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# FRACTAL STRUCTURE OF THE INTERPLANETARY MAGNETIC FIELD

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### ABSTRACT

Under some conditions, time series of the interplanetary magnetic field strength and components have the properties of fractal curves. Magnetic field measurements made near 8.5 AU by Voyager 2 from June 5 to August 24, 1981 were self-similar over time scales from  $\sim 20$  sec to  $\sim 3 \times 10^5$  sec, and the fractal dimension of the time series of the strength and components of the magnetic field was  $D = 5/3$ , corresponding to a power spectrum  $P(f) \propto f^{-5/3}$ . Since the Kolmogorov spectrum for homogeneous, isotropic, stationary turbulence is also  $f^{-5/3}$ , the Voyager 2 measurements are consistent with the observation of an inertial range of turbulence extending over approximately four decades in frequency. Interaction regions probably contributed most of the power in this interval. As an example, one interaction region is discussed in which the magnetic field had a fractal dimension  $D = 5/3$ .

## 1. Introduction

The interplanetary magnetic field is variable on many time scales, and spectral analysis of the time series for both the magnitude and the components frequently shows a power law spectrum  $f^{-\alpha}$  over at least one decade in frequency, with a slope depending of the frequency range and the time interval that is analyzed (see e.g., the reviews by Childers and Russell, 1972; Barnes, 1979; Burlaga, 1972; and Behannon and Burlaga, 1981). For example, Sari and Ness (1969) found  $f^{-2}$  for  $2.8 \times 10^{-4} \text{ Hz} < f < 1.6 \times 10^{-2} \text{ Hz}$  at 1 AU at a time when discontinuities in the interplanetary magnetic field were prominent; Burlaga and Goldstein (1984) found  $f^{-5/3}$  for  $10^{-6} \text{ Hz} \leq f \leq 10^{-4} \text{ Hz}$  in a turbulent "transient flow system" between 4.1 AU and 5.2 AU; and Goldstein et al. (1984) found  $f^{-1}$  for  $2 \times 10^{-5} \leq f \leq 10^{-6}$  at 1 AU, which is presumably a signal representing the generation of disturbances at the sun. Burlaga and Goldstein (1984) and Burlaga et al. (1985) found that the turbulence extends to increasingly low frequencies at larger distances as the interaction regions increase in size.

Spectra are usually computed by either the "fast-fourier transform" method or the Blackman-Tukey method. As discussed below, a curve with a single power law spectrum is self-affine, and it has the properties of a "fractal curve" across the frequency range covered by the law (Mandelbrot, 1975a,b, 1977, and 1967). It is relatively simple to compute the fractal dimension  $D$  of such a curve, and this dimension is very simply related to the index of a power law spectrum. It will be shown that under some conditions the interplanetary magnetic field has the properties of a fractal curve, and calculation of its fractal dimension is an efficient and economical alternative to spectral analysis.

## 2. Variance, Spectra and Fractal Dimension

Mandelbrot (1975, 1977) has discussed a function that is a generalization of the function representing Brownian motion, which he calls "the fractional

Brownian function". This is a Gaussian scalar function  $B(t)$  which has zero mean and a variance given by

$$(\delta B)^2 = \langle [B(t_2) - B(t_1)]^2 \rangle = (t_2 - t_1)^{2H}, \quad (1)$$

where  $H$  is a constant between 0 and 1. It reduces to the Brownian function when  $H = 1/2$ . For stationary time series with  $t_2 = t_1 + \tau$  the function  $B(t)$  has a power law spectrum  $P_B(f) \propto f^{-\alpha}$  where

$$\alpha = 2H + 1 \quad (2)$$

(Panchev, 1971). The case  $H = 1/3$  corresponds to "Kolmogorov variance" and the well-known "Kolmogorov spectrum",  $f^{-5/3}$ , which describes inertial range turbulence in an incompressible fluid. The case when  $H = 1/2$ , corresponds to an  $f^{-2}$  spectrum, and the curve  $B(t)$  represents ordinary Brownian motion. This is related to Burger's turbulence and to a Poisson field resulting from an infinite number of discontinuities (planes) whose positions, orientations, and intensities are given by three infinite sequences of mutually independent random variables (Mandelbrot, 1975). A Poisson field describes the distribution of tangential discontinuities in certain regions of the solar wind (Burlaga, 1972; Sari and Ness, 1969). The case  $H = 0$  corresponds to an  $f^{-1}$  spectrum, which has many interesting applications (Montrol and Schlesinger, 1982), but this case must be treated with care, because it is valid only as a limit.

The function  $B(t)$  can be viewed geometrically as a curve which has structure on every scale and which is "statistically self-affine". "Statistically self-affine" means that each part can be considered a reduced scale image of the whole, i.e.,  $h^{-2H}B(ht)$  is statistically identical to  $B(t)$ , and (1) is invariant under the transformation  $t \rightarrow ht$  and  $\delta B \rightarrow h^H \delta B$ . In this paper  $B(t)$  represents measurements of the magnitude of the interplanetary magnetic field or any one of its components, and the "scale"  $\tau$  is determined by the averaging interval that we choose. In particular, the time series

representing  $B(t)$  is approximated by a histogram, where the width of each bar is  $\tau$  and the height of each bar is the average value of  $B(t)$  between  $t = t_k$  and  $t_k + \tau$ , which we denote by  $\bar{B}(t_k)$ . The "length" of the curve defined by the histogram over some interval  $0 \leq t \leq T_0$

(where  $T_0 = N\tau$  and  $N$  is an integer) is  $L(\tau) = \sum_{k=1}^N |\bar{B}(t_k + \tau) - \bar{B}(t_k)|$ ,

neglecting the constant horizontal part. Since we shall discuss measurements made at different distances from the sun, we normalize the curve by the mean "spiral magnetic field strength"  $B_p(R) = 4.75 (1 + R^2)^{1/2} / R^2$ , where  $R = R(t_k)$  is the distance from the sun measured in AU at the time  $t_k$  (Burlaga et al. (1984)). Thus, the "length" of the curve  $B(t)$  is approximately

$$L(\tau) = \sum_{k=1}^N \frac{|\bar{B}(t_k + \tau) - \bar{B}(t_k)|}{B_p(t_k)} \quad (3)$$

This length is a function of  $\tau$ , and for statistically self-affine curves

$$L(\tau) = L_0 \tau^{-S}, \quad (4)$$

where  $L_0$  and  $S$  are constants. For curves whose variance is given by (1) with  $t_2 - t_1 = \tau$ ,  $L(\tau) \propto N(\delta B^2)^{1/2} \propto (T_0/\tau) \tau^H \propto \tau^{H-1}$ , so that  $S = 1-H$ . Mandelbrot (1977) introduced the number

$$D = S + 1 = 2-H. \quad (5)$$

For smooth rectifiable curves  $L(\tau)$  must be constant, hence  $S = 0$  and  $D = 1$ , and the number  $D$  is equal to the topological dimension. For statistically self-affine curves in a plane,  $S > 0$  and  $D$  is a fraction  $1 < D < 2$ ; such curves are called "fractals" and  $D$  is called the "fractal dimension" (Mandelbrot, 1977).  $D$  is related to the Hausdorff-Besicovitch dimension or

"capacity" of the set of elements defined by  $|\bar{B}(t_k + \tau) - \bar{B}(t_k)| / \bar{B}_p(t_k)$ ,  
 $k = 1 \dots N = T_0/\tau$  (Farmer et al., 1983).

Another way of looking at this analysis is to say that we are estimating the "structure function"  $D(\tau) = \langle [B(t + \tau) - B(t)]^2 \rangle$  of  $B(t)$  for small lags  $\tau$ . Panchev (1971) shows that  $D(\tau) = 2 [ \langle B^2(t) \rangle - R(\tau) ] = 2 [R(0) - R(\tau)]$ , where  $R(\tau)$  is the correlation function  $R = \langle B(t + \tau) B(t) \rangle$  and  $R(0)$  is the variance of  $B(t)$ . The approximation  $D(\tau) \propto \tau^{-\alpha}$  discussed above is equivalent to approximating  $R(0) - R(\tau)$  by the power law  $\tau^{-\alpha}$ . In effect, we are fitting  $R(\tau)$  by a power law, which can be accomplished by using averages of  $B(t)$ . Note that we used  $|B(t + \tau) - B(t)|$  as our measure instead of  $(B(t + \tau) - B(t))^2$ , but the slopes derived from (4) are not sensitive to the choice between the two metrics.

The interplanetary magnetic field is a vector field which is a function of position  $(x, y, z)$  and time. We regard it as a set of scalar fields  $B_i$ ,  $i = 0, 1, 2, 3$  representing the magnitude and the  $x, y, z$  components of  $\underline{B}$ , respectively. Most interplanetary measurements are made near the ecliptic plane, and it is convenient to think of the magnitude or a component of the magnetic field at any instant as a function  $B_i(x, y)$  defined on the ecliptic which represents a surface or "landscape". When the 1-dimensional section of this surface, say  $B_i(x, y_0)$ , is a fractional Brownian function of  $x$ , the "landscape"  $B_i(x, y)$  is a "fractionally Brownian surface" in the 3-dimensional space  $(B_i, x, y)$ . The section has fractal dimension  $D = 2 - H = 1 + S$ ; this is the fractal dimension that we refer to below. Thus, for Kolmogorov turbulence,  $H = 1/3$ ,  $S = 2/3$  and  $D = 5/3$ . It is not possible to measure  $B_i(x, y_0)$  at any instant, but the solar wind carries the magnetic field pattern past a fixed observer. When the magnetic field changes slowly during the interval under consideration, or when the field is statistically stationary, the statistical properties of the observed  $B_i(t)$  are the same as those of  $B_i(x, y_0)$ .



### 3. Observations of Inertial Range Turbulence

As Voyager 2 approached Saturn, near 9 AU, high-resolution measurements of the interplanetary magnetic field were made from June 5 to August 24, 1981, and the data coverage was  $\approx 80\%$ . Since the plasma is convected past the observer, one can approximate  $B_1(x, y_0)$  by the observed  $B_1(t)$ . Using the set of 9.6 sec averages of the measurements of the magnetic field  $B_1$  as our basic curve (histogram), we compute averages of the data successively over intervals  $\tau_j = 2^j \times 9.6$  sec,  $j = 1 \dots 15$ , thereby obtaining 15 curves  $B_{1j}(t)$ , one for each  $\tau_j$ . The logarithm of the "length" of each curve,  $\log_{10} L_1(\tau_j)$ , was computed using (3), and this was plotted versus  $\log_{10} \tau_j$  in Figure 1. This procedure was carried out for the magnetic field magnitude  $B_0$  ( $i = 0$ ) and for each of the components of the magnetic field  $B_i$ ,  $i = 1, 2, 3$  ( $B_1 \equiv B_X$  is the  $X_{HG}$  component,  $B_2 \equiv B_Y$  is the  $Y_{HG}$  component and  $B_3 = B_Z$  is the  $Z_{HG}$  component in the heliographic coordinate system described by Burlaga, 1985). Figure 1 shows that for each of the curves  $B_0(t)$ ,  $B_1(t)$ ,  $B_2(t)$ , and  $B_3(t)$ , the points lie very close to a straight line, which shows that  $L_1(\tau) \propto \tau^{-S}$ . Thus the magnetic field is "self-affine" or "fractal" over more than 4 decades in time scale. All of the lines have the same slope,  $-S$ , where  $S = 2/3$ , corresponding to curves with fractal dimension  $D = 5/3$ . Thus, in this interval, the magnitude and components of the interplanetary magnetic field behave like fractal curves (specifically, fractional Brownian functions) with fractal dimension  $D = 5/3$ .

Referring to the discussion in Section 2, a fractal dimension of  $5/3$  and  $S = 2/3$  correspond to  $H = 1/3$ , or "Kolmogorov variance", and this is related to a power spectrum of the form  $k^{-5/3}$ , which represents the inertial range of stationary homogeneous turbulence. In this sense our results are consistent with the observation of the inertial range of turbulence in the interplanetary magnetic field. Kolmogorov spectra ( $k^{-5/3}$  in wave number space and  $f^{-5/3}$  in frequency space, where length is related to time  $t$  by  $L = V_{SW} t$ ,  $V_{SW}$  being the solar wind speed) have previously been reported over  $\approx 2$  decades in frequency, but the fractal method used here has made it possible to show that the

inertial range can extend over more than four decades in frequency, from  $f = 5 \times 10^{-2}$  Hz to  $f = 3 \times 10^{-6}$  Hz.

#### 4. Turbulence in an Interaction Region

Beyond several AU, the strength of the interplanetary magnetic field is typically either significantly higher than average for a few days (an "interaction region") or lower than average for a few days (a "rarefaction region"). Most of the power in the magnetic field during the interval described in Section 3 was probably from the interaction regions (see Burlaga et al., 1985). In this section we shall discuss an interaction region observed at 9.3 AU.

An interaction region moved past Voyager 2 in the interval from July 10, hr 13 to July 14, hr 0, 1981. High resolution measurements of the magnetic field strength and each of the components were averaged to over 13 different intervals,  $\tau_j = 2^j \times 9.6$  sec,  $j = 1 \dots 13$ , to obtain 13 "histograms" for each of functions  $B_i(t)$ ,  $i = 0, 1, 2, 3$ , and the "length"  $L_i(\tau_j)$  of each component was computed from (3). Plots of  $\log_{10} L_i(\tau_j)$  vs  $\log_{10} \tau_j$  for  $i = 0, 1, 2, 3$  are shown in Figure 2. Again, we find that  $L_i(\tau_j) \propto \tau_j^{-S}$ , indicating that the magnetic field strength and each of its components are approximately self-affine and have the properties of fractal curves. The slope of each of the four lines is  $-2/3$ , giving  $S = 2/3$ ,  $H = 1/3$ , and  $D = 5/3$ . Thus, from (2), the fractal dimension of the magnetic field profiles is that which corresponds to a Kolmogorov ( $f^{-5/3}$ ) spectrum, and in this sense the magnetic field in this interaction region at 9.3 AU is turbulent with an inertial range spectrum for  $5 \times 10^{-2}$  Hz  $\lesssim f \lesssim 2 \times 10^{-5}$  Hz.

#### 5. Discussion

We have shown that in some cases the components and magnitude of the interplanetary magnetic field have properties of fractal curves. A fractal curve is equivalent to a function with a power law spectrum,  $f^{-\alpha}$ , and the

fractal dimension of the curve is related to the exponent  $\alpha$ . There are at least two advantages in thinking of the magnetic field from a geometrical point of view. First, it is an economical and powerful approach. We used  $\sim 10^6$  data points to obtain the results in Figure 1. Determining the fractal dimension from  $L(\tau)$  requires  $\sim N$  operations, and these are carried out sequentially, implying reduced memory requirements. This is in essence an approximation of the structure function of the data. In order to compute such a spectrum with the standard FFT algorithm, one needs much more sophisticated algorithms and must take care in dealing with data gaps. The simplicity and economy of the fractal method allows one to scan large amounts of data for fractal behavior and it allows one to identify power laws associated with turbulence or other phenomena over a large ( $> 10^4$ ) range of frequencies. Second, the fractal method allows one to visualize the "texture" or "topography" of the magnetic field. Given the fractal dimension of an interplanetary magnetic field measurement, which is a 1-dimensional section of a "fractionally Brownian surface" (see Section 2), one knows the fractal dimension of the surface (assuming homogeneity), and the techniques described by Mandelbrot can be used to visualize the surface. For example, turbulent interplanetary fields with  $H = 1/3$  can be visualized as shown in Plates 211, 213, 215, 222 and 233 in Mandelbrot (1977).

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### Figure Captions

Figure 1      The fractal lengths of the curves representing the magnetic field magnitude and components as a function of time, computed using various averaging intervals  $\tau$ . Lines with a slope of  $-2/3$  are drawn through the points. The interval contains many compression and rarefaction regions.

Figure 2      Fractal lengths as in Figure 1 for a compression region at 9.3 AU.

# VOYAGER 2

JUNE 5 - AUG. 24, 1981

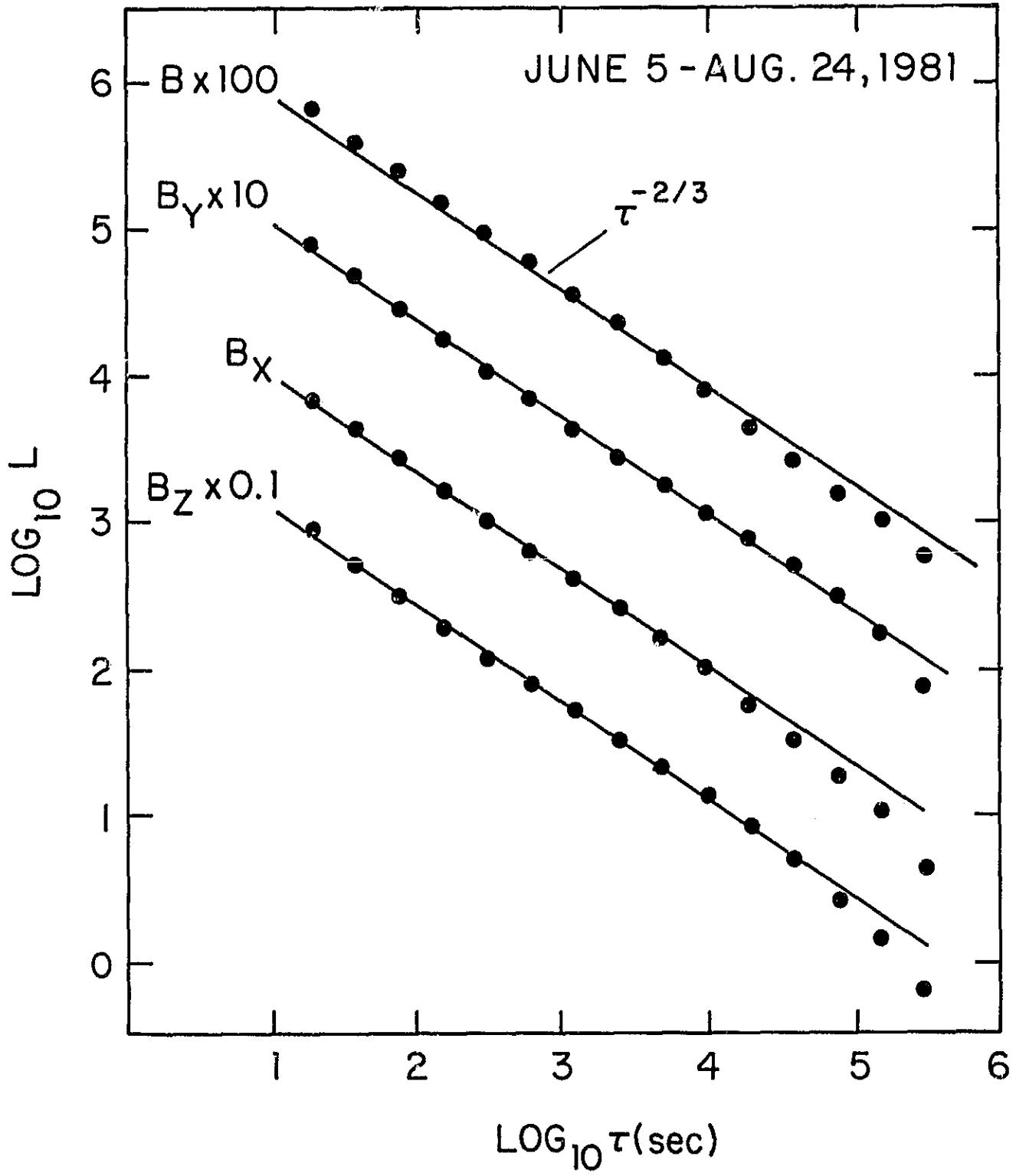


Figure 1

# VOYAGER 2

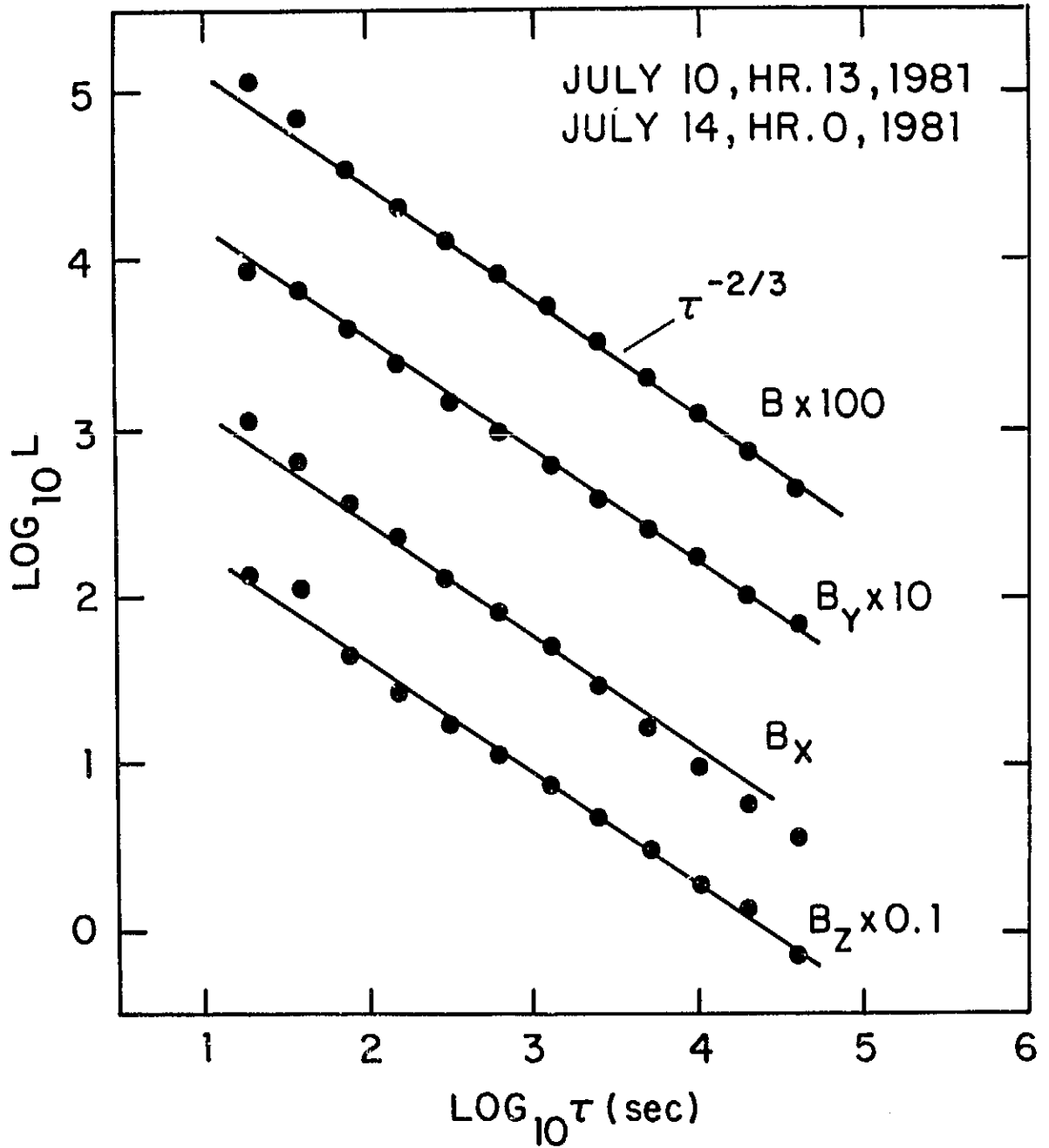


Figure 2



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