Fractional Order Systems in Industrial Automation—A Survey

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***Abstract*—This paper describes an emerging tool for industry: fractional order systems. Conventional understanding of the notion of derivative and integral uses integer orders and our sense is mature in their physical interpretations. Derivatives or integrals of fractional orders are generalizations of the concept containing the classical cases and solutions based on fractional order operators utilize the full flexibility offered by the mathematical definitions. The interest of the industry to fractional order systems lie in the fact that complicated modules can be simplified significantly and practical applications can be diverse. This paper describes linear and nonlinear cases with necessary stability and performance considerations for the benefit of a practicing engineer exploiting informatics in industry.**

***Index Terms*—Fractional order control, industrial automation.**

I. INTRODUCTION

THE BIRTH OF fractional calculus goes back to 1695, with a letter from Leibniz to L'Hôpital, asking the meaning of derivative of order 1/2. For a few centuries, the developments in the calculus of fractional mathematics have remained in theory yet with the advances in the high-speed computing technology, the operators of fractional domain has become visible in applications covering a wide range from all disciplines.

The field of industry is a special application domain for fractional order systems as the demanding performance expectations with uncertainties make it a challenge to come up with models that are useful, or control modules that can alleviate the difficulties in various forms. The blending of the overall performance notions with wireless technologies, security issues, networked controls, and communications make the physics of the problem a distributed one and tackling of which needs versatile tools that have the highest possible level of flexibility, [1]–[4]. A common feature in all these resources is the fact that the differentiation and integration, or shortly differintegration, of quantities are performed in integer order, i.e., $\mathbf{D} := \mathrm{d}/\mathrm{d}t$ for the differentiation with respect to t and $\mathbf{I} := \mathbf{D}^{-1}$ for integration over t in the traditional sense. A significantly different branch of mathematics, called fractional calculus, suggests operators $\mathbf{D}^{\beta} := \mathrm{d}^{\beta}/\mathrm{d}t^{\beta}$ with $\beta \in \Re$, [5], and it becomes possible to write $\mathbf{D}f = \mathbf{D}^{1/2}\mathbf{D}^{1/2}f$. The operator $\mathbf{D}$ with the positive values of β describes differentiation, while negative values indicate integration. Expectedly, Laplace and Fourier transforms in fractional calculus are available to exploit in the design process, involved with s^{β} or $(jw)^{\beta}$ generic terms, respectively. Indeed, a lossy transmission line, heat conduction process, neutron flux dynamics in a nuclear reaction or telegraph equation are all governed by fractional differential equations and the motivation of a systems and control engineer is to exploit these tools in a way to obtain better performance in industry.

As we know from courses on systems theory, a Bode plot for a real and rational transfer function has asymptotes of 20 dB/dec for every real zero and −20 dB/dec for every real pole. The fractional order realizations remove this limitation and high-order systems can be represented by fractional operators more compactly. For example, dynamics governed by an integer order transfer function $\mathcal{H}(s) = (5.406s^4 + 177.6s^3 + 209.6s^2 + 9197s + 0.0145)/(s^5 + 88.12s^4 + 279.2s^3 + 33.3s^2 + 1.927s + 0.0002276)$ can be approximated by a fractional order integrator $\mathcal{H}_f(s) = 1/s^{0.7}$ over the frequency range [0.01, 100] rad/s. This enables to obtain an arbitrary frequency response by a number of noninteger order components scheduled appropriately over the relevant frequency spectrum. Clearly, such a design freedom can be useful not only in the field of control systems but also in all areas involving digital signal processing, [6]–[9].

Fractional order control offers more degrees of freedom to designers to meet a predefined set of performance criteria. Order selection for differentiation and integration in a proportional, integral plus derivative (PID) controller is an example to this. Many successful outcomes have appeared in the literature on linear control applications. Recently, there has been a dramatic increase in the number of research outcomes regarding the theory and applications of fractional order systems and control, [10]–[12]. PID controllers are considered in [13] and [14], stability considerations are elaborated in [15]–[18], Kalman filtering is studied in [19], state-space models and approaches are handled in [12], [20], and [21], root locus technique is discussed in [22], applications involved with the partial differential equations are focused in [23] and [24], discrete time issues are tackled in [10]–[12] and [19], an introductory work considering SMC of a double integrator is elaborated in [25] and so on.

In this paper, after describing the industrial perspective and the needs, we first present the preliminaries of fractional order systems and control by defining the structural properties such as controllability and observability, and dynamic qualities such as stability. Next, we consider the PID control methods in noninteger order domain. Nonlinear cases are elaborated in the titles of sliding mode control in Section V, backstepping control in Section VI and adaptive control in Section VII. The links in

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between the logic, heuristics, experience, and intelligence are discussed in Section VIII and an application example is given before the concluding words.

II. An Industrial Perspective and the Needs

The industrial applications exploiting the expertise of systems and control engineering has a significantly wide spectrum covering the open-loop characterization of processes to develop high-performance feedback mechanisms, tuning of controllers via expert knowledge and computer tools, adaptive and self-tuning mechanisms taking care of the changes in the process dynamics, nonlinear, robust and optimal control policies to meet a predefined set of performance criteria, fault tolerant schemes, and safety critical applications. Some of the sectors benefiting from these are information technologies covering networking, security and industrial buses, sensor and transducer manufacturing processes, chemical process industries, robot manufacturing industries, energy industries processing all forms of resources and energy transmission industries, marine and automotive sectors, metal forming industries and so on.

The ultimate expectation in all such branches of industry is to maintain the production, while keeping the cost at minimum possible level. In essence, the production process is also subject to manufacture a product that can compete with its rivals and such a production process can only be developed via components that are versatile and that offer more than necessary degrees of freedom. The needs in this respect are to have modules that have low cost in terms of money or time, low-computational complexity, high manufacturability (i.e., no extraordinary requirements), high reliability, low-mean time between failures, adaptability with low cost for similar applications, understandability, and small distance from standard practice.

Looking at the sectors and their needs, in the sequel, we propose fractional order systems to those experimenting with real-time data to build systems that respond quickly and that offer more degrees of freedom to exploit.

III. Fundamentals of Fractional Order Systems and Control

The two popular definitions of fractional order differintegration are by Riemann–Liouville and Caputo. Though both of them produce the same results, Caputo's definition is more suitable for the control systems engineering.

Caputo's definition of the fractional order differentiation is given in (1), where $\beta \in \Re^+$ is the order of the differentiation. According to the definition in (1), let m be an integer and $m-1<\beta<m$ is satisfied. With such a value of m, βth order derivative of a function of time, say $\sigma(t)$ has the Laplace transform given in (2)

$$\mathbf{D}^\beta \sigma(t) = \frac{1}{\Gamma(m-\beta)} \int_0^t \frac{\mathbf{D}^m \sigma(\tau)}{(t-\tau)^{\beta+1-m}} \mathrm{d}\tau, \quad m-1 \le \beta < m \tag{1}$$

$$\int_0^\infty e^{-st}\mathbf{D}^\beta \sigma(t)\mathrm{d}t = s^\beta S(s) - \sum_{k=0}^{m-1} s^{\beta-k-1}\mathbf{D}^k \sigma(0) \tag{2}$$

where $\Gamma(\beta) = \int_0^t e^{-t}t^{\beta-1}\mathrm{d}t$ is the Gamma function and $S(s) := \int_0^\infty e^{-st}\sigma(t)\mathrm{d}t$.

From a control engineer's perspective, if a system is at rest initially, i.e., all initial conditions are zero, the operator $\mathbf{D}^\beta$ acting in time domain has a counterpart s^β in s-domain and the transfer function of a system described by a fractional order differential equation as given in (3) can be obtained as given by (4), where $a_k, b_k \in \Re$ and $\alpha_k, \beta_k \in \Re^+$

$$\begin{aligned}(a_n\mathbf{D}^{\alpha_n} + a_{n-1}\mathbf{D}^{\alpha_{n-1}} + \ldots + a_1\mathbf{D}^{\alpha_1} + a_0)y(t) \\ = (b_m\mathbf{D}^{\beta_m} + b_{m-1}\mathbf{D}^{\beta_{m-1}} + \ldots + b_1\mathbf{D}^{\beta_1} + b_0)u(t)\end{aligned} \tag{3}$$

$$\frac{Y(s)}{U(s)} = \frac{a_n s^{\alpha_n} + a_{n-1}s^{\alpha_{n-1}} + \ldots + a_1 s^{\alpha_1} + a_0}{b_m s^{\beta_m} + b_{m-1}s^{\beta_{m-1}} + \ldots + b_1 s^{\beta_1} + b_0}. \tag{4}$$

Similarly, one could define the affine nonlinear systems of fractional order in state space, as in (5)

$$\mathbf{D}^\beta \mathbf{x} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u \tag{5}$$

where u is the control input, $\mathbf{f}$ and $\mathbf{g} \neq 0$ are the vector functions of the system-state $\mathbf{x}$. When the system under interest is a linear one, as in (6) and (7)

$$\mathbf{D}^\beta \mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \quad \mathbf{x} \in \Re^n \tag{6}$$

$$y = \mathbf{C}\mathbf{x} + Du \tag{7}$$

the transfer function characterizing the relation between $Y(s)$ and $U(s)$, the Laplace transforms of the output and input respectively, is as given in (8). The solution of the homogeneous case $(u = 0)$ is obtained as given in (9), in which $E_\beta(\mathbf{A}t^\beta)$ is the Mittag–Leffler function defined as $E_\beta(\mathbf{A}t^\beta) := \sum_{k=0}^\infty ((\mathbf{A}t^\beta)^k / \Gamma(1+\beta k))$ and denoted by $\mathbf{\Phi}(t)$, [10]–[12]

$$H(s) = \mathbf{C}(s^\beta \mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D \tag{8}$$

$$\mathbf{x}(t) = E_\beta(\mathbf{A}t^\beta)\mathbf{x}(0) = \mathbf{\Phi}(t)\mathbf{x}(0). \tag{9}$$

The full solution of the fractional state equation in (6) and the output equation in (7) is as given by (10)

$$y(t) = \mathbf{C}\mathbf{\Phi}(t-t_0)\mathbf{x}(t_0) + \mathbf{C}\int_0^t \mathbf{\Phi}(t-\tau)\mathbf{B}u(\tau)\mathrm{d}\tau + Du(t). \tag{10}$$

Controllability and observability conditions are similar to the integer order case and these are given in (11) and (12), respectively

$$\mathbf{W}_c = (\mathbf{B} \quad \mathbf{AB} \quad \ldots \quad \mathbf{A}^{n-1}\mathbf{B}) \quad \mathrm{rank}(\mathbf{W}_c) = n \tag{11}$$

$$\mathbf{W}_o = \begin{pmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{n-1} \end{pmatrix}, \quad \mathrm{rank}(\mathbf{W}_o) = n. \tag{12}$$

Finally, in this section, it is useful to define the stability conditions for fractional order dynamic system representations. Denoting λ_i as an eigenvalue of the matrix $\mathbf{A}$, the system in (6) and (7) is said to be stable if the condition in (13) is satisfied by all eigenvalues of $\mathbf{A}$. For the transfer function representation in (8),

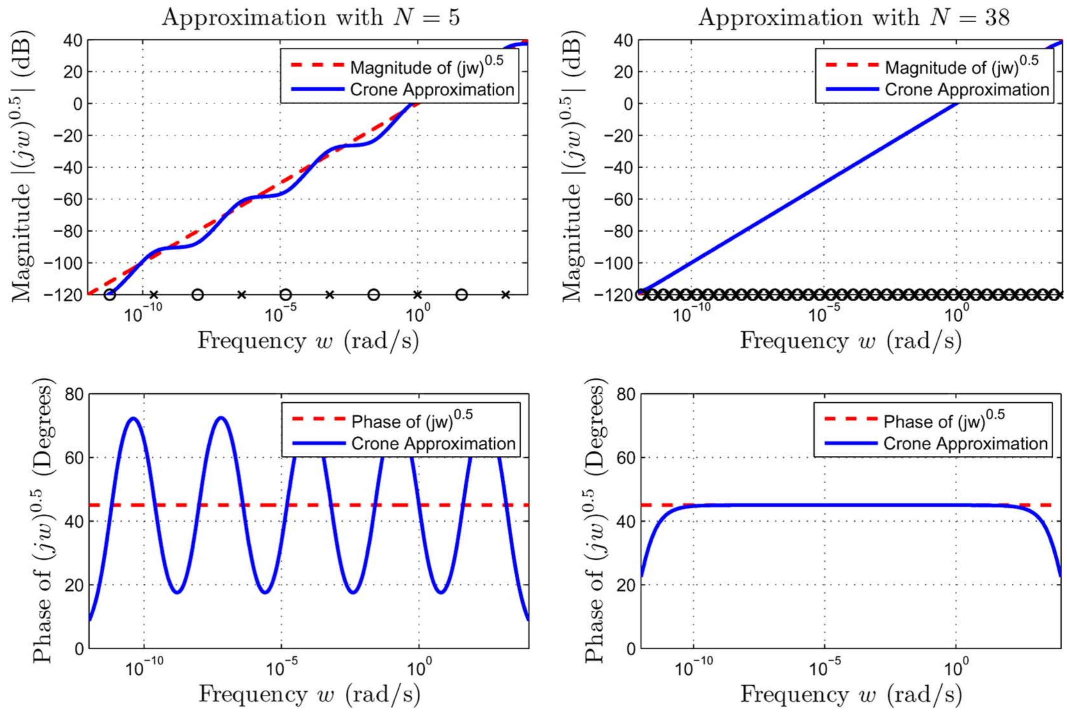

Fig. 1. Left subplots: Crone approximation to the operator $\mathbf{D}^{\beta}$, $\beta = 0.5$, $w_{\min} = 10^{-12}$ rad/s, $w_{\max} = 10^{4}$ rad/s, and $N = 5$. Right subplots: Crone approximation to the operator $\mathbf{D}^{\beta}$, $\beta = 0.5$, $w_{\min} = 10^{-12}$ rad/s, $w_{\max} = 10^{4}$ rad/s, and $N = 38$.

λ_is correspond to the poles and the same condition applies for stability

$$\arg(\lambda_i) > \beta\frac{\pi}{2}, \quad i = 1, 2, \ldots, n. \tag{13}$$

It is straightforward to see that in the integer order case ($\beta = 1$), the stability condition above describes the open left half s-plane for the stability. An in depth discussion on these issues can be found in [15], [17], and [20].

A fundamental issue with the fractional order systems is to realize the fractional differintegration operators in real time. Some results on this issue are reported in [8]. A frequently followed approach is to approximate these operators via integer order components as defined in (14)

$$\mathbf{D}^{\beta} := s^{\beta} \approx K\frac{\prod_{k=1}^{N} 1 + \frac{s}{w_{pk}}}{\prod_{k=1}^{N} 1 + \frac{s}{w_{zk}}}. \tag{14}$$

A widely used approximation is the Crone method approximating the s^{β} term as given above. Crone method adjusts the gain K such that when $w = 1$ rad/s the magnitude of the expression coincides to 0 dB level. Here, N is the order of the approximation and the Crone algorithm determines the pole and zero locations in such a way that the approximation is optimum over the chosen frequency band. In the left subplots of Fig. 1, the approximation order N is equal to 5 and in the right subplots, N is 38. The magnitude and phase plots are given and it is seen that as N increases a better fit is obtained at the cost of increasing the computational intensity. For both cases, the pole and zero locations prescribed by the Crone algorithm are marked along the frequency axis of the magnitude plots.

In the literature, there are other approximations postulating alternative algorithms to distribute the poles and zeros; Carlson, Matsuda, high/low-frequency continued fraction approximations are just to name a few. A detailed discussion is given in [26].

IV. PI$^{\lambda}$D$^{\mu}$ Control

The value of PID control scheme in the industrial practice is high and its role is critical in most cases. The interpretability and comprehensibility of the scheme makes it a natural choice when considered with the peripherals automatically tuning the parameters without external intervention. Fractional order version of the scheme is defined by the following transfer function:

$$C(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu}. \tag{15}$$

Clearly, for $\lambda = 1$ and $\mu = 1$, we obtain the standard integer order setting having three degrees of freedom (dof) only via the gains k_p, k_i, and k_d, yet the module in (15) has five parameters to determine, and through which we are able to force five independent specifications to meet. Assuming that this controller is placed in front of a $G(s)$ in a unity feedback loop, first specification can be on phase margin as it is tightly coupled with the robustness of the control system. The equations defining the phase margin are $20\log|C(w_{gc})G(w_{gc})| = 0$ dB and $\arg(C(w_{gc})G(w_{gc})) = -\pi + \varphi_{pm}$, where w_{gc} is the gain crossover frequency, and φ_{pm} stands for the phase margin. Secondly, forcing a flat magnitude response $|G(jw)C(jw)|$ around the gain crossover frequency can be another specification to be met by a properly set controller. Ensuring this can be achieved by equating the derivative $\mathrm{d}(\arg(C(jw)G(jw)))/\mathrm{d}w$ to zero when $w = w_{cg}$, [27]. Meeting such a constraint would make the closed-loop control system robust against variations in the gain of $G(s)$. As a third specification, a controller is supposed to introduce noise rejection property in the high frequencies, which can be achieved through setting a critical frequency, say w_h, beyond which the magnitude of the transfer function $T := CG/(1+CG)$, which corresponds to the complementary sensitivity function, is less than a preselected level. Then, we consider the capability of good output disturbance rejection that entails forcing an upper bound (M) on the magnitude of the sensitivity function below a predefined frequency (w_s), i.e., we have

$$20\log|S(jw)|_{w\le w_s} = 20\log\left|\frac{1}{1+C(jw)G(jw)}\right|_{w\le w_s} \le M \text{ dB}. \quad (16)$$

Finally, to obtain zero steady-state error, the controller $C(s)$ is designed so that it contains an integral component.

Clearly forcing such a set of specifications require searching for an appropriate set of parameters k_p, k_i, k_d, λ, and μ. Despite deriving the necessary set of equations from the set of constraints above is one way to fix the parameters, it necessitates the knowledge of model order, dead time, poles and zeros *a priori*. If the prior information about the process to be controlled is limited, autotuning becomes an elegant alternative as discussed in [27] exploiting relay tests and [28] postulating phase shapers.

V. Sliding Mode Control

Sliding Mode Control is a widely studied robust control scheme that has a switching nature. The state of the process under control is guided towards a predefined attracting subspace of the state space such that the trajectories on it display a desired behavior. The phase lasting until the hitting of a trajectory to the switching subspace is called reaching phase, while the motion thereafter is called sliding mode. The latter phase exhibits certain degrees of robustness against disturbances and variations in the process parameters and this result is called the invariance property. In this section, the control scheme is adapted to fractional order case and some results on stability are emphasized.

Consider the nth order fractional dynamic system given in (5) and define the switching function as

$$\sigma = \mathbf{\Lambda}(\mathbf{x}-\mathbf{r}) \quad (17)$$

where $\mathbf{\Lambda}$ is a design parameter making the sliding manifold defined by $\sigma = 0$ a stable subspace whose stability can be determined by using (13). Practically, this means that the nominal plant model is a linear one, while the process is indeed nonlinear. Choose $0 < \beta < 1$, for which the Caputo and Riemann–Liouville definitions of the fractional integration coincide, and let $\mathbf{r}$ be the vector of differentiable command signals. The goal of the reaching law approach is to obtain $\mathbf{D}^\beta\sigma = -k\,\mathrm{sgn}(\sigma)$ for some $k > 0$. For $\beta = 1$, this would correspond to $\dot\sigma = -k\,\mathrm{sgn}(\sigma)$ that ensures $\sigma\dot\sigma < 0$ if $\sigma \neq 0$. Clearly, this is the time derivative of the Lyapunov function $V = (1/2)\sigma^2$ and the physical meaning of enforcing such a subdynamics is to render the sliding manifold an attractor and once the error vector gets trapped to it, the motion thereafter takes place in the vicinity of the sliding hypersurface. Before generalizing such a result, one has to prove that the mechanism works also for the cases where the order of differentiation is not an integer. To show this, differentiate $\mathbf{D}^\beta\sigma = -k\,\mathrm{sgn}(\sigma)$ at the order $-\beta$, this would leave σ alone, and differentiate at order unity to obtain $\dot\sigma$. These steps are given as in (18) and the resulting expression is in (19), [8], [25]

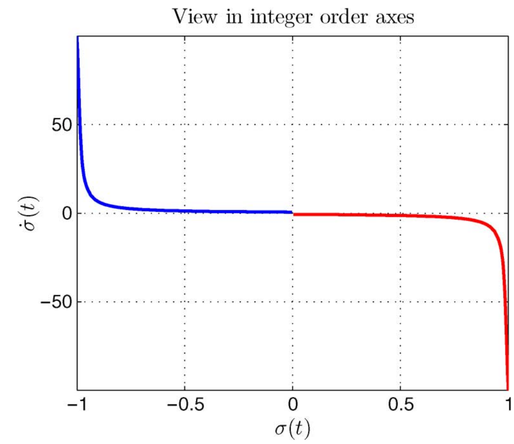

Fig. 2. For $\mathbf{D}^\beta\sigma = -k\,\mathrm{sgn}(\sigma)$, reaching the sliding manifold from both sides.

$$\mathbf{D}^1\left(\mathbf{D}^{-\beta}(\mathbf{D}^\beta\sigma)\right) = -k\mathbf{D}^1\left(\mathbf{D}^{-\beta}\mathrm{sgn}(\sigma)\right) \quad (18)$$

$$\dot\sigma = -k\mathbf{D}^{1-\beta}\mathrm{sgn}(\sigma). \quad (19)$$

Since $0 < \beta < 1$, we have $\mathrm{sgn}(\mathbf{D}^{1-\beta}\mathrm{sgn}(\sigma)) = \mathrm{sgn}(\sigma)$, and forcing $\mathbf{D}^\beta\sigma = -k\,\mathrm{sgn}(\sigma)$ makes the locus described by $\sigma = 0$ a global attractor. To demonstrate this, the reaching dynamics $\mathbf{D}^\beta\sigma = -k\,\mathrm{sgn}(\sigma)$ is solved numerically and two sample trajectories are shown over the pair of axes σ and $\dot\sigma$. This is deliberate as it is straightforward to interpret the prescribed motion in the integer order axis settings. In Fig. 2, the solution of $\mathbf{D}^\beta\sigma = -k\,\mathrm{sgn}(\sigma)$ is shown for $\beta = 0.5$, $\sigma(0) = 1$, and $\sigma(0) = -1$. Clearly, the initial push toward the switching manifold is excessive and it gradually decreases as σ gets closer to zero.

It is straightforward to demonstrate that choosing $\mathbf{D}^\beta\sigma = -k\,\mathrm{sgn}(\sigma) - p\sigma$ with $p > 0$ has the same effect in the reaching dynamics of that in integer order design. Since $p\sigma = p|\sigma|\mathrm{sgn}(\sigma)$, we have the following relation in between the quantities $\dot\sigma$ and $\mathrm{sgn}(\sigma)$:

$$\begin{aligned}\dot\sigma &= -k\mathbf{D}^{1-\beta}\mathrm{sgn}(\sigma) - p\mathbf{D}^{1-\beta}\left(|\sigma|\mathrm{sgn}(\sigma)\right)\\ &= -\mathbf{D}^{1-\beta}\left((k+p|\sigma|)\,\mathrm{sgn}(\sigma)\right).\end{aligned} \quad (20)$$

Due to the relation $\mathrm{sgn}(\mathbf{D}^{1-\beta}\mathrm{sgn}(\sigma)) = \mathrm{sgn}(\sigma)$, the reaching dynamics governed by the above expression will create a stronger push from both sides of the switching manifold. In other words, the attraction strength of the switching manifold is higher for any σ with $p \neq 0$ than that with $p = 0$.

For a fixed σ, the increasing values of p creates larger $\dot\sigma$ values and this leads to quicker approaching to the locus characterized by $\sigma = 0$. If one wants to choose a Lyapunov function like $V = (1/2)\sigma^2$ and takes the βth order derivative of it, according to the Leibniz's rule of differentiation, the result is $\mathbf{D}^\beta V = \sum_{k=0}^{\infty}(\Gamma(1+\beta)/\Gamma(1+k)\Gamma(1-k+\beta))\mathbf{D}^k\sigma\mathbf{D}^{\beta-k}\sigma$, which requires the manipulation of infinitely many terms. This clearly does not allow inferring the attractiveness of $\sigma = 0$ deduced either from $\sigma\mathbf{D}^\beta\sigma < 0$ or from $\mathbf{D}^\beta\sigma = -k\,\mathrm{sgn}(\sigma) - p\sigma$.

Due to the definition in (1), we have the equality $\sigma\mathbf{D}^\beta\sigma = (\sigma/\Gamma(1-\beta))\int_0^t(\mathbf{D}\sigma(\tau)/(t-\tau)^\beta)\mathrm{d}\tau$. This relation stipulates that for $\sigma > 0$, $\mathbf{D}\sigma$ (the first derivative of σ) must be negative to have $\sigma\mathbf{D}^{\beta}\sigma < 0$, or alternatively, for $\sigma < 0$, $\mathbf{D}\sigma$ (the first derivative of σ) must be positive to have $\sigma\mathbf{D}^{\beta}\sigma < 0$. Therefore, for closed-loop stability forcing $\sigma\mathbf{D}^{\beta}\sigma < 0$ via an appropriately designed control law is sufficient. This explanation of inferring the stability shows that the stability requirement $\sigma\dot{\sigma} < 0$ (or $\sigma\mathbf{D}\sigma < 0$) of the integer order design is forced naturally if $\sigma\mathbf{D}^{\beta}\sigma < 0$ is forced. This is a major contribution of the current paper. To summarize, a control law ensuring $\sigma\mathbf{D}^{\beta}\sigma < 0$ also ensures $\sigma\dot{\sigma} < 0$ and the closed-loop system becomes stable. Having these in mind, taking the βth order derivative of (17) yields

$$\mathbf{D}^{\beta}\sigma = \mathbf{\Lambda}(\mathbf{D}^{\beta}\mathbf{x} - \mathbf{D}^{\beta}\mathbf{r}) = \mathbf{\Lambda}\left(\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u - \mathbf{D}^{\beta}\mathbf{r}\right). \quad (21)$$

Equating the above expression to $-k\mathrm{sgn}(\sigma) - p\sigma$ and solving for u lets us have the following control signal:

$$u = \frac{-\mathbf{\Lambda}\mathbf{f}(\mathbf{x}) + \mathbf{\Lambda}\mathbf{D}^{\beta}\mathbf{r} - k\mathrm{sgn}(\sigma) - p\sigma}{\mathbf{\Lambda}\mathbf{g}(\mathbf{x})} \quad (22)$$

where it is necessary to have $\mathbf{\Lambda}\mathbf{g}(\mathbf{x}) \neq 0$. With such a control law deduced from a nominal model, what would be the response if the model in (5) is a nominal representation of a plant containing uncertainties $\Delta\mathbf{f}(\mathbf{x})$ and $\Delta\mathbf{g}(\mathbf{x})$ such as the one below

$$\mathbf{D}^{\beta}\mathbf{x} = (\mathbf{f}(\mathbf{x}) + \Delta\mathbf{f}(\mathbf{x})) + (\mathbf{g}(\mathbf{x}) + \Delta\mathbf{g}(\mathbf{x}))\,u. \quad (23)$$

Inserting (22) into (23) and computing $\mathbf{D}^{\beta}\sigma$ we get the dynamics in (24).

$$\mathbf{D}^{\beta}\sigma = -\left(1 + \frac{\mathbf{\Lambda}\Delta\mathbf{g}(\mathbf{x})}{\mathbf{\Lambda}\mathbf{g}(\mathbf{x})}\right)(k\mathrm{sgn}(\sigma) + p\sigma) + \frac{\mathbf{\Lambda}\Delta\mathbf{g}(\mathbf{x})}{\mathbf{\Lambda}\mathbf{g}(\mathbf{x})}\mathbf{\Lambda}\left(\mathbf{D}^{\beta}\mathbf{r} - \mathbf{f}(\mathbf{x})\right) + \mathbf{\Lambda}\Delta\mathbf{f}(\mathbf{x}) \quad (24)$$

- If there are no uncertainties, i.e., $\Delta\mathbf{f}(\mathbf{x}) = 0$ and $\Delta\mathbf{g}(\mathrm{x}) = 0$, then we have $\mathbf{D}^{\beta}\sigma = -k\,\mathrm{sgn}(\sigma) - p\sigma$, which is desired to observe the sliding regime after hitting the sliding hypersurface.
- If $\Delta\mathbf{g}(\mathbf{x})$ is zero and the columns of $\Delta\mathbf{f}(\mathbf{x})$ =are in the range space of $\mathbf{g}(\mathbf{x})$, then $\mathbf{D}^{\beta}\sigma = -k\,\mathrm{sgn}(\sigma) - p\sigma + \mathbf{\Lambda}\Delta\mathbf{f}(\mathbf{x})$. This case further requires the hold of the condition in (25) for maintaining $\sigma\mathbf{D}^{\beta}\sigma < 0$

$$k > |\mathbf{\Lambda}\Delta\mathbf{f}(\mathbf{x})|. \quad (25)$$

- If there are nonzero uncertainty terms, then (24) is valid and the designer needs to set k and p carefully to maintain the attractiveness of the subspace defined by $\sigma = 0$. The conditions in (26) and (27) are needed to maintain $\sigma\mathbf{D}^{\beta}\sigma < 0$

$$\left|\frac{\mathbf{\Lambda}\Delta\mathbf{g}(\mathbf{x})}{\mathbf{\Lambda}\mathbf{g}(\mathbf{x})}\right| < 1 \quad (26)$$

$$\left(1 + \frac{\mathbf{\Lambda}\Delta\mathbf{g}(\mathbf{x})}{\mathbf{\Lambda}\mathbf{g}(\mathbf{x})}\right)k > \left|\frac{\mathbf{\Lambda}\Delta\mathbf{g}(\mathbf{x})}{\mathbf{\Lambda}\mathbf{g}(\mathbf{x})}\mathbf{\Lambda}(\mathbf{D}^{\beta}\mathbf{r} - \mathbf{f}(\mathbf{x})) + \mathbf{\Lambda}\Delta\mathbf{f}(\mathbf{x})\right|. \quad (27)$$

Here, we assume that columns of $\Delta\mathbf{f}(\mathbf{x})$ and $\Delta\mathbf{g}(\mathbf{x})$ are in the range space of $\mathbf{g}(\mathbf{x})$, i.e., the uncertainties are matched uncertainties. If the matching conditions are not satisfied, the closed-loop performance will be degraded to some extent and the degree of this is determined by the functional details embodying the plant dynamics.

It is straightforward to show that the first hitting to the switching subspace occurs when $t = t_h$, where $t_h = (|\sigma(0)|\Gamma(\beta+1)/k)^{1/\beta}$.

VI. Backstepping Control

Backstepping technique has been a frequently used nonlinear control technique that is based on the definition of a set of intermediate variables and the procedure of ensuring the negativity of Lyapunov functions that add up to build a common control Lyapunov function for the overall system. Due to this nature, the backstepping technique is applicable to a particular yet wide class of systems, which includes most mechanical systems, biochemical processes etc. The technique has successfully been implemented in the field of robotics as one of the state variables is of type position and the other is of type velocity [29]. Consider the system

$$\begin{aligned} x_1^{(\beta_1)} &= x_2 \\ x_2^{(\beta_2)} &= f(x_1, x_2) + g(x_1, x_2)u \end{aligned} \quad (28)$$

where x_1 and x_2 are the state variables, $0 < \beta_1, \beta_2 < 1$ are positive fractional differentiation orders, $f(x_1, x_2)$ and $g(x_1, x_2)$ are known and smooth functions of the state variables and $g(x_1, x_2) \neq 0$. Define the following intermediate variables of backstepping design:

$$\begin{aligned} z_1 &:= x_1 - r_1 - A_1 \\ z_2 &:= x_2 - r_2 - A_2 \end{aligned} \quad (29)$$

where $A_1 = 0$ and $r_1^{(\beta_1)} = r_2$.

Let z be the variable of interest and choose the Lyapunov function given by (30)

$$V = \frac{1}{2}z^2. \quad (30)$$

We know from the previous section that if $zz^{(\beta)}$ ensures $z\dot{z} < 0$ for $0 < \beta < 1$. Now, we will formulate the backstepping control technique for the plant described by (28) by repetitively checking the quantities $z_1 z_1^{(\beta_1)}$ and $z_1 z_1^{(\beta_1)} + z_2 z_2^{(\beta_2)}$ as explained next.

Step 1) Check $z_1 z_1^{(\beta_1)}$

$$\begin{aligned} z_1 z_1^{(\beta_1)} &= z_1\left(x_1^{(\beta_1)} - r_1^{(\beta_1)}\right) \\ &= z_1(x_2 - r_2) \\ &= z_1(z_2 + r_2 + A_2 - r_2) \\ &= z_1(z_2 + A_2). \end{aligned} \quad (31)$$

Step 2) With $k_1 > 0$, choose $A_2 = -k_1 z_1$, this would let us have

$$z_1 z_1^{(\beta_1)} = -k_1 z_1^2 + z_1 z_2 \quad (32).$$

Step 3) Check $z_1 z_1^{(\beta_1)} + z_2 z_2^{(\beta_2)}$

$$\begin{aligned} & z_1 z_1^{(\beta_1)} + z_2 z_2^{(\beta_2)} \\ & = -k_1 z_1^2 + z_1 z_2 + z_2 \left(x_2^{(\beta_2)} - r_2^{(\beta_2)} - A_2^{(\beta_2)} \right) \\ & = -k_1 z_1^2 + z_2 \left(x_2^{(\beta_2)} - r_2^{(\beta_2)} - A_2^{(\beta_2)} + z_1 \right) \\ & = -k_1 z_1^2 + z_2 \left(f + gu - r_2^{(\beta_2)} - A_2^{(\beta_2)} + z_1 \right). \end{aligned} \tag{33}$$

Step 4) Force $z_1 z_1^{(\beta_1)} + z_2 z_2^{(\beta_2)} = -k_1 z_1^2 - k_2 z_2^2$, $k_2 > 0$, this requires

$$f + gu - r_2^{(\beta_2)} - A_2^{(\beta_2)} + z_1 := -k_2 z_2. \tag{34}$$

Step 5) Solve for u

$$u = -\frac{1}{g(x_1, x_2)} \left(f(x_1, x_2) - r_2^{(\beta_2)} + k_1 z_1^{(\beta_2)} + z_1 + k_2 z_2 \right). \tag{35}$$

It is possible to generalize the above procedure for higher order systems of the form

$$\begin{aligned} x_i^{(\beta_i)} &= x_{i+1}, \quad i = 1, 2, \ldots, q-1 \\ x_q^{(\beta_q)} &= f(x_1, x_2, \ldots, x_q) + g(x_1, x_2, \ldots, x_q) u \end{aligned} \tag{36}$$

and we obtain the control law

$$u = -\frac{1}{g} \left(f - r_q^{(\beta_q)} - A_q^{(\beta_q)} + z_{q-1} + k_q z_q \right) \tag{37}$$

where $k_q > 0$ and

$$A_1 = 0, \quad z_0 = 0 \tag{38}$$

$$A_{i+1} = -k_i z_i + A_i^{(\beta_i)} - z_{i-1}, \quad i = 1, 2, \ldots, q-1 \tag{39}$$

and the result of applying the control law in (37) is as below

$$\sum_{i=1}^{q} z_i z_i^{(\beta_i)} = -\sum_{i=1}^{q} k_i z_i^2. \tag{40}$$

Ensuring the negativeness of the right-hand side of (40) is equivalent to ensuring the negativity of $\sum_{i=1}^{q} z_i \dot{z}_i$, and the trajectories in the coordinate system spanned by $z_1, z_2, \ldots, z_q$ converge the origin.

VII. Adaptive Control

Adaptive control has been a good alternative for the industrial applications where the process parameters change and the controller needs to adapt itself automatically to new operating conditions. This capability is called adaptiveness and fractional calculus enters the picture in designing noninteger order adaptation laws or choosing noninteger order reference models [8]. A widely used adaptive control structure is based on a model and is called Model Reference Adaptive Control (MRAC), which is depicted in Fig. 3.

The underlying assumption is that the changes in the process parameters are slower than other changes in the closed-loop system. Parameter adjustment mechanism exploits the difference between the model output (y_m) and the process response (y) and utilizes the gradient rule to modify the parameters of the control law. This is stated as given below

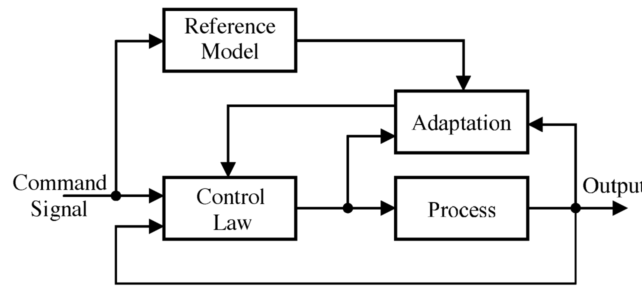

Fig. 3. Structure of the MRAC scheme.

$$\mathbf{D}^{\beta} \phi = -\eta \frac{\partial J}{\partial \phi} = -\eta e \frac{\partial e}{\partial \phi} \tag{41}$$

where ϕ is a generic parameter of the control law, $e := y - y_m$ is the instantaneous model following error and $J := (1/2)e^2$ is the instantaneous performance measure. Clearly, if $\beta = 1$ in (41), we obtain the conventional update laws. Stability considerations and an example considering fractional order reference model is presented in [8]. The benefit of utilizing fractional order setting in MRAC is to observe a shorter transient regime compared to the classical case and this might be critical in applications requiring high speed in response.

As an illustrative example, consider the process transfer function given by $\mathcal{P}(s) = Y(s)/U(s) = b/s^{0.7} + a$ and the reference model $\mathcal{Q}(s) = Y_m(s)/U_c(s) = b_m/s^{0.7} + a_m$. Denote $\mathcal{L}$ as the Laplace transform and choose the control law $u = \theta_1 u_c + \theta_2 y$, where $U(s) = \mathcal{L}\{u(t)\}$ and $U_c(s) = \mathcal{L}\{u_c(t)\}$. According to the rule in (41), we obtain the following update laws to tune θ_1 and θ_2:

$$\theta_1^{(\beta_1)} = -\eta e \left(\frac{b_m}{\mathbf{D}^{0.7} + a_m} u_c \right) \tag{42}$$

$$\theta_2^{(\beta_2)} = -\eta e \left(\frac{b_m}{\mathbf{D}^{0.7} + a_m} y \right). \tag{43}$$

In Table I, the settings of the simulation are given and in Fig. 4, the time evolution of the relevant variables are shown. For the chosen value of the adaptation gain (η), we see a slow convergence yet the speed of the response could be increased by increasing the value of this parameter. According to the top subplot Fig. 4, the process output follows the model output, the middle subplot depicts the applied control signal and in the bottom subplot, the evolution of the adaptable parameters are displayed. The values of β_1 and β_2 are chosen 0.8 to demonstrate the overall performance in the cases where differentiation orders are all noninteger.

This simple yet descriptive example shows that the design steps of the adaptive control framework can be implemented for the processes that are too complicated in integer order domain yet simple in fractional orders and the toolkit in fractional order representations is mature enough to employ in industrial applications.

TABLE I
SIMULATION SETTINGS

Parameter	Value
β_1	0.8
β_2	0.8
a	1.6
b	0.6
a_m	1
b_m	1
$y(0)$	1
$y_m(0)$	0
$\theta_1(0)$	1
$\theta_2(0)$	-1
η	0.05

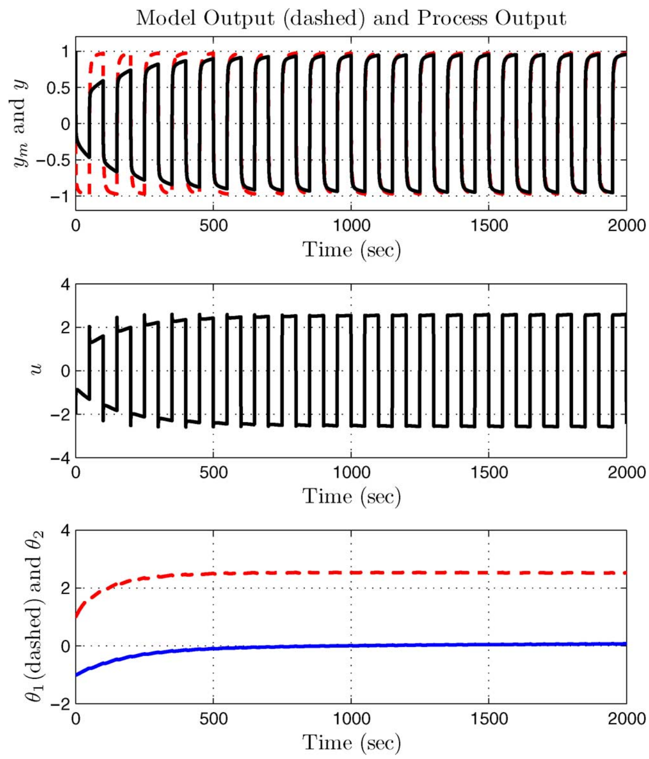

Fig. 4. Simulation results. Process output and the model output (top), applied control signal (middle), time evolution of the adjustable parameters (bottom).

VIII. LINKS TO COMPUTATIONAL INTELLIGENCE

The term *computational intelligence* is often used to refer to connectionist structures like neural networks, rule-based modules in the form of fuzzy inference systems, support vector machines or systems using a blend of these structures with heuristics, evolutionary computation or methods adopted from artificial intelligence. The 1990s were the years of resurgence for these novel methodologies as their computational needs were excessive for the computers of the 1980s and tuning laws were not optimized. Today, the lightweight and cheap hardware for collecting data at high speeds and optimum processing of them in very high-speed CPUs make these versatile tools good assets for industrial applications that are involved with real time, typically multidimensional, possibly nonlinear, and sometimes sparsely or multirate sampled. As discussed previously within the context of adaptive control, intelligent systems have the flexibility to adapt themselves to new operating conditions and use of fractional calculus in adaptation of parameters is another alternative speeding up the learning process. In [5], a fractional order adaptation scheme is proposed for an adaptive neurofuzzy inference system model that runs as a control module for a two dof manipulator, and in [30] another adaptation law is considered for sliding mode control and an adaptive linear element is used as the controller for tuning, which is the mechanism making the neural models dynamic.

In [31], Hopfield neural network is analyzed by replacing the capacitors with fractors and recognition problem is considered. This second line of research deals with the realization of neurodynamic systems by utilizing noninteger order components, whose terminal equations are, involved with noninteger orders of the operator $\mathbf{D}$. In the future, the representations that are not constrained to integer orders are expected to dominate the application domains as exemplified in the next section.

IX. A DESIGN EXAMPLE: CEMENT MILL CONTROL

Maintaining the quality of the ground product with the increasingly demanding cement standards has been a core issue in the industry of cement producers. Some parameters like the strength after a certain period of time, percent sulfate content, percent tricalcium aluminate content or fineness of the cement material determine to what extent the final product satisfies the desired specifications. Obtaining a consistent fineness and quality, on the other hand, depend heavily on the control and optimization approach utilized on-site. Without loss of generality, the design and implementation of control schemes in cement milling processes is typically involved with the selection of several feed rates as the control variables and these schemes aim to maintain a desired load on the mill. Therefore, the clarification of operational properties of milling process has been an interesting research problem encountered in the field. For this reason, the dynamic representation of cement milling processes relate the variables like mill load, product flow rate, tailing flow rate and some other system parameters in a nonlinear fashion, consequently, the synthesis of an appropriate command and control mechanism entails tools offering design flexibility and novel toolkits to handle nontraditional representations.

Control of cement milling processes has been the focus of a number of research studies. The approaches postulated in the area of nonlinear control have extensively been applied. Particularly, the model used in this paper has constituted a prime example due to the inextricably intertwined relations among the variables involved. The model has three state variables and two control inputs, despite its representational simplicity, the dynamics is quite complex, and a good control performance can only be achieved if a suitable coordination between the two control inputs can be established and maintained.

The dynamic model of the system is described by three coupled and nonlinear differential equations as given in (44)–(46). In this representation, z is the mill load, y_f is the product flow rate, and y_r is the tailings flow rate. These three variables are the states of the system. On the other hand, ξ is the output flow rate of the mill and d denotes the relative hardness of the material inside the mill with respect to the nominal one, which is unity.

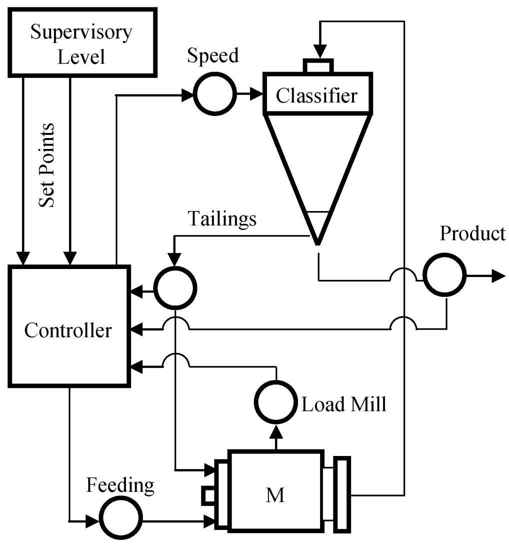

Fig. 5. Schematic diagram of the cement milling circuit.

The system has two control inputs, denoted by u, feed flow rate and, v, classifier speed

$$z^{(\beta_1)} = -\xi(z,d) + u + y_r \tag{44}$$
$$T_f y_f^{(\beta_2)} = -y_f + (1-\alpha(z,v,d))\,\xi(z,d) \tag{45}$$
$$T_r y_r^{(\beta_3)} = -y_r + \alpha(z,v,d)\xi(z,d) \tag{46}$$

where the functions $\alpha(z,v,d) = \xi(z,d)^l v^b/(K_\alpha + \xi(z,d)^l v^b)$ and $\xi(z,d) = \max(0, -dk_{\phi 1}z^2 + k_{\phi 2}z)$, where $l = 0.8$ and $b = 4$. In the above, $K_\alpha = 570^l 170^b((570/450) - 1)$, $k_{\phi 1} = 0.1116\ (\text{tons} \times \text{h})^{-1}$, $k_{\phi 2} = 16.50\ \text{h}^{-1}$, $T_f = 0.3$ h, $T_r = 0.01$ h, and $d = 1$. A schematic representation of the process is depicted in Fig. 5, and a detailed description of the system dynamics and results regarding the experimental verification can be found in [31] and [32].

The control problem is to enforce the system states by appropriately altering the two control inputs. However, it can easily be shown that the designer can choose two of the three state variables independently as the behavior of the third state variable will be determined upon the selection of other two. It is emphasized in [31] that the choice of y_f and y_r may lead to unachievable values for ξ and it is suggested that keeping y_f and z under control would be a suitable approach. In this paper, we adopt the same reasoning and proceed parallel to this idea.

In [31] and [32], the model is used with $\beta_1 = \beta_2 = \beta_3 = 1$, i.e., the dynamics in (44)–(46) are constructed via integer order differential equations. In this paper, we will consider $\beta_1 = 0.8$, $\beta_2 = 0.7$, and $\beta_3 = 0.5$ case. Obviously, a process model having such a fractional order components would require the knowledge of fractional order systems and control and in the sequel, due to the space limit, we will discuss only the sliding mode control of the cement milling process. Consider the stable reference model given by (47)–(49), where r_z and r_{y_f} are externally supplied command signals driving the reference model and $\xi_r(z_r,d) = \max(0, -dk_{\phi 1}z_r^2 + k_{\phi 2}z_r)$

$$z_r^{(\beta_1)} = -z_r + r_z \tag{47}$$
$$T_f y_{f_r}^{(\beta_2)} = -y_{f_r} + r_{y_f} \tag{48}$$
$$T_r y_{r_r}^{(\beta_3)} = -y_{r_r} - r_{y_f} + \xi_r(z_r,d). \tag{49}$$

The control laws in (50)–(52) with $K_1 > 0$, $K_2 > 0$ enforce the response of the process to that of the reference model. We further modify the control laws to the those given in (53)–(55) to consider the effects of noise in the observations and changes in the hardness parameter d

$$u = \xi(z,d) - y_r + r_z - z_r - K_1\text{sgn}(z - z_r) \tag{50}$$
$$\zeta = \frac{y_f - \xi(z,d) + r_{y_f} - y_{f_r} - K_2\text{sgn}\left(y_f - y_{f_r}\right)}{\xi(z,d)} \tag{51}$$
$$v = \left(\frac{\zeta K_\alpha}{(1-\zeta)\xi(z,d)^l}\right)^{\frac{1}{n}} \tag{52}$$
$$\begin{aligned} u =& \xi(z+n_z, d+nd) - (y_r + n_{y_r}) + r_z - z_r \\ &- K_1\text{sgn}(z + n_z - z_r) \end{aligned} \tag{53}$$
$$\begin{aligned} \zeta =& \frac{1}{\xi(z+n_z, d+n_d)}\big(y_f + n_{y_f} - \xi(z+n_z, d+n_d) \\ &+ r_{y_f} - y_{f_r} \\ &+ -K_2\text{sgn}\left(y_f + n_{y_f} - y_{f_r}\right)\big) \end{aligned} \tag{54}$$
$$v = \left(\frac{\zeta K_\alpha}{(1-\zeta)\xi(z+n_z, d+n_d)^l}\right)^{\frac{1}{n}} \tag{55}$$

where the Gaussian noise sequences n_z, n_{y_f}, and n_{y_r} are zero-mean and they have magnitude less than five with a probability very close to zero. The variation in hardness parameter is embedded into the system by selecting The relative material hardness parameter has been chosen as $d(t) = 1 + 0.34\sin(2\pi t/20) + 0.005\sin(20\pi t/90)$, which has the slowly changing component to simulate the changes in relative hardness in the material and high-frequency component to simulate the small magnitude noise. Initially, $y_f(0) = 100$, $y_r(0) = 5$, and $z(0) = 50$, on the other hand, the reference model states have initially been set as $y_{f_r}(0) = 120$, $y_{r_r}(0) = 0$, and $z_r(0) = 40$. These values have been selected according to the typical values that appear in the cited references. We set $N = 25$ for realizing the fractional order operators numerically and chose the frequency band 10^{-4} to 10^4 rad/s as the spectrum over which the fractional approximation is to be carried out. 90 h of simulation is performed with these settings and the results of the simulation are illustrated in Fig. 6. The results on the left column indicate that the process states follow the state values prescribed by the reference model (shown dashed in the plots) and the tracking precision under the considered noise and uncertainty scenarios is very good. In the right column of the figure, the applied control signals and the change of the relative material hardness parameter are depicted. The control signals seem to have high-frequency components due to the discontinuities in the control laws yet the duration of the experiment is 90 h and such fluctuations are fairly in the acceptable range of ordinary actuation periphery. Nevertheless, one can smooth out the sign functions to obtain smoother control signals yet the price paid for this will be to introduce a boundary layer around the switching subspace.

The demonstrated example shows that the control laws of the classical systems and control theory can be generalized in such a

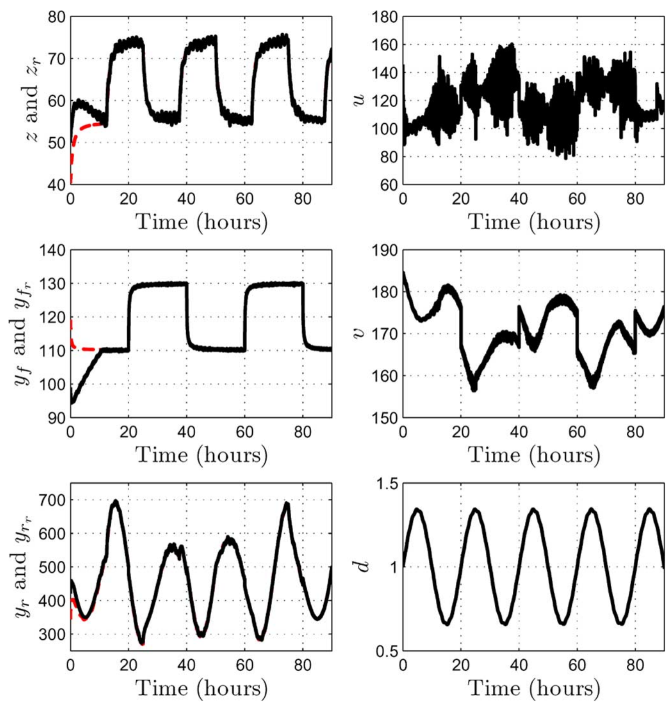

Fig. 6. State tracking results and the applied control signals for $K_1 = K_2 = 1$.

way that they can be applied to systems having noninteger order derivative terms.

X. Conclusion

This paper considers the fractional order systems and control methods within the context of industrial automation. The expectations of the industrial applications are demanding and oftentimes the conventional solutions to problems are so complicated that the manufacturing of the goods based on standard approaches is not feasible. Restricting ourselves to the realm of control systems, fractional order models and controllers provide precision and they are able to represent any real order of system perfectly. From this point-of-view, a high-order model containing integer order differentiators or integrators can fairly be approximated by fractional order modules that are mathematically tractable and representationally simple.

This paper has considered the fractional order versions of PID controller, sliding mode controller, backstepping and adaptive controllers as well as the links to computational intelligence are described. Though the literature of the control systems technology has a lot more than what could be presented here but the goal of this paper is to motivate the practicing engineers in industry to try solutions based on fractional order models.

Toward this goal, a cement milling circuit is chosen as the test bed and a fractional order version of the nonlinear model is adopted as the process to be investigated. The control laws were defined in such a way that the process states follow those of a reference model, where both of these systems are fractional order. The findings in the simulations seem very promising to motivate real-time experimentations in industrial applications.

References

[1] A. K. Jana, "A hybrid FLC-EKF scheme for temperature control of a refinery debutanizer column," *IEEE Trans. Ind. Inform.*, vol. 57, no. 1, pp. 25–35, Jan. 2010.
[2] H. Schweppe, A. Zimmermann, and D. Grill, "Flexible on-board stream processing for automotive sensor data," *IEEE Trans. Ind. Inform.*, vol. 57, no. 1, pp. 81–92, Jan. 2010.
[3] C. Huang, Y. Bai, and X. Liu, "H-infinity state feedback control for a class of networked cascade control systems with uncertain delay," *IEEE Trans. Ind. Inform.*, vol. 57, no. 1, pp. 62–72, Jan. 2010.
[4] P.-E. P. Odiowei and Y. Cao, "Nonlinear dynamic process monitoring using canonical variate analysis and kernel density estimations," *IEEE Trans. Ind. Inform.*, vol. 57, no. 1, pp. 36–45, Jan. 2010.
[5] M. Ö. Efe, "Fractional fuzzy adaptive sliding mode control of a 2 DOF direct drive robot arm," *IEEE Trans. Syst., Man, Cybern., Part B: Cybern.*, vol. 28, no. 6, pp. 1561–1570, Dec. 2008.
[6] C.-C. Tseng, "Design of variable and adaptive fractional order FIR differentiators," *Signal Process.*, vol. 86, no. 10, pp. 2554–2566, 2006.
[7] A. W. Lohmann, D. Mendlovic, Z. Zalevsky, and R. G. Dorsch, "Some important fractional transformations for signal processing," *Opt. Commun.*, vol. 125, no. 1–3, pp. 18–20, 1996.
[8] C. A. Monje, Y. Q. Chen, B. Vinagre, D. Xue, and V. Feliu, "Fractional order systems and control-fundamentals and applications," in *Advanced Industrial Control Series*. Berlin, Germany: Springer-Verlag, Oct. 2010.
[9] M. Pineda-Sanchez, M. Riera-Guasp, J. A. Antonino-Daviu, J. Roger-Folch, J. Perez-Cruz, and R. Puche-Panadero, "Diagnosis of induction motor faults in the fractional Fourier domain," *IEEE Trans. Instrum. Meas.*, vol. 59, no. 8, pp. 2065–2075, Aug. 2010.
[10] K. B. Oldham and J. Spanier, *The Fractional Calculus*. New York: Academic Press, 1974.
[11] I. Podlubny, *Fractional Differential Equations*, 1st ed. New York: Elsevier, 1998.
[12] S. Das, *Functional Fractional Calculus for System Identification and Controls*, 1st ed. New York: Springer, 2008.
[13] C. Zhao, D. Xue, and Y.-Q. Chen, "A fractional order PID tuning algorithm for a class of fractional order plants," in *Proc. IEEE Int. Conf. Mech. Autom.*, Niagara Falls, Canada, Jul. 2005.
[14] I. Podlubny, "Fractional-order systems and (PID-mu)-D-lambda-controllers," *IEEE Trans. Automatic Control*, vol. 44, no. 1, pp. 208–214, 1999.
[15] D. Matignon, "Stability results for fractional differential equations with applications to control processing," *Comput. Eng. Syst. Appl.*, pp. 963–968, 1996.
[16] D. Matignon, "Stability properties for generalized fractional differential systems," *ESAIM Proc. Fractional Differential Syst., Models, Methods, Appl.*, vol. 5, pp. 145–158, 1998.
[17] Y.-Q. Chen, H.-S. Ahna, and I. Podlubny, "Robust stability check of fractional order linear time invariant systems with interval uncertainties," *Signal Process.*, vol. 86, pp. 2611–2618, 2006.
[18] E. Ahmed, A. M. A. El-Sayed, and H. A. A. El-Saka, "On some Routh-Hurwitz conditions for fractional order differential equations and their applications in Lorenz, Rssler, Chua and Chen systems," *Phys. Lett. A*, vol. 358, pp. 1–4, 2006.
[19] D. Sierociuk and A. D. Dzielinski, "Fractional Kalman filter algorithm for the states, parameters and order of fractional system estimation," *Int. J. Appl. Math. Comput. Sci.*, vol. 16, no. 1, pp. 129–140, 2006.
[20] M. D. Ortigueira, "Introduction to fractional linear systems. Part 1: Continuous time case," *IEE Proc. Vis. Image, Signal Process.*, vol. 147, no. 1, pp. 62–70, 2000.
[21] H.-F. Raynaud and A. Zerganoh, "State-space representation for fractional order controllers," *Automatica*, vol. 36, no. 7, pp. 1017–1021, July 2000.
[22] F. Merrikh-Bayat and M. Afshar, "Extending the root-locus method to fractional order systems," *J. Appl. Math.*, 2008, Article ID 528934.
[23] M. M. Meerschaert and C. Tadjeran, "Finite difference approximations for two-sided space-fractional partial differential equations," *Appl. Numerical Math.*, vol. 56, pp. 80–90, 2006.
[24] I. Podlubny, A. Chechkin, T. Skovranek, Y.-Q. Chen, and B. M. Vinagre Jara, "Matrix approach to discrete fractional calculus II: Partial fractional differential equations," *J. Comput. Phys.*, vol. 228, pp. 3137–3153, 2009.
[25] B. M. Vinagre and A. J. Calderon, "On fractional sliding mode control," in *Proc. 7th Portuguese Conf. Autom. Control (CONTROLO'06)*, Lisbon, Portugal, Sep. 2006.

[26] D. Valerio, "Ninteger v. 2.3 fractional control toolbox for MatLab," 2005.
[27] C. A. Monje, B. M. Vinagre, V. Feliu, and Y. Q. Chen, "Tuning and auto-tuning of fractional order controllers for industry applications," *Control Eng. Practice*, vol. 16, pp. 798–812, 2008.
[28] Y. Q. Chen, K. L. Moore, B. M. Vinagre, and I. Podlubny, "Robust PID controller autotuning with a phase shaper," in *Proc. 1st IFAC Workshop on Fractional Differentiation and Its Applications*, Bordeaux, France, 2004.
[29] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design*. New York: Wiley, 1995.
[30] M. Ö. Efe and C. Kasnakoğlu, "A fractional adaptation law for sliding mode control," *Int. J. Adaptive Control Signal Process.*, vol. 22, no. 10, pp. 968–986, Dec. 2008.
[31] V. Van Breusegem, L. Chen, G. Bastin, V. Wertz, V. Werbrouck, and C. De Pierpont, "An industrial application of multivariable linear quadratic control to a cement mill circuit," *IEEE Trans. Ind. Appl.*, vol. 32, pp. 670–677, 1996.
[32] L. Magni, G. Bastin, and V. Wertz, "Multivariable nonlinear predictive control of cement mills," *IEEE Trans. Control Syst. Technol.*, vol. 7, no. 5, pp. 502–508, 1999.

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