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# FRACTURE MECHANICS OF CONCRETE STRUCTURES

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## MARKOV PROCESS MODEL FOR RANDOM GROWTH OF CRACK WITH R-CURVE

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Due to the statistical nature of material properties, crack growth is a random process. This process may be described by the Markov chain model (see Fig. 1). However, model must exhibit R-curve behavior. The basic relation for the Markov chain model (Bogdanoff and Kozin, 1985) is

$$\underset{\sim}{p}_x = \underset{\sim}{p}_0 \underset{\sim}{P}^X \tag{1}$$

in which $\underset{\sim}{p}_0$ is the initial state probability vector, $\underset{\sim}{p}_0 = \{\pi_1, \pi_2, \dots \pi_{B-1}, 0\}^T$, $\sum \pi_j = 1$, in which $\pi_j$ = Prob(damage state j is initially occupied). We assume $\pi_1 = 1$, with other $\pi_j = 0$, which means the crack (or damage) always starts from state 1; $\underset{\sim}{p}_x$ is the damage state probability, $\underset{\sim}{p}_x = \{p_x(1), p_x(2), p_x(B)\}^T$, where $p_x(j)$ = Prob(damage state j is occupied at stress level X); $\underset{\sim}{P}$ is the probability transition matrix,

$$\underset{\sim}{P} = \begin{bmatrix} p_1 & q_1 & 0 & \dots\dots 0 \\ 0 & p_2 & q_2 & \dots\dots 0 \\ 0 & 0 & p_3 & \dots\dots 0 \\ \dots\dots & \dots\dots & \dots & \dots \\ 0 & 0 & 0 \dots p_{B-1} & q_{B-1} \\ 0 & 0 & 0 \dots 0 & 1 \end{bmatrix} \tag{2}$$

where $p_i$ = probability of remaining in the state i during one loading step, and $q_i$ = probability that in one loading step the damage moves from state i to state i+1. The present model is a unit-jump, discrete-variable and state-dependent stationary process. $p_i$ and $q_i$ can be determined from the deterministic relation of stress and crack length and the deviation of this relation. The deterministic relation for the nominal stress may generally be written in the form:

$$\bar{X} = \frac{\sqrt{R(a - a_0)E_c}}{\sqrt{\pi a}\; F(a/d)} \tag{3}$$

where $\bar{X}$ represents the mean nominal stress (which is proportional to the applied load), $E_c$ is initial elastic modulus, $R(a - a_0)$ is the R-curve, $F(a/d)$ is a geometry dependent function, available in handbooks (e.g. Tada, 1983), a is the current crack length, and $a_0$ is the initial crack (or notch) length. The R-curve can be obtained from the size effect law proposed by Bažant (Bažant and Kazemi, 1990), calibrated by size effect measurements.

The variance at state j may be expressed approximately as a linear function of the crack length a,

$$\sigma_j^2 = \frac{(a_j - a_0)}{(a_{max} - a_0)} \sigma_{max}^2 \tag{4}$$

where $a_{max}$ can be obtained from Eq. 3. $\sigma_{max}^2$, representing the variance of the peak load, may be considered to be size independent, because the random scatter is mainly related to the size of the fracture process zone during the loading process and at ultimate state the fracture process zone size is almost independent of the structure size.

The formula for any state j can be derived from Eqs. 3,4

$$B_j = \frac{(\bar{X}_j - \bar{X}_{j-1})^2}{(\bar{X}_j - \bar{X}_{j-1}) + (\sigma_j^2 - \sigma_{j-1}^2)} + B_{j-1} \tag{5}$$

$$r_j = \frac{\bar{X}_j - \bar{X}_{j-1}}{B_j - B_{j-1}} - 1$$

where $p_i = r_i/(1+r_i)$, and $q_i = 1/(1+r_i)$.

Consider, now, a notched three-point-bend beam specimen of high strengtconcrete as an example. The details of the test can be found in Gettu (1990). The R-curve obtained from the peak loads is shown in Fig. 2. Fig. 3 shows the probability at each damage state and nominal stress. One can see that, for example, at loading level 61 (almost the peak load) the probability for the occurrence of the damage state 61 (almost the failure state) is very high, more than 90%. On the other hand, the probabilities for the occurrence of the lower damage states, 1 - 50, at the same loading level are almost zero, which is true in reality.

An advantage of present model is that the sample curve can be easily simulated by the computer. In this manner, the scatter band and the trend of damage evolution can be seen. Fig. 4 shows the sample curves of the relation between the crack extension and the loading level. One can clearly see that the generated sample curves represent the observed test curves quite well. This means that the present model can characterize the probabilistic structure for the entire loading history from the initial state up to the failure load.

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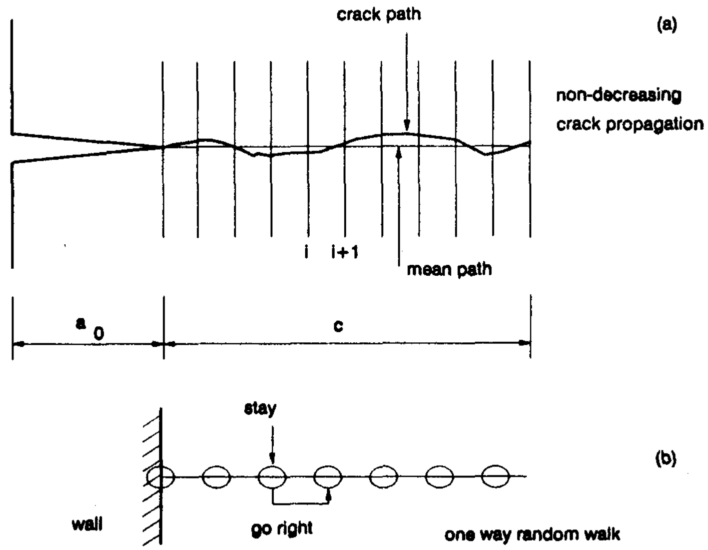


Fig. 1 One way random walk model

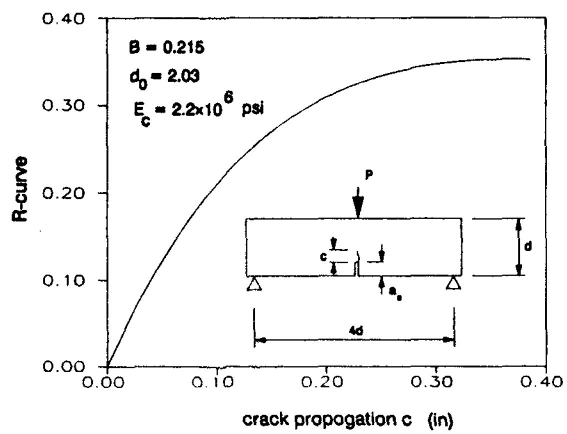


Fig. 2 R-curve

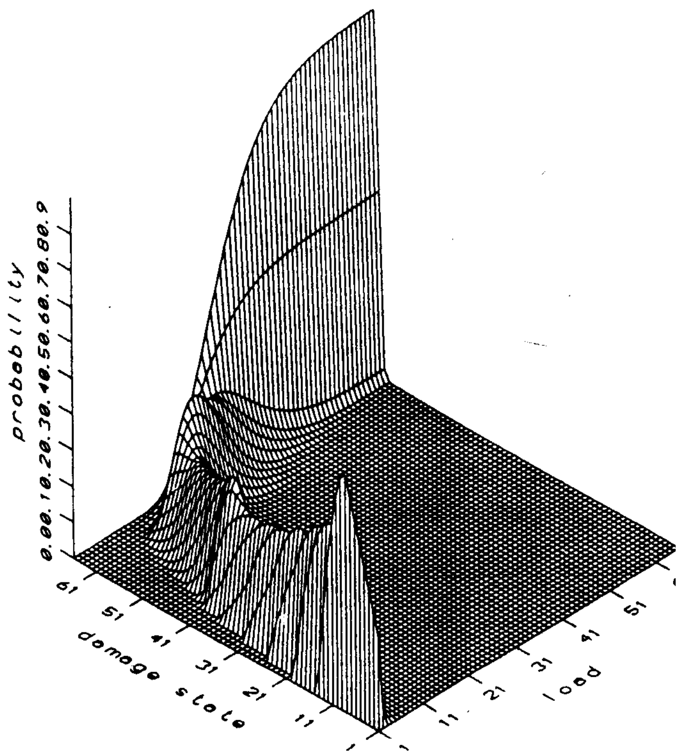


Fig. 3 Probability - loading level - damage state

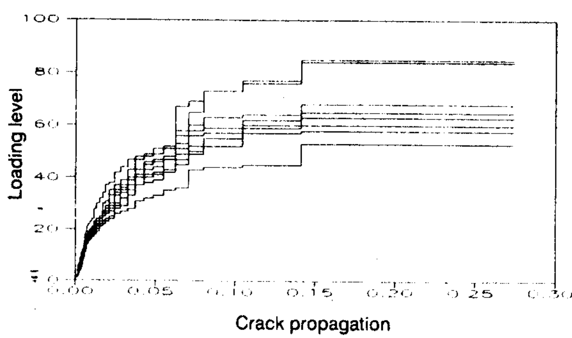


Fig. 4 Samples of load - crack propagation