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FRACTIONAL WAVEGUIDE WITH IMPEDANCE WALLS

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Abstract—Fractional solutions of a parallel plate waveguide originally with impedance walls have been derived and fractional impedance of the guiding walls have been investigated. Two distinct ranges of wall impedance have been found in which fractional impedance behaves in opposite ways. For $0 < \alpha < 1$, the fractional impedance is inductive in range 1 and is capacitive in range 2, where α is fractional parameter. For $1 < \alpha < 2$, the fractional impedance is capacitive in range 1 and is inductive in range 2. At the boundary of the two ranges, the fractional impedance is independent of α and is resistive. This behavior is periodic with period $\alpha = 2$.

1. INTRODUCTION

Fractional derivative/integrals are mathematical operators involving differentiation/integration to arbitrary non-integer orders. These operators, possess interesting mathematical properties and have been studied in the field of fractional calculus [1]. Engeta applied the tools of fractional calculus in various problems of electromagnetic fields and waves, and obtained interesting results. These results highlight certain notable features and promising potential applications of fractional operators in electromagnetic theory. Fractionalization of such operators has led us to novel solutions, interpretable as "fractional solutions", for certain electromagnetic problems [2–5]. An interesting and useful work done by Engheta is fractionalization of curl operator [2]. Mathematical recipe to fractionalize a linear operator is available in [2, 5]. Some interesting works are reported in [6, 7]. Problem of implementation of fractional order electric potential has been addressed in [7]. Debnath deals with recent applications of fractional calculus in science and engineering [8].

Recently, many authors have been interested in exploring the fractional dual solutions for various problems [9–24], while waveguides with impedance walls have been analyzed in [25–30]. In this work we have analyzed the fractional solutions of a parallel plate waveguide with impedance walls. In Section 2, the waveguide with impedance walls is studied and fractional solutions have been derived in Section 3. In Section 4, transverse impedance of the fractional guide is studied and the plots are discussed in Section 5. The paper is concluded in Section 6.

2. IMPEDANCE WALLS PARALLEL PLATE WAVEGUIDE

Consider a parallel plate waveguide consisting of two plates of finite impedance Z_w separated by a dielectric medium with constitutive parameters ϵ and μ. The separation between the two parallel plates is b. One plate is located at $y = 0$, while other plate is located at $y = b$. The plates are assumed to be of infinite extent. As a general scheme, we solve the Helmholtz equation only for the axial component and use these solutions to write the transverse components [11, 12, 17]. Let us suppose that a TM wave ($H_z = 0$) is propagating in z-direction. Solution for the electric field can be obtained by vector Helmholtz equation as

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0$$

For exponential z-dependence e.g., $\exp(\gamma z)$, above equation can be simplified in terms of tangential part of ∇ as

$$\nabla_t{}^2 \mathbf{E} + h^2 \mathbf{E} = 0$$

where

$$h^2 = k^2 + \gamma^2$$

For parallel plates structure of infinite dimension along x-axis, we can ignore the x-dependence and hence electric field can be describe by following differential equation

$$\frac{d}{dy}^2 \mathbf{E} + h^2 \mathbf{E} = 0$$

general solution of this equation for longitudinal component can be written as

$$E_z(y) = A \cos(hy) + B \sin(hy) \tag{1a}$$

Applying the impedance boundary conditions

$$E_z|_{(y=0)} = Z_w H_x|_{(y=0)} \qquad \text{(IBC-1)}$$
$$E_z|_{(y=b)} = -Z_w H_x|_{(y=b)} \qquad \text{(IBC-2)}$$

Using these boundary conditions in Equation (1a) and making use of the Maxwell equations, we can solve for the relation of the constants A, B and as well as eigen value h as

$$A = FB$$

where $F = (\frac{Z_w}{\eta})(\frac{-jk}{h})$.

h can be obtained from the solution of the following eigen value equation

$$h = \frac{1}{b}\tan^{-1}\left(\frac{F}{1+F}\right).$$

Using this relation we can write the electric and magnetic field components as

$$E_z(y) = B[F\cos(hy) + \sin(hy)] \qquad (1b)$$
$$E_y(y) = B\left(\frac{\gamma}{h}\right)[F\sin(hy) - \cos(hy)] \qquad (1c)$$
$$\eta H_x(y) = B\left(\frac{-jk}{h}\right)[F\sin(hy) - \cos(hy)] \qquad (1d)$$
$$E_x(y) = H_y(y) = H_z(y) = 0$$

Re-introducing the z-dependance $\exp(-\gamma z)$, Equations (1b)–(1d) can be arranged for total electric and magnetic fields as a combination of two TEM plane waves bouncing back and fourth obliquely between the two plates as

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 \qquad (2a)$$
$$\eta\mathbf{H} = \eta\mathbf{H}_1 + \eta\mathbf{H}_2 \qquad (2b)$$

where $(\mathbf{E}_1, \mathbf{H}_1)$ are the electric and magnetic fields associated with one plane wave and are given below

$$\mathbf{E}_1 = \frac{B}{2}(F-j)\left\{\hat{z} - \frac{j\gamma}{h}\hat{y}\right\}\exp(jhy - \gamma z) \qquad (3a)$$
$$\eta\mathbf{H}_1 = \hat{x}\left(\frac{-k}{h}\right)\frac{B}{2}(F-j)\exp(jhy - \gamma z) \qquad (3b)$$

while electric and magnetic fields $(\mathbf{E}_2, \mathbf{H}_2)$ associated with second plane wave and are given below

$$\mathbf{E}_2 = \frac{B}{2}(F+j)\left\{\hat{z} + \frac{j\gamma}{h}\hat{y}\right\}\exp(-jhy - \gamma z) \tag{4a}$$

$$\eta\mathbf{H}_2 = \hat{x}\left(\frac{k}{h}\right)\frac{B}{2}(F+j)\exp(-jhy - \gamma z) \tag{4b}$$

3. FRACTIONAL PARALLEL PLATE WAVEGUIDE HAVING IMPEDANCE WALLS

Fields $\mathbf{E}_1$ and $\mathbf{H}_1$ given by Equation (3) are related through the Maxwell equations as

$$\begin{aligned}
\nabla \times \mathbf{E}_1 &= -j\omega\mu\mathbf{H}_1 \\
(jh\hat{y} - \gamma\hat{z}) \times \mathbf{E}_1 &= -j\omega\mu\mathbf{H}_1 \\
\frac{1}{(jk)}(-jh\hat{y} + \gamma\hat{z}) \times \mathbf{E}_1 &= \eta\mathbf{H}_1 \\
\mathbf{k}_1 \times \mathbf{E}_1 &= \eta\mathbf{H}_1 \qquad (5a)
\end{aligned}$$

Similarly

$$\begin{aligned}
\frac{1}{(jk)}(-jh\hat{y} + \gamma\hat{z}) \times \eta\mathbf{H}_1 &= -\mathbf{E}_1 \\
\mathbf{k}_1 \times \eta\mathbf{H}_1 &= -\mathbf{E}_1 \qquad (5b)
\end{aligned}$$

where $\mathbf{k}_1 = \frac{1}{(jk)}(-jh\hat{y} + \gamma\hat{z})$.

Fields $\mathbf{E}_2$ and $\mathbf{H}_2$ in Equations (4) are also related through Maxwell equation as given below

$$\begin{aligned}
\nabla \times \mathbf{E}_2 &= -j\omega\mu\mathbf{H}_2 \\
-(jh\hat{y} + \gamma\hat{z}) \times \mathbf{E}_2 &= -j\omega\mu\mathbf{H}_2 \\
\frac{1}{(jk)}(jh\hat{y} + \gamma\hat{z}) \times \mathbf{E}_2 &= \eta\mathbf{H}_2 \\
\mathbf{k}_2 \times \mathbf{E}_2 &= \eta\mathbf{H}_2 \qquad (6a)
\end{aligned}$$

Similarly

$$\begin{aligned}
\frac{1}{(jk)}(jh\hat{y} + \gamma\hat{z}) \times \eta\mathbf{H}_2 &= -\mathbf{E}_2 \\
\mathbf{k}_2 \times \eta\mathbf{H}_2 &= -\mathbf{E}_2 \qquad (6b)
\end{aligned}$$

where $\mathbf{k}_2 = \frac{1}{(jk)}(jh\hat{y}+\gamma\hat{z})$. It may be noted that $|\mathbf{k}_1| = |\mathbf{k}_2| = 1$. It may also be deduced from above expressions that for set of fields $(\mathbf{E}_1, \mathbf{H}_1)$, the operator $\left(\frac{1}{jk}\nabla\times\right)$ is equivalent to cross product operator $(\mathbf{k}_1\times)$ while for set of fields $(\mathbf{E}_2, \mathbf{H}_2)$, the operator $\left(\frac{1}{jk}\nabla\times\right)$ is equivalent to cross product operator given by $(\mathbf{k}_2\times)$. It is also obvious that if $(\mathbf{E}_1, \eta\mathbf{H}_1)$ is one set of solutions to Maxwell's equation then other set of solutions to Maxwell's equations is $(\eta\mathbf{H}_1, -\mathbf{E}_1)$. Similarly if $(\mathbf{E}_2, \eta\mathbf{H}_2)$ is one set of solutions to Maxwell's equation then other set of solutions to Maxwell's equations is $(\eta\mathbf{H}_2, -\mathbf{E}_2)$. Our interest is to determine the fields which may be regarded as intermediate step of the field $(\mathbf{E}, \eta\mathbf{H})$ and $(\eta\mathbf{H}, -\mathbf{E})$, that is, new set of solutions $(\mathbf{E}_{fd}, \eta\mathbf{H}_{fd})$. For this purpose solutions sets $(\mathbf{E}_{ifd}, \eta\mathbf{H}_{ifd})$ with $i = 1, 2$ are required. $(\mathbf{E}_{ifd}, \eta\mathbf{H}_{ifd})$ may be obtained by using the following relations

$$\mathbf{E}_{ifd} = \frac{1}{(jk)^\alpha}\left[(\nabla\times)^\alpha \mathbf{E}_i\right] \tag{7a}$$

$$\eta\mathbf{H}_{ifd} = \frac{1}{(jk)^\alpha}\left[(\nabla\times)^\alpha \eta\mathbf{H}_i\right], \qquad i = 1, 2 \tag{7b}$$

Solutions $(\mathbf{E}_{fd}, \eta\mathbf{H}_{fd})$ may be obtained by linear combination of $(\mathbf{E}_{1fd}, \eta\mathbf{H}_{1fd})$ and $(\mathbf{E}_{2fd}, \eta\mathbf{H}_{2fd})$, that is

$$\mathbf{E}_{fd} = \mathbf{E}_{1fd} + \mathbf{E}_{2fd} \tag{8a}$$

$$\eta\mathbf{H}_{fd} = \eta\mathbf{H}_{1fd} + \eta\mathbf{H}_{2fd} \tag{8b}$$

In order to determine the fractional dual solutions $(\mathbf{E}_{ifd}, \eta\mathbf{H}_{ifd})$, the eigenvalues and eigenvectors of the two cross product operators $(\mathbf{k}_1\times, \mathbf{k}_2\times)$ are required. Eigenvectors and eigenvalues of the operator $(\mathbf{k}_1\times)$ are

$$\mathbf{A}_1 = \frac{1}{\sqrt{2}}\left[\hat{x} - \frac{\gamma}{k}\hat{y} - j\frac{h}{k}\hat{z}\right], \qquad a_1 = j$$

$$\mathbf{A}_2 = \frac{1}{\sqrt{2}}\left[\hat{x} + \frac{\gamma}{k}\hat{y} + j\frac{h}{k}\hat{z}\right], \qquad a_2 = -j$$

$$\mathbf{A}_3 = -j\frac{h}{k}\hat{y} + \frac{\gamma}{k}\hat{z}, \qquad a_3 = 0$$

Fields $(\mathbf{E}_1, \mathbf{H}_1)$ may be expressed in terms of the eigenvectors of the operator, that is

$$\mathbf{E}_1 = [P\mathbf{A}_1 + Q\mathbf{A}_2 + R\mathbf{A}_3]\exp(jhy - \gamma z) \tag{9}$$

where the coefficients are given below

$$P = \frac{B}{2\sqrt{2}}\left(\frac{jk}{h}\right)(F-j)$$
$$Q = -\frac{B}{2\sqrt{2}}\left(\frac{jk}{h}\right)(F-j)$$
$$R = 0$$

The expression for $\mathbf{E}_{fd}$ is obtained by applying fractional curl operator on vector $\mathbf{E}$. Fractionalization of curl operator means fractionalization of the equivalent cross product operator. Fractionalization of cross product operator means fractionalization of eigenvalues of the operator. Fractionalizing the eigenvalues of the operator yields

$$\mathbf{E}_{1fd} = [(a_1)^\alpha P\mathbf{A}_1 + (a_2)^\alpha Q\mathbf{A}_2 + (a_3)^\alpha R\mathbf{A}_3]\exp(jhy-\gamma z) \qquad (10)$$

Solutions to the Maxwell equations, which may be regarded as intermediate step between the solutions set $(\mathbf{E}_1, \eta\mathbf{H}_1)$ and solutions set $(\eta\mathbf{H}_1, -\mathbf{E}_1)$ are given by

$$\begin{aligned}\mathbf{E}_{1fd} &= (\mathbf{k}_1\times)^\alpha\,\mathbf{E}_1\\ &= \frac{B}{2}\left(\frac{jk}{h}\right)(F-j)\left[j\sin\left(\frac{\alpha\pi}{2}\right)\hat{x} - \frac{\gamma}{k}\cos\left(\frac{\alpha\pi}{2}\right)\hat{y}\right.\\ &\quad\left. -j\frac{h}{k}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(jhy-\gamma z) \qquad (11a)\end{aligned}$$

$$\begin{aligned}\eta\mathbf{H}_{1fd} &= (\mathbf{k}_1\times)^\alpha\,\eta\mathbf{H}_1\\ &= \frac{B}{2}\left(\frac{jk}{h}\right)(F-j)\left[j\cos\left(\frac{\alpha\pi}{2}\right)\hat{x} + \frac{\gamma}{k}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y}\right.\\ &\quad\left. +j\frac{h}{k}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(jhy-\gamma z) \qquad (11b)\end{aligned}$$

Eigenvectors and eigenvalues of the operator $(\mathbf{k}_2\times)$ are

$$\mathbf{A}_1 = \frac{1}{\sqrt{2}}\left[\hat{x} - \frac{\gamma}{k}\hat{y} + j\frac{h}{k}\hat{z}\right], \qquad a_1 = j$$
$$\mathbf{A}_2 = \frac{1}{\sqrt{2}}\left[\hat{x} + \frac{\gamma}{k}\hat{y} - j\frac{h}{k}\hat{z}\right], \qquad a_2 = -j$$
$$\mathbf{A}_3 = j\frac{h}{k}\hat{y} + \frac{\gamma}{k}\hat{z}, \qquad a_3 = 0$$

Fields may be expressed in terms of eigenvectors of the operator $\mathbf{k}_2\times$

$$\mathbf{E}_2 = [P\mathbf{A}_1 + Q\mathbf{A}_2 + R\mathbf{A}_3]\exp(-jhy-\gamma z) \qquad (12)$$

where coefficients are

$$
\begin{aligned}
P &= \frac{B}{2\sqrt{2}}\left(\frac{-jk}{h}\right)(F+j) \\
Q &= -\frac{B}{2\sqrt{2}}\left(\frac{-jk}{h}\right)(F+j) \\
R &= 0
\end{aligned}
$$

Solutions to the Maxwell equations, which may be regarded as intermediate step between the solutions set $(\mathbf{E}_2, \eta\mathbf{H}_2)$ and solutions set $(\eta\mathbf{H}_2, -\mathbf{E}_2)$ are given by

$$
\begin{aligned}
\mathbf{E}_{2fd} &= (\mathbf{k}_2\times)^{\alpha}\,\mathbf{E}_2 \\
&= \frac{B}{2}\left(\frac{-jk}{h}\right)(F+j)(-1)^{\alpha}\left[-j\sin\left(\frac{\alpha\pi}{2}\right)\hat{x} - \frac{\gamma}{k}\cos\left(\frac{\alpha\pi}{2}\right)\hat{y}\right. \\
&\quad \left. +j\frac{h}{k}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(-jhy-\gamma z) \qquad (13a) \\
\eta\mathbf{H}_{2fd} &= (\mathbf{k}_2\times)^{\alpha}\,\eta\mathbf{H}_2 \\
&= \frac{B}{2}\left(\frac{-jk}{h}\right)(F+j)(-1)^{\alpha}\left[j\cos\left(\frac{\alpha\pi}{2}\right)\hat{x} - \frac{\gamma}{k}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y}\right. \\
&\quad \left. +j\frac{h}{k}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(-jhy-\gamma z) \qquad (13b)
\end{aligned}
$$

Solutions to the Maxwell equations, which may be regarded as intermediate step between the solutions set $(\mathbf{E}, \eta\mathbf{H})$ and solutions set $(\eta\mathbf{H}, -\mathbf{E})$ may be obtained by substituting results by (11) and (13) in (8) and are given below

$$
\begin{aligned}
\mathbf{E}_{fd} &= B\left(\frac{jk}{h}\right)\exp\left(\frac{-j\alpha\pi}{2}\right)\exp(-\gamma z) \\
&\quad \left[j\sin\left(\frac{\alpha\pi}{2}\right)\left\{F\cos\left(hy+\frac{\alpha\pi}{2}\right)+\sin\left(hy+\frac{\alpha\pi}{2}\right)\right\}\hat{x}\right. \\
&\quad -\left(\frac{j\gamma}{k}\right)\cos\left(\frac{\alpha\pi}{2}\right)\left\{F\sin\left(hy+\frac{\alpha\pi}{2}\right)-\cos\left(hy+\frac{\alpha\pi}{2}\right)\right\}\hat{y} \\
&\quad \left.-\left(\frac{jh}{k}\right)\cos\left(\frac{\alpha\pi}{2}\right)\left\{F\cos\left(hy+\frac{\alpha\pi}{2}\right)+\sin\left(hy+\frac{\alpha\pi}{2}\right)\right\}\hat{z}\right] \qquad (14a) \\
\eta\mathbf{H}_{fd} &= B\left(\frac{-k}{h}\right)\exp\left(\frac{-j\alpha\pi}{2}\right)\exp(-\gamma z) \\
&\quad \left[j\cos\left(\frac{\alpha\pi}{2}\right)\left\{F\sin\left(hy+\frac{\alpha\pi}{2}\right)-\cos\left(hy+\frac{\alpha\pi}{2}\right)\right\}\hat{x}\right.
\end{aligned}
$$

$$- \left(\frac{j\gamma}{k}\right) \sin\left(\frac{\alpha\pi}{2}\right) \left\{F \cos\left(hy + \frac{\alpha\pi}{2}\right) + \sin\left(hy + \frac{\alpha\pi}{2}\right)\right\} \hat{y}$$
$$+ \left(\frac{jh}{k}\right) \sin\left(\frac{\alpha\pi}{2}\right) \left\{F \sin\left(hy + \frac{\alpha\pi}{2}\right) - \cos\left(hy + \frac{\alpha\pi}{2}\right)\right\} \hat{z}\Bigg] \qquad (14b)$$

It may be noted that for $\alpha = 0$

$$\mathbf{E}_{fd} = \mathbf{E}$$
$$\eta \mathbf{H}_{fd} = \eta \mathbf{H}$$

For $\alpha = 1$

$$\mathbf{E}_{fd} = \eta \mathbf{H}$$
$$\eta \mathbf{H}_{fd} = -\mathbf{E}$$

Which shows that both the fractional fields satisfy the duality principle.

4. TRANSVERSE IMPEDANCE OF FRACTIONAL WAVEGUIDE

Transverse impedance of the guide can be defined as

$$Z_{fdzx} = \frac{E_{fdz}}{H_{fdx}}$$
$$= j\eta \left(\frac{h}{k}\right) \frac{\left\{F \cos\left(hy + \frac{\alpha\pi}{2}\right) + \sin\left(hy + \frac{\alpha\pi}{2}\right)\right\}}{\left\{F \sin\left(hy + \frac{\alpha\pi}{2}\right) - \cos\left(hy + \frac{\alpha\pi}{2}\right)\right\}} \qquad (15a)$$
$$Z_{fdxz} = -\frac{E_{fdx}}{H_{fdz}}$$
$$= j\eta \left(\frac{k}{h}\right) \frac{\left\{F \cos\left(hy + \frac{\alpha\pi}{2}\right) + \sin\left(hy + \frac{\alpha\pi}{2}\right)\right\}}{\left\{F \sin\left(hy + \frac{\alpha\pi}{2}\right) - \cos\left(hy + \frac{\alpha\pi}{2}\right)\right\}} \qquad (15b)$$

where $F = (\frac{Z_w}{\eta})(\frac{-jk}{h})$.

Equation (15) can be analyzed for normalized impedance $z_w = \frac{Z_w}{\eta}$ of the fractional guiding surface e.g., $y = 0$ as

$$z_{fdzx} = j \left(\frac{h}{k}\right) \frac{\left\{z_w \left(\frac{-jk}{h}\right) \cos\left(\frac{\alpha\pi}{2}\right) + \sin\left(\frac{\alpha\pi}{2}\right)\right\}}{\left\{z_w \left(\frac{-jk}{h}\right) \sin\left(\frac{\alpha\pi}{2}\right) - \cos\left(\frac{\alpha\pi}{2}\right)\right\}} \qquad (16a)$$

$$z_{fdxz} = j\left(\frac{k}{h}\right)\frac{\left\{z_w\left(\frac{-jk}{h}\right)\cos\left(\frac{\alpha\pi}{2}\right)+\sin\left(\frac{\alpha\pi}{2}\right)\right\}}{\left\{z_w\left(\frac{-jk}{h}\right)\sin\left(\frac{\alpha\pi}{2}\right)-\cos\left(\frac{\alpha\pi}{2}\right)\right\}} \qquad (16b)$$

For the limiting cases, Equations (16a) and (16b) can be analyzed as

$$\alpha = 0 \Rightarrow \quad z_{fdzx} = -z_w, \qquad z_{fdxz} = -z_w\left(\frac{k}{h}\right)^2$$

$$\alpha = 1 \Rightarrow \quad z_{fdzx} = -\left(\frac{h}{k}\right)^2\frac{1}{z_w}, \quad z_{fdxz} = -\frac{1}{z_w}$$

For PEC walls i.e., $z_w = 0$

$$\alpha = 0 \Rightarrow \quad z_{fdzx} = 0, \quad z_{fdxz} = 0$$

$$\alpha = 1 \Rightarrow \quad z_{fdzx} = \infty, \quad z_{fdxz} = \infty$$

which is in agreement with our previous work. For intermediate values of α, Equations (16a) and (16b) have been discussed in the next section.

5. RESULTS AND DISCUSSION

Fractional impedances given in Equations (16a) and (16b) have been plotted for a mode having specific value of k/h (e.g., $\frac{k}{h} = 2$ in this case) in the range of $0 \le \alpha \le 4$. The plots are given in Figures 1–4. Fractional impedance have been found periodic with the period $\alpha = 2$. As given in the previous section, impedance value at $\alpha = 0$ is converted into the admittance at $\alpha = 1$. After that it reverses and goes back to the original value at $\alpha = 2$.

Figure 1 shows the plot of real part of Z_{fdxz} verses α. It can be observed that it has zero value for all values of α. As the the value of the normalized impedance, z_w, of the wall increases from zero, a sharp negative peak appears at $\alpha = 1,\ 3$ which broadens and the magnitude of peak value decreases with increasing z_w in the range $0 < z_w < \frac{h}{k}$. At $z_w = \frac{h}{k}$ it becomes independent of α and assumes a non zero constant value. In the range $z_w > \frac{h}{k}$, a negative peak starts appearing near $\alpha = 0,\ 2,\ 4$ and the width of the peak decreases while its magnitude increases with increasing value of z_w.

Plot of reactive part of Z_{fdxz} has been shown in Figure 2. This shows the oscillatory behavior of the imaginary part around zero value of reactance such that it reverses the polarity at integer values of α. At $z_w = 0$, it has infinite value at $\alpha = 1,\ 3$, and zero value at $\alpha = 0,\ 2,\ 4$.

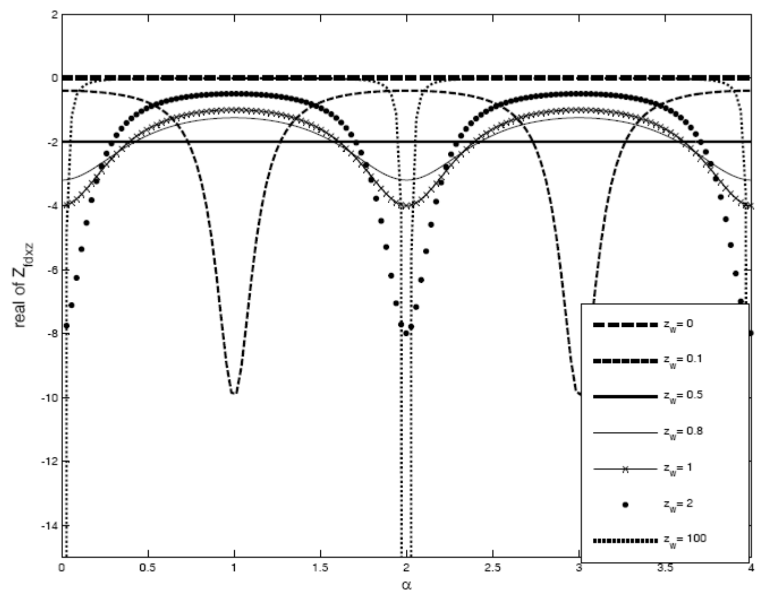

Figure 1. Real of Z_{fdxz} vs. α.

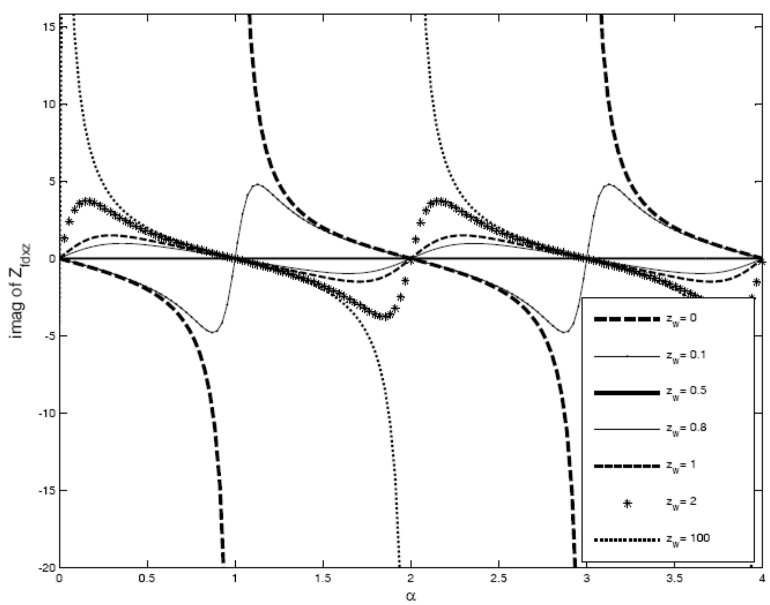

Figure 2. Imaginary of Z_{fdxz} vs. α.

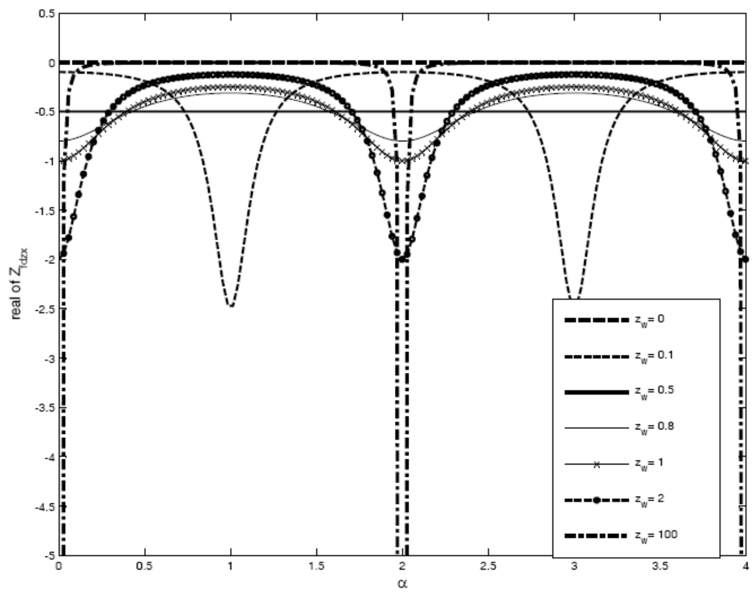

Figure 3. Real of Z_{fdzx} vs. α.

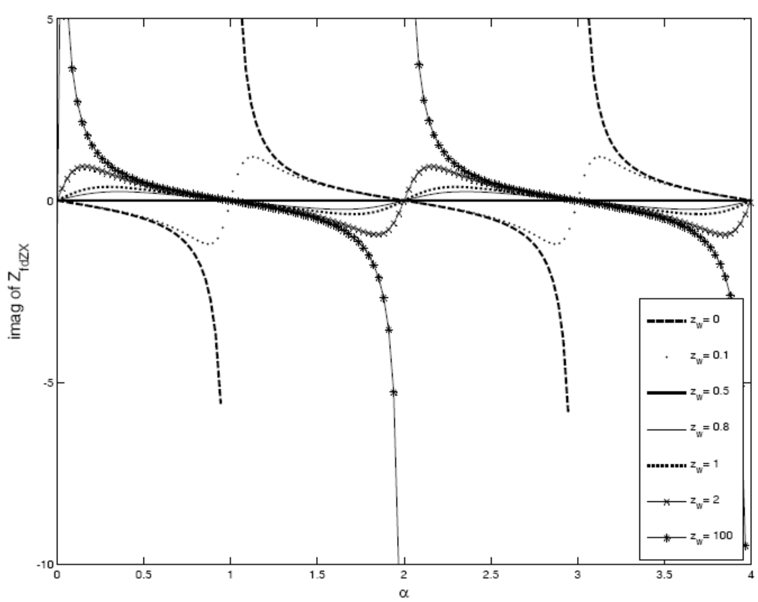

Figure 4. Imaginary of Z_{fdxz} vs. α.

In the range $0 < z_w < \frac{h}{k}$, the reactance is negative with decreasing absolute value for increasing value of z_w for $0 < \alpha < 1$ while behavior is reverse for $1 < \alpha < 2$. At $z_w = \frac{h}{k}$, reactance becomes zero for all values of α. In the range, $z_w > \frac{h}{k}$, the reactance becomes positive and increases with increasing value of z_w for $0 < \alpha < 1$ and reverses for $1 < \alpha < 2$.

Figures 3 and 4 are the plots of real and imaginary values of Z_{fdzx}. These figures show the behavior similar to Z_{fdxz} with difference only in magnitude.

6. CONCLUSION

For an impedance walls fractional parallel plate waveguide, two distinct ranges of the ordinary impedance have been found in which fractional impedance of the guiding walls behaves in different ways depending upon the value of the fractional parameter α. The behavior is periodic with period $\alpha = 2$. For $0 < \alpha < 1$, the impedance of walls is inductive for $0 \leq z_w < \frac{h}{k}$ and is capacitive for $z_w > \frac{h}{k}$. For $1 < \alpha < 2$, the impedance of walls is capacitive for $0 \leq z_w < \frac{h}{k}$ and is inductive for $z_w > \frac{h}{k}$. For $z_w = \frac{h}{k}$, the impedance is independent of α and is resistive.

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