

FRACTIONAL CALCULUS AND WAVES IN LINEAR VISCOELASTICITY

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ERRATA CORRIGENDA (July 2012)

Cuiusvis hominis est errare, nullius nisi insipientis in errore perseverare.
Marcus Tullius Cicero (*Oratio Philippica Duodecima*).

Many thanks to all those who send corrections or detect misprints and omissions! This will prove highly valuable in preparing the next edition/printing.

- **p. 17, Ch. 1.** I. Liouville, I. Hadamard, I.E. Littlewood, I Spanier would read: J. Liouville, J. Hadamard, J.E Littlewood, J. Spanier.

- **p. 31, Ch. 2.** $G(t) \equiv G_g \equiv G_e = 1/m$ would read $G(t) \equiv G_g \equiv G_e = m$.

- **p. 60, Ch. 3.** In the RHS of Eq. (3.11) $s^{1-\nu}$ would be replace by $1/s^{1-\nu}$, namely:

$$\delta(t) \div 1 \Rightarrow \frac{t^{-\nu}}{\Gamma(1-\nu)} \div \frac{1}{s^{1-\nu}}, \quad (3.11)$$

- **p. 41, Ch. 2.5.** The sentence on the time-spectral function must read as follows: For the sake of convenience we shall omit the suffix to denote any one of the two spectra; we shall refer to $R(\tau)$ as the *time-spectral function* in \mathbb{R}^+ , with the supplementary normalization condition $\int_0^\infty R(\tau) d\tau = 1$ if the integral of $R(\tau)$ in \mathbb{R}^+ is convergent.

- **p. 62, Ch. 3.** The R.H.S of Eq. (3.20a) must read as follows:

fractional anti-Zener model :

$$\left[1 + a_1 \frac{d^\nu}{dt^\nu}\right] \sigma(t) = \left[b_1 \frac{d^\nu}{dt^\nu} + b_2 \frac{d^{(\nu+1)}}{dt^{(\nu+1)}}\right] \epsilon(t), \quad (3.20a)$$

- **p. 154, Ch. 6.** Footnote 3: tuentieth would read twentieth

- **p. 163, App. A.** Eq. (A.23) would read:

$$\int_0^\infty e^{-zt^\mu} t^\nu - 1 dt = \frac{1}{\mu} \frac{\Gamma(\nu/\mu)}{z^{\nu/\mu}} = \frac{1}{\nu} \frac{\Gamma(1 + \nu/\mu)}{z^{\nu/\mu}}, \quad (\text{A.23})$$

- **p. 186, App. B.** 2-3 line, read: Eq. (B.58) is valid under the restriction $-\pi < \arg(z) < \pi$.

- **p. 187, App. B.** 8 line, read: **Taylor series.**

- **pp. 188-189, App. B.** After Eq (B.71b) we have improved the presentation of the Airy functions as follows:

Asymptotic representations for real variable ($z = x \in \mathbb{R}$).

$$Ai(x) \sim \begin{cases} \frac{1}{2\sqrt{\pi}} x^{-1/4} \exp\left(-2x^{3/2}/3\right), & x \rightarrow +\infty, \\ \frac{1}{\sqrt{\pi}} |x|^{-1/4} \sin\left(|x|^{3/2}/3 + \pi/4\right), & x \rightarrow -\infty. \end{cases} \quad (\text{B.72a})$$

$$Bi(x) \sim \begin{cases} \frac{1}{\sqrt{\pi}} x^{-1/4} \exp\left(2x^{3/2}/3\right), & x \rightarrow +\infty, \\ \frac{1}{\sqrt{\pi}} |x|^{-1/4} \cos\left(|x|^{3/2}/3 + \pi/4\right), & x \rightarrow -\infty. \end{cases} \quad (\text{B.72b})$$

Graphical representations for real variable ($z = x \in \mathbb{R}$).

We present the plots of the Airy functions with their derivatives on the real line in Figs B.5 and B.6.

As expected from their relations with the Bessel functions, see Eqs. (B.69a) and (B.69b), and from their asymptotic representations, see Eqs (B.72a), (B.72b), we note from the plots that for $x > 0$ the functions $Ai(x)$, $Bi(x)$ are monotonic ($Ai(x)$ is exponentially decreasing, $Bi(x)$ is exponentially increasing), whereas for $x < 0$ both of them are oscillating with a slowly diminishing period and an amplitude decaying as $|x|^{-1/4}$. These changes in behaviour along the real line are the most noteworthy characteristics of the Airy functions.

For a survey on the applications of the Airy functions in physics we refer the interested reader to [Vallé and Soares (2004)].

- **p. 204, App. D.** Eq. (D.5a) would read:

$$\mathcal{E}_\nu(z) = \int_1^\infty \frac{e^{-zu}}{u^\nu} du, \quad \nu \in \mathbb{R}. \quad (D.5a)$$

- **p. 207, App. D.** Eq. (D.18b) would read:

$$\mathcal{L}\{f_2(t); s\} = \frac{1}{s} \log \left(\frac{1}{s} + 1 \right), \quad \operatorname{Re} s > 0, \quad (D.18b)$$

- **p. 208, App. D.** Please read: After the previous proofs, the proof of (D.19b) is trivial.

- **p. 223, App. E.** In the RHS of Eq. (E.45) β must be replaced by β^n , namely:

$$\mathcal{E}_\alpha(\beta, t) := t^\alpha \sum_{n=0}^\infty \frac{\beta^n t^{n(\alpha+1)}}{\Gamma[(n+1)(\alpha+1)]}, \quad t \geq 0. \quad (E.45)$$

- **p. 226, App. E.** In the LHS of Eq. (E.66) γ must be replaced by $-\gamma$, namely:

$$(1+z)^{-\gamma} = \sum_{n=0}^\infty \frac{\Gamma(1-\gamma)}{\Gamma(1-\gamma-n)n!} z^n = \sum_{n=0}^\infty (-1)^n \frac{\Gamma(\gamma+n)}{\Gamma(\gamma)n!} z^n. \quad (E.66)$$

- **p. 226, App. E.** Eq. (E.68) would read:

$$e_{\alpha,\beta}^\gamma(t; \lambda) := t^{\beta-1} E_{\alpha,\beta}^\gamma(-\lambda t^\alpha) \text{ CM iff } \begin{cases} 0 < \alpha, \beta, \gamma \leq 1 \\ \alpha \gamma \leq \beta. \end{cases} \quad (E.68)$$

- **p. 238, App. F.** In line 6 read:

$$\frac{1}{\Gamma(\zeta)} = \frac{1}{2\pi i} \int_{Ha} e^u u^{-\zeta} du, \quad \zeta \in \mathbb{C},$$

- **p. 249, App. F.** The sentence in the middle page would read: For the series approach, let us expand the Laplace transform in series of positive powers of s and formally invert term by term.

- **p. 251, App. F.** The sentence on the absolute moments of order δ would read: The *absolute moments* of order $\delta > -1$ of the Wright M -function in \mathbb{R}^+ are finite and turn out to be

- **p. 252, App. F.** In the first line of Eq. (F.34) insert in the Fourier integral $\exp i\kappa x$, namely:

$$\begin{aligned}\mathcal{F}\left[\frac{1}{2}M_\nu(|x|)\right] &:= \frac{1}{2}\int_{-\infty}^{+\infty} e^{i\kappa x} M_\nu(|x|) dx \\ &= \int_0^\infty \cos(\kappa x) M_\nu(x) dx = E_{2\nu}(-\kappa^2).\end{aligned}\tag{F.34}$$

- **p. 253, App. F.** Eq. (F.36) would read:

$$L_\alpha^\theta(x, t) \div \exp\left[-t\psi_\alpha^\theta(\kappa)\right] \Longleftrightarrow L_\alpha^\theta(x, t) = t^{-1/\alpha} L_\alpha^\theta(x/t^{1/\alpha}),\tag{F.36}$$

so the exponent α must be replaced by $1/\alpha$.

- **p. 254, App. F.** The Equation without number after (F.37) would read:

$$\left[L_\alpha^0(x, t/n)\right]^{*n} := L_\alpha^0(x, t/n) * L_\alpha^0(x, t/n) * \dots * L_\alpha^0(x, t/n)$$

- **p. 259, App. F.** The last reference would read: [Mainardi *et al.* (2010)]

- **p. 262, Bibliography.** In the references Agrarwal (2000), (2001, (2002), (2003) you would read: Agrawal

- **p. 268, Bibliography.** In the reference to Becker read: (1925) and in the reference to Becker and Doring read *Ferromagnetismus*.

- **p. 272, Bibliography.** In the reference after Butzer, P.L., Kilbas, A.A., and Trujillo, J.J. (2003), which appears without authors, you would add: Butzer, P.L. and Westphal, U. (1975).

- **p. 273, Bibliography.** The reference to Cafagna and Grassi (2009) would be updated as follows:

Cafagna, D. and Grassi, G. (2009). Fractional-order chaos: a novel four-wing attractor in coupled Lorenz systems, *Int. Journal of Bifurcation and Chaos*, **19** No 10, 3329–3338. .

- **p. 279, Bibliography.** The reference Debnath, L. (2003b) would read:

Debnath, L. (2003b). Recent developments in fractional calculus and its applications to science and engineering, *Internat. Jour. Math. and Math. Sci.* **54**, 3413–3442.

- **p. 292, Bibliography.** Before Gorenflo, R. and Rutman, R. (1994) add the reference:

Gorenflo, R., Mainardi, F. and Srivastava, H.M. (1998). Special functions in fractional relaxation-oscillation and fractional diffusion-wave phenomena, in Bainov, D. (Editor), *Proceedings VIII International Colloquium on Differential Equations, Plovdiv 1997*, VSP (International Science Publishers), Utrecht, pp. 195–202.

- **p. 303, Bibliography.** The two references to Kiryakova would be updated as follows:

Kiryakova, V. (2010a). The special functions of fractional calculus as generalized fractional calculus operators of some basic functions, *Computers and Mathematics with Applications*, **59** No 3, 1128–1141.

Kiryakova, V. (2010b). The multi-index Mittag-Leffler functions as important class of special functions of fractional calculus, *Computers and Mathematics with Applications*, **59** No 5, 1885–1895.

- **p. 311, Bibliography.** This reference is updated:

Mainardi, F., Mura, A. and Pagnini, G. (2010). The M -Wright function in time-fractional diffusion processes: a tutorial survey, *Int. Journal of Differential Equations* Vol. 2010, Article ID 104505, 29 pages. [E-print <http://arxiv.org/abs/1004.2950>]

- **p. 324, Bibliography.** In two references to Rossikhin the year was missed. They would read:

Rossikhin, Yu.A. (1970). *Dynamic problems of linear viscoelasticity connected with the investigation of retardation and relaxation spectra*, PhD Dissertation, Voronezh Polytechnic Institute, Voronezh. [in Russian]

Rossikhin, Yu.A. (2010). Reflections on two parallel ways in the progress of

fractional calculus in mechanics of solids, *Appl. Mech. Review* **63**, 010701/1–12.

- **p. 325, Bibliography.** The references to Rossikhin et al. (2010) would be updated:

Rossikhin, Yu.A., Shitikova, M.V. and Shcheglova, T.A. (2010). Analysis of free vibrations of a viscoelastic oscillator via the models involving several fractional parameters and relaxation/retardation times, *Computers and Mathematics with Applications* **59**, 1727–1744.

- **p. 346, Index.** Riemann-Liouville fractional integral, 2,230 in only one line.