Volume : 3 | Issue : 6 | June 2014 • ISSN No 2277 - 8179

Free Convection Boundary-Layer Flow of A Nanofluid Past A Vertically Stretching Sheet with Momentum, Thermal and Solutal Slip

Mathematics

KEYWORDS : Nanofluid fluid, Free convection, velocity slip, thermal slip and Solutal slip

M. Subhas Abel	Department of Mathematics, Gulbarga University, Gulbarga-585 106, Karnataka, India
Monayya Mareppa	Department of UG and PG Studies in Mathematics, Government College Yadgir-585 201, Karnataka, India
Mahesha Narayana	School of Mathematical Sciences, University of KwaZulu-Natal Private Bag X01, Scottsville 3209, Pietermaritzburg, South Africa.
Jagadish V Tawade	Department of Mathematics, Bheemanna Khandre Institute of Technology Bhalki-585 328, Karnataka, India Corresponding author

ABSTRACT

Boundary-layer flow of a nanofluid past a stretching sheet with momentum, thermal, and solutal, slip conditions have been investigated numerically by Runge-Kutta shooting technique. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. A similarity solution is presented which depends primarily on Prandtl number Pr, Lewis number Le, Brownian motion number Nb, and thermophorsis number Nt. The variation of the reduced Nusselt number and reduced Sherwood number with Nb and Nt for various values of Pr and Le is presented in tabular and graphical forms. Comparison with published results is presented.

1. INTRODUCTION:

During the last many years, the study of boundary layer flow and heat transfer has achieved a lot of success because of its large number of applications in Science and Engineering disciplines. Some of these applications include materials manufactured by the polymer extrusion, drawing of copper wires, continuous stretching of plastic films, artificial fibres, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning or electronic chips, and many others. A large number of researchers engaged with this area of investigation, for many years.

In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching. After the pioneering work by Sakiadis [1], a large amount of literature is available on boundary layer flow of Newtonian and non-Newtonian fluids over linear and nonlinear stretching surfaces [2–10]. However, only a limited attention has been paid to the study of boundary layer flow of nano fluids over a stretching surface.

Most conventional heat transfer fluids, such as water, ethylene glycol, and engine oil, have limited capabilities in terms of thermal properties, which, in turn, may impose, serve restrictions in many thermal applications. On the other hand, most solids, in particular, metals, have thermal conductivities muchhigher, say, by one to three orders of magnitude, compared with that of liquids. Hence, one can then expect that fluid containing solid particles may significantly increase its conductivity. Many of the publications on nanofluids are about understanding of their behaviors so that they can be utilized, where straight heat transfer enhancement is paramount as in many industrial applications, nuclear reactors, transportation, electronics as well as biomedicine and food (see Ding et al. [11]). Nanofluid is asmart fluid, where the heat transfer capabilities can be reduced or enhanced at will. These fluids enhance thermal conductivity of the base fluid enormously, which is beyond the explanation of any existing theory. They are also very stable and have no additional problems, such as sedimentation, erosion, additional pressure drop and non-Newtonian behavior, due to the tiny size of nano elements and the low volume fraction of nano elements required for conductivity enhancement. Much attention has been paid in the past decade to this new type of composite material because of its enhanced properties and behavior associated with heat transfer, mass transfer, wetting and spreading and antimicrobial activities, and the number of publications related to nanofluids increases in an exponential manner. The enhanced thermal behavior of nanofluids could provide a basis for an enormous innovation for heat transfer intensification, which is of major importance to a number of industrial sectors including transportation, power generation, micro-manufacturing, thermal therapy for cancer treatment, chemical and metallurgical sectors, as well as heating, cooling, ventilation and air-conditioning. Nanofluids are also important for the production of nanostructured materials, for the engineering of complex fluids, as well as for clean fluid.

They have shown that even with small volumetric fraction of nanoparticles (usually less than 5%) the thermal conductivity of base liquid increased by 10-50% with remarkable improvement in heat transfer co-efficient [see references (12-14)]

These authors discussed about the convective transport in nanofluids [see references (15-18)]. They studied the natural convective flow of a nanofluids over a vertical plate and their similarity analysis is identical with four parameters governing the transport process , namely a lewis number , a Buoyancy- ratio Number Gr, a Brownian motion number Nb, and a thermophoresis number Nt , A.V. Kuznetsov, D.A. Nield [19]. The study of boundary layer flow of a nanofluid past a stretching sheet with a constant surface temperature, has been considered by these author[20].

The objective of the present study is to analyze the development of the free convective steady boundary layer flow, heat transfer and nanoparticle volume fraction over a vertically stretching surface in a nanofluid. A similarity solution is presented. This solution depends on prandtl number Pr, Lewis number Le, Grashof number Gr, Brownian motion number Nb thermophoresis number Nt, momentum slip parameter, thermal slip parameter, solutal slip parameter. The dependency of the local Nusselt and local Sherwood numbers on these four parameters is numerically investigated, To our best of Knowledge, the results of the paper are new and they have not been published before.

2. MATHEMATICAL FORMULATION

We consider a steady incompressible laminar two dimensional boundary layer flow of a nano fluid past a vertical stretching sheet with linear velocity. A steady uniform stress leading to equal and opposite forces is applied along *x*-axis, so that sheet is stretched keeping origin fixed. A uniform magnetic field of strength B_0 is imposed along the *y*-axis. The schematic of the physical configuration is displayed in Fig. 1. Under the usual boundary layer approximation, the basic governing equations can be taken in the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\chi\left(T-T_\infty\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

Here *x* and *y* are respectively the directions along and perpendicular to the surface, *u*, *v* are the velocity components along the *x* and *y* directions respectively, *T* is the temperature of liquid, *C* is the nanoparticle fraction and other terms have their usual meanings as given in the nomenclature. Equations (1) - (4) are supplemented with the following boundary conditions

$$\begin{aligned}&u-u_w(x)=L\frac{\partial u}{\partial y},\quad v=0,\quad T-T_w(x)=k_1\frac{\partial T}{\partial y},\quad C-C_w(x)=k_2\frac{\partial C}{\partial y}\quad \text{at}\quad y=0,\\&u\to 0,\quad T\to T_\infty,\quad C\to C_\infty\quad \text{as}\quad y\to\infty,\end{aligned} \tag{5}$$

Where,

$$u_w(x)=ax,\qquad T_w(x)=T_\infty+b\left(\frac{x}{l}\right)^2\quad \text{and}\quad C_w(x)=C_\infty+c\left(\frac{x}{l}\right)^2$$

Using the following similarity transformations

$$\left.\begin{aligned}&u=axf'(\eta),\quad v=-(a\upsilon)^{\frac{1}{2}}f(\eta),\quad \eta=\left(\frac{a}{\upsilon}\right)^{\frac{1}{2}}y,\\&\theta(\eta)=\left(\frac{T-T_\infty}{T_w-T_\infty}\right),\quad \phi(\eta)=\left(\frac{C-C_\infty}{C_w-C_\infty}\right),\end{aligned}\right\} \tag{6}$$

equations (2) – (4) can be written as

$$f'''=ff''-f'^2-Gr\theta, \tag{7}$$

$$\theta''+\Pr\left(f\theta'+N_b\phi'\theta'+N_t\,\theta'^2\right)=0, \tag{8}$$

$$\phi''+Lef\phi'+\frac{N_t}{N_b}\theta''=0. \tag{9}$$

Here we note that the continuity equation (1) holds automatically with the definitions given in (6). Using equation (6) the boundary conditions (5) can be written as

$$\begin{aligned}&f(0)=0,\quad f'(0)=1+\beta f''(0),\quad \theta(0)=1+\gamma\theta'(0),\quad \phi(0)=1+\delta\phi'(0),\\&f'(\infty)\to 0,\quad \theta(\infty)\to 0,\quad \phi(\infty)\to 0.\end{aligned}$$

Here $N_b\to 0$ implies that there is no thermal transport due to the buoyancy effects formulated as a result of nano particle concentration gradients. The non-dimensional parameters appearing in equations (7) – (10) are the Grashof number *Gr*, Magnetic parameter *M*, Prandtl number *Pr*, the Brownian motion parameter N_b, the thermophoretic parameter N_t, the Lewis number *Le* and the slip parameter γ and they are defined respectively as follows:

$$Gr=\frac{g\chi\left(T_w-T_\infty\right)}{a^2x},\qquad \Pr=\frac{\upsilon}{\alpha},\qquad N_b=\frac{\tau D_B\left(C_w-C_\infty\right)}{\upsilon},$$

$$N_t=\frac{\tau D_T\left(T_w-T_\infty\right)}{\upsilon T_\infty},\qquad Le=\frac{\upsilon}{D_B}$$

$$\beta=L\sqrt{\frac{a}{\nu}},\gamma=k_1\sqrt{\frac{a}{\nu}},\delta=k_2\sqrt{\frac{a}{\nu}} \tag{11}$$

3. NUMERICAL SOLUTION

The non-linear differential equations (7) to (9) with boundary conditions (10) are solved numerically by the shooting technique that uses fourth order Runge-Kutta and Newton-Raphson methods. The non-linear differential equations are first decomposed into to a system of first order differential equations

(12)

$$\left.\begin{aligned}&\frac{df_0}{d\eta}=f_1,\\&\frac{df_1}{d\eta}=f_2,\\&\frac{df_2}{d\eta}=f_1^2-f_0f_2-Gr\theta_0,\\&\frac{d\theta_0}{d\eta}=\theta_1,\\&\frac{d\theta_1}{d\eta}=-\Pr\left(f_0\theta_1+N_b\phi_1\theta_1+N_t\theta_1^2\right)=\theta_2,\\&\frac{d\phi_0}{d\eta}=\phi_1,\\&\frac{d\phi_1}{d\eta}=-\left[Le\,f_0\,\phi_1+\left(\frac{N_t}{N_b}\right)\theta_2\right],\end{aligned}\right\}$$

with the boundary conditions

$$\left.\begin{aligned}&f_1(0)=1+\beta f_2(0),\quad \theta(0)=1+\gamma\theta_1(0),\quad \phi(0)=1+\delta\phi_1(0).\\&f_2(\infty)=0,\quad \theta_0(\infty)=0\,,\quad \phi_0(\infty)=0.\end{aligned}\right\} \tag{13}$$

In order to integrate equations (8), (9), (10), with boundary conditions (11), We require values of $f'(0)$ and $\theta(0)$,and $\phi(0)$, but no such values are available at the boundary. The important factor of shooting method is to choose the appropriate values of η_∞ .In order to determine η_∞ for boundary value problem stated by equations, (8) to (11), we start with some guess values for some particular set of parameters to obtain $f'(0)$, $\theta(0)$, and $\phi(0)$. The solution procedure is repeated with another large value of η_∞ until two successive values of $f'(0)$,$\theta(0)$ and $\phi(0)$ differ only by the specified significant digit. The last value of η_∞ is finally chosen to be the most appropriate value of the limit η_∞ for that particular set of parameters. The value of η_∞ may change for another set of physical parameters. Once the finite value of η_∞ is determined then the integration is carried out. We compare the calculated values of $f',\theta,\ and\phi(0)$ at $\eta=10$ (say) with the given boundary conditions $f'(10)=0$,

$\theta(10)=0, and\ \phi(10)=10.$ and adjust the estimated values of $f'(0)$,$\theta(0)\ and\ \phi(0)$, to give a better approximation for the solution. We take the series of values of $f'(0)$,$\theta(0)\ and\phi(0)$, and apply the fourth order Runge-Kutta method with step size 0.01. The above procedure is repeated until we get the results upto the desired degree of accuracy, 10^{-6}.

4. RESULTS AND DISCUSSION.

To provide a physical insight into the problem, comprehensive numerical computations are carried out for various values of the parameters that describe the flow, heat and mass transfer characteristics with nanoparticles. The parameters Gr Pr ,Le, Nb ,Nt , β, γ, δ, are involved in the final form of the mathematical model. The problem can be extended on many directions, but the primary consideration, is the effects of slip flow, free convection and nanoparticle volume fraction. In order to bring out the salient features of the flow, heat and mass transfer characteristics, the numerical results are presented in Figs. 2–11,and in Tables 1–3.

Volume : 3 | Issue : 6 | June 2014 • ISSN No 2277 - 8179

Equations 8 -10 with boundary conditions (11) have been solved numerically for various values of the parameters Gr Pr ,Le, Nb ,Nt , β , γ , δ ,using Runge-Kutta shooting method, which is stable and convergent .The most crucial factor of this numerical solution is to choose the appropriate finite value of η_∞ Thus, the asymptotic boundary conditions given by (13) were replaced by a comparatively large value $\eta_{\max} = 10$,for the similarity variable $\eta_{\max}$. The choice of $\eta_{\max} = 10$ ensured that all numerical solutions approached to the asymptotic values correctly. It is worth mentioning to consider that the choice of a large value for $\eta_{\max}$ is an important point that is often overlooked in publications on boundary layer flows.

The result for the reduced Nusselt number -$\theta'(0)$ are compared with those of Wang[22] Gorla and Sidawi[21] and Khan and Pop[20] in **Table 1**. It is observed that comparison shows good agreement for each value of Pr. Hence it is justified that our results are correct to the best of our knowledge.

The variation of reduced Nusselt number (Nur)and Sherwood number (shr) with Nb and Nt for Pr =10, Le =10 are presented in tables 2(a) and 2(b).

It is observed from the above tables that, Nur is a decreasing function and Shr is an increasing function , for the parameters Pr, Le, Nb, and Nt and our results match very well with the results of Khan and Pop [20]. Further, as in Kuznetsov and Nield [18], simple linear multiple regression estimations Nur_{est} and Shr_{est} of the reduced Nusselt number and reduced Sherwood number are also obtained, which incorporate the effects of Brownian motion parameter Nb and thermophoresis parameter Nt. These linear regression estimations can be written as

$Nur_{est} = Nur + C_b Nb + C_t Nt$;

$Shr_{est} = Shr + C_b Nb + C_t Nt$ (14)

The results of these estimations with the regression coefficients and the maximum relative errors ε and γ are calculated as ε =|(Nur_{est} – Nu)/ Nu | and γ =| (Shr_{est} – Sh)/Sh |, see Kuznetsov and Nield [11]. Further it is noticed that for most practical purposes, the simple linear regression formulas in Equation (14) is enough to justify the resultant analysis.

Table 1: Comparison of results for the reduced Nusselt Number -$\theta'\theta'(0)$

Pr	Present results Nb=Nt=0	Wang[22]	Gorla and Sidawi[21]	Khan and Pop[20]
0.07	0.087151	0.0656	0.0656	0.0663
0.2	0.172349	0.1691	0.1691	0.1691
0.7	0.453918	0.4539	0.5349	0.4539
2	0.911358	0.9114	0.9114	0.9113
7	1.89542	1.8954	1.8905	1.8954
20	3.354154	3.3539	3.3539	3.3539
70	6.459435	6.4622	6.4622	6.4621

(a) Variation of Nur with Nb and Nt for Pr=5

Nb=0.1		Nb=0.2		Nb=0.3		Nb=0.4		Nb=0.5	
Nt	-Nur	Nt	-Nur	Nt	-Nur	Nt	-Nur	Nt	-Nur
0.1	1.52571	0.1	1.49294	0.1	1.50153	0.1	1.48506	0.1	1.46862
0.2	1.51757	0.2	1.50151	0.2	1.48507	0.2	1.46867	0.2	1.45234
0.3	1.50137	0.3	1.48505	0.3	1.46114	0.3	1.45233	0.3	1.43601
0.4	1.48503	0.4	1.46113	0.4	1.45228	0.4	1.43601	0.4	1.41987
0.5	1.46862	0.5	1.45228	0.5	1.43607	0.5	1.41987	0.5	1.40375

(b) Variation of Shr with Nb and Nt when Pr=5

Nb=0.1		Nb=0.2		Nb=0.3		Nb=0.4		Nb=0.5	
Nt	Shr	Nt	Shr	Nt	Shr	Nt	Shr	Nt	Shr
0.1	2.232335	0.1	1.068073	0.1	0.433837	0.1	0.308759	0.1	0.233694
0.2	2.883622	0.2	1.401455	0.2	0.906707	0.2	0.65933	0.2	0.510934
0.3	4.303617	0.3	2.102545	0.3	1.361134	0.3	1.001745	0.3	0.781566
0.4	5.689995	0.4	2.772253	0.4	1.819651	0.4	1.335961	0.4	1.045893
0.5	7.042962	0.5	3.455619	0.5	2.260107	0.5	1.662334	0.5	1.303739

Fig 2, depicts the plot of velocity profile f' versus, dimensionless distance η for different values of Grashof number Gr, and it is noticed that velocity increase, with the increase of Grashof number Gr, physically Gr > 0 means heating of fluid or cooling boundary Surface and Gr< 0, means cooling of fluid or heating of the surface and Gr = 0 implies absence of free convection current .

Fig 3, projects the effect of slip parameter β on the transverse and axial velocity profiles. It is readily seen that the amount of slip $1 - f_\eta(0)$ increases monotonically with β from the no-slip situation $\beta = 0$ towards full slip as $\beta \to \infty$. The latter limiting case implies that the frictional resistance between the cooling liquid and the stretching sheet is eliminated, and the stretching of the sheet does no longer impart any motion to the cooling liquid. Clearly, β has a substantial effect on the flow of the liquid past stretching sheet .This slip parameter is a function of the local Reynolds number, the local Knudsen number, and the tangential momentum accommodation coefficient representing the fraction of the molecules reflected diffusively at the surface. As the slip parameter increases, the slip velocity increases and the wall shear stress decreases. These results match with the conclusions reached in other recent studies.

The effect of Pr on temperature profile is exhibited in **fig 4.** It is noticed from fig 4,that temperature decrease with increasing values of Pr . An increase in Pr reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity .Fluids with lower prandtl number passes higher conductivities (and thicker thermal boundary layer structures) so that heat can diffuse from wall faster than higher Pr fluids so the property of prandtl number is to control the rate of cooling in conducting flows .

Fig 5 depicts that as the thermal slip parameter γ , increases temperature in the thermal boundary layer is progressively enhanced. There is sudden overshoot in temperature between

γ = 0.6, and γ =0.7,where It can also be seen that the thickness of the thermal boundary increases as the flow becomes more rarefied.

The effect of Le on nanoparticle volume fraction profile is shown in fig(6) for constant values of Pr, Gr, Nt, Nb. It is observed from fig. 6, that as Le increases nanoparticle volume fraction profile decreases .It is to be noticed that increasing values of Le, lowers the concentration of the nanoparticle in the boundary layer region which in turn results in reducing heat transfer rates . Hence the value of Le must be maintained at minimum value to have a situation of conducive for cooling of the stretching sheet. Further it is noticed that the effect of Le numbers on the dimensionless mass transfer rates is that increase in dimensionless mass transfer rates is monotonic for larger Le numbers.

Figs. 7 and 8 shows the variation in nanoparticle volume fraction profile vs Nb and Nt for the selected values of other parameters . It is clear from fig. 7, that there is decrease nanoparticle volume fraction profile with the increase in Nt. Like dimensionless heat transfer rates, the change in the mass transfer rates is higher for smaller values of the parameter Nb and decreases with the increase in the parameter Nb. However, for large values of Pr numbers, the dimensionless mass transfer rates increase with the increase in Nt and decrease in Nb.

The effects of β, γ, δ, on nanoparticle volume fraction profile is shown in figs, 9, 10 and 11 for constant values of Pr, Gr, Nt, Nb respectively and it is noticed that as β increases nanoparticle volume fraction profile increases, which means that there is thickening of concentration boundary layer thickness , resulting in enhancement of fluid temperature where as the values of γ and δ increases, keeping values of all the other parameters fixed, it is noticed that nanoparticle volume fraction profile decreases, which attributes to the fact that, there is thinning of concentration boundary layer thickness, resulting in reduction of fluid temperature.

5. CONCLUSION:

The problem of laminar free convective flow resulting from the stretching of a flat, vertically surface in a nanofluid with,Momentum, thermal and solutal slip conditions have been investigated numerically. The model used for the nanofluid incorporates primarily the effects of Brownian motion, thermophoresis, momentum slip, thermal slip, and solutal slip parameters respectively.A similarity solution is presented which depends on the Prandtl number Pr, Lewis number Le, Brownian motion parameter Nb and thermophoresis parameter Nt. The variation of the reduced Nusselt and reduced Sherwood numbers with Nb and Nt for various values of Pr and Le is presented in tabular form.We can find that the inclusion of the slip parameters change the wall drag force greatly. Linear regression estimations of the reduced Nusselt and reduced Sherwood numbers are also obtained in terms of Brownian motion and thermophoresis parameters.

The following are the important results of the present investigation:

1. The effect of Grashof number and momentum slip parameter is to enhance velocity in the boundary layer, where as thermal slip parameter enhances temperature of the fluid in the thermal boundary layer region.
2. The Grashof number accelerates the fluid velocity in boundary layer region.
3. Because of the Brownian and thermophoresis effects, a jump in the heat transfer rate is observed in case of nanofluid which points to the fact that nanofluids are better suited for effect cooling of the stretching sheet.
4. Prandtl number decreases thermal boundary layer thickness and Lewis number decreases the nanoparticle concentration, in the boundary layer. Hence, these parameters are to be kept at their minimum for better cooling conditions.
5. The effect of momentum slip is to increase nanoparticle concentration, in concentration boundary layer, where as the effect of thermal slip and solutal slip is to reduce nanoparticle concentration, in concentration boundary layer.

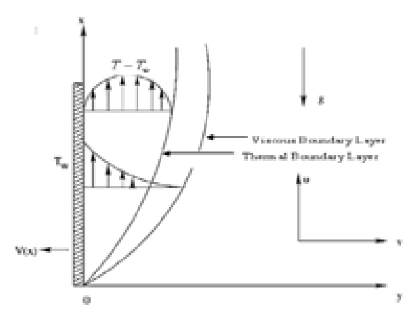

Fig 1. Schematic diagram of a nano fluid flow over a vertical stretching sheet

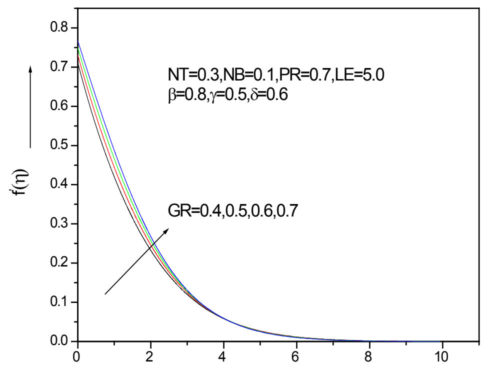

Fig - 2
The behaviour of velocity profile for various values of Gr.

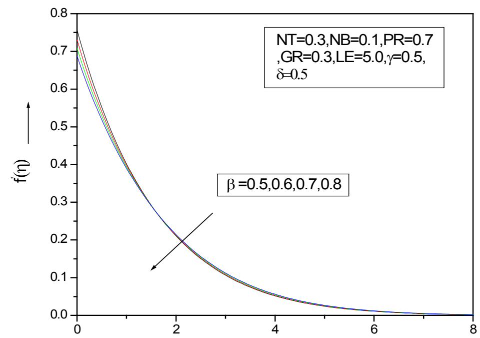

fig-3
The behaviour of velocity profile for various values ofβ

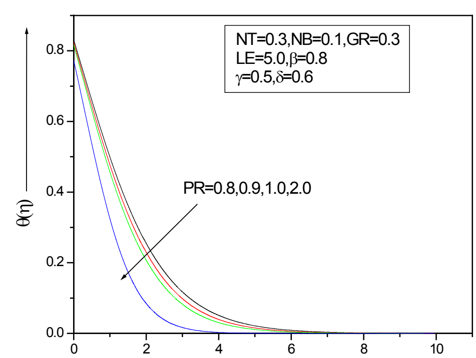

Fig - 4
The behavior of temperature profile for different values of Pr.

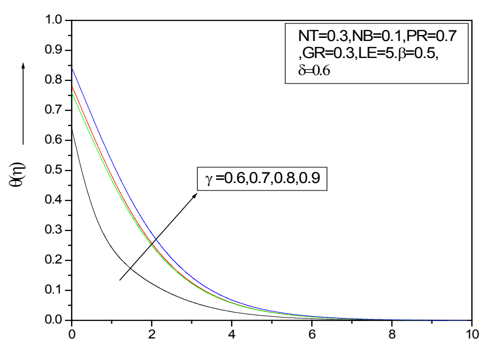

Fig- 5
The behaviour of temperature profile for various values of γ

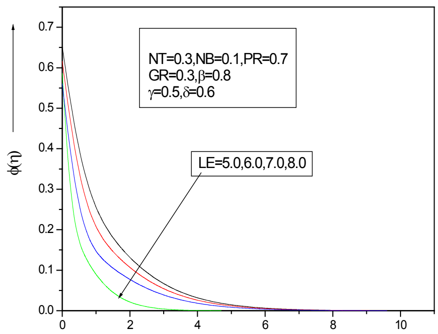

fig-6

The behaviour of nanoparticle volume fraction profile for various values of LE

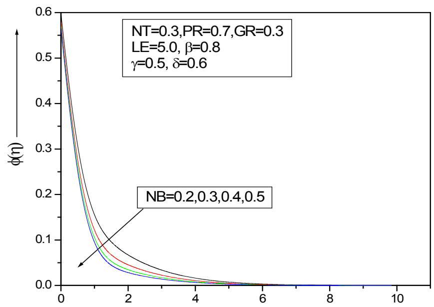

fig-7 η

The behaviour of Nano particle volume fraction profile for various values of NB

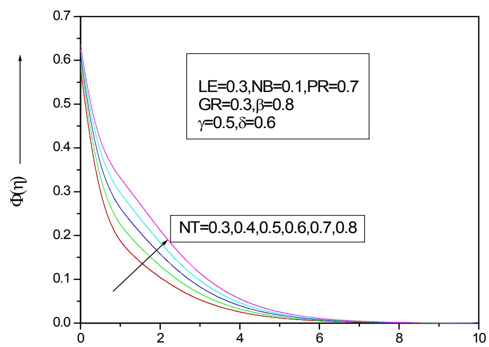

Fig - 8 η

The behaviour of Nano particle volume fraction profile for various values of NT

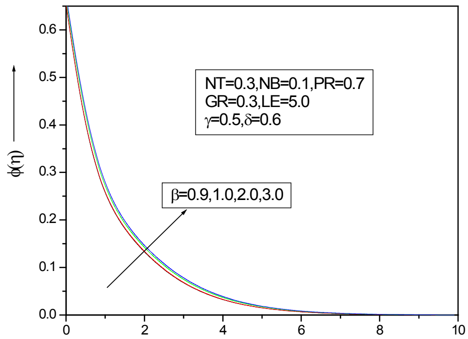

Fig-9 η

The behaviour of Nanoparticle volume fraction profile for various values of β

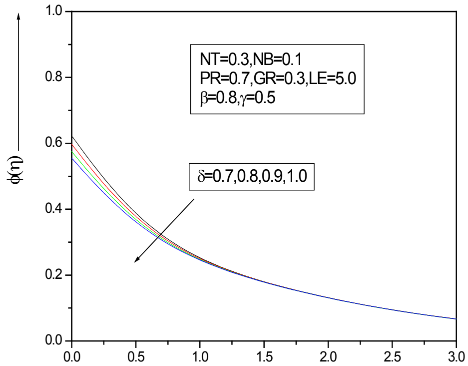

Fig - 11 η

The behaviour of Nanoparticle volume fraction profile for various values of δ

Acknowledgement:
Authors are thankful to the reviewers for their useful comments and suggestions. One of the author Dr. Jagadish Tawade wishes to thank Bharat Ratna Prof. C.N.R.Rao, Hon'ble Chairman, Dr. S. Anant Raj Consultant, Prof. Roddam Narasimha, Hon'ble member VGST, Department of IT, BT S & T, GoK, India, for supporting this work under Seed Money to Young Scientists for Research (F.No.VGST/P-3/ SMYSR/ GRD-286 / 2013-14).

REFERENCE

1. B.C. Sakiadis, Boundary layer behavior on continuous solid surfaces: I Boundary layer equations for two dimensional and axisymmetric flow, AIChE J. 7 (1961) 26–28 | 2. I.C. Liu, Flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to a transverse magnetic field, Int. J. Heat Mass Transfer 47 (2004) 4427–4437. | 3. S.K. Khan, M. Subhas Abel, M. Sonth Ravi, Viscoelastic MHD flow heat and mass transfer over a porous stretching sheet with dissipation of energy and stress work, Int. J. Heat Mass Transfer 40 (2003) 47–57. | 4. R. Cortell, Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet, Phys. Lett. A 357 (2006) 298–305. | 5. B.S. Dandapat, B. Santra, K. Vajravelu, The effects of variable fluid properties and thermocapillarity on the flow of a thin film on an unsteady stretching sheet, Int. J. Heat Mass Transfer 50 (2007) 991–996. | 6. S.Nadeem, A.Hussain, M.Khan, HAM solutions for boundary layer flow in the region of the stagna -tion point towards a stretching sheet, Commun. Nonlinear Sci.Numer.Simulat. 15(2010)475–481 | 7. N. Bachok, A. Ishak, Flow and heat transfer over a stretching cylinder with prescribed surface heat flux Malays, J. Math. Sci. 4 (2010) 159–169. | 8. N.Bachok, A.Ishak, R.Nazar, Flow and heat transfer over an unsteady stretching sheet in a micropolar fluid with prescribed surface heat flux, Int.J.Math.Mod.Meth.Appl. Sci.4(2010)167–176 | 9. N.Bachok,A.Ishak, I.Pop,On the stagnation point flow towards stretching sheet with homogeneous - heterogeneous reactions effects, Commun. Nonlinear Sci. Numer. Simulat. 16 (2011) 4296–4302. | 10. N. Bachok, A. Ishak, R. Nazar, Flow and heat transfer over an unsteady stretching sheet in a micropolar fluid, Meccanica 46 (2011) 935–942 | 11. Y. Ding, H. Chen, L. Wang, C.-Y. Yang, Y. He, W. Yang, W.P. Lee, L. Zhang, R. Huo,Heat transfer intensification using nanofluids, Kona 25 (2007) 23–38. | 12. S. Das, temperature dependence of thermal conductivity enhancement for nanofluids, J. Heat Trnsfer 125 (2003) 567-574. | 13. B.C. Pak, Y. Cho , Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles, Exp. Heat Transfer 11 (1998) 151-170. | 14. Y. Xuan, Q. Li, Investigation on convective heat transfer and flow features of nanofluids, J. Heat Transfer 125 (2003) 151-155. | 15. V. Trisaksri, S. Wongwises, Critical review of heat transfer characteristics of nanofluids, Renew. Sust. Energ. Rev. 11 (2007) 512-523. | 16. X.-Q. Wang, A.S. Mujumdar, Heat transfer characteristics of nanofluids: a review. Int. J. Therm. Sci. 46(2007) 1-19. | 17. J.A. Eastman, S.R. Phillpot, S. U .S. Choi, P. Keblinski, Thermal transport in nanofluids, Annu. Rev. Mater. Res. 34 (2004) 219-246. | 18. S. Kakac, A. Pramaumjaroenkij, Review of convective heat transfer enhancement with nanofluids, Int. J. Heat Transfer. 52 (2005) 3187-3196. | 19. A.V. Kuznetsov, D.A. Nield, Natural convective boundary- layer flow of a nanofluids past a vertical plate, Int. J.Therm. Sci. 49 (2010) 243-247. | 20. W.A. Khan, I.Pop, Boundary-layer flow of a nanofluid past a stretching sheet, Int. J. Heat Mass Transf. 53 (2010) 2477-2483. | 21. R.S.R. Gorla, I. Sidawi, Free convection on a vertical stretching surface with suction and blowing, Appl. Sci. Res. 52 (1994) 247–257. | 22. C.Y. Wang, Free convection on a vertical stretching surface, J. Appl. Math. Mech. (ZAMM) 69 (1989) 418–420. |