Frame Consistency: Computing with Causal Explanations

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Abstract

This paper presents a computational model for reasoning with causal explanations of observations within the framework of Abductive Event Calculus (AEC). The model is based on abductive reasoning based on the notions of ”deserts” and ”oases” on the time line. Our work is motivated from the need to recover from the inconsistency that can arise when observations of fluents are added to the narrative of a domain description. We study how such observations can be assimilated via abductive explanations in order to render the domain *frame consistent*. Typically, such explanations would involve non-ground events whose time of occurrence can only be constraint within some interval. We present some notions of minimal commitment for such explanations and study how we can reason and compute with these explanations once they have been chosen and added to the theory. The computational model proposed can be readily implemented by exploiting, in a modular way, any of the different computational models for Abductive Logic Programming or for Answer Set Programming, augmented, again in a modular way, by suitable forms of temporal constraint solving.

Introduction

Agents operating within open environments need to reason about actions and change with partial information. As a result, it is possible that the narrative information, about event occurrences and fluent properties at certain time points (or situations), can lead to inconsistency as we accumulate these observations in our theory. In many cases, a way to recover from such an inconsistency is to add to the theory extra event occurrences that have not been observed but that would render the theory consistent if indeed they had occurred.

The choice of such *hypothetical* events can be guided strongly by the fluent properties that have been observed in the given narrative. Several works (e.g. (Shanahan 1997)) have suggested that we can draw such hypothetical events as *explanations* of the observed properties. This is particularly appropriate when the inconsistency arises from the frame persistence of a property, f, from one time point (or situation) to a later one at which the property has been observed not to hold. Indeed, this form of *frame inconsistency*, as we might call it, can be resolved by assuming that an (unknown) event has occurred which has the effect of $neg(f)$, thus explaining the observation of the falsity of the property. A framework well suited for capturing this notion of *causal explanations* is that of the Abductive Event Calculus (Eshghi 1988; Denecker, Missiaen, & Bruynooghe 1992; Shanahan 2000) where abductive explanations can be employed to recover from frame inconsistency.

In this paper, we study how to reason and compute with such abductive explanations once the reasoner has chosen an appropriate explanation. In particular, we highlight the important need for causal explanations to be as least committal as possible. As a result these will typically contain assumed *interval events* whose time of occurrence is not fully specified but rather it is non-deterministically constrained. Reasoning with such ”open” events and computing this reasoning becomes the focus of our work in this paper.

The abductive approach that we are adopting is used both for the formulation of the semantic notions of explanations and the reasoning with them and also for the computational model and implementation of this reasoning. In order to make this computational model more effective we exploit the central frame persistence property that the state of affairs remain unchanged over periods of time, called *deserts*, where no events have occurred allowing us to concentrate only on representative time points, *oases* points. The non-deterministic nature of interval events in the explanations makes these notions complicated but still viable. The computational model can be implemented using, in a modular way, any one of the recent systems of Abductive Logic Programming (ALP) or of Answer Set Programming (ASP) by exploiting the close correspondence (Satoh & Iwayama 1992) between ALP and ASP.

The rest of this paper is organized as follows. Section 2 presents background material on the (form of the) abductive event calculus that we will use. Section 3, formulates the semantic notions of frame consistency, explanations and reasoning with interval actions. Section 4 presents the computational model for reasoning with abductive explanations with its formal results and some implementation details. Section 5 concludes with a brief discussion of related work and plans for future work.

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$$
\begin{array}{rcl}
holds\_at(F,T) & \leftarrow & happens(A,T_1), T_1 < T, initiates(A,T_1,F), not\ clipped(T_1,F,T). \\
holds\_at(F,T) & \leftarrow & observed(F,T_1), T_1 \leq T, not\ clipped(T_1,F,T). \\
holds\_at(F,T) & \leftarrow & assume\_holds(F,0), not\ clipped(0,F,T). \\
holds\_at(neg(F),T) & \leftarrow & happens(A,T_1), T_1 < T, terminates(A,T,F), not\ declipped(T_1,F,T). \\
holds\_at(neg(F),T) & \leftarrow & observed(neg(F),T_1), T_1 \leq T, not\ declipped(T_1,F,T). \\
holds\_at(neg(F),T) & \leftarrow & assume\_holds(neg(F),0), not\ declipped(0,F,T). \\
clipped(T_1,F,T_2) & \leftarrow & happens(A,\ T), terminates(A,T,F), T_1 \leq T < T_2. \\
declipped(T_1,F,T_2) & \leftarrow & happens(A,T), initiates(A,T,F), T_1 \leq T < T_2.
\end{array}
$$

Figure 1: Domain independent knowledge

Extended Abductive Event Calculus

We will represent theories of Reasoning about Actions and Change (RAC) in an extended framework of Abductive Event Calculus (AEC) (Eshghi 1988; Denecker, Missiaen, & Bruynooghe 1992; Shanahan 2000) drawn by translating into Abductive Logic Programming (ALP) a simplified form of the language $\mathcal{E}$ (Kakas & Miller 1997b). The reason for choosing ALP as the underlying representation framework for RAC is twofold:

- abduction will facilitate the RAC reasoning and its computation in open domains with incomplete information about the fluents of the domain.
- abductive reasoning will also help us formalize notions of frame consistency and that of explanations of observations where now abduction is applied to incomplete information on the events that might have occurred in the world[1].

RAC theories, are therefore ALP theories of the form $KB_{TR} = \langle P_{TR}, A_{TR}, IC_{TR} \rangle$ where P_{TR} is a normal logic program, A_{TR} is a set of abducible predicates, and IC_{TR} is a set of integrity constraints. The latter specify which sets of assumptions drawn form A_{TR} are allowed, thus ruling out some hypothetical worlds from those modeled in P_{TR}. As observed later on, P_{TR} are locally stratified programs that always have a unique stable model.

In the following, $f, f_1, \ldots (neg(f), neg(f_1), \ldots)$ represent positive (negative) fluent literals, $a, a_1, ...$ events and $t, t_1, ...$ time points (which we will assume are natural numbers), while $T, T_1, ..., F, F_1, ..., A, A_1, ...$ are variables ranging over time points, fluents and events, respectively. A RAC theory $KB_{TR} = \langle P_{TR}, A_{TR}, IC_{TR} \rangle$ can be separated into a domain independent and a domain dependent part.

Domain Independent Knowledge. Figure 1, shows the general domain independent Event Calculus axioms in P_{TR} that formalize how fluents are caused by events and how they persist forwards in time. The predicate $holds\_at(f,t)$ represents the fact that the fluent f holds at the time point t. This may happen either because an event occurred in the past initiating the fluent, or the fluent has been observed in the past, or the fluent is assumed (abduced) to hold initially. We assume that 0 is the initial time point, where no event occurs. The predicate $assume\_holds(FluentLiteral, TimePoint)$ is the only abducible predicate in A_{TR} expressing the fact that we may have incomplete information on fluents. Note that for a restricted class of theories it is sufficient to allow (as we will in this paper) such assumptions at the initial time point 0 only.

The domain independent part of any KB_{TR} theory also contains in I_{TR} an integrity constraint, shown first in Figure 2(c), that enforces the general consistency on the theory (and any abducible assumptions) that a fluent and its negation can not hold at the same time point.

Domain Dependent Knowledge. Figure 2(a), shows the form of the general domain dependent knowledge in a theory KB_{TR}. These are facts and rules in P_{TR} modeling the specific world into consideration: $observed(f,0)$ sets initial conditions of the world, rules for $initiates$ and $terminates$ state how events affect fluents and under which preconditions. I_{TR} can also contain domain dependent constraints as shown in Figure 2(c) stating that the fluents literals $L_1, ..., L_n$ can not all hold together at the same time. Note that here these are represented as denials but any other classically equivalent form is also suitable.

Narrative Knowledge, KB_0. The domain dependent part of the knowledge contains also, within P_{TR}, a narrative part, denoted by KB_0. This has the form shown in Figure 2(b), to describe specific state of affairs. It represents the perceived state of the world in terms of fluents observed to hold at a (ground) time, and events that have happened at a given (ground) time. This information may result only in a partial view of the world, which may be an inconsistent view due to lack of information, as described Section 3. We will make the simplifying assumption that narratives do not contain concurrent events that at the same time initiate and terminate a fluent.

The above formulation of a RAC framework deals only with the frame problem. We can easily extend this to handle also the *ramification problem* by adopting a standard solution, e.g (Thielscher 1997; Kakas & Miller 1997a; Kakas, Miller, & Toni 2001), to incorporate causal ramification statements in the domain dependent part of the domain description. Hence to express the causal relationship between the set of fluent literals, $l_1, ..., l_n$, and the fluent literal, l, we can have statements of the form:

$$
generated(l,T) \leftarrow generated(l_i,T), holds\_at(l_1,T+1), ..., holds\_at(l_n,T+1).
$$

[1] ALP also provides a natural framework in which to compute explanations but we will not be concerned with this issue in this paper. See for example (Shanahan 2000; Kakas, Michael, & Mourlas 2000) where planning in the AEC can form a basis for computing explanations.

$$\begin{array}{rcl} initiates(a,T,f) & \leftarrow & holds\_at(f_1,T),\ldots,holds\_at(f_k,T). \\ terminates(a,T,f) & \leftarrow & holds\_at(f_1,T),\ldots,holds\_at(f_k,T). \\ observed(f,0). & & \end{array}$$

(a)

$$\begin{array}{l} happens(a,t). \\ observed(l,t). \end{array}$$

(b)

$$\begin{array}{rl} holds\_at(F,T), holds\_at(neg(F),T) & \Rightarrow \bot \\ holds\_at(L_1,T),...,holds\_at(L_n,T) & \Rightarrow \bot \end{array}$$

(c)

Figure 2: Domain dependent knowledge, Narration and Integrity Constraints

for any $i : 1 \leq i \leq n$. The base case of the $generate$ predicate is given (in the domain independent part) by:

$$\begin{array}{l} generated(F,T) \leftarrow happens(A,T), initiates(A,T,F). \\ generated(neg(F),T) \leftarrow happens(A,T), \\ \qquad\qquad terminates(A,T,F). \end{array}$$

We can then replace appropriately in the domain independent axioms (see Figure 1) all occurrences of $happens(A,T), initiates(A,T,F)$ by $generated(F,T)$, and those of $happens(A,T), terminates(A,T,F)$ by $generated(neg(F),T)$.

As we will see below, the assumptions that we will adopt on the form of explanations and the type of reasoning with these will not be affected directly by ramification statements - the issue is somewhat orthogonal - and therefore we will not give any further details on this.

As regards the qualification problem the situation is different. This problem is not currently understood well enough (with the notable exception of (Thielscher 2001)) to be able to address it in our framework and the underlying computational model that we are developing. In addition, the qualification problem has an intricate link with the problem of frame consistency and hence further study is needed for their integration especially when we wish to do this at the computational level.

Given a theory KB_{TR} we can reason about what fluents may or may not hold at different time points as follows.

Definition 1 ($\models^{cred}_{TR}$). *Given a (ground) fluent $f[t]$, a theory KB_{TR}* credulously entails *the fluent, $KB_{TR} \models^{cred}_{TR} f[t]$, iff there exists a set Δ of (ground) atoms in the predicates in A_{TR} such that*

$$P_{TR} \cup \Delta \models_{LP} holds\_at(f,t) \quad \textit{and} \quad P_{TR} \cup \Delta \models_{LP} I_{TR},$$

with $\models_{LP}$ a semantics of normal LP, which we choose to be the stable model semantics (i.e. $P \models_{LP} C$ means classical truth of the clause C in a stable model of P).

We note here that the form of the logic programs, P_{TR}, are locally stratified (a level mapping can be constructed based on the time of the $holds\_at$ predicate at the head of the rules - see e.g. (Shanahan 1997)) and hence $P_{TR} \cup \Delta$ always has a unique stable model. Note that when we have ramification statements for this local stratification property to hold we also need to impose that these statements are acyclic on the ground fluent literals that they related.

Based on this credulous form of reasoning, $\models^{cred}_{TR}$, we can define a skeptical entailment, $\models^{skep}_{TR}$, as follows:

Definition 2 ($\models^{skep}_{TR}$). *Given a (ground) fluent $f[t]$, a temporal reasoning theory KB_{TR}* skeptically entails *the fluent, $KB_{TR} \models^{skep}_{TR} f[t]$, iff*

$$KB_{TR} \models^{cred}_{TR} f[t] \quad \textit{and} \quad KB_{TR} \not\models^{cred}_{TR} \overline{f}[t],$$

where $\overline{f}$ is the (idempotent) negative instance of f.

Example 1. Let us consider a variation of the well-known example of the parked car (Kautz 1986) with the following domain dependent knowledge together with an empty KB_0:

$$\begin{array}{l} initiates(park(Car),T,parked(Car)) \leftarrow \\ \qquad holds\_at(free\_place,T). \\ terminates(park(Car),T,free\_place). \end{array}$$

Without any information about the current state of the world, we can conclude credulously, for any time point now, that $KB_{TR} \models^{cred}_{TR} holds\_at(free\_place, now)$, since it is possible to assume $assume\_holds(free\_place, 0)$, without violating any constraint. Instead reasoning skeptically, $KB_{TR} \not\models^{skep}_{TR} holds\_at(free\_place, now)$, given the fact that it is also possible to assume that the place is not available, and then conclude that $KB_{TR} \models^{cred}_{TR} holds\_at(neg(free\_place), now)$.

Note that above we have made several simplifying assumptions on the form of the RAC theories and the queries for their temporal projection. Amongst these are the fact the assumption that a given theory is consistent and that the events in the narrative refer to specific ground times. We will see in the next section how we need to relax these two assumptions and extend suitably the reasoning.

Frame Consistency and Causal Explanations

Any RAC theory KB_{TR} with an empty narrative is always consistent provided that its integrity constraints I_{TR} are classically consistent at every time point. When the narrative KB_0 is non-empty though the theory may become inconsistent.

We can distinguish two types of inconsistency: *classical inconsistency* where KB_0 together with the integrity constraints I_{TR} alone is inconsistent and *frame inconsistency* where KB_0 is classically consistent with I_{TR} (at every time point) but becomes inconsistent (at some time point(s)) when we consider the whole KB_{TR} thus allowing the persistence of fluents from one time point to another. In this paper we will concentrate on the problem of frame inconsistency.

A *frame consistent* theory KB_{TR} is then a theory such that there exists a set of assumptions Δ such that $P_{TR} \cup \Delta \models_{LP} I_{TR}$. This notion of frame (in)consistency can easily be reformulated as follows.

Definition 3 (Frame consistency). *Let be KB_{TR} be a temporal reasoning theory, Obs the subset of KB_0 $\{observed(l,t)|observed(l,t) \in KB_0\}$, and $KB'_{TR} = KB_{TR} \setminus Obs$. Then KB_{TR} is* frame consistent *iff $KB'_{TR} \models^{cred}_{TR} Obs$.*

This means that a frame consistent theory is such that it is possible to find at least one set of assumptions, Δ, on the abducible predicate $assume\_holds$ (i.e. a possible state of affairs) that satisfies, under the general frame persistence laws, the integrity constraints of the theory coherently with the given observations on fluents.

Example 2. Adding to the theory KB_{TR} of Example 1 a narrative KB_0 containing both $observed(free\_place, 3)$ and $observed(neg(free\_place), 3)$ makes it classically inconsistent. On the other hand, adding a narrative containing both $observed(free\_place, 0)$ and $observed(neg(free\_place), 10)$ makes it frame inconsistent.

Intuitively speaking, while the first narrative appears intrinsically contradictory, the second one is typically explained by assuming that in between 0 and 10 someone has occupied the parking space, i.e. an unknown event has occurred that can render the theory frame consistent. This frame inconsistency resolving event comes as an explanation of (some of) the observations in the hitherto inconsistent narrative. Indeed, the above formulation of frame consistency motivates (as already proposed in the literature (Shanahan 1989; 1997)) the use of *causal explanations* of the observations as a general method to recover from frame inconsistency.

The basic notion of a causal explanation is formalized as follows. Within the framework of our AEC we extend the set of abducible predicates A_{TR} to a new set, A^{Ext}_{TR}, that contains also the predicate $assume\_happens(EventType, TimePoint)$. This would allow us to make abductive hypotheses that events have occurred. The extended set of abducible hypotheses, A^{Ext}, now has also sentences of the form $assume\_happens(e, T^*)$ where e is an event constant and T^* is a time point or an existentially quantified variable. Given this extension and the bridge rule:

$$happens(A,T) \leftarrow assume\_happens(A,T).$$

we have the following definition (the extended theory is referred to as KB^{Ext}_{TR}).

Definition 4. *Let KB_{TR} be a temporal reasoning theory, Obs^s a chosen (fluent) set of observations of this and $KB'_{TR} = KB_{TR} \setminus Obs^s$. Then an* explanation, *$\langle E(Obs^{inc}), C\rangle$, of Obs^s is a set of abducibles $E(Obs^s) \subseteq A^{Ext}_{TR}$, together with an associated set of temporal constraints C, on the time variables appearing in $E(Obs^s)$ such that, for every assignment π of these variables into the natural numbers satisfying C, (written $C \models \pi$)*

$$KB'^{Ext}_{TR} \cup E(Obs^s)\pi \models^{cred}_{TR} Obs^s.$$

Clearly, if a theory is frame consistent then for any subset of its observations there exists an explanation which does not contain any hypotheses on $assume\_happens$. Note also that we can use instead a stronger notion of explanation where we require that the observations are a skeptical consequence of the theory KB'^{Ext}_{TR} when extended with the explanation.

There are two important issues to consider when generating such causal explanations: (i) how are the observations selected and (ii) which of the many possible explanations are more significant. We will only address these briefly in this paper and concentrate more on how we reason and compute this reasoning once we have chosen an explanation.

With regards to the first question clearly the subset Obs^s selected should be such that $KB'_{TR} = KB_{TR} \setminus Obs^s$, is frame consistent. In practice, observations may be collected in some sequence and hence it may be relatively easy to notice when the theory first becomes frame inconsistent and attribute this to the last observations. This does not necessarily mean that it is the last observations that need explaining but it is a good heuristic for how to choose the observations to be explained.

There are several "qualities" that we may require from our causal explanations. Apart from the fact that we want the time of the assume events to be constraint to be before that of the observations, hence the name *causal*, the other major property that we would require is that the explanation is *least committal*, in the sense that it tries to minimize any extra conclusions that it would impose on the theory. There are two general restrictions on the explanations that we can adopt for this. First we introduce in the language for each fluent constant, f, two abstract event types $start(f)$ and $stop(f)$ with the simple effect rules:

$$initiates(start(f), T, f).$$
$$terminates(stop(f), T, f).$$

and restrict our $assume\_happens$ hypotheses to refer only to these. In other words we allow only $assume\_happens$ hypotheses of the form $assume\_happens(stop(f), T^*)$ or $assume\_happens(start(f), T^*)$ for any ground fluent f and T^* a time point or an existentially quantified variable. This means that our explanations will not cause any other new effects apart from the ones we are trying to explain (and their ramifications if we have such statements in the theory).

Example 3. Consider again Example 2 in which the narrative KB_0 contains both $observed(free\_place, 0)$ and $observed(neg(free\_place), 10)$.

The last observation can be explained by assuming $assume\_happens(stop(free\_place), T^*)$, with T^* constrained to be between 0 and 9. Note that this explanation does not say exactly how the $free\_space$ was terminated, e.g. by an event of parking or by a no-parking sign etc., thus been least committal.

Secondly, we can impose a notion of *minimality* on the explanations particularly with the way that the unknown time of the occurrence of the assumed events is constrained. Informally, minimality means that explanations are composed of a minimal number of assumptions, whose time constraints are as general as possible.

Definition 5 (<<). *Given an explanation $\langle E(Obs^s), C\rangle$ and f a fluent, let $N(E, start(f))$ be the number of $assume\_happens(start(f), T')$*

in $E(Obs^s)$ *and* $N(E, stop(f))$ *the number of* $assume\_happens(stop(f), T')$ *in* $E(Obs^s)$. *Then an explanation,* $\langle E(Obs^s), C\rangle$ *is* smaller than *another one,* $\langle E(Obs^s)', C'\rangle$*, denoted by* $\langle E(Obs^s), C\rangle << \langle E(Obs^s)', C'\rangle$*, iff*

- *for all fluents* f*,* $N(E, X(f)) < N(E', X(f))$ *(with* $X = start$ *and* $X = stop$*)*[2]*, or*
- *for all fluents* f*,* $N(E, X(f)) = N(E', X(f))$*, and* $\forall\ \pi$ *time assignments:* $C' \models \pi \Rightarrow C \models \pi$*, i.e. all the admissible time points for the bigger explanation are also admissible for the smaller one.*

Definition 6. *An explanation* $\langle E(Obs^s), C\rangle$ *is* minimal *for* Obs^s *iff there exists no explanation* $\langle E(Obs^s)', C'\rangle$ *such that* $\langle E(Obs^s)', C'\rangle << \langle E(Obs^s), C\rangle$.

Given these two restrictions on the explanations, in many cases (but not always) we can have a unique minimal explanation for an observation on a fluent f consisting of only *one* $stop(f)$ or $start(f)$ event with the interval constraint for this event maximal.

Example 4. The frame consistent narrative $KB_0 = \{observed(free\_place, 0).\ \ happens(park(my\_car), 3).\}$ of Example 2 becomes frame inconsistent when updated with $observed(neg(parked(my\_car)), 10)$ as in (Kautz 1986). A causal explanation of the last observation, which is minimal, is:

$\langle assume\_happens(stop(parked(my\_car)), T), T \in [4, 9]\rangle$.

Example 5. Starting again from Example 2, suppose that it is known that initially there is not space, and hence the attempt to park at 3 has not the desired effect (as confirmed by the later observation at 10):

$$KB_0 = \begin{array}{l}\{\ observed(neg(free\_place, 0)).\\ \ \ happens(park(my\_car), 3).\\ \ \ observed(neg(parked(my\_car)), 10).\ \}\end{array}$$

However, the latest $observed(free\_place, 2)$ makes the theory frame inconsistent and *both* the facts $free\_space$ at 2 and $neg(parked(my\_car))$ at 8 must be explained. A minimal explanation is

$$\begin{array}{l}\langle\ \{assume\_happens(start(free\_space), T1),\\ \ \ \ assume\_happens(stop(parked(my\_car)), T2)\},\\ \ \ \ \{T1 \in [0, 1], T2 \in [4, 9]\}\ \rangle.\end{array}$$

Finally, explanation selection could ultimately depend on domain specific criteria such as heuristic preferences or *crucial experiments* (see e.g. (Sattar & Goebel 1991)) namely actively looking for further observations that would discriminate between different explanations.

Reasoning with Explanations

From this point onward in this paper we will assume that an explanation is chosen that recovers frame consistency and that the reasoner will adopt this as part of its theory. Until and if further information invalidates this the events in the explanation are treated equivalently as the known events. How do we then reason we these *interval events* whose time is not fixed to a specific ground time point?

[2]Several variations of this condition are possible.

An assignment π that fulfills the constraint store C, i.e $C \models \pi$, associated to interval events in an extended narrative, is called *placement*. Reasoning then needs to consider all allowed placements thus covering all possible interleavings of these events between them and with the existing ground narrative.[3] Placing interval events according to different placements can lead to completely different scenarios, even if the placements fulfill the same total order. This is due to the way in which placed events interfere with the existing one and the fluents that hold or do not hold in that specific time point. We will see below in section 4 how we can employ computational techniques that can reduce significantly the number of placements that need to be considered in our reasoning.

In this context, we interpret credulous reasoning as the existence of a placement of the interval events in our extended theory which allows the fluent to be credulously proved against the grounded theory resulting from the placement. Skeptical reasoning, instead, is interpreted as the fact that all the placements fulfilling the constraints allow the fluent to be skeptically proved against the grounded theory. Intervals are represented as constraints, in terms of the binary operators $\leq$ and $<$ defined over ground time points and existentially quantified variables. For instance $\{s \leq T, T \leq e\}$ stands for $T \in [s, e]$. Constraints, in a constraint store, can also relate different variables, e.g. $\{s \leq T_i, T_i \leq e, T_i \leq T_j\}$.

Definition 7 (KB^+_{TR}). *A temporal reasoning theory is an* extended *theory, written* KB^+_{TR}*, if its narrative contains a set of non-ground predicates and a temporal constraint store*

$$\begin{array}{ll}\exists\, T_1 \ldots, T_n.\ C(T_1, \ldots, T_n). & happens(a_1, T_1).\\ & \vdots\\ & happens(a_n, T_n).\end{array}$$

such that the variables $T_1, \ldots, T_n$ *are quantified from the outside over all the constraints in* $C(T_1, \ldots, T_n)$*. A predicate* $happens(a, T)$*. is called an* interval event.

The application of a placement to an interval event is given by $happens(a, T)\pi = happens(a, T\pi)$, and it transforms an extended theory KB^+_{TR} into a ground KB_{TR} one. The formal notion of entailment for KB^+_{TR} in terms of that for theories KB_{TR} with ground narratives is defined as follows.

Definition 8 ($\models^{cred}_{TR^+}$). *Given a theory* KB^+_{TR}*, a placement* π *and a ground fluent literal* $f[t]$*, then*

$$KB^+_{TR} \models^{cred}_{TR^+} f[t] \quad \Leftrightarrow \quad \exists\, \pi\ KB^+_{TR}\pi \models^{cred}_{TR} f[t].$$

Hence a credulous conclusion now depends on the extra dimension of a placement π as well as the existence of a set of assumptions Δ from A_{TR}. Note that the set of abducibles does not contain $assume\_happens$ now whose role ends with the generation and selection of an explanation.

[3]The problem of determining (all) the placements that fulfill the constraint set C can be addressed with efficient constraint solving techniques, like the ones presented in (Dechter, Meiri, & Pearl 1991; VanBeek 1992).

Building on credulous reasoning, as for KB_{TR} theories, we define skeptical conclusions as follows where all possible placements need to be considered.

Definition 9 ($\models^{skep}_{TR^+}$). *Given a theory KB^+_{TR} and a ground fluent literal $f[t]$, then*

$$KB^+_{TR} \models^{skep}_{TR^+} f[t] \quad \Leftrightarrow \quad \forall\, \pi\ KB^+_{TR}\pi \models^{cred}_{TR} f[t]\ \wedge$$
$$KB^+_{TR}\pi \not\models^{cred}_{TR} \overline{f[t]}.$$

where π is a placement and $\overline{f[t]}$ is the (idempotent) negation of $f[t]$.

Example 6. Consider Example 4, with the frame consistent narrative:

$$KB_0 = \begin{array}{l} \{\ observed(free\_place, 0). \\ \ \ happens(park(my\_car), 3). \\ \ \ observed(neg(parked(my\_car)), 10). \\ \ \ happens(stop(parked(my\_car)), T).\{T \in [4, 9]\}\ \}. \end{array}$$

In this theory $holds\_at(neg(parked(my\_car)), time)$ holds skeptically for any time $time \in [10, \infty)$. But for $time \in [5, 9]$ we can show credulously both $holds\_at(parked(my\_car), time)$ and also $holds\_at(neg(parked(my\_car)), time)$, since for any of the two fluent literals and a given time point, a placement that entails it exists. Hence none of literals can be proved skeptically at these times.

Computing with Explanations

We will now assume that theories are frame consistent where possibly a chosen causal explanation has been added to it to render it so. We will then develop a computational model for such theories based on the underlying abductive reasoning of the ALP framework in which these theories are expressed. We start with the notion of *deserts and oases*, introduced in (Kakas, Miller, & Toni 2001), to help us compute with theories that have a ground narrative and then we generalise the model to narratives that can contain interval events with their existentially quantified variables and temporal constraints over them.

Deserts and Oases. Given a (consistent) theory with a ground narrative, the properties that hold within a *desert*, i.e. an interval without event occurrences, are identical at all the time points in the desert. This will allows us to restrict our attention to a finite set of time points called *oasis* points and hence ground (partially) the theory when computing with this. The next Propositions 1 allows us to ground the integrity constraints of the theory.

Definition 10. *Given a narrative, KB_0 and a ground fluent literal $f[t_{n+1}]$ (corresponding to the query $holds\_at(f, t_{n+1})$), the relative* time line, *TL, is the (maximal) totally ordered sequence of ground instants*

$$TL = [0 = t_0, t_1, ..., t_n, \{t_{n+1}\}],$$

where $\forall\ i \in [1, \ldots, n]\ \exists\ happens(a, t_i) \in KB_0$. (Note $\{t_{n+1}\}$ is added to the time line iff t_{n+1} is a time instant beyond the last event in KB_0).
Each interval $[t_i + 1, t_{i+1}]$ is called a desert, *and each point t_i is an* oasis.

Proposition 1. *Given a theory KB_{TR}, a fluent literal $f[t]$, a desert $[t_i + 1, t_{i+1}]$ in the relative time line, and a set $\Delta \subset A_{TR}$ then $\forall\ t \in [t_i + 1, t_{i+1}]$:*

$$P_{TR} \cup\ \Delta \models_{LP} holds\_at(f, t)$$
$$\Leftrightarrow$$
$$P_{TR} \cup\ \Delta \models_{LP} holds\_at(f, t_i + 1)$$

$$P_{TR} \cup \Delta \models_{LP} I_{TR}(t) \Leftrightarrow P_{TR} \cup \Delta \models_{LP} I_{TR}(t_i + 1)$$

where $I_{TR}(t_i + 1)$ is obtained from I_{TR} by grounding all the universally quantified (time) variables to the value $t_i + 1$.

Proposition 2. *Given a theory KB_{TR}, a fluent literal $f[t]$, t_i the oases, and a set $\Delta \subset A_{TR}$ of abducibles then:*

$$P_{TR} \models^{cred}_{TR} holds\_at(f, t)$$
$$\Leftrightarrow$$
$$\exists\ \Delta \quad P_{TR} \cup \Delta \models_{LP} holds\_at(f, t)\ \wedge$$
$$\forall\ i = 0, ..., n \quad P_{TR} \cup \Delta \models_{LP} I_{TR}(t_i + 1),$$

Basic computational model. Given Proposition 2, we can build a computational model for credulously, and hence also skeptically, reasoning since the right-hand side has a procedural interpretation under an ALP abductive proof procedure. This computational model can be described simply as in Figure 3.

Example 7. Given the narrative of Example 2 $KB_0 = \{observed(free\_place, 0).\ happens(park(my\_car), 3).\}$, and the query $holds\_at(parked(my\_car), 6)$, the oasis points are $\{1, 4, 7\}$ and the ground integrity constraints are:

$$\begin{array}{l} holds\_at(F, 1), holds\_at(neg(F), 1) \rightarrow \bot \\ holds\_at(F, 4), holds\_at(neg(F), 4) \rightarrow \bot \\ holds\_at(F, 7), holds\_at(neg(F), 7) \rightarrow \bot \end{array}$$

In this settings the query can then be credulously and skeptically proved.

We note that we can also use this computational model to answer non-ground queries. Lack of space prevents us from giving the details. In the example above, the existentially quantified query $holds\_at(parked(my\_car), T)$, $\exists\ T \in [2, 5]$ (as well as its universally quantified version) can be examined, by considering the extremes of the interval $[2, 5]$ as extra oases points and checking that the ground query holds at one of (all) the oases falling within the interval $[2, 5]$. The query holds at $T = 4$, and so the existential query succeeds but the universal one fails since the query at $T = 3$, the new oasis 2 (plus one), does not hold.

Essential Placements. According to Definition 8, entailment for theories with interval events in their narratives depends on the existence of a placement, i.e. an ordering of the interval events that makes the narrative ground. The basic computational model introduced above can be extended to theories KB^+_{TR} that include interval events by considering only a restricted class of *essential placements*. For simplicity of presentation, we present the credulous case only, on which the skeptical one is straightforwardly defined.

An effective computational model for KB^+_{TR} requires us to extend the partition of the time line into deserts and oases to include as oases the extremes of the intervals of interval events and the time-points of the observations. This is so because the placement of an event relatively to the query time point and the observations can change the state of affairs.

$query\_credulous\_TR(\langle P_{TR}, A_{TR}, I_{TR}\rangle, KB_0, holds\_at(f,t), AbducedPredicates) \leftarrow$
$extract\_oases(KB_0, t, Oases),$
$instantiate\_constraints(I_{TR}, Oases, OI_{TR}),$
$prove\_ALP(\langle P_{TR}, A_{TR}, OI_{TR}\rangle, holds\_at(f,t), AbducedPredicates).$
$prove\_ALP(\langle P_{TR}, A_{TR}, OI_{TR}\rangle, KB_0, holds\_at(f,t), Delta) \leftarrow$ Calls any abductive proof procedure (a simple ALP proof procedure for ground abduction suffi ces for the basic model).

Figure 3: Basic computational model

Definition 11. *Given a query literal $f[t]$ and an extended narrative containing a set of interval events referring to intervals $[s_j, e_j]$, the relative* extended time line ETL *is the (maximal) totally ordered sequence*

$$ETL = [0 = t_0, t_1, ..., t_n],$$

where $\forall\, i\ (\exists\, observed(f, t_i) \in KB_0 \vee \exists\, happens(a, t_i) \in KB_0 \vee t_i = t \vee \exists\, j\ t_i = s_j \vee t_i = e_j)$.

We now give the definition of *essential placement* and show how proofs can be computed by considering only essential placements, which are finite in number. We will denote by $\bigcirc$ the (commutative and associative) composition operator for substitutions of assignments and by ϵ the empty substitution.

Definition 12 (Essential placement). *Given a theory KB^+_{TR}, in which m interval events (i.e. m temporal variables) occur with the associated temporal constraints C, a ground fluent literal $f[t]$, and the relative extended time line $ETL = [0 = t_0, t_1, ..., t_n]$, then a placement π is* essential *iff $\pi = \bigcirc_i \pi_i$, with $i \in [0, n]$, and*

1. $\forall\, i, j \in [0, n]\ dom(\pi_i) \cap dom(\pi_j) = \emptyset\ \wedge$ $\forall\, i \in [1, m]\ \pi(T_i) \in [0, \infty)$,
2. $C \models \pi$,
3. $\forall\, i\ \pi_i = \epsilon\ \vee$ $\exists\, k, o\ \pi_i : [T_{i_1}, \ldots, T_{i_k}] \rightarrow [t_i + 1, \ldots, t_i + 1 + o]$, *with* $o \leq k\ \wedge\ t_i + o \leq t_{i+1}$.

Intuitively speaking, the definition requires that an essential placement can be partitioned in n placements, one for each desert, such that

1. they are disjoint and π is total over time variables of interval events in KB^+_{TR},
2. π fulfills the partial order induced by temporal constraints in KB^+_{TR}, and
3. each π_i either
   - is the empty substitution (no events are mapped in that desert), or
   - maps, in any order, a set of k temporal variables into $o \leq k$ contiguous time points starting at the beginning ($t_i + 1$) of the desert, and not spilling out into the next desert.

It is easy to see that the definition of essential placement encompasses all the possible orders in which interval events can be placed at the beginning of a desert, respecting the temporal constraints C, but without imposing any further ordering on the placement of interval events (which hence can be interleaved in all the possible ways that respect time constraints).

Extended Computational Model. The definition of essential placement satisfies the following proposition, directly leading to the computational model for extended theories.

Proposition 3. *Let KB^+_{TR} be a theory, $f[t]$ a ground fluent literal, π a placement, and $\tilde{\pi}$ an essential placement, then*

$$\exists\, \pi\ KB^+_{TR}\pi \models^{cred}_{TR} f[t] \quad \Leftrightarrow \quad \exists\, \tilde{\pi}\ KB^+_{TR}\tilde{\pi} \models^{cred}_{TR} f[t]$$

Proposition 3 gives directly a computational model for credulously reasoning with interval events, $\models^{cred}_{TR^+}$, as defined in Definition 8, by restricting attention to essential placement. The prolog-like specification of Figure 4 provides a procedure which, given an extended theory and a fluent, proves the fluent by building, if any, an essential placement that gives a ground narrative under which the fluent is entailed. Again this exploits in a modular way any underlying ALP proof procedure.

This extended computational model contains two non-deterministic choices:

i a total order of temporal variables fulfilling the minimal partial order, MPO, of C

ii a partition of the totally ordered time variables into deserts,

Note that i. enforces constraint fulfillment, and i. plus ii. determine the parameters k and o of point 3.(b) of Definition 12. Then, values are assigned to variables defining the actual placement, which by construction is an essential placement.

Given a sound and complete underlying ALP proof procedure this computational model for $KB^+_{TR}$is itself sound and complete. This is formalized in Figure 4 for credulous reasoning. For skeptical reasoning we have analogous results. Note also that quantified queries can be treated similarly (as in the basic computational model) but are not detailed here.

Definition 13 ($\vdash^{cred}_{TR^+}$). *Given a non-ground theory KB^+_{TR}, the* computational model *for $\models^{cred}_{TR^+}$, indicated as $\vdash^{cred}_{TR^+}$, is defined as follows:*

$$KB_{TR} \vdash^{cred}_{TR^+} f[t]$$
$$\Leftrightarrow$$
$$query\_credulous\_extended\_TR(KB^+_{TR}, holds\_at(f,t))$$
succeeds.

Theorem 1 ($\models^{cred}_{TR^+} \Leftrightarrow \vdash^{cred}_{TR^+}$). *Assuming that the abductive proof procedure used in $query\_credulous\_extended\_TR/2$ is correct and complete, the computational model $\vdash^{cred}_{TR^+}$ is correct and complete with respect to $\models^{cred}_{TR^+}$:*

$$KB^+_{TR} \models^{cred}_{TR^+} f[t] \ \Leftrightarrow\ KB^+_{TR} \vdash^{cred}_{TR^+} f[t].$$

$query\_credulous\_extended\_TR(KB^+_{TR}, holds\_at(f,t), AbducedPredicates) \leftarrow$
$extract\_extended\_oases(KB_0, t, ETL)$,
$extract\_minimal\_partial\_order(ETL, C, MPO)$,
$generate\_a\_total\_order(MPO, TotalOrder)$,
$generate\_a\_partition\_of\_total\_order\_over\_oases(TotalOrder, ETL, TOPartition)$,
$generate\_an\_essential\_placements\_from\_a\_partition(TOPartition, EssentialPlacement)$,
$apply\_essential\_placement(KB^+_{TR}, EssentialPlacement, GroundTheory)$,
$prove\_ALP(GroundTheory, holds\_at(f,t), AbducedPredicates)$.

Figure 4: Extended computational model

Example 8 illustrates a case of non-ground theories and an existential query for the credulous and skeptical case.

Example 8. [Computing with explanations] Let us reconsider Example 6, where KB_0 =

$\{observed(free\_place, 0).\ happens(park(my\_car), 3).$
$observed(neg(parked(my\_car)), 10).\}$

has been extended with a (minimal and causal) explanation of the last observation:

$\langle assume\_happens(stop(parked(my\_car)), T'),\ T' \in [4, 9]\rangle$.

The extended time line, considering the existential query $holds\_at(parked(my\_car, T)), \{T \in [4, 20]\}$, is $[0, 3, 4, 9, 10, 20]$. It also includes the extremes of the query interval. The partial order induced by the constraints $4 \leq T,\ T \leq 20$ and $4 \leq T',\ T' \leq 9$ does not impose any ordering between the two variables, then the following three total orders can be generated in order to define the possible essential placements.

$\mathbf{T' < T}$ There are four essential placements fulfilling this total order and the set of constraints: $\tilde{\pi}_1 = \{T' = 5, T = 6\}$, $\tilde{\pi}_2 = \{T' = 5, T = 10\}$, $\tilde{\pi}_3 = \{T' = 5, T = 11\}$, and $\tilde{\pi}_4 = \{T' = 5, T = 21\}$. Let us consider $\tilde{\pi}_3$, and the corresponding ground narrative:

$\{observed(free\_place, 0).\ happens(park(my\_car), 3).$
$observed(neg(parked(my\_car)), 10).$
$happens(stop(parked(my\_car)), 5)\}$

In this case, the query $holds\_at(parked(my\_car, 11)$, which has been made ground, can not be credulously (and hence also skeptically) proved. Similarly, for the cases of $\tilde{\pi}_1$, $\tilde{\pi}_2$ and $\tilde{\pi}_4$.

$\mathbf{T' = T}$ The only possible essential placement is $\tilde{\pi}_5 = \{T' = 5, T = 5\}$:

$\{observed(free\_place, 0).\ happens(park(my\_car), 3).$
$observed(neg(parked(my\_car)), 10).$
$happens(stop(parked(my\_car)), 5)\}$

In this case the query $holds\_at(parked(my\_car, 5))$, now ground, can be credulously proved against the ground theory Hence, the essential placement $\tilde{\pi}_5$ allows the existentially quantified query to be proved against a non-ground theory, according to the definition of $\models^{cred}_{TR^+}$.

$\mathbf{T' > T}$ Again there is only one essential placement, $\tilde{\pi}_6 = \{T' = 6, T = 5\}$:

$\{observed(free\_place, 0).\ happens(park(my\_car), 3).$
$observed(neg(parked(my\_car)), 10).$
$happens(stop(parked(my\_car)), 6)\}$

against which, the query $holds\_at(parked(my\_car, 5))$ can be proved credulously.

On the other hand, as we have seen that it is not the case that for all the possible essential placements the correspondence query holds, e.g. the case of $\tilde{\pi}_3$, it follows that the existentially quantified query does not hold skeptically.

Related work and Conclusions

We have presented a computational model for reasoning with causal explanations in RAC theories. In particular, we have shown how this computational model can be applied to reasoning with interval events, where their time of occurrence cannot be known exactly, that arise naturally in such explanations.

This work was motivated by the need to address the problem of frame inconsistency in open problem domains where the information available in the theory is incomplete. The problem of frame inconsistency and the proposal to use causal explanations to address it is a relatively old one. The stolen car scenario (Kautz 1986) and the Stanford murder mystery (Baker 1989) exposed the problems in a simple way. Several works have addressed the problem in terms of causal explanations for assimilating observations. In particular, the works of (Shanahan 1989; Denecker, Missiaen, & Bruynooghe 1992) base this on the Abductive Event Calculus within the Abductive Logic Programming framework as we have in this paper.

Clearly, our work is also strongly related to the problem of planning and in particular abductive planning where there has been a lot of activity, e.g. (Eshghi 1988; Missiaen, Bruynooghe, & Denecker 1995; Shanahan 2000; Kakas, Michael, & Mourlas 2000; Finzi, Pirri, & Reiter 2000). But planning relates more to the problem of computing the explanations rather than reasoning with them once they are found. Typically, in planning such reasoning is not done until after the plan has been executed when we reason with the ground events of the executed planned actions and not the interval events originally in the plan.

Reasoning with explicit narratives is at the heart of the Event Calculus framework and has also been studied within several other frameworks that are based on the Situation Calculus (see e.g. (Pinto & Reiter 1995; Baral, Gelfond, & Provetti 1997)) by suitably extending their ontologies to include the occurrence of actions. With the exception of (Shanahan 1997) we are not aware of other work which attempts to addresses in some detail the problem of reasoning and computing with interval events. This work uses circumscription to formalize the reasoning but does not address in a systematic way, as we have, the formulation of a general computational model to compute this in a viable way. In fact, many recent works on RAC concentrate on other issues and avoid the problem of reasoning with interval actions by assuming that the given theories are frame consistent and that the known events are the only events that have occurred in the world. Our work puts an emphasis on providing a viable computational model for narratives with interval events linking this with temporal constraint solving.

An important characteristic of our computational model is that this is a general model which can exploit in a *modular* way any given computational method or system for Abductive Logic Programming (ALP). Through the equivalence between the ALP framework (for normal logic programs) and Answer Set Programming this modular exploitation of general systems can be extended also to the use of ASP systems. We are currently implementing our model using an ALP system and using this to provide the temporal reasoning module of an autonomous agent (Kakas *et al.* 2003a; 2003b). A full implementation requires the integration with a constraint solver to handle the time constraints on interval events. This is currently under investigation as is also the possibility of implementing our model using ASP systems and using this as a testbed for comparison between ALP and ASP systems in order to examine the problem conditions best suited for each approach.

Another important problem for future work is the study of the link between the problem of frame inconsistency, as we have addressed in this paper, with the qualification problem (Thielscher 2001) where we cannot assume that once an event has occurred its effects will necessarily be generated. This opens several interesting possibilities least of all being the basic problem of deciding when it is appropriate to attribute the apparent frame inconsistency to an unknown event (and hence look for causal explanations) or to the failure of an already executed action to generate its effect.

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