

Fractional-order integral and derivative controller for temperature profile tracking

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Abstract. This paper establishes a new strategy to tune a fractional order integral and derivative (ID) controller satisfying gain and phase margins based on Bode's ideal transfer function as a reference model, for a temperature profile tracking. A systematic analysis resulting in a non-linear equation relating user-defined gain and phase margins to the fractional order controller is derived. The closed-loop system designed has a feature of robustness to gain variations with step responses exhibiting a nearly iso-damping property. This paper aims to apply the analytical tuning procedure to control the heat flow systems at selected points in Quanser experimental platform. Thus, the main purpose of this paper is to examine performances of two different fractional order controllers in temperature profile tracking. From experimental comparisons with the traditional PI/PID controller based on Ziegler–Nichols' tuning method, it will be shown that the proposed methodologies are specifically beneficial in controlling temperature in time-delay heat flow systems.

Keywords. Temperature control; fractional-order integral derivative control; heat flow.

1. Introduction

This paper also briefly reviews fractional M_s constrained integral gain optimization (FMIGO) (Varsha Bhambhani *et al* 2008), which is a proportional and integral (PI)-type control scheme, and then applies this method to temperature profile tracking. Temperature control arises in many engineering fields. For example, in cryogenic applications John *et al* (1985), the precision spatial temperature control via spatial heating and spatial temperature sensing is important, and in process industry (Tsai & Lu 1998), the most common control task is to achieve

the precise temperature profile. There are basically three types of temperature control tasks: temperature set point regulation, temperature profile tracking, and temperature uniformity control. For temperature set-point control, single loop commercial off-the-shelf temperature controllers can usually be used. Temperature profile tracking is to raise the temperature according to a prescribed temperature time history, which is required for applications such as precision heat treatment for material, batch chemical reactors, etc. The temperature uniformity control is to achieve uniformly spatially distributed temperature profile. The objective of this paper is to design a controller for temperature profile tracking at the spatially distributed places; thus, to regulate temperatures at given sensor locations to ensure that the temperatures be close to the desired values.

For temperature control, usually, On/Off control, proportional control, and traditional PID controller have been widely used (see <http://www.omega.com/prodinfo/temperature/controllers.html>). In this paper, as a new control scheme for temperature regulation, we suggest using fractional-order ID controller for a more accurate temperature profile tracking of the spatially distributed heat flow. An actual experimental task is conducted using the heat flow equipment (HFE) of QUANSER (www.quanser.com). This paper consists of the following sections. Section 2 briefly summarizes the role of integer order PID controller in temperature control and section 3 lists basic definitions in fractional-order calculus and merits of fractional-order controller. This is followed by in depth description and derivation of proposed analytical tuning method in section 4. Then, section 5 outlines detailed descriptions of the HFE platform, system analysis, and design of a fractional controller based on the analytical tuning approach. In section 6 an extensive comparison of existing integer order solutions and fractional order solutions is presented. Finally, section 7 concludes this paper with some remarks on the achieved results and ideas for future work.

2. Temperature control

In temperature control, it is difficult to find the response time constant and hence on/off control scheme is usually used so as to regulate the output temperature within a dead band. However, in On/Off control, the output will be oscillatory around a set-point; so an accurate temperature profile is not achieved. For a more accurate temperature regulation, closed-loop feedback control schemes are required. Most popular control scheme is PID control, because it does not require the plant model and practically it is easily implementable. In traditional PID control, the proportional term, integral term, and derivative term have different effects on the heat flow, temperature, and speed. For temperature control, it is usually recommended to use full PID control, but with accurately-tuned-control gains. As an example of successful industrial applications, see <http://www.w-dhave.inet.co.th/index/>. To see various PID tuning methods for temperature control, refer to http://www.lakeshore.com/pdf_files/Appendices/LSTC_appendixF_1.pdf and “<http://www.omega.com/temperature/Z/pdf/z115-117.pdf>”. For a reliable temperature control, however there are some basic environmental requirements. The heater should apply enough power and temperature sensors should be spatially distributed in appropriate places. Tuning the controller means that we select the proportional, integral, and derivative gains with a particular purpose. In fact, tuning PID gains for the temperature control requires some physical interpretation about the system. It is necessary to understand the effects of the proportional, integral, and derivative terms to the system. In temperature control, individual PID gain has the following characteristics:

2.1 Proportional gain

It requires more power proportional to the error between sensor temperature and the desired trajectory profile. The proportional gain is used for On/Off control. That is, when the output is within the proportional band, the power is off; but when it is out of the dead band, the power is on. If the temperature is below a set point, the output will be on longer; if the temperature is too high, the output will be off longer (<http://www.omega.com/prodinfo/temperaturecontrollers.html>).

2.2 Integral gain

It provides a control signal that is proportional to the accumulated error. So, this integral term is for slow mode reaction and forces the steady-state error to zero for a step response. In temperature control, it adjusts the temperature to set point after stabilization.

2.3 Derivative gain

It provides the control force proportional to the rate of change of the output error. So, this derivative term is for fast mode reaction and yields large signal with the high-frequency control errors and with the rise or fall of system temperature.

From a literature survey, it is shown that there have been numerous applications of PID controller or fuzzy/neural network-based PID controller for the temperature control of various engineering objects. As some examples, Peter Galan showed that a fuzzy logic for enhancing PID controller is necessary for the satisfactory temperature profile tracking of injection moulding processes (<http://www.manufacturing.net/ctl/article/CA408369.html>) and Dihac *et al* (1992) used PID controller for a rapid thermal processor control. Lin *et al* (1999) proposed a neural fuzzy inference network for the temperature control of a water bath system and compared the performance with the PID control. Juang and Chen (2003) proposed TSK-type recurrent neural fuzzy network based on the direct inverse control configuration, which does not require a priori knowledge of the plant order and Ramos *et al* (2005) used PID controller to control the temperature of the bath. However, there has been no trial of using fractional-order PID controller for the temperature control of spatially distributed systems. In the next section, we briefly summarize fractional-order $I^\alpha D^\beta$ control and its benefits in temperature control.

3. Fractional-order integral and derivative control

After Newton & Leibniz discovered calculus in the 17th century, fractional-order calculus has been studied as an alternative calculus in mathematics (Debnatho 2004). As claimed in (Chen 2004), fractional order calculus will play an important role in mechatronic and biological systems. Fractional order dynamic system and controls are relatively new research areas in control engineering. From early 90's, there has been steadily research in these areas as shown in (Lurie 1994, Podlubny 1999, Oustaloup *et al* 1995, Oustaloup *et al* 1996, Raynaud & Zergainoh 2000, Vinagre & Chen 2002, Machado 2002, Ortigueira & Machado 2003, Lazarevic & Debeljković 2005) even though some pioneering works can be traced back to (Manabe 1960, Manabe 1961, Oustaloup 1981, Axtell 1990). Traditional PID control method is a most popular control approach where integrator and derivative are integer order. Recently, in fractional order calculus community, a trend of using non-integer integrator or non-integer derivative for the accurate profile tracking in controlled-output has appeared, which is so-called fractional-order PID control. In the following subsections, we briefly review this fractional-order PID control (Ahn *et al* 2008).

3.1 Fractional-order calculus

In this paper, fractional integral is defined as:

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f(\tau) d\tau, \quad t > 0, \quad \alpha \in \mathbf{R}^+ \quad (1)$$

and for the fractional derivative, Caputo derivative is used, which is defined as:

$$D^\alpha f(t) = \frac{d^\alpha f(t)}{dt^\alpha} = \frac{1}{\Gamma(\alpha - n)} \int_0^t \frac{f^{(n)}(\tau)}{(t - \tau)^{\alpha+1-n}} d\tau, \quad (n - 1) < \alpha \leq n \quad (2)$$

where Euler's Gamma function is given as

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt, \quad x > 0 \quad (3)$$

with the special case when $x = n$:

$$\Gamma(n) = (n - 1)(n - 2) \cdots (2)(1) = (n - 1)!$$

Now, considering all the initial conditions to be zero, which is a typical assumption in fractional order derivative and fractional integral (Wen *et al* 2008), and using the Laplace transformation, we have $s^{-\mu} F(s) = s^{-\mu} \mathcal{L}\{f(t)\} = \mathcal{L}\{I^\alpha f(t)\}$, $\mu = \alpha$, and $s^\lambda F(s) = s^\lambda \mathcal{L}\{f(t)\} = \mathcal{L}\left\{\frac{d^\lambda f(t)}{dt^\lambda}\right\}$. Then, the fractional PID controller can be written as (Podlubny 1999):

$$C(s) = K_p + K_i s^{-\mu} + K_d s^\lambda. \quad (4)$$

If we take $\mu = \lambda = 1$, then we obtain the classic PID controller; with $\mu = 0$, it is the PD controller, and if $\lambda = 0$, it is the PI controller. Also we can consider fractional order $I^\alpha D^\beta$ controller which is an special case of fractional PID controller with $K_p = 0$.

3.2 Merit of using fractional-order controller

The idea of using fractional-order controllers for the dynamic system control is well-addressed in (Oustaloup *et al* 1995, Podlubny *et al* 1997, Oustaloup 1995). Generalized fractional-order PID controller was proposed by Podlubny *et al* (1996). Advantages of using fractional-order PID controller have been introduced in a number of publications. In (Monje *et al* 2005), it was claimed that, out of the following specifications:

- (i) No steady-state error,
- (ii) Phase margin and gain crossover frequency specifications,
- (iii) Gain margin and phase crossover frequency specifications,
- (iv) Robustness to variations in the gain of the plant,
- (v) Robustness to high frequency noise,
- (vi) Good output disturbance rejection, five specifications can be met by the closed-loop system, because the fractional-order PID controller (4) has five tuning parameters (i.e. $K_p, K_i, K_d, \lambda, \alpha$).

In (Vinagre 2002), some observations about fractional-order PID control were given. Particularly, by varying α from 1 to ∞ , it was shown that there could be a constant increment

in the slope of the magnitude curve varying between -20 db/dec and 0 db/dec and could be a constant delay in the phase response varying between $-\frac{\pi}{2}$ and 0 . Similarly, by changing λ from zero to 1, we can change the amount of phase lead and the slope of magnitude response. Vinagre *et al* (2000) provided frequency domain analysis to illustrate the superiority of the fractional-order PID controller applied to both the fractional dynamic system and the integer dynamic system. (Monje *et al* 2005) claimed that fractional-order PID controller is an adequate controller for the fractional-order mathematical models and it is less sensitive to shifts of parameters of a controlled-system and to variations of parameters of the controller. Particularly, in it was illustrated that the fractional-order PID controller is a suitable way for the control of the fractional system. Hwang *et al* (2002) proposed the fractional-order band-limited compensator, which has the similar response as the fractional-order PD controller but has a less sensitivity to the high frequency noise. Leu *et al* (2002) provided a differential evolution algorithm to search for optimal fractional-order PID parameters to meet the phase margin and the gain margin specifications. Thus, fractional-order controller could be beneficial for temperature control since there are often variations in parameters in heat flow systems, and most of desired specifications, which are not readily achieved simultaneously by traditional PID controller, can be ensured by fractional-order closed-loop dynamics.

4. $I^\alpha D^\beta$ gain tuning

If the system model is given, it is easy to design a controller to control the system. However, it is not easy to design an appropriate controller for a system whose transfer function is time varying and is highly dependent on the environmental change like temperature control. Hence, it is actually quite tough to find a transfer function of the output temperature change from the power input, because the temperature of the environment is time-dependent. Furthermore, as shown in Petras & Vinagre (2002), the transient unit response of the heat solid system could be fractional-order and as noted in (Aoki *et al* 2005) also, heat transfer coefficients may be fractional or non-integer order. Thus, the use of fractional-order differential model may approximate well the time-dependent behaviour of conductive systems of complex geometries with convective heat transfer. In fact, since thermal loads involve thermal diffusion, a half-order controller represents a better match to the physics of the plant to be controlled (Bohannon 2006). Therefore, for the temperature control, it is not a good idea to design a controller with fixed integer gains, and since the response of the temperature output from the power input could be fractional, the traditional integer-order controller may not provide an accurate temperature profile tracking. Thus, this paper proposes using the fractional-order $I^\alpha D^\beta$ controller for the accurate temperature profile tracking, which will be experimentally demonstrated by the Quanser HFE. However, note that it is very difficult to find an accurate model of transfer function of the output temperature change from the power input; thus, we use an approximated integer-order transfer function for heat flow, while a fractional-order $I^\alpha D^\beta$ controller is used to regulate the temperature profile. Let the plant be represented by a first order plus time delay (FOPTD) transfer function as:

$$G(s) = \frac{K}{Ts + 1} e^{-Ls}, \tag{5}$$

where K is the static gain or steady state gain of the system, L is the time-delay and T is the time-constant of the system. These can be computed from an open loop step response of the

system. The fractional controller is represented by $C(s)$ as:

$$C(s) = K_i \frac{Ts + 1}{s^\alpha} \quad (6)$$

which can be rewritten as

$$C(s) = K_i \left(\frac{1}{s^\alpha} + Ts^{1-\alpha} \right) \quad (7)$$

The realization of the above fractional order controller can be done by various methods. In this research, we use a rational approximation method in implementing s^α (Podlubny *et al* 2002). In our implementation, we use Carlson's iteration method. Given a fractional transfer function $H(s) = (G(s))^p$, where p is a fractional number, we define $p = 1/q$ and $m = q/2$ in each iteration. Starting from the initial value $H_0(s) = 1$, an approximated rational function is obtained in the form:

$$H_i(s) = H_{i-1}(s) \frac{(q-m)H_{i-1}(s)^2 + (q+m)G(s)}{(q+m)H_{i-1}(s)^2 + (q-m)G(s)}. \quad (8)$$

We use three iterations to approximate s^α . The above equation (7) reveals the fractional integral derivative nature of the controller. The open-loop transfer function is given by:

$$C(s)G(s) = K_i K \frac{e^{-Ls}}{s^\alpha}, \quad 1 < \alpha < 2. \quad (9)$$

The gain and phase responses of the open loop transfer function are calculated as:

$$|C(j\omega)G(j\omega)| = \frac{K_i K}{\omega^\alpha} \quad (10)$$

$$\phi = -L\omega - \alpha \frac{\pi}{2}. \quad (11)$$

Let us suppose that the user-defined gain margin A_m and phase margin ϕ_m , are given as:

$$|C(j\omega_p)G(j\omega_p)| = \frac{1}{A_m} \quad (12)$$

$$\phi_m = \pi + \angle C(j\omega_g)G(j\omega_g), \quad (13)$$

where ω_p and ω_g are the phase and gain crossover frequencies of the open-loop system.

On substitution, the gain relations are:

$$\frac{K K_i}{\omega_g^\alpha} = 1 \quad (14)$$

$$\frac{\omega_p^\alpha}{K K_i} = A_m \quad (15)$$

and the phase relations are:

$$\phi_m = \pi - L\omega_g - \alpha \frac{\pi}{2} \quad (16)$$

$$\pi = \alpha \frac{\pi}{2} + L\omega_p. \quad (17)$$

Therefore, we obtain:

$$A_m = \left(\frac{\omega_p}{\omega_g}\right)^\alpha \tag{18}$$

$$\frac{\omega_p}{\omega_g} = \frac{\pi - \alpha \frac{\pi}{2}}{\pi - \phi_m - \alpha \frac{\pi}{2}}, \tag{19}$$

which yields

$$A_m = \left(\frac{\pi - \alpha \frac{\pi}{2}}{\pi - \phi_m - \alpha \frac{\pi}{2}}\right)^\alpha. \tag{20}$$

Thus, it is shown that given gain and phase margins desired, the above equation (20) can be solved for fractional order α using numerical classical approach in MATLAB. However as shown in figure 1, given desired ϕ_m and A_m , there exists α in some restricted area. For example, if $\phi_m = 1.4$ and $A_m = 0.01$, then there does not exist a solution α . Thus, there is an area that ensures existence of solutions. Therefore, we need to select A_m and ϕ_m in an allowed area as shown in figure 1. Though this is a limitation of our new method, it reveals a better performance than existing methods as addressed in the next section. Once the value of α is obtained, the corresponding values of $(\omega_g, \omega_p, K_i, K_d)$ are computed as:

$$\omega_g = \frac{\pi - \phi_m - \alpha \frac{\pi}{2}}{L} \tag{21}$$

$$\omega_p = \frac{\pi - \alpha \frac{\pi}{2}}{L} \tag{22}$$

$$K_i = \frac{\omega_p^\alpha}{K A_m} = \frac{\omega_g^\alpha}{K} \tag{23}$$

$$K_d = T K_i \tag{24}$$

5. Experiment of HFE

Quanser HFE system (see figure 2) consists of a duct equipped with a heater and a blower at one end and three temperature sensors located along the duct. The power delivered to the heater is controlled using an analog signal. For the analog signal generation and measurement, we use Quanser analog input/output libraries via MATLAB/Simulink/RTW. The fan speed of HFE is controlled using an analog signal. Fan speed is measured using a tachometer and is an input signal. Figure 2 shows the set of test equipment: computer, software, AD/DA converter and QUANSER HFE. HFE system includes built-in power module, analog signals for fan speed and power; an onboard tachometer to design speed control; and fast settling platinum temperature transducers (3 sensors along the duct) to measure the temperature.

This section summarizes the results of open-loop step response analysis. The fan (VF) and heater (VQ) voltages are applied in the range [0–5 Volts] and the temperature readings (degrees) at three sensor positions are recorded. The heating operation takes effect only when $VQ > 3VDC$. Figure 3, 4 and 5 are the open loop step response of sensor 1, 2 and 3 at varying

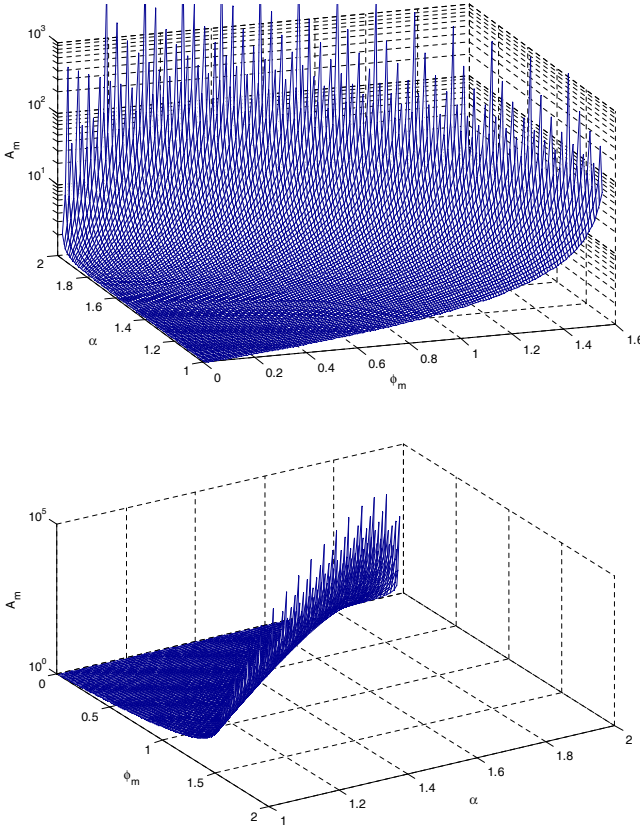


Figure 1. Relation between A_m , ϕ_m , and α in (20).

heater (VQ) voltages but constant fan (VF) voltage of 5 VDC. The open loop response shows an initial delay, indicating that it takes a finite time for the sensors to detect the change in the temperature.

A FOPTD model at a desired sensor is given by:

$$\frac{T_n}{V_q} = \frac{K}{Ts + 1} e^{-Ls}, \tag{25}$$



Figure 2. The set of equipment for heat flow test.

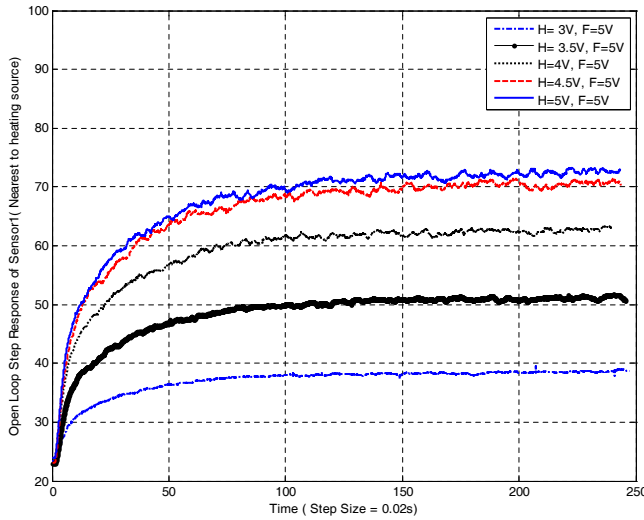


Figure 3. Sensor 1 (Fan command with constant voltage = 5V, Heat command with varying voltage).

where K is the open loop gain given by $(\text{Max}-\text{Min})/VQ$ in (C/V) and Min is the temperature inside the tube before the experiment begins or the room temperature, and Max is the maximum temperature reached after 60 seconds. This is, strictly speaking, not the steady state value as the temperature is increasing. However, the rate of increase considerably slows down after 20 seconds. We now define the following parameters:

- L : The delay in the response in seconds.
- T : The time constant is given by the time it takes to reach 63% of the change in temperature or the time it takes for the response to reach $\text{Min} + 0.63 \times (\text{Max} - \text{Min})$.
- τ : The relative dead time of the response given by $L/(L + T)$.

Experiments were conducted to study the open loop step response of three sensors first at constant fan voltage (5V) and varying heater voltage ranging between 3–5V. The above

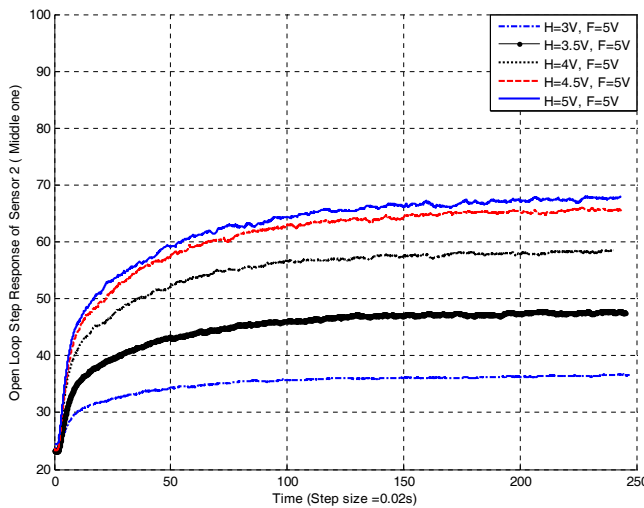


Figure 4. Sensor 2 (Fan command with constant voltage = 5V, Heat command with varying voltage).

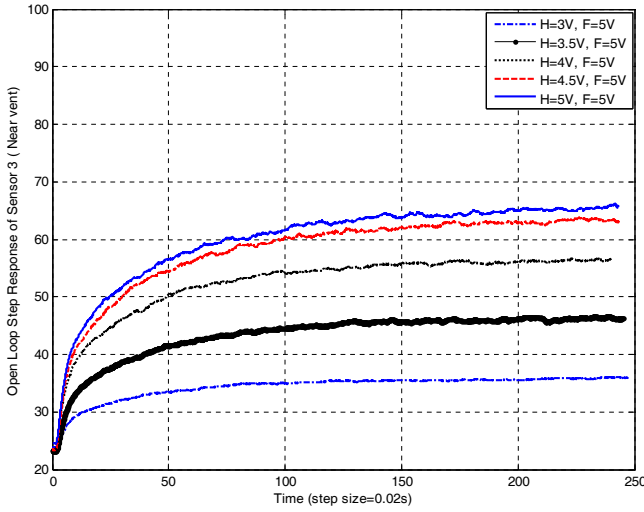


Figure 5. Sensor 3 (Fan command with constant voltage = 5V, Heat command with varying voltage).

experiment was repeated for constant heater voltage (5V) and varying fan voltage ranging between 3 and 5V. The step response values obtained are listed in figure 6. First, the heating-effect is analyzed by keeping the fan speed constant. It is seen that as the distance of the sensors from heater increases, the corresponding gain values of the first order transfer function models of the sensors decrease and delay time increase; i.e. $K_1 > K_2 > K_3$ and $D_1 < D_2 < D_3$ where K_1, K_2 and K_3 are the static gain values and D_1, D_2 and D_3 are the delay values of the first order transfer function models of sensor 1, sensor 2 and sensor 3. This is shown in figures 7 and 8. There is no trend in time constant of the three sensors with distance. However for every sensor, time constant increases as the heater voltage increases. This is shown in figure 9.

Cooling-effect is comparatively different from heating effect. This is shown in figures 10, 11 and 12. As shown in the plots, gain values decrease as the distance from the fan increases or fan voltage increases, i.e. $K_1 > K_2 > K_3$ whereas the delay values show an increase with distance from the blower; i.e. $D_1 < D_2 < D_3$. Also there is an increase in time constant values of sensors with increase in distance from the fan, i.e. $T_1 < T_2 < T_3$. Here T_1, T_2 and T_3 are time constant of sensor 1, sensor 2 and sensor 3. Thus one can see that dynamics of HFE is really very complex and permits the need of a better controller than a simple integer

VF=5V									
VQ	Sensor T1			Sensor T2			Sensor T3		
	Kp	T	L	Kp	T	L	Kp	T	L
3	5.6888	11.36	.36	4.7936	6.7	.64	4.582	7.34	.86
3.5	8.2525	28.32	.28	7.0526	22.64	.54	6.6968	25.46	.78
4	9.87	35.22	.18	8.5393	32.88	.5	8.0449	38.32	.64
4.5	10.7428	47.46	.2	9.4678	48.66	.4	9.0012	52.38	.7
5	9.8883	46.6	.18	8.7164	45.98	.54	8.3452	52.38	.66
VQ=5V									
VF	Sensor T1			Sensor T2			Sensor T3		
	Kp	T	L	Kp	T	L	Kp	T	L
3	11.2507	62.72	.22	10.1325	65.74	.5	9.6783	67.34	.72
3.5	10.5719	50.92	.24	9.3853	51.86	.54	8.858	53.72	.74
4	10.4352	49.52	.14	9.1851	49.7	.46	8.814	56.36	.7
4.5	10.0348	44.98	.38	8.858	49.1	.78	8.2915	50.26	.98
5	9.8151	44.84	.22	8.6529	48.72	.44	8.1353	48.68	.62

Figure 6. Step response data from open loop heat flow experiment.

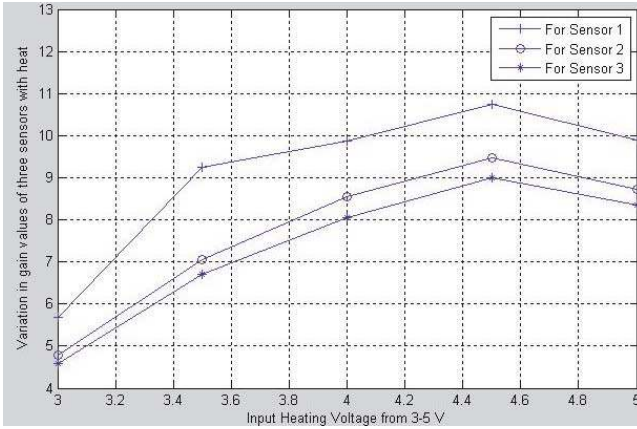


Figure 7. Heating Effect: Variation of static gain of sensors with distance.

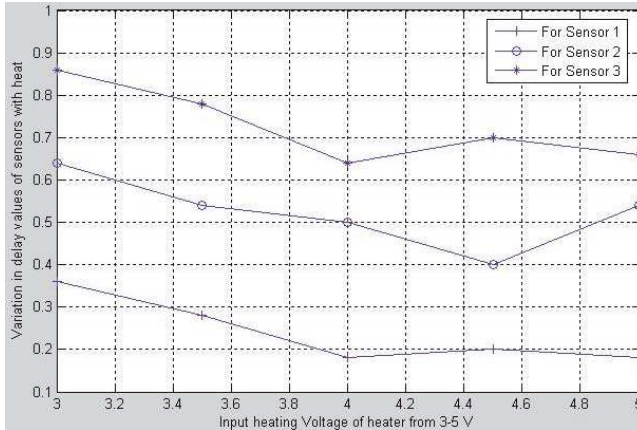


Figure 8. Heating Effect: Variation of delay of sensors with distance.

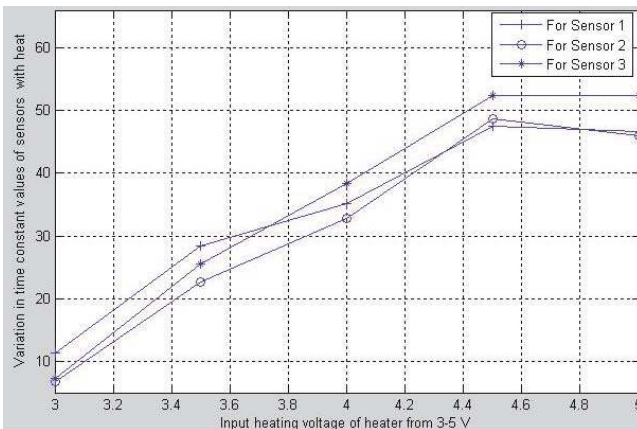


Figure 9. Heating Effect: Time constant of sensors with distance.

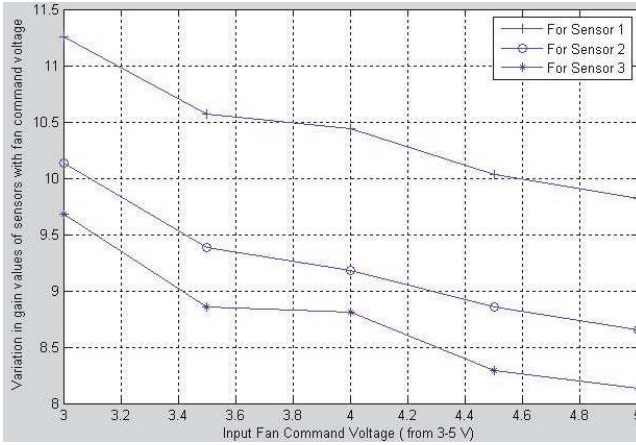


Figure 10. Cooling Effect: Variation of static gain of sensors with distance.

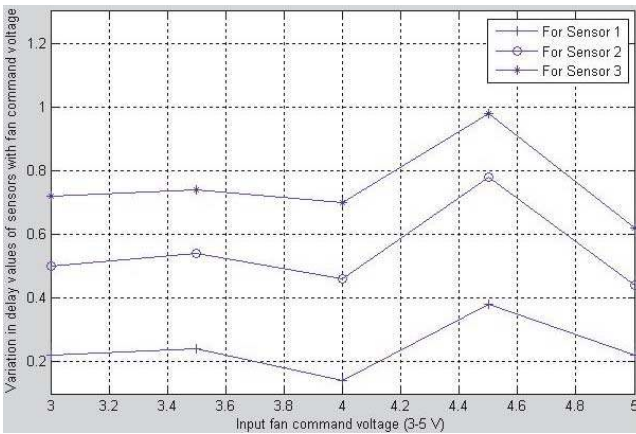


Figure 11. Cooling Effect: Variation of delay of sensors with distance.

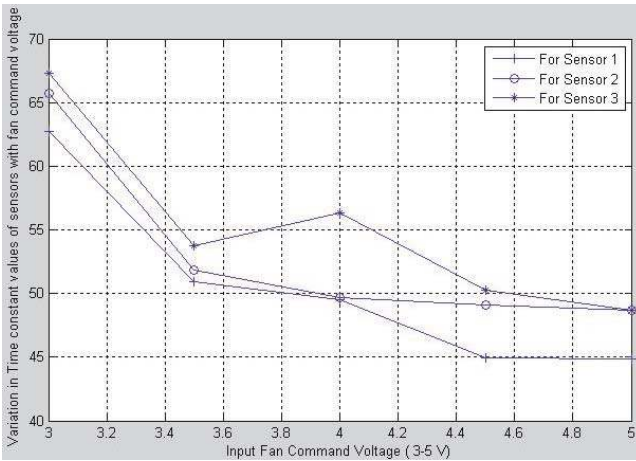


Figure 12. Cooling Effect: Variation of time constant of sensors with distance.

order PI/PID controller. Next section aims at justifying the efficiency of the new method over already existing integer and fractional order controllers.

6. Experimental tests

In this section, an extensive comparison of existing integer order solutions and fractional order solutions is made. For integer order case both PI and PID controllers are considered based on Ziegler–Nichols’ tuning method. The Ziegler–Nichols’ tuning formulae for a PI controller is given as:

$$K_p = \frac{0.9T}{KL} \tag{26}$$

$$K_i = \frac{K_p}{3L} \tag{27}$$

For a PID controller the gain values are:

$$K_p = \frac{1.2T}{KL} \tag{28}$$

$$K_i = \frac{K_p}{2L} \tag{29}$$

$$K_d = K_p \frac{L}{2}. \tag{30}$$

The other controller compared in this paper is a fractional PI controller based on M_s constrained FO-PI controller (FMIGO) tuning method (Varsha Bhambhani *et al* 2008) outlined as:

$$C(s) = K_p^* + \frac{K_i^*}{s^\alpha}, \tag{31}$$

where K_p^* and K_i^* and α values are given by:

$$K_p^* = \frac{0.2978}{K(\tau + .000307)} \tag{32}$$

$$K_i^* = T \frac{.8578}{\tau^2 - 3.402\tau + 2.405} \tag{33}$$

$$\alpha = \begin{cases} 0.7, & \text{if } \tau < 0.1 \\ 0.9, & \text{if } 0.1 \leq \tau < 0.4 \\ 1.0, & \text{if } 0.4 \leq \tau < 0.6 \\ 1.1, & \text{if } \tau \geq 0.6 \end{cases} \tag{34}$$

As explained in (Bhaskaran *et al* 2007), FMIGO is a tuning method for fractional-order controller on the base of M_s constrained integral optimization (MIGO), which was developed in (Astrom *et al* 1998; Astrom *et al* 2002). The motivation of MIGO is to improve upon the

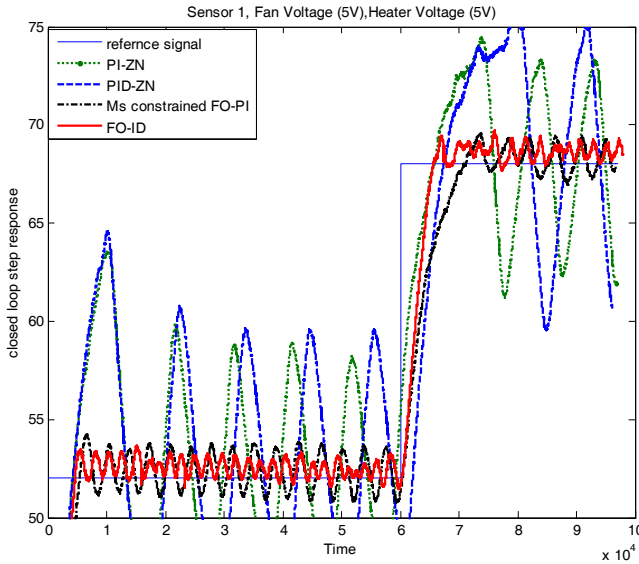


Figure 13. Controller performances for sensor 1 in real time.

Ziegler–Nichols’ tuning rules to overcome two major drawbacks: (i) very little process information was taken into account as the rules were based on the two parameter characterization of the system dynamics based on step response data and (ii) the quarter amplitude damping design method exhibits very poor robustness. To overcome these drawbacks, in (Astrom *et al* 1998; Astrom *et al* 2002), a new criterion for developing tuning method for the PI controllers based on robust loop shaping is employed. The FMIGO is a generalized version of MIGO method aiming at obtaining the gains of the PI^α at any given fractional order α (Varsha Bhambhani *et al* 2008).

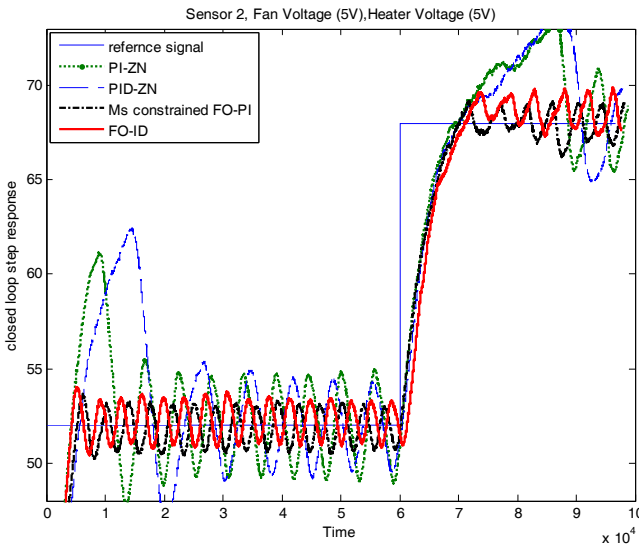


Figure 14. Controller performances for sensor 2 in real time.

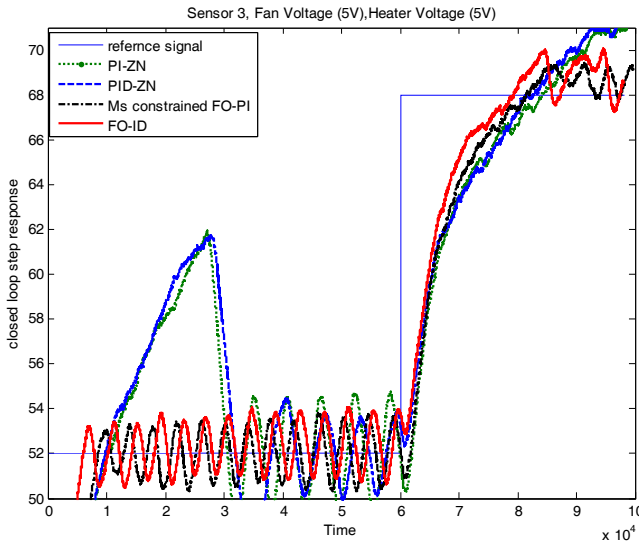


Figure 15. Controller performances for sensor 3 in real time.

The controller performances for all the three sensors are compared in real time as shown in figures 13, 14 and 15. As is clear from the figures, the fractional controllers outperform simple integer order PI/PID controller. The new controller (fractional ID controller) is tuned better in comparison to M_s constrained FO-PI controller (FMIGO) for sensor 1 whereas for sensor 2 and sensor 3 both the fractional order controllers perform more or less the same. The same results in MATLAB Simulink environment are shown in figure 16, which is the controller performances for sensor1. The simulation and actual lab results showed some mismatches indicating that the linear model was unable to model the system nonlinearities. The

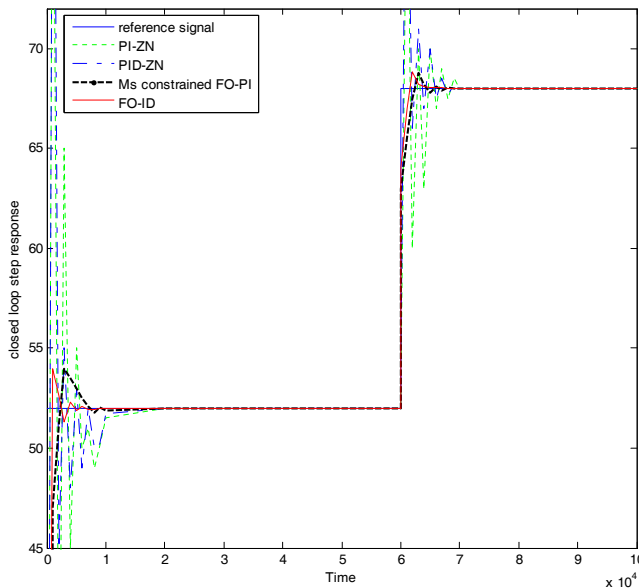


Figure 16. Controller performances for sensor1 in Simulink.

discrepancy between simulation and experimental results may be due to employing integer-order model of the heat flow (5) for tuning fractional-order controller (7). It is noticeable that fractional order controllers could be regarded as interpolations of conventional PI, PD, and PID controllers. Fuzzy and neural network controllers may have a similar effect as fractional order controllers. However, fuzzy PID controllers or neural network PID controllers try to find optimal control gains using input and output mappings (Malki *et al* 1997, Tang *et al* 2001, Wang *et al* 2007). Thus, though fuzzy PID and neural network PID could be considered as optimal tuning methods of traditional integer order controllers when the control structure is fixed, they cannot represent fractional order effects.

7. Conclusions

The objective of this paper is to introduce a novel analytical tuning method for a fractional integral derivative ($I^\alpha D^\beta$) controller (FO-ID) and apply the results to analyse the heat flow in HFE module. The performance of new FO-ID controller was compared to the integer-order PI/PID controller and FO-PI controller. Our observation is that since the HFE systems have very small relative dead time, a full integrator is not necessary. The FO-PI controller gives a strong competition to FO-ID controller tuned by new analytical method for some cases. For FO-ID method introduced, the resulting closed loop system has the desirable feature of being robust to gain variations with step responses exhibiting a nearly iso-damping property. Extensive simulation results are included to illustrate the simple yet practical nature of the developed new tuning rules. However as admitted in the previous section, there is a discrepancy between simulation and experimental results due to inaccurate modelling of the temperature system. In our future research efforts, we would like to consider a model uncertainty problem by employing a robust fractional order controller, which is to be researched further in this field.

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