Free Convection Flow From an Inclined Porous Plate Embedded in a Porous Medium in the Presence of Diffusion Thermo Effect, Heat Source, and Chemical Reaction

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Abstract: An analytical solution to the problem of magnetohydrodynamics free convection flow past an inclined porous plate considering the heat source, diffusion thermo effect, and chemical reaction is obtained. The governing equations for velocity, temperature, and concentration are evaluated using the perturbation method. The effect of relevant physical parameters like porosity parameter, heat source, magnetic parameter, radiation parameter, solutal Grashof number, thermal Grashof number, Dufour effect, chemical reaction on the velocity, temperature, concentration profiles, skin friction, mass flux, and heat flux are demonstrated and discussed graphically. It is noticed that velocity and concentration decrease under the effect of the chemical reaction.

Keywords: viscous dissipation; inclined plate; mass diffusivity; perturbation technique.

Nomenclature:

$\vec{q}$ Fluid Velocity (m/s);

B_0 Strength of magnetic field (weber/m^2);

u_0 velocity of plate (m/s);

v_0 Suction velocity (m/s);

T Fluid temperature (K);

q_r Radioactive heat flux (W/m^2);

M Magnetic parameter;

Pr Prandtl number;

Du Dufour number;

E Eckert number;

D_M Mass dissusivity (m^2/s);

K_T Thermal diffusion ratio;

R Chemical reaction;

F Radiation parameter;

Gr Thermal Grashof number;

Gm Modified Grashof number;

S heat source;

T_w wall temperature (K);

T_m Mean fluid temperature (K);

T_∞ Fluid temperature far away from the plate (K);

C Species concentration (mol/m^3);

C_w Wall concentration (mol/m^3);

C_p Specific heat at const. pressure (J/kg.K);

C_∞ Concentration far away from the plate (mol/m^3);

u Dimensionless velocity;

κ Thermal conductivity (W/m^2.K);

g Gravitational acceleration;

σ Electrical conductivity (s/m);

ν Kinematic viscosity (m^2/s);

ρ Fluid density (kg/m^3);

μ Coefficient of viscosity (kg/m.s);

β_c Volumetric coefficient of thermal expansion with concentration (1/K.mol);

β Volumetric coefficient of thermal expansion (1/K);

K *Porosity parameter* ;
Sc *Schmidt number* ;
I *Absorption coefficient at plate* ;

p *Pressure* (N/m^2);
$\vec{B}$ *Magnetic flux density vector* ;

1. Introduction

MHD is a term used to describe the study of electrically conducting fluid motion in the presence of a magnetic field. MHD has wide application in many scientific fields such as geophysics, plasma, astrophysics, the medical field, etc. Many authors contributed to the current version of MHD, including Alfven [1], Shercliff [2], Cowling [3], *etc*. The problem of heat and mass transfer via porous media is useful in geothermal energy extraction and geothermal reservoirs. Several authors scholars have discussed these topics. Some of them are Rani *et al.* [4], Javed *et al.* [5], Manjula and Chandra Sekhar [6], which have studied the fluid flow in the existence of viscous dissipation over an inclined plate. In a system where both heat and mass transfer exist simultaneously, it has been observed that composite gradients and temperature differences cause energy flux.

In the presence of viscous dissipation and heat absorption, Ali *et al.* [7] explored an unsteady magneto-hydrodynamic (MHD) convection flow of a double-diffusive, viscoelastic fluid across an inclined permeable plate. Palani and Arutchelvi [8] investigated the effects of viscous dissipation on a magneto-hydrodynamic nanofluid in a permeable medium subjected to homogeneous heat and mass flux along a plate inclined at an angle. The effects of viscous dissipation and a heat source or sink on the magneto-hydrodynamic laminar boundary layer flow of a Jeffrey fluid past a vertical plate are investigated by Hillary and Shateyi [9]. In the presence of an induced magnetic field, Kumar *et al.* [10] examined the effects of thermal radiation and magnetic fields on a viscous dissipative free convection fluid flow across an inclined infinite plate. Kumar *et al.* [11] are a group of researchers who have worked on a number of different projects. The main objective of this work is to understand how thermal radiation reacts with magnetic fields when a viscous dissipative free convection fluid flow passes over an inclined infinite plate in the presence of an induced magnetic field.

Diffusion thermo effect is the effect of energy flux generator due to composite gradient on the flow. The experimental investigation of the Dufour effect on isotope separation in a gaseous mixture of N$_2$ and H$_2$ was first performed by Charles L. Dufour in 1873. Several authors have performed a model study on the Dufour as well as diffusion thermo effect in different thermal and solutal conditions. Dursunkaya and Worek [12], Ahmed [13], Ahmed and Sengupta [14], Ahmed [15], Hossain *et al.* [16], Mythreye and Balamurugan [17] are a few of them. On steady MHD flow over a vertical porous plate with viscous dissipation, radiation, and Ohmic heating, the chemical reaction and Dufour effects are investigated by Balamurugan *et al.* [18]. K. Balamunugan *et al.* [19] have analyzed the viscous dissipation, and ohmic heating effect on heat and mass transfer flow in the presence of thermal diffusion, radiation effect, and later include diffusion thermo. Zyauddin and Kumar [20] have explained the radiation effect on a moving inclined porous plate on unsteady MHD heat and mass transfer flow in the presence of chemical radiation effect.

The impact of heat and mass transfer on an unsteady conducting upper convected Maxwell fluid (UCM) flow across a stretching surface with porous medium, and the higher-order chemical reaction was investigated by Reddy and Pallavarapu [21]. Alsenafi and Ferdows [22] studied the dynamics of a nanofluid along a plane in two dimensions, uniformly, incompressibly, and freely. Around the porous media, the plate is oriented upward. Kumar et

al. [23] explored thermal diffusion and diffusion thermo impacts on MHD using an analytical solution. In the presence of thermal radiation, aligned magnetic field, and chemical reaction, Casson fluid flows past an oscillating inclined plate inserted through porous media. Oyekunle and Agunbiade [24] reviewed the effects of various fluid physical quantities on unsteady MHD slip flow over a permeable vertical plate, including diffusion-thermo, thermal diffusion, thermal radiation, viscous dissipation, and inclined magnetic field. In the presence of a magnetic field and the radiation effect, Jahir et al. [25] explored unsteady two-dimensional heats and mass transfer free convection flow around a vertical plate immersed in porous media. Quader and Alam [26] addressed the combined Soret and Dufour effects in the presence of Hall current and continuous heat flux to investigate unsteady MHD natural convective heat and mass transfer flow through a semi-infinite vertical porous plate in a rotating system. The Soret and Dufour effects on MHD unsteady fluid flow across an accelerating inclined vertical plate with thermal radiation and heat source were investigated by Shankar and Yanala [27]. In the presence of a heat source and a chemical reaction, Dash and Mishra [28] investigated free convective heat and mass transfer of non-conducting micropolar fluid flow over an infinitely inclined sliding porous plate.

2. Mathematical Analysis

We consider a viscous incompressible electrically conducting fluid flow over a semi-infinite inclined porous plate embedded in a porous medium at an acute angle α to the vertical direction, which is subjected to thermal and concentration buoyancy effects. The chemical process is assumed to be first-order homogenous. The concentration C_w and temperature T_w of the wall are maintained at a constant. The physical coordinates (x', y') are set so that the x'-axis is parallel to the motion direction and the y'-axis is perpendicular to it. Using the standard Boussinesq approximations, the flow is governed by the following system of equations:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1.1}$$

$$\rho v' \frac{\partial u'}{\partial y'} = \mu \frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' - \frac{\mu u'}{K^*} + g\rho\beta(T - T_\infty)\cos\alpha + g\rho\beta_c(C - C_\infty)\cos\alpha \tag{1.2}$$

$$\rho C_p v' \frac{\partial T}{\partial y'} = \kappa \frac{\partial^2 T}{\partial y'^2} - Q(T - T_\infty) - \frac{\partial q_r}{\partial y'} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 + \sigma B_0^2 u'^2 + \frac{\rho D_M K_T}{C_s}\frac{\partial^2 C}{\partial y'^2} \tag{1.3}$$

$$v' \frac{\partial C}{\partial y'} = D_M \frac{\partial^2 C}{\partial y'^2} - \bar{K}(C - C_\infty) \tag{1.4}$$

The corresponding boundary conditions:

$$At \quad y' = 0 ; u' = 0 \ , \ T = T_w \ , \ C = C_w \tag{1.5}$$

$$As \quad y' \to \infty ; u' \to 0 \ , \ T \to T_\infty \ , \ C \to C_\infty \tag{1.6}$$

Consider an optically thin fluid, that has a low flow density, and a radioactive heat flux of:

$$\frac{\partial q_r}{\partial y'} = 4(T \to T_\infty)I \tag{1.7}$$

From (1.1) we have $v' = -v_0$ where, suction velocity is directed toward the plate, as shown by the negative sign.

The non-dimensional quantities:

$$y = \frac{v_0 y'}{v}, \quad u = \frac{u'}{v_0}, \quad Gr = \frac{\rho \beta v (T_w - T_\infty)}{v_0^{\,3}}, \quad Gm = \frac{\rho \beta_c v (C_w - C_\infty)}{v_0^{\,3}}, \quad K = \frac{K^* v_0^{\,2}}{v^2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad R = \frac{\bar{K}v}{v_0^{\,2}}, \quad F = \frac{4vI}{\rho C_p v_0^{\,2}}, \quad S = \frac{Qv}{\rho C_p v_0^{\,2}}, \quad \Pr = \frac{\mu C_p}{\kappa}, \quad E = \frac{v_0^{\,2}}{C_p (T_w - T_\infty)},$$

$$Sc = \frac{v}{D_M}, \quad Du = \frac{D_M K_T (C_w - C_\infty)}{v T_m (T_w - T_\infty)}, \quad M = \frac{\sigma B_0^{\,2} v}{\rho v_0^{\,2}} \tag{2.1}$$

Substituting dimensionless quantities (2.1) in equation (1.2), (1.3) and (1.4) we get:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \left(M + \frac{1}{K} \right) u = -Gr \cos \alpha \theta - Gm \cos \alpha \phi \tag{3.1}$$

$$\frac{\partial^2 \theta}{\partial y^2} + \Pr \frac{\partial \theta}{\partial y} + S \Pr \theta - F \Pr \theta = -\Pr E \left(\frac{\partial u}{\partial y} \right)^2 - \Pr M E u^2 - \Pr Du \frac{\partial^2 \phi}{\partial y^2} \tag{3.2}$$

$$\frac{\partial^2 \phi}{\partial y^2} + Sc \frac{\partial \phi}{\partial y} - R Sc \phi = 0 \tag{3.3}$$

With boundary conditions:

$$At \quad y = 0; \quad u = 0, \quad \theta = 1, \quad \phi = 1 \tag{4.1}$$

$$As \quad y \to 0; \quad u \to 0, \quad \theta \to 0, \quad \phi \to 0 \tag{4.2}$$

4. Method of Solution

To solve governing equations (3.1)-(3.3) are non-linear, and to obtain a solution, we expand in powers of Eckert number assuming to be small. Therefore, we can express it as a regular perturbation series:

$$u(y) = u_0 + \varepsilon u_1 + o\left(\varepsilon^2\right) \tag{5.1}$$

$$\theta(y) = \theta_0 + \varepsilon \theta_1 + o\left(\varepsilon^2\right) \tag{5.2}$$

$$\phi(y) = \phi_0 + \varepsilon \phi_1 + o\left(\varepsilon^2\right) \tag{5.3}$$

Using (5.1) in equations (3.1), (3.2), and (3.3) for reducing PDEs into ODEs and equating the coefficient of similar terms and neglecting terms of $o\left(\varepsilon^2\right)$, then we obtained the following

Zeroth-order equations are:

$$u_0'' + u_0' - \left(M + \frac{1}{K} \right) u_0 = - Gr \cos \alpha \theta_0 - Gm \cos \alpha \phi_0 \tag{6.1}$$

$$\theta_0'' + \Pr \theta_0' - \Pr \left(S - F \right) \theta_0 = - \Pr Du \phi_0'' \tag{6.2}$$

$$\phi_0'' + Sc \phi_0' - RSc \phi_0 = 0 \tag{6.3}$$

First-order equations are:

$$u_1'' + u_1' - \left(M + \frac{1}{K} \right) u_1 = - Gr \cos \alpha \theta_1 - Gm \cos \alpha \phi_1 \tag{6.4}$$

$$\theta_1'' + \Pr \theta_1' - \Pr \left(S - F \right) \theta_1 = - \Pr u_0'^2 - \Pr M u_0^2 - \Pr Du \phi_1'' \tag{6.5}$$

$$\phi_1'' + Sc \phi_1' - RSc \phi_1 = 0 \tag{6.6}$$

With boundary conditions:

$$At \quad y = 0; \quad u_0 = 0, \ u_1 = 0, \theta_0 = 1, \ \theta_1 = 0, \ \phi_0 = 1, \phi_1 = 0 \tag{7.1}$$

$$As \quad y \to \infty; \quad u_0 = 0, \ u_1 = 0, \ \theta_0 = 0, \ \theta_1 = 0, \ \phi_0 = 0, \ \phi_1 = 0 \tag{7.2}$$

4.1. Solutions of zeroth-order equations.

The solutions of (6.1) - (6.3) with the boundary conditions (7.1) and (7.2) we get:

$$u_0 = A_6 e^{m_3 y} + A_3 e^{m_2 y} + \left(A_4 + A_5 \right) e^{m_1 y} \tag{8.1}$$

$$\theta_0 = A_2 e^{m_2 y} + A_1 e^{m_1 y} \tag{8.2}$$

$$\phi_0 = e^{m_1 y} \tag{8.3}$$

4.2. Solutions of first-order equations.

The solutions of (6.4) - (6.6) with the boundary conditions (7.1) and (7.2) we get:

$$u_1 = A_{21} e^{m_3 y} + A_{14} e^{m_2 y} + A_{15} e^{2 m_3 y} + A_{16} e^{2 m_2 y} + A_{17} e^{2 m_1 y} + A_{18} e^{(m_2 + m_3) y}$$
$$+ A_{19} e^{(m_1 + m_2) y} + A_{20} e^{(m_1 + m_3) y} \tag{8.4}$$

$$\theta_1 = A_{13} e^{m_2 y} + A_7 e^{2 m_3 y} + A_8 e^{2 m_2 y} + A_9 e^{2 m_1 y} + A_{10} e^{(m_2 + m_3) y} + A_{11} e^{(m_1 + m_2) y} + A_{12} e^{(m_1 + m_3) y} \tag{8.5}$$

$$\phi_1 = 0 \tag{8.6}$$

Substituting the solutions (8.1) - (8.6) in (5.1) we get the resultant following equations:

$$u(y) = A_6 e^{m_3 y} + A_3 e^{m_2 y} + \left(A_4 + A_5 \right) e^{m_1 y} + E \left(A_{21} e^{m_3 y} + A_{14} e^{m_2 y} + A_{15} e^{2 m_3 y} + A_{16} e^{2 m_2 y} \right.$$
$$\left. + A_{17} e^{2 m_1 y} + A_{18} e^{(m_2 + m_3) y} + A_{19} e^{(m_1 + m_2) y} + A_{20} e^{(m_1 + m_3) y} \right) \tag{9.1}$$

$$\theta(y) = A_2 e^{m_2 y} + A_1 e^{m_1 y} + E\left\{A_{13} e^{m_2 y} + A_7 e^{2m_3 y} + A_8 e^{2m_2 y} + A_9 e^{2m_1 y}\right.$$

$$\left. + A_{10} e^{(m_2+m_3)y} + A_{11} e^{(m_1+m_2)y} + A_{12} e^{(m_1+m_3)y}\right\} \tag{9.2}$$

$$\phi(y) = e^{m_1 y} \tag{9.3}$$

4.3. Skin friction.

Skin friction is defined by Newton's law. The co-efficient of skin friction for is given by:

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= m_3 A_6 + m_2 A_3 + m_1(A_4 + A_5) + E\left\{m_3 A_{21} + m_2 A_{14} + 2m_3 A_{15} + 2m_2 A_{16}\right.$$

$$\left. + 2m_1 A_{17} + (m_2 + m_3)A_{18} + (m_1 + m_2)A_{19} + (m_1 + m_3)A_{20}\right\} \tag{10.1}$$

4.4. Nusselt number.

Nusselt number is given by:

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$= -\left[m_2 A_2 + m_1 A_1 + E\left\{m_2 A_{13} + 2m_3 A_7 + 2m_2 A_8 + 2m_1 A_9 + (m_2 + m_3)A_{10}\right.\right.$$

$$\left.\left. + (m_1 + m_2)A_{11} + (m_1 + m_3)A_{12}\right\}\right] \tag{10.2}$$

4.5. Sherwood number.

Sherwood number is given by:

$$Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = -m_1 \tag{10.3}$$

5. Results and Discussion

To determine the problem's physical scope, the effects of several parameters such as porosity parameter K, heat source parameter S, solutal Grashof number Gm, Hartmann number M, Dufour effect, thermal Grashof number Gr, radiation parameter F, chemical reaction parameter R on velocity distribution, temperature, concentration field, Skin friction, Nusselt number, Sherwood number have been analyzed graphically, which are depicted in Figures 1-22.

The effects of Du, F, Gr, Gm, S, K, and R on velocity profiles are presented in Figures 1-7. Figure 1 depicts a significant increase in fluid velocity due to the Dufour effect. The velocity field is subjected to the influence of radiation parameter F, as displayed in Figure 2; for raising the value of Gr or Gm, the fluid's velocity increases, shown in Figures 3 and 4. That is, fluid motion is accelerated due to thermal and solutal buoyancy forces. The effect of the

heat source parameter S and the porosity parameter K on velocity is shown in Figures 5 and 6. The effect of a chemical reaction on the velocity field is depicted in Figure 7. It is seen that the velocity field falls under the influence of chemical reaction parameters. That is, the consumption of species drops the fluid velocity to a considerable extent.

Figures 8-13 depict the evolution of temperature profiles. Figure 8 shows how the thermal diffusion effect raises the temperature profile. Further in Figures 9 and 10, it can be seen that the temperature of the fluid falls as raising the values of Prandtl number and radiation. Figures 11-13 display the effect of the heat source parameter, Schmidt number, and chemical reaction parameter R on fluid temperature. The fluid temperature rises as the heat source parameter, the Schmidt number, and the chemical reaction parameter rise. That is, fluid temperature decreases for increasing mass diffusivity. Figures 14 and 15 represent concentration field vs. y under the effects of Schmidt number and Chemical reaction parameter R. The concentration of the fluid drops for increasing both Sc or R. In other words, concentration increases for increasing mass diffusivity.

Effects of magnetic parameter, modified Grashof number, thermal Grashof number on skin friction are displayed in Figures 16-18. As increasing the value of the magnetic parameter, skin friction reduces, as shown in Figure 16. Figures 17 and 18 show that viscous drag increases under the effect of thermal and solutal buoyancy force.

Figures 19, 20, and 21 illustrate the behavior of Nusselt numbers for numerous values of Gm, Gr, and Sc. The Nusselt number increases under the impact of Gm, as shown in Figure 19. Figure 20 shows that the Nusselt number drops for tiny Gr at first, but after a critical point, the behavior trend reverses. Figure 21 shows that the Nusselt number decreases as the Schmidt number enlarges. As a result, when mass diffusivity is high, the heat transfer rate at the plate decreases. Figure 22 shows how the Sherwood number rises as the chemical reaction parameter rises.
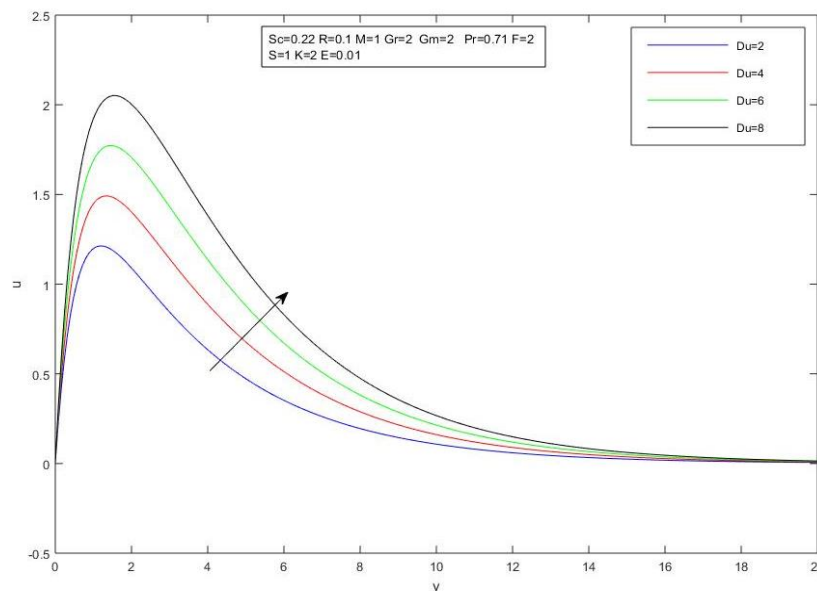

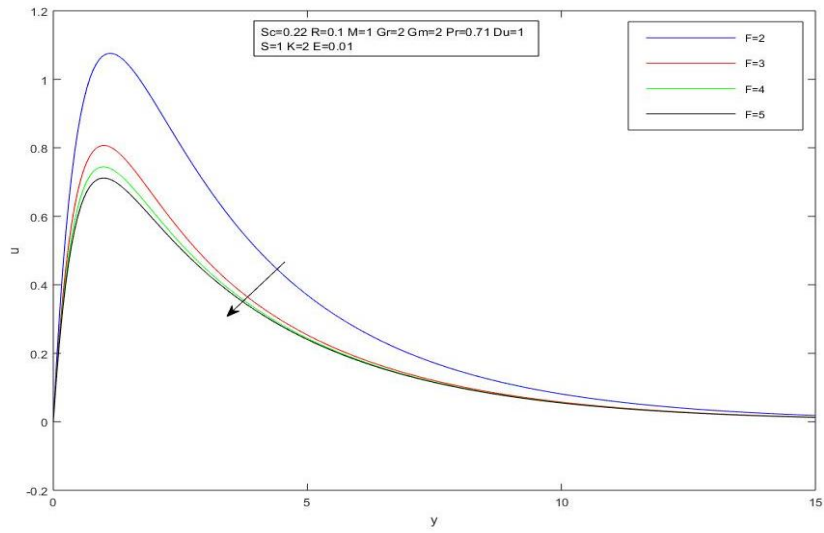**Figure 1**. Velocity profile for variations in Dufour number.

Figure 2. Velocity profile for variations in radiation parameter.
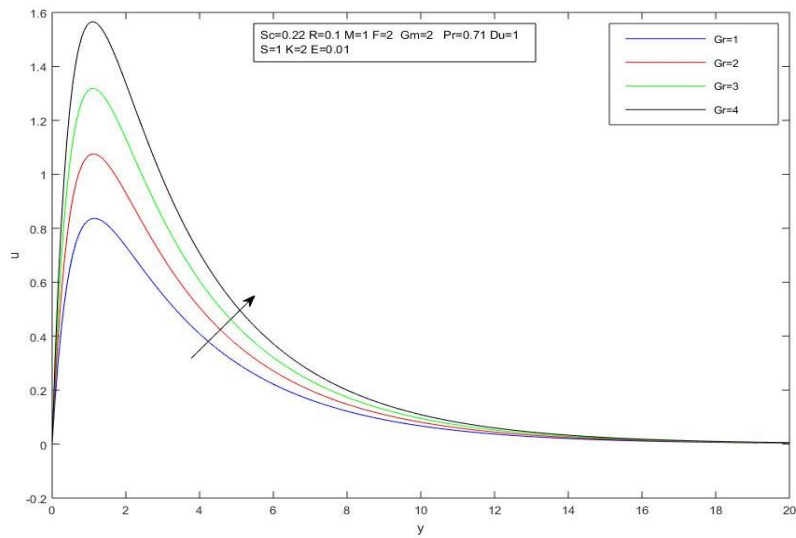

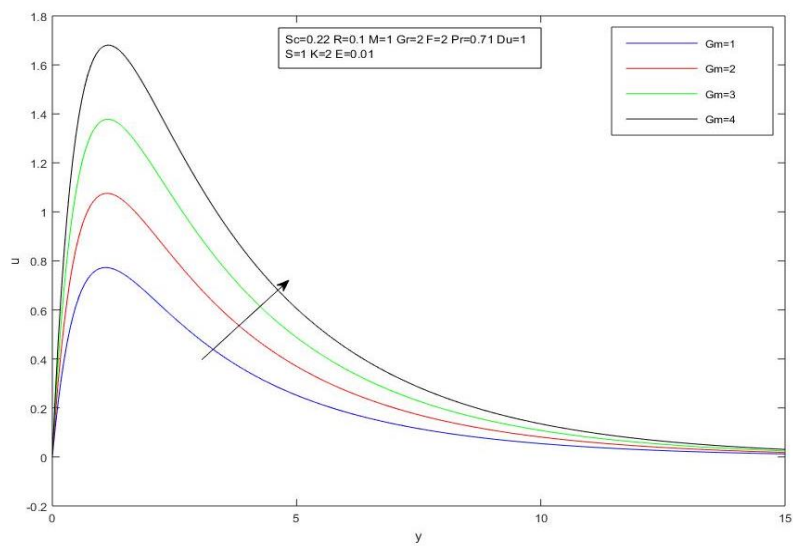**Figure 3**. Velocity profile for variations in thermal Grashof number.

Figure 4. Velocity profile for variations in solutal Grashof number.
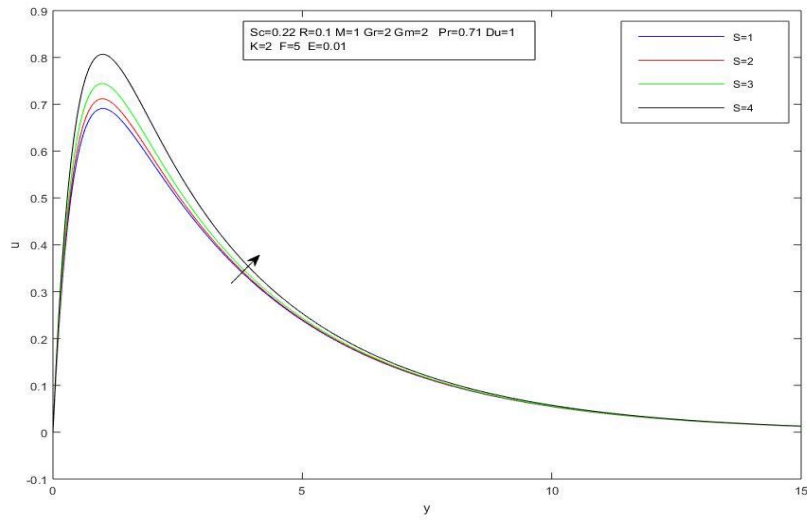
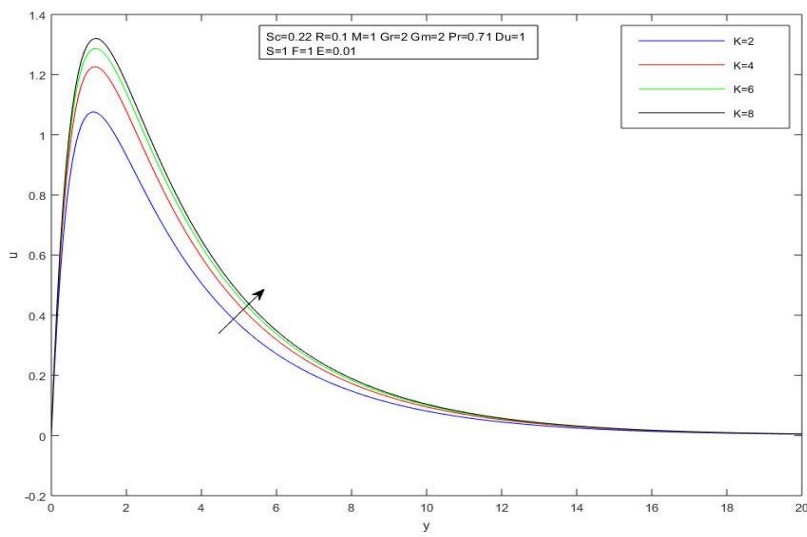**Figure 5**. Velocity profile for variations in the heat source.

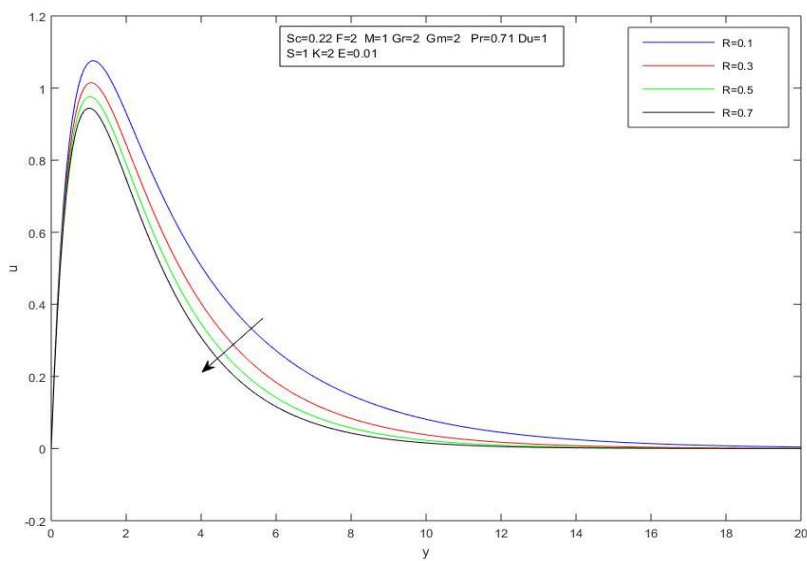**Figure 6**. Velocity profile for variations in porosity parameter.

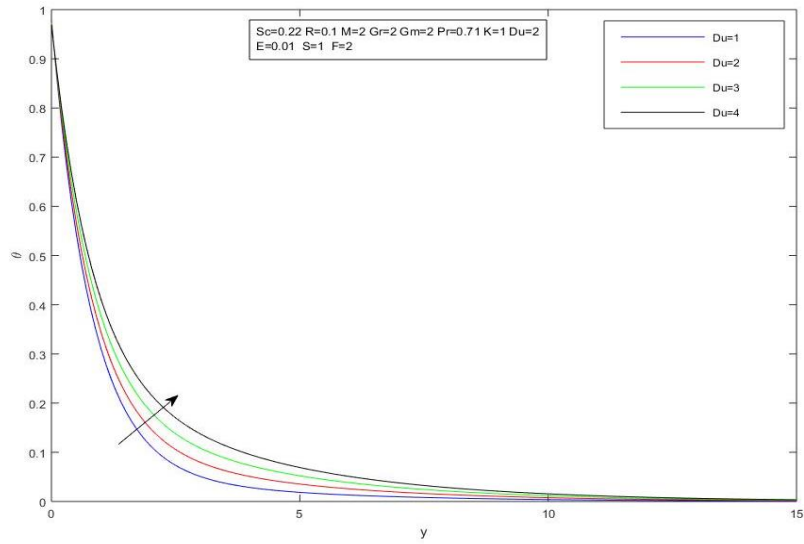**Figure 7**. Velocity profile for variations in chemical reaction parameters.

Figure 8. Temperature profile for variations in Dufour number.
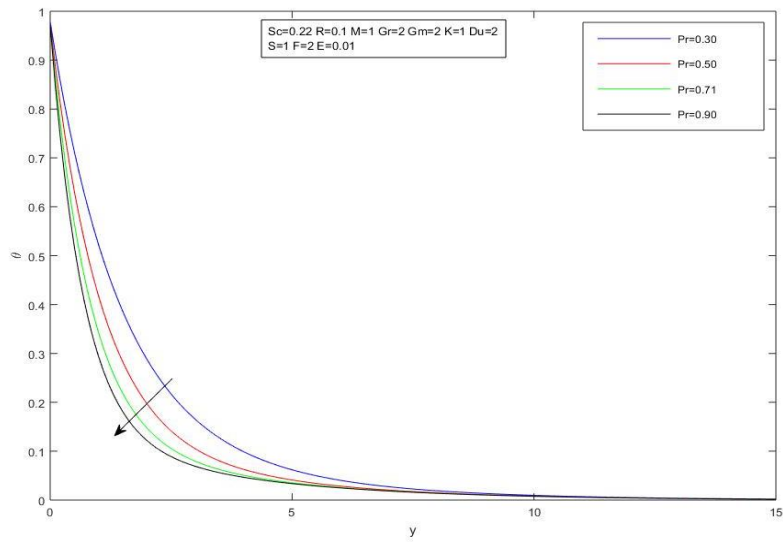
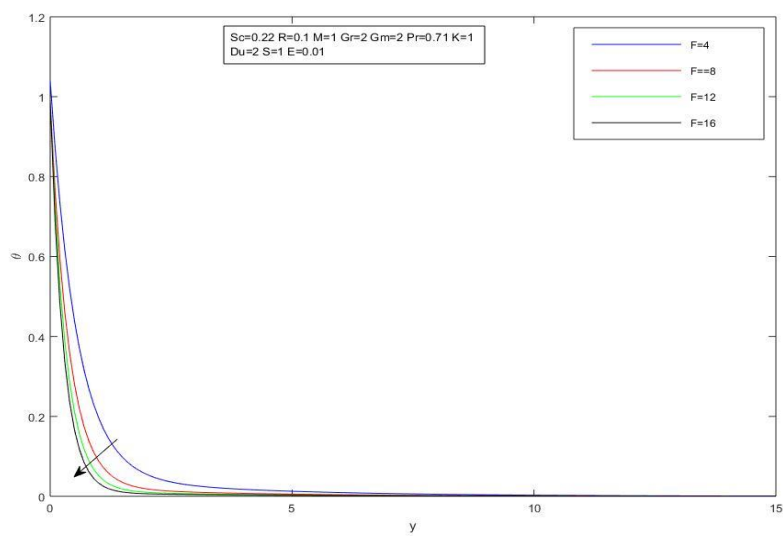**Figure 9**. Temperature profile for variations in Prandtl number.

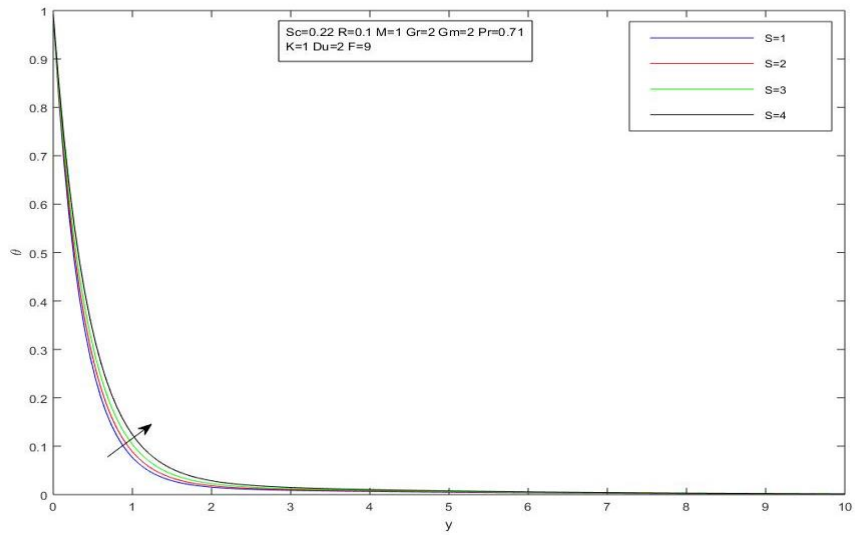**Figure 10**. Temperature profile for variations in radiation parameter.

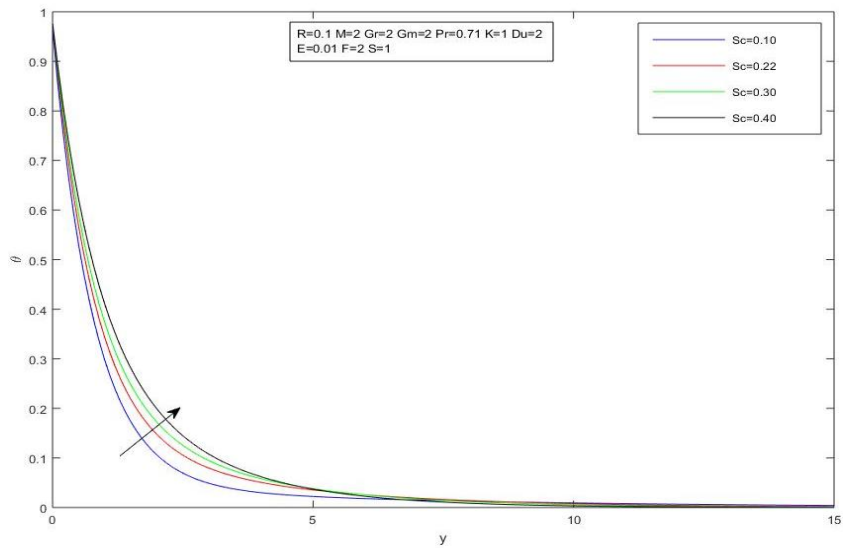**Figure 11**. Temperature profile for variations in heat source parameter.

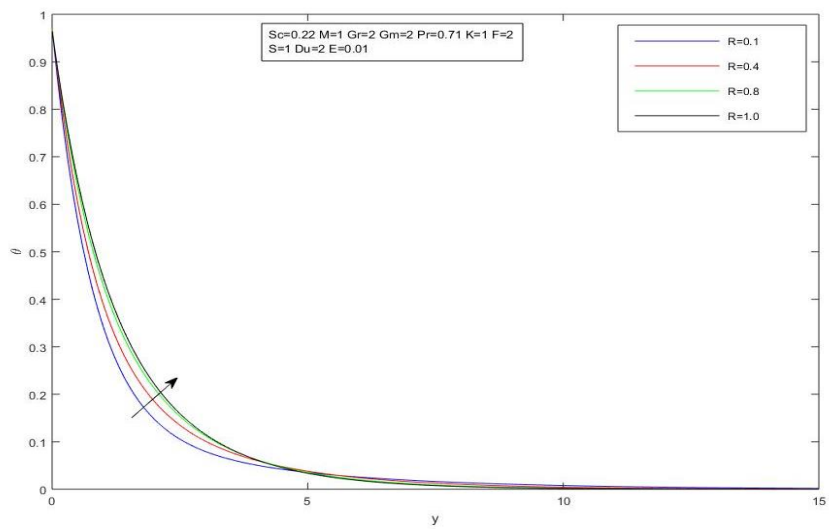**Figure 12**. Temperature profile for variations in Schmidt number.

Figure 13. Temperature profile for variations in chemical reaction parameters.
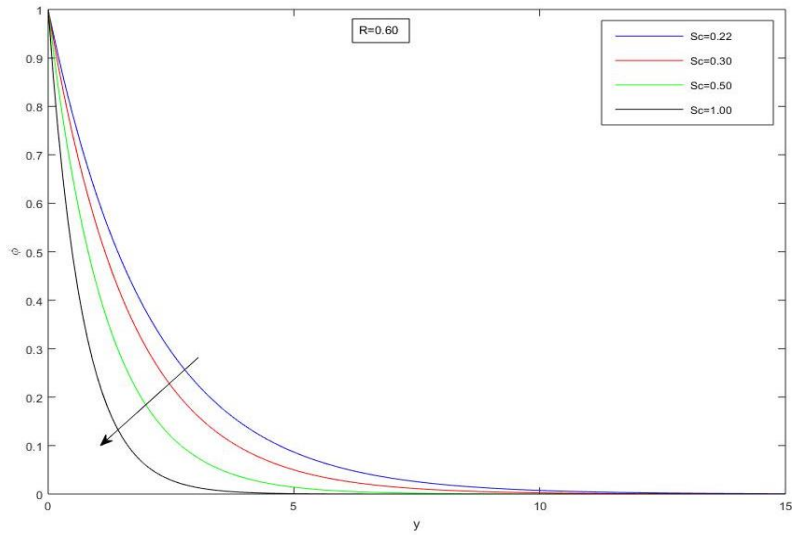
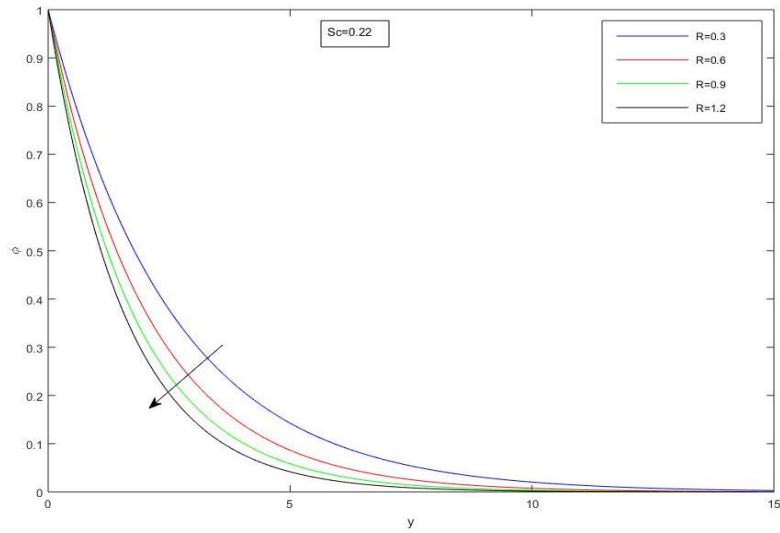**Figure 14**. Concentration profile for variations in Schmidt number.

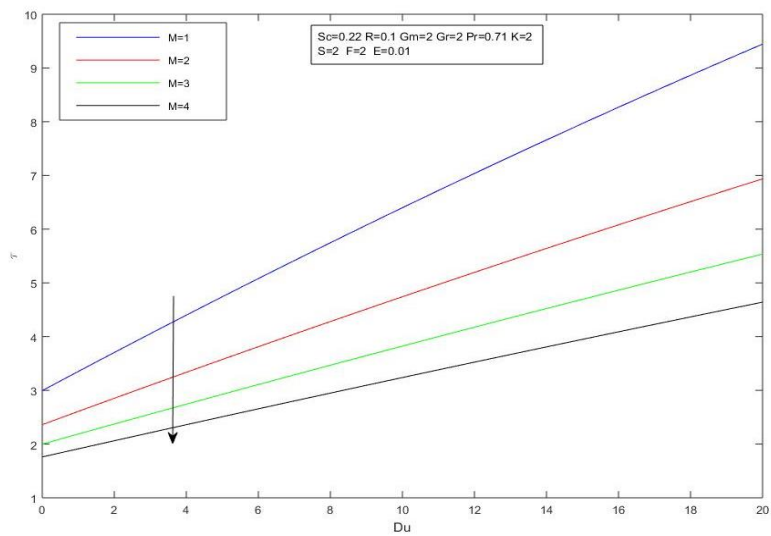**Figure 15**. Concentration profile for variations in chemical reaction parameters.

Figure 16. Skin friction for variations in magnetic parameters.
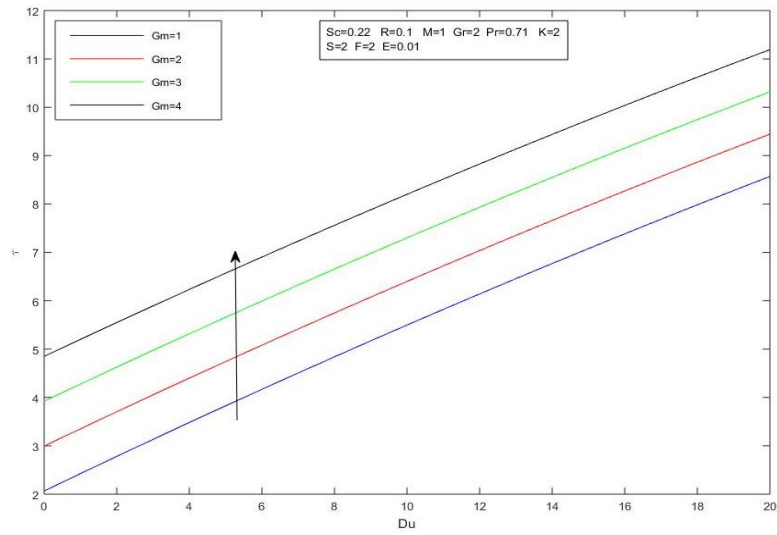
Figure 17. Skin friction for variations in modified Grashof number.

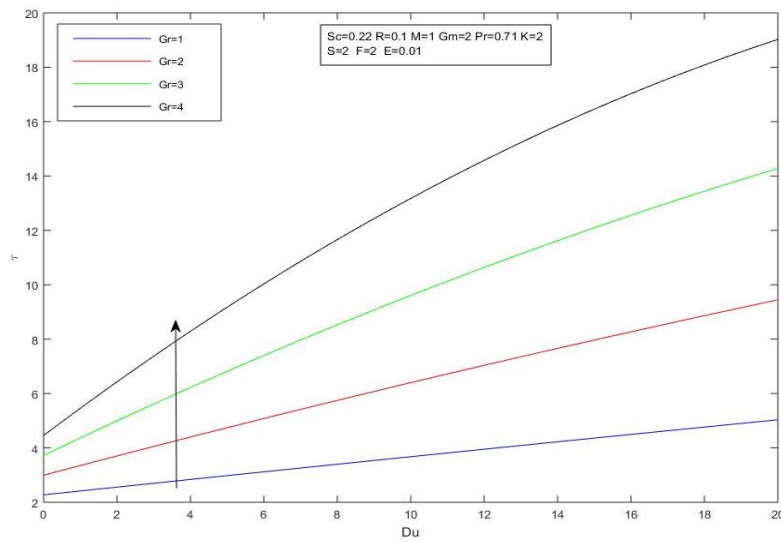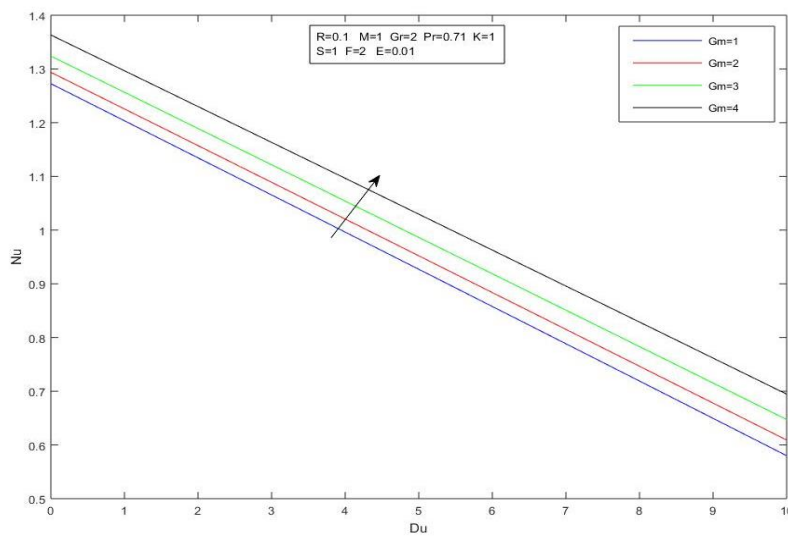**Figure 18**. Skin friction for variations in Grashof number.

Figure 19. Nusselt number for variations in modified Grashof number.
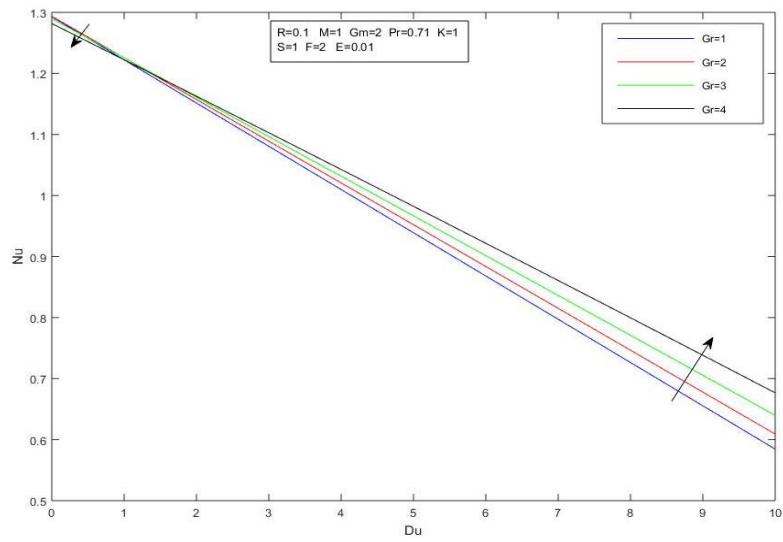
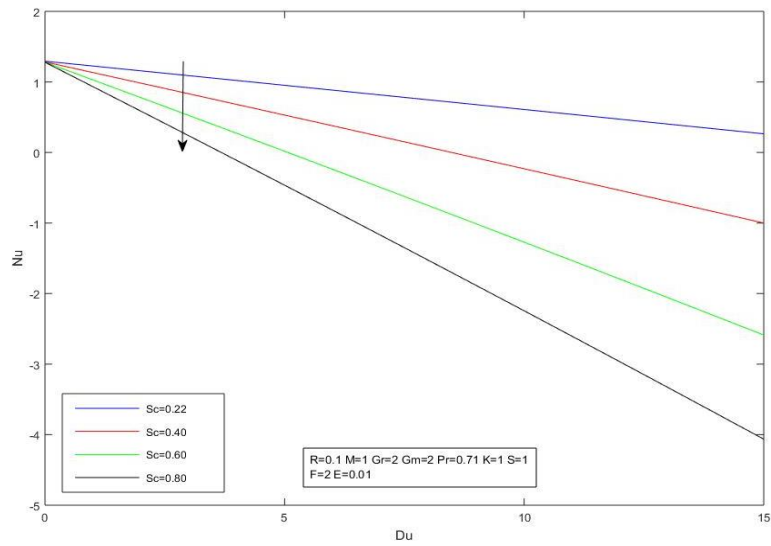**Figure 20**. Nusselt number for variations in Grashof number.

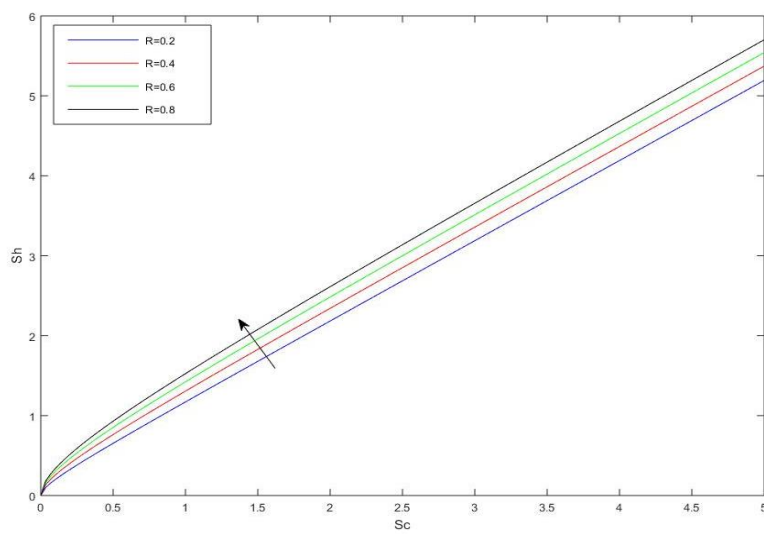**Figure 21**. Nusselt number for variations in Schmidt number.

Figure 22. Sherwood number for variations in Chemical reaction parameter.

6. Conclusions

Several important conclusions can be drawn. Diffusion thermal effects cause a significant increase in fluid velocity. Velocity field falls under the effect of Chemical radiation R. Fluid temperature rise with Dufour effect. Fluid temperature increases due to the rise of heat source parameter R and Schmidt number, *i.e.,* for increasing mass diffusivity, fluid temperature decreases. Viscous drag on the plate rises due to buoyancy force.

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Conflicts of Interest

The authors declare no conflict of interest.

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Appendix

$$m_1 = -\frac{Sc + \sqrt{Sc^2 + 4RSc}}{2}, \quad m_2 = -\frac{\Pr + \sqrt{\Pr^2 + 4\Pr(F-S)}}{2}, \quad m_3 = -\frac{1 + \sqrt{1 + 4(M + \frac{1}{K})}}{2},$$

$$A_1 = -\frac{\Pr Du m_1^2}{m_1^2 + m_1 \Pr - \Pr(F-S)}, \quad A_2 = 1 - A_1, \quad A_3 = -\frac{Gr\cos\alpha A_2}{m_2^2 + m_2 - \left(M + \frac{1}{K}\right)},$$

$$A_4 = -\frac{Gr\cos\alpha A_1}{m_1^2 + m_1 - \left(M + \frac{1}{K}\right)}, \quad A_5 = -\frac{Gm\cos\alpha}{m_1^2 + m_1 - \left(M + \frac{1}{K}\right)}, \quad A_6 = -\left(A_3 + A_4 + A_5\right),$$

$$A_7 = -\frac{\Pr\left(M + m_3^2\right)A_6^2}{4m_3^2 + 2m_3 \Pr - \Pr(F-S)}, \quad A_8 = -\frac{\Pr\left(M + m_2^2\right)A_3^2}{4m_2^2 + 2m_2 \Pr - \Pr(F-S)}$$

$$A_9 = -\frac{\Pr\left(M + m_1^2\right)\left(A_4 + A_5\right)^2}{4m_1^2 + 2m_1 \Pr - \Pr(F-S)}, \quad A_{10} = -\frac{2\Pr\left(M + m_2 m_3\right)A_3 A_6}{\left(m_2 + m_3\right)^2 + \left(m_2 + m_3\right)\Pr - \Pr(F-S)},$$

$$A_{11} = -\frac{2\Pr\left(M + m_1 m_2\right)A_3\left(A_4 + A_5\right)}{\left(m_1 + m_2\right)^2 + \left(m_1 + m_2\right)\Pr - \Pr(F-S)}, \quad A_{12} = -\frac{2\Pr\left(M + m_1 m_3\right)A_6\left(A_4 + A_5\right)}{\left(m_1 + m_3\right)^2 + \left(m_1 + m_3\right)\Pr - \Pr(F-S)},$$

$$A_{13} = -\left(A_7 + A_8 + A_9 + A_{10} + A_{11} + A_{12}\right), \quad A_{14} = -\frac{Gr\cos\alpha A_{13}}{m_2^2 + m_2 - \left(M + \frac{1}{K}\right)},$$

$$A_{15} = -\frac{Gr\cos\alpha A_7}{4m_3^2 + 2m_3 - \left(M + \frac{1}{K}\right)}, \quad A_{16} = -\frac{Gr\cos\alpha A_8}{4m_2^2 + 2m_2 - \left(M + \frac{1}{K}\right)} \quad A_{17} = -\frac{Gr\cos\alpha A_9}{4m_1^2 + 2m_1 - \left(M + \frac{1}{K}\right)},$$

$$A_{18} = -\frac{Gr\cos\alpha A_{10}}{\left(m_2 + m_3\right)^2 + \left(m_2 + m_3\right) - \left(M + \frac{1}{K}\right)}, \quad A_{19} = -\frac{Gr\cos\alpha A_{11}}{\left(m_1 + m_2\right)^2 + \left(m_1 + m_2\right) - \left(M + \frac{1}{K}\right)},$$

$$A_{20} = -\frac{Gr\cos\alpha A_{12}}{\left(m_1 + m_3\right)^2 + \left(m_1 + m_3\right) - \left(M + \frac{1}{K}\right)}, \quad A_{21} = -\left(A_{14} + A_{15} + A_{16} + A_{17} + A_{18} + A_{19} + A_{20}\right).$$