonymous of a rather homogeneous distribution, whereas the lower the value of *D*, the more the mass is aggregated at different scales. A fractal dimension close to 0.0 corresponds to a mass concentrated in one point. A value of 1.0 corresponds to a line, but also represents a threshold in fractals: when $D < 1.0$, the structure is necessarily composed of an unconnected set of points. Such a situation is typical for Fournier dusts (cf. Figure 6). $D \geq 1.0$ refers to structures that are either Fournier dusts, or consist of a unique cluster, highly fragmented, like Sierpinski carpets. One method for estimating *D* is used in this paper: the correlation. In the **correlation analysis,** the texture is not modified: we simply count the number of occupied sites (pixels) that lie within a square of base length ε of each occupied site and then compute their mean number. The procedure is repeated for other values of ε. With this method, we get information about the so-called "second-order" effects, i.e. we test the mean neighbourhood scaling behaviour. This method turns out to be quite reliable. For each window, a measure of mean density of the built-up surface is also computed.

3.c Empirical results

- ***Fractal dimension and window type***

We expect *D* to vary according to the window type. Table 2 gives the main descriptive statistics related to *D* and Figure 7 shows the histograms for three types of window definition. The mean value computed for each sample of windows is given for information; a mean value of *D* has indeed little or no mathematical sense (see De Keersmaecker, Frankhauser and Thomas, 2003 for a discussion). The gliding window (Type A) is like a grid applied to the studied area, without taking into account the reality of the real world. Hence, very small values of *D* are observed: they correspond to windows with just a few scattered farms or houses (buildings). This leads to values close to 0 (a point). For Type B this is less the case (only one value of *D* smaller than 1.0). Given the size of the windows and their definition, values of *D* smaller than 1 are never observed in Type C.

Given the history and the heterogeneity of the built-up area in Brabant Wallon (Section 3.a), **each way of defining a window (A to C) generates different fractal results.** If this should not always be true for highly urbanised studied area (fully urban), one has to be careful when interpreting fractal analyses of complex periurban realities: the way of defining the window generates biases in the results … and most of the time the researcher does not have the choice of the definition of the window!

We are aware that the present results are impossible to compare with any other published for other cities, for other data sets: *D* depends on the data used, on the way of defining the window and also on the size of the window. These parameters are often not controlled for in the literature. Further attention should be later put on that point. We know that *D* values computed for cities range from 1.28 to 1.93; these values are often obtained on cities seen as a whole, as large aggregates on which fractal measurements have been performed. In our case, we use detailed measurements of spatial distribution for implementing adequately the description of the morphology of the suburbs of Brussels. Let us here remind that for the city of Brussels and for a gliding window *D* values vary between a minimum value of 1.34 and a maximum of 1.96 (De Keersmaecker, Frankhauser and Thomas, 2003). Hence, some values observed in the city centre (low or high) are quite close to those observed in the peripheries.

	n	*Minimum*	*Maximum*	*Mean*
Type A	151	**0,347**	1,779	1,366
Type B	89	**0,956**	1,833	1,525
Type C	26	1,206	1,822	1,576

Table 2: Fractal dimension *D* (minimum, maximum and mean value) computed by correlation for several ways of defining the windows.

Figure 7: Histogram of *D* values for each window type (A, B and C).

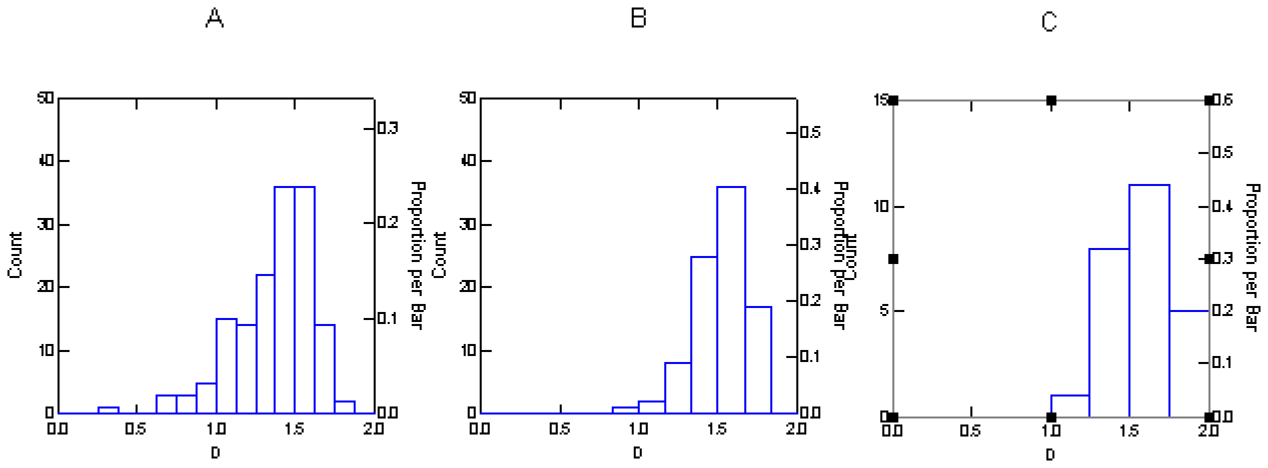
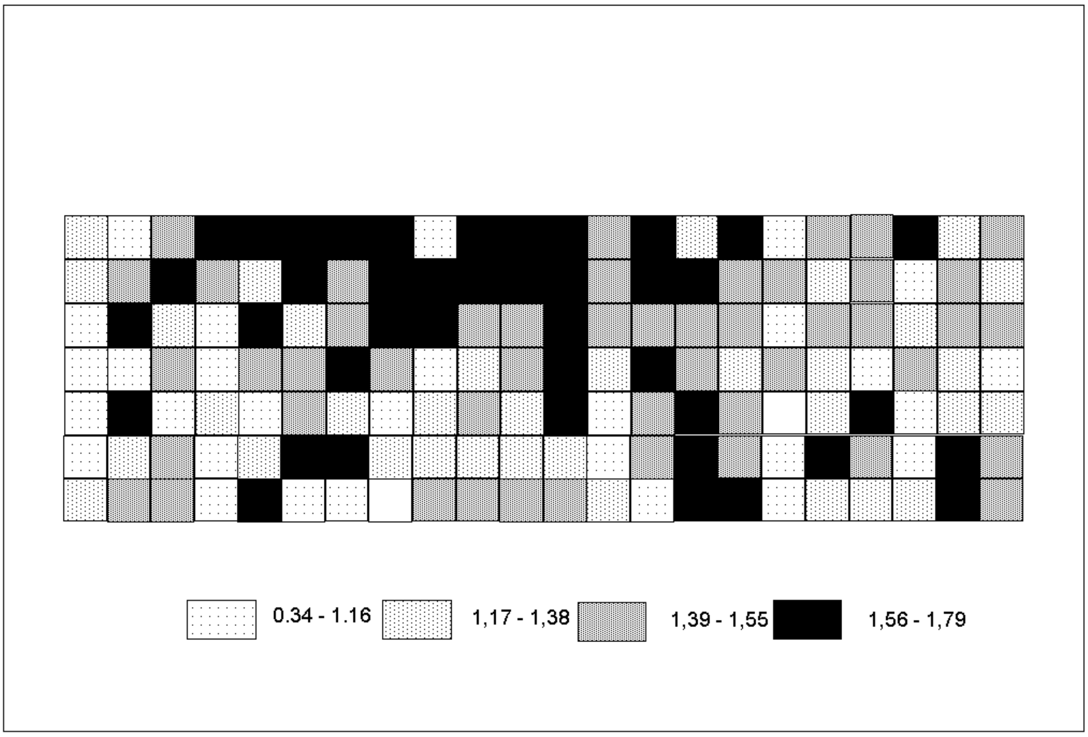


Figure 8(a): Choropleth mapping of Type A windows *(still missing)*



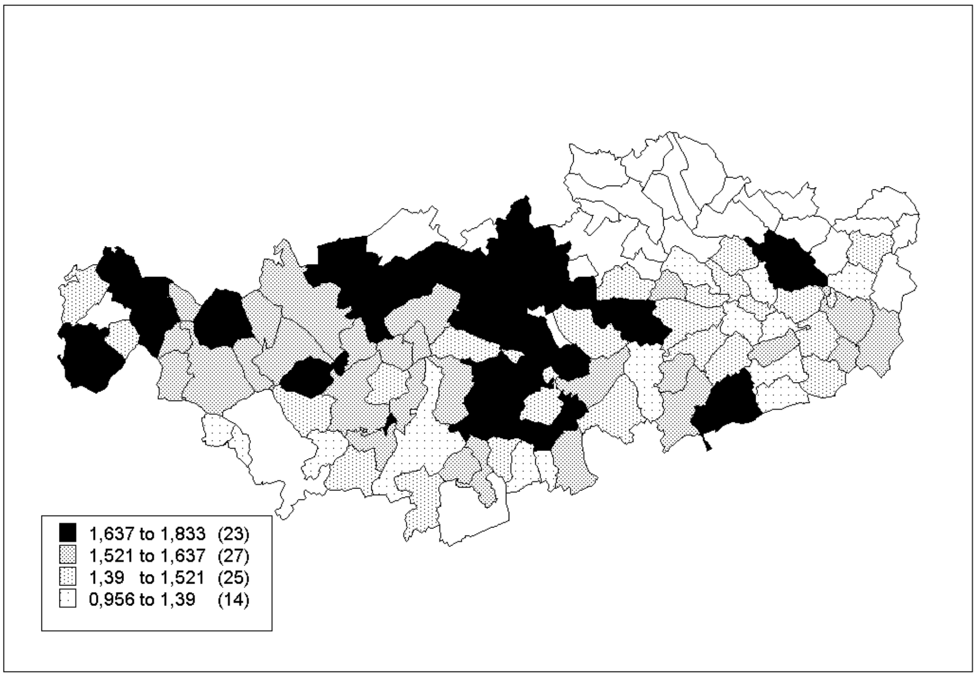


Figure 8(b): Choropleth mapping of *D* for Type B windows

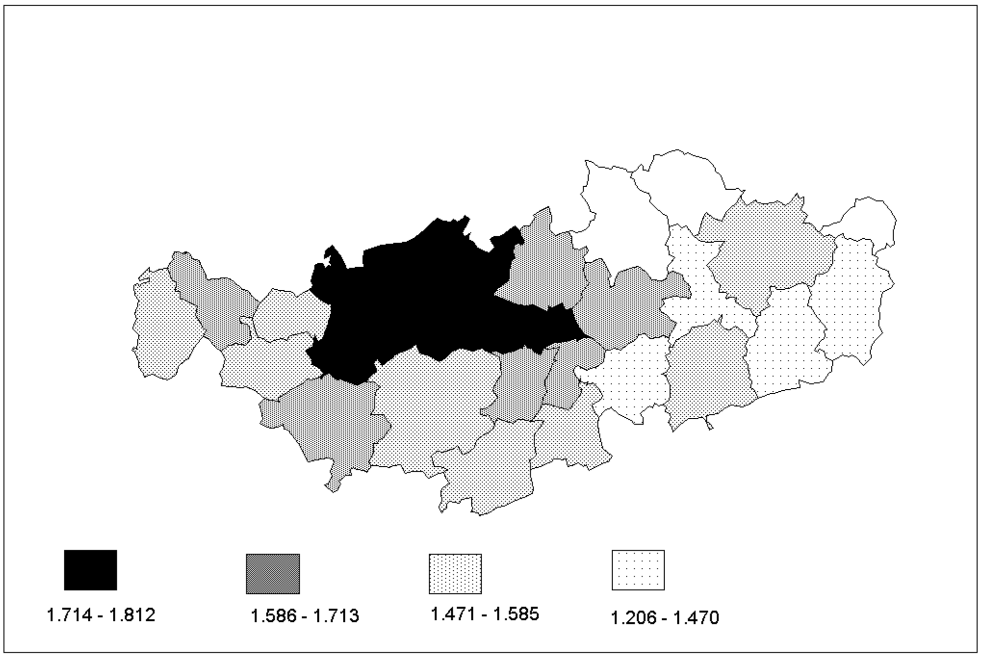
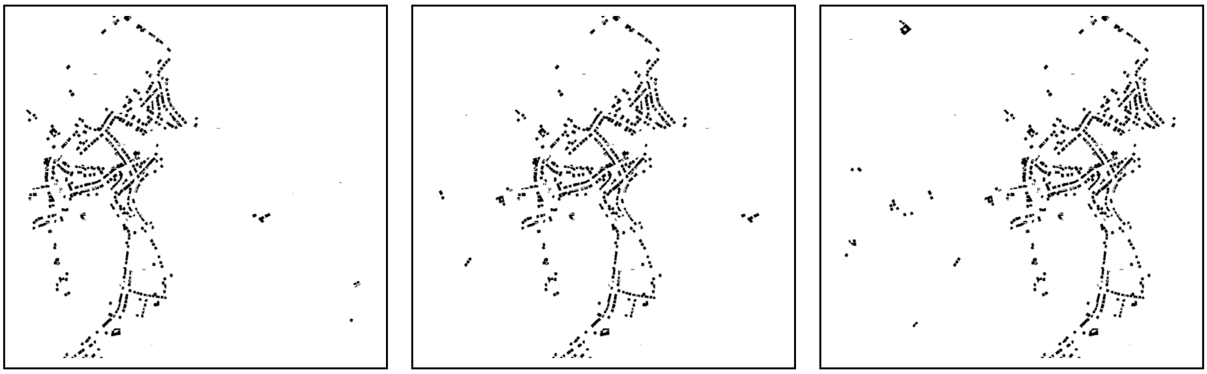


Figure 8(c): Choropleth mapping of Type C windows

Figures 8(a) to 8(c) are choropleth maps of *D* for the three types of windows. They are a two dimensional representation of the results reproduced in Table 2 and Figure 7. We see that if *D* values (Table 2) differ with window definition, the relative positions of the individual values generally translates the same geographical realities. The fractal dimension is higher in the vicinity of Brussels: the compactness of the urban clusters increases when the distance to the city (Brussels) decreases; as expected, *D* is also higher is the former urban concentrations (small towns such as Nivelles; old industry areas in the western part). Globally, the classification of the *D* values illustrates a centre-periphery structure, slightly « disrupted » by functional and morphological realities: history matters! This was put forward by former traditional multivariate analyses (see e.g. Halleux, Derwael and Mérenne, 1998).

Let us here add that, when defining window types B and C, a centre had to be defined. We've chosen the administrative centre of each commune. Several trials were done for other definitions of the centroid (Everyone knows that this is not always an easy task for geographers). Figure 9 gives one example. Three different positions of the centroid are adopted and corresponding fractal dimensions computed. Variations in *D* are insignificant compared to those already mentioned in Table 2.



$D = 1.6182$ $D = 1.6188$ $D = 1.6114$

Figure 9: Three ways of defining the centroid.

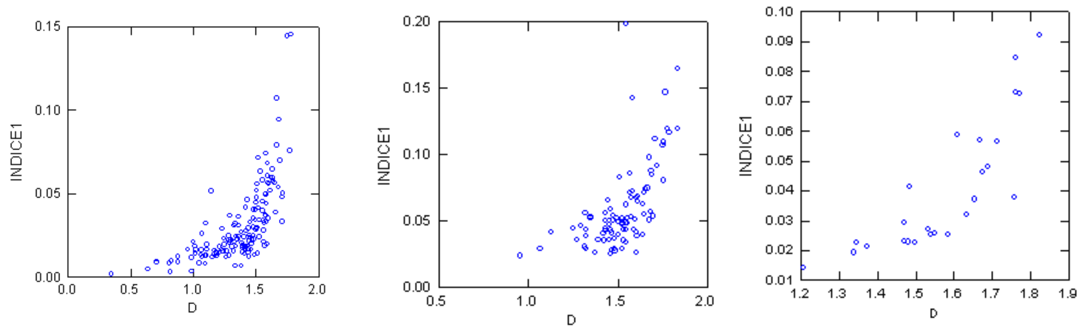
- ***Fractal dimension and built-up density***

Given the theoretical expectations of Section 2 and the spatial distributions observed in Figure 8(a) to 8(c), we suspect that *D* is not only a measure of density; it is more than that. If fractal dimension was a "simple" synonym of density, the correlation would be close to 1.0. Table 3 gives for each type of window (A to C) the Pearson correlation coefficient computed between the values of *D* and two ways of defining the density (built-up density = number of built-up pixels / total of the window). Let us here add that the population density is the number of inhabitants per square kilometer in the commune to which the window is associated. Table 3 confirms the results formerly obtained by Batty and Kim (1992) and Longley and Mesev (2002), but deepens them. Figure 10 illustrates the relationships through correlograms. We see that – as expected in Section 2 – **(1) changing the size of the reference area affects the density values; thus results for density refer strictly to the chosen scale of analysis and (2) there exists an exponential relationship between mean density and fractal dimension and this is only true for fixed sized reference surfaces (Type A).** Indeed, according to the above-discussed theoretical considerations, for type A the set of points is not very dispersed and the empirical relationship between mean density end fractal dimension shows an exponential shape. Obviously the prefactor *a* seems not to play an important role and the regularity of the curve seems to confirm that the fractal law suits well for characterizing the morphology of the analysed urban patterns. On the contrary, points are rather dispersed in type B and C, where the size of the windows varies and thus mean density may no longer be compared.

This kind of results is to be interpreted fractally but it also pertains to **M.A.U.P.** (Modifiable Areal Unit Problem) in spatial analysis: shape and size of the basic spatial units affect the statistical results. One really has to be careful about this kind of bias, whatever the statistical method used. In this particular case it is coupled with a theoretical problem

	n	*Built-up density*	*Population density**
Type A	151	0,653	-
Type B	89	0,609	0,573
Type C	26	0,820	0,716

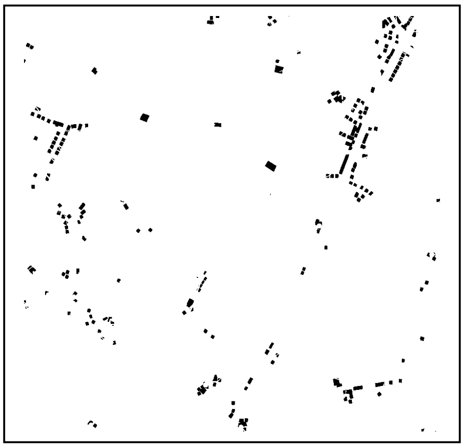
Table 3: Pearson correlation coefficients between *D* and two density measures.



Type A ($n = 151$) Type B ($n = 89$) Type C ($n = 25$)

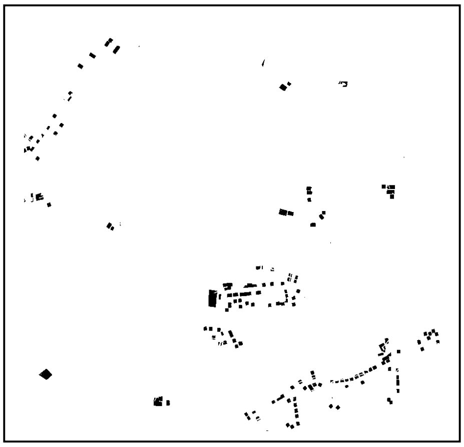
Figure 10: Correlogram between *D* and built-up density for three types of window definitions

As final proof for validating the true sense of *D* in periurban morphologies, let us now limit ourselves to a fixed size of windows (Type A) and select some windows having the same built-up density values. From Figure 10 and from Section 2, we can expect that *D* can be quite different and hence for a given value of built-up density, we can have quite different spatial patterns. Figure 11 gives some examples. For a same value of density, fractal dimension is different and this variation can be explained by the level of organisation of the built-up area. **Fractal dimension of built-up surfaces is far from being equal their density**.

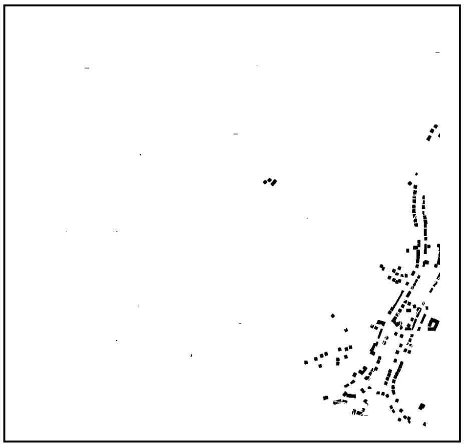


D = 1.013, density = 1.85

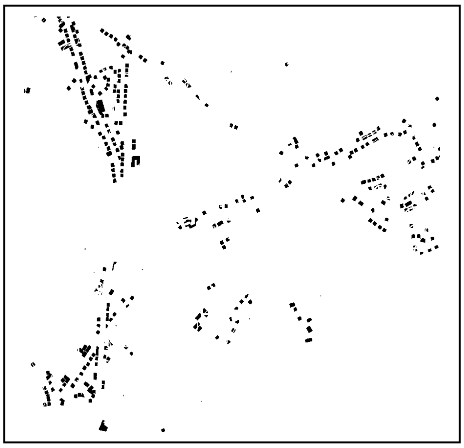
D = 1.431, density = 1.86



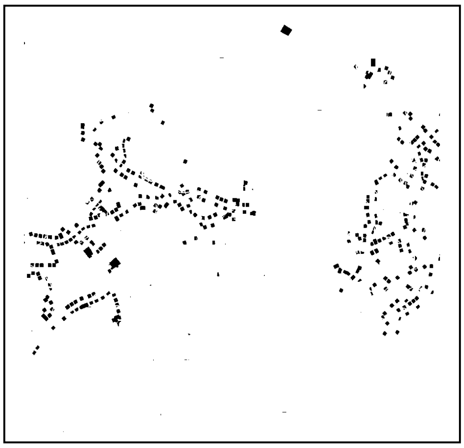
D = 1.059, density = 1.32



D = 1.487, density = 1.33



D = 1.360, density = 2.44



D = 1.478 , density = 2.45

Figure 11: Examples of built-up areas and *D* values controlling the size of the window (Type A) and the built-up density

5. Conclusions

This paper analyses the urban morphology of the built-up surfaces in the periphery of a European city (Brussels – 1 million inhabitants) by means of fractal dimension. If limited to one city, our empirical results are validated by a geometrical/theoretical demonstration. Our main results can be summarised in two points.

First, we pursued the analysis of the relationship between density and fractal dimension conducted by Batty and Kim (1992). Both measures are interesting in terms of urban analysis; fractal dimension considers morphology, internal structure of the built-up areas while mean density gives a rough idea of the occupation of the surface. We here particularly showed that for a given size of basic spatial units (windows), the relationship between density and fractal dimension is – without doubt – exponential. This demonstration is done in theoretical as well as purely empirical terms. As far as we know, this is quite novel in urban fractal analysis.

Second, we knew that fractal dimension was useful for understanding urban structure; in this paper we showed that it is also interesting in analysing periurban realities and hence in urban sprawl modelling. Fractal dimension discriminates quite well the periurban space: the compactness of the urban clusters increases when the distance to the city (Brussels) decreases; as expected, the classification of the D values illustrates a centre-periphery structure, slightly « disrupted » by more traditional functional and morphological realities (history matters!). Our fractal results can be interpreted similarly to former multivariate traditional analyses (see e.g. Halleux, Derwael and Mérenne, 1998; De Keersmaecker, Frankhauser and Thomas, 2004).

As in most paper, critics and questions are more numerous when writing the conclusion. Even if this is not our first paper in fractal analysis, we are aware that we only studied one city and that many choices were done due to data and methodological constraints. However, the theoretical section (Section 2) here validates the empirical results (Section 3); this strengthens the empirical results. We intend at pursuing the analyses about Brussels (centre and periphery). We first would like to find a way of statistically inferring on fractal measures; second to analyse the relationship between the observed differences in fractal dimensions and the structural, socio-economical and town-planning realities. A third and last objective would be to develop more morphological indices and link then to those obtained with fractal measurements Both tasks are not easy. Let us here finally remind that this paper is limited to the analysis of the usefulness of the fractal dimension in studying periurban morphology; we don't aim at giving an explanatory power to the fractal approach; we just want to show that fractal analysis could further be a quite interesting technique for reproducing / modelling the urban sprawl as already initiated by two papers (Cavailhès, Frankhauser, Peeters and Thomas, 2002 and 2004).

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