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# Free Space Optical System Performance for a Gaussian Beam Propagating Through Non-Kolmogorov Weak Turbulence

Italo Toselli, Larry C. Andrews, Ronald L. Phillips, and Valter Ferrero

**Abstract**—Atmospheric turbulence has been described for many years by Kolmogorov’s power spectral density model because of its simplicity. Unfortunately several experiments have been reported recently that show Kolmogorov theory is sometimes incomplete to describe atmospheric statistics properly, in particular in portions of the troposphere and stratosphere. It is known that free space laser system performance is limited by atmospheric turbulence. In this paper we use a non-Kolmogorov power spectrum which uses a generalized exponent instead of constant standard exponent value  $11/3$  and a generalized amplitude factor instead of constant value  $0.033$ . Using this spectrum in weak turbulence, we carry out, for a Gaussian beam propagating along a horizontal path, analysis of long term beam spread, scintillation, probability of fade, mean signal to noise ratio and mean bit error rate as variation of the spectrum exponent. Our theoretical results show that for alpha values lower than  $\alpha = 11/3$ , but not for alpha close to  $\alpha = 3$ , there is a remarkable increase of scintillation and consequently a major penalty on the system performance. However when alpha assumes values close to  $\alpha = 3$  or for alpha values higher than  $\alpha = 11/3$  scintillation decreases leading to an improvement on the system performance.

**Index Terms**—Atmospheric turbulence, beam spreading, bit error rate (BER), fade, non Kolmogorov spectrum, scintillation, signal to noise ratio (SNR), structure function.

## I. INTRODUCTION

FOR A long time, the structure function has been modeled according to Kolmogorov’s power spectrum of refractive index fluctuations which is widely accepted and has been applied extensively in studies of optical and radio wave propagation in the atmosphere. However, recent experimental data from space-based stellar scintillation, balloon-borne *in-situ* temperature, and ground-based radar measurements indicate that turbulence in the upper troposphere and stratosphere deviates from predictions of the Kolmogorov model [5], [9], [10]. Further development of the turbulent theory of passive scalar transfer has shown that although the Kolmogorov spectrum is generally correct, it constitutes only one part of the more general behavior of

passive scalar transfer in a turbulent flow [6]. In fact, Euler equation has two integrals of motion, and not simply energy. The second integral of motion proved to be the magnitude called helicity or mean helicity. It plays an important role in the non-Kolmogorov turbulent fluctuations; depending on the relationship between the parameters of the velocity field and of the passive scalar field various types of spectra with different alpha value can arise. Some anomaly behavior [3] seems to occur when the atmosphere is extremely stable because under such condition the turbulence is no longer homogeneous in three dimensions since the vertical component is suppressed. It has been shown [4] that for such two dimensional turbulence, the rate of the energy cascade from larger to smaller scales is reduced and Kolmogorov turbulence is not fully developed.

In addition anisotropy in stratospheric turbulent inhomogeneities has been experimentally investigated [5], [8], [11], [12] and turbulence spectrum was investigated by Lidar measurements [15]. The experimental results show the various strata and layers in the vertical turbulence profiles. It is shown that the power law exponent of the structure function is different from the cases of purely Kolmogorov. Finally, A. Tunick paper [18] showed recently experimental results on non-Kolmogorov turbulence even at low altitude (rooftop elevation) for lasercom systems that traverse complex inhomogeneous propagation paths.

We must accept de facto that turbulence is still an unsolved problem in classical physics, and the scientific community must persist in doing more simulations, measurements and experiments [7].

It is very important, therefore, to find new models more general than Kolmogorov spectrum in order to describe experimental data also in non-Kolmogorov turbulence. In this work, we present a theoretical spectrum model which reduces to one type of Kolmogorov only for a particular case of its exponent: the standard value  $11/3$ . The exponent can assume all the values between the range 3 to 4. Using this new spectrum, following the same procedure already used from Andrews, Phillips, *et al.* [1], [2], we have analyzed the impact of the exponent’s variation on long term beam spread, scintillation index, probability of fade, mean signal to noise ratio and mean bit error rate for horizontal path, that is for constant value of the refractive index structure parameter. We have done this analysis for a Gaussian beam propagating in weak turbulence. The same analysis both for plane wave and spherical wave has been reported in [13] and in [16]. Angle of arrival for plane wave and spherical wave in non Kolmogorov turbulence has been analyzed in [14].

We used in our approach the Rytov method because we supposed to be in weak turbulence, in fact the Rytov method, as re-

ported in [1], can not be used in strong turbulence conditions. In strong turbulence conditions we need to use parabolic equation method which leads to correct results for all turbulence conditions (weak, moderate and strong). However, parabolic equation method, in weak turbulence, leads to equivalent results with the Rytov method only for first-order and second-order field moments but only asymptotic results have been obtained at present for the fourth-order field moments. The paper [19]–[21] show a method based on parabolic equation and modal decomposition to carry out up to second-order moment of the field (two-frequency mutual coherence function), but they do not analyze the fourth-order field moment and the scintillation index.

## II. NON KOLMOGOROV SPECTRUM

The basic power-law spectrum of Kolmogorov is defined by

$$\Phi_n(\kappa) = 0.033 \cdot C_n^2 \cdot \kappa^{-11/3} \quad (1)$$

where  $C_n^2$  is the refractive-index structure parameter. The validity of the Kolmogorov spectrum is restricted to the inertial range although in some analyses it is extended to all spatial wave numbers. Here we examine a more general power spectrum model that describes non-Kolmogorov atmospheric turbulence in which the power law exponent 11/3 is allowed to deviate somewhat from this value.

We assume that in an atmosphere exhibiting non-Kolmogorov turbulence the structure function for the index of refraction is given by

$$D_n(r) = \beta \cdot C_n^2 \cdot r^\gamma \quad (2)$$

where  $\gamma$  is the power law which reduces to 2/3 in the case of conventional Kolmogorov turbulence. Here,  $\beta$  is a constant equal to unity when  $\gamma = 2/3$ , but otherwise has units  $m^{-\gamma+2/3}$ . Following same procedure reported in [1], the corresponding power-law spectrum associated with structure function takes the form

$$\Phi_n(\kappa, \alpha) = A(\alpha) \cdot \tilde{C}_n^2 \cdot \kappa^{-\alpha}, \quad \kappa > 0, \quad 3 < \alpha < 4 \quad (3)$$

where  $\alpha = \gamma + 3$  is the spectral index or power law,  $\tilde{C}_n^2 = \beta \cdot C_n^2$  is a generalized structure parameter with units  $m^{-\gamma}$ , and  $A(\alpha)$  is defined by

$$A(\alpha) = \frac{1}{4\pi^2} \Gamma(\alpha - 1) \cos\left(\frac{\alpha\pi}{2}\right), \quad 3 < \alpha < 4 \quad (4)$$

and the symbol  $\Gamma(x)$  in the last expression is the gamma function. When  $\alpha = 11/3$ , we find that  $A(11/3) = 0.033$ , and the generalized power spectrum reduces to the conventional Kolmogorov spectrum in (1). Also, when the power law approaches the limiting value  $\alpha = 3$ , the function  $A(\alpha)$  approaches zero. Consequently, the refractive-index power spectral density vanishes in this limiting case.

## III. LONG TERM BEAM SPREAD

The first important quantity that shows total average beam spot size radius on the receiver lens is the *long term beam spread*. It can be written as the sum of three terms: diffraction limited beam spreading, beam spreading due to small turbulence scales and beam wander which can be described by the

variance of the instantaneous center of the beam in the receiver plane.

The analytical form of *long term beams spread* for a Gaussian beam wave is [1]

$$W_e^2 = \langle W_{LT}^2(\alpha) \rangle = W^2 \cdot [1 + \langle T(\alpha) \rangle] \quad (5)$$

where  $W$  is the diffraction limited spot size radius and  $\langle T(\alpha) \rangle$  is the term which includes small scale beam spreading and beam wander atmospheric effects.

To carry out *long term beams spread* analysis we need to calculate the  $\langle T(\alpha) \rangle$  term.

For horizontal path the parameter  $C_n^2$  that appears inside the relation  $\tilde{C}_n^2 = \beta \cdot C_n^2$  is constant. Following same formula reported in [1] but using the non Kolmogorov spectrum in (3), we carry out

$$\begin{aligned} T(\alpha) &= 4\pi^2 k^2 L \cdot \left( \int_0^1 \int_0^\infty \kappa \cdot \phi_n(\alpha, \kappa) d\kappa d\xi \right. \\ &\quad \left. - \int_0^1 \int_0^\infty \kappa \cdot \phi_n(\alpha, \kappa) \exp\left(-\frac{\Lambda L \kappa^2 \xi^2}{k}\right) d\kappa d\xi \right) \\ &= -2\pi^2 \cdot A(\alpha) \cdot \frac{1}{\alpha - 1} \cdot \Gamma\left(1 - \frac{\alpha}{2}\right) \\ &\quad \cdot \Lambda^{\alpha/2-1} \cdot \tilde{C}_n^2 \cdot k^{3-\alpha/2} \cdot L^{\alpha/2} \\ &= 0.25 \cdot \frac{\alpha}{\alpha - 1} \cdot \left[ \sin\left(\alpha \cdot \frac{\pi}{4}\right) \right]^{-1} \cdot \tilde{\sigma}_R^2(\alpha) \cdot \Lambda^{\alpha/2-1} \quad (6) \end{aligned}$$

where  $\xi = 1 - z/L$  ( $z$  is the propagation distance),  $\Lambda = 2L/kW^2$  and we have defined a non Kolmogorov Rytov variance by the plane wave scintillation index in non Kolmogorov weak turbulence [17]

$$\begin{aligned} \tilde{\sigma}_R^2(\alpha) &= -8\pi^2 \cdot A(\alpha) \cdot \frac{1}{\alpha} \cdot \Gamma\left(1 - \frac{\alpha}{2}\right) \\ &\quad \cdot \sin\left(\alpha \cdot \frac{\pi}{4}\right) \cdot \tilde{C}_n^2 \cdot k^{3-\alpha/2} \cdot L^{\alpha/2}. \quad (7) \end{aligned}$$

It is interesting to observe that for  $\alpha = 11/3$  we obtain the particular case of the Kolmogorov spectrum already reported in [1].

At this point, we plot in Fig. 1 the *long term beam spread* as a function of  $\alpha$  for a particular horizontal case, we take

$$\begin{aligned} L &= 1 \text{ km}; \quad \tilde{C}_n^2 = 7 \cdot 10^{-14} m^{-\alpha+3} \\ \lambda &= 1.55 \text{ } \mu\text{m}; \quad W_0 = 0.01 \text{ m} \end{aligned}$$

where  $W_0$  is the spot radius at the transmitter and we take a collimated beam at the transmitter.

We deduce from Fig. 1 that if alpha decreases from  $\alpha = 11/3$  (excluding alpha values close to 3), then *long term beam spread*  $W_e$  increases up to a maximum value. At this point the curve changes its slope because of the term  $A(\alpha)$  that assumes very low values. In addition it is shown that if alpha increases from  $\alpha = 11/3$ , then the *long term beam spread*  $W_e$  decreases down to a minimum value. At this point the curve changes its slope, because the term  $\Gamma(1 - \alpha/2)$  assumes high values close to its asymptote as  $\alpha \rightarrow 4$ . The obvious physical interpretation of alpha approaching 3 is that turbulence tends to vanish. On the

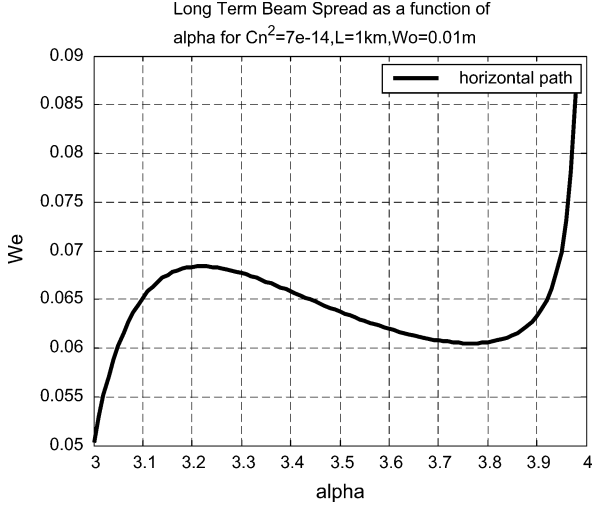


Fig. 1. Long term beam spread as a function of alpha for horizontal path.

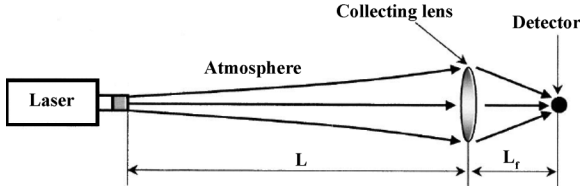


Fig. 2. Propagation geometry for a Gaussian beam originating at distance  $L$  to the left of a thin Gaussian lens.

other extreme, the physical interpretation of alpha approaching 4 is that phase effects dominate in the form of random tilts, which generate beam wander.

#### IV. SCINTILLATION

Another important parameter that is necessary to calculate the system performance is the scintillation index. In our analysis, we include aperture averaging effects of the receiver aperture, so we carry out the flux variance in the plane of the detector at a short distance  $L_f$  behind the collecting lens. We illustrate such a system below in Fig. 2.

To describe the beam characteristics at the input plane and at the front plane of the Gaussian lens, we use two sets of non-dimensional beam parameters. We assume the transmitted Gaussian beam at the input plane has finite radius  $W_0$  and phase front radius of curvature given by  $F_0$ . Thus, we have

$$\begin{aligned} \text{at the transmitter } (z = 0) : \Theta_0 &= 1 - \frac{L}{F_0} \\ \Lambda_0 &= \frac{2L}{kW_0^2} \\ \text{at the Gaussian lens } (z = L) : \Theta_1 &= \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2} \\ \Lambda_1 &= \frac{\Lambda_0}{\Theta_0^2 + \Lambda_0^2}. \end{aligned}$$

Following the same procedure as discussed in [1] for the standard Kolmogorov spectrum, but this time using a non Kolmogorov spectrum and introducing the collecting lens diameter

$D_G$ , our analysis for a Gaussian beam leads to (neglecting beam wander effects)

$$\begin{aligned} \sigma_I^2(\alpha, D_G) &= 4\pi^2 \cdot k^{3-\alpha/2} \cdot L \\ &\cdot \text{Re} \int_0^1 \int_0^\infty \kappa \cdot \Phi_n(\kappa, \alpha) \cdot \exp \left\{ -\frac{L \cdot \kappa^2}{k(\Lambda_1 + \Omega_G)} \right. \\ &\cdot \left. \left[ (1 - \bar{\Theta}_1 \xi)^2 + \Lambda_1 \Omega_G \xi^2 \right] \right\} \\ &\cdot \left\{ 1 - \exp \left[ -j \frac{L \kappa^2}{k} \left( \frac{\Omega_G - \Lambda_1}{\Lambda_1 + \Omega_G} \right) \right. \right. \\ &\cdot \left. \left. \times \xi (1 - \bar{\Theta}_1 \xi) \right] \right\} d\kappa d\xi \\ &= 4\pi^2 \cdot k^2 \cdot L \cdot A(\alpha) \cdot \tilde{C}_n^2 \cdot \Gamma \left( 1 - \frac{\alpha}{2} \right) \\ &\cdot \left[ \frac{k}{L} \cdot (\Lambda_1 + \Omega_G) \right]^{1-\alpha/2} \\ &\cdot \left\{ \int_0^1 \left[ (1 - \bar{\Theta}_1 \cdot \xi)^2 + \Lambda_1 \Omega_G \xi^2 \right]^{\alpha/2-1} d\xi \right. \\ &- \text{Re} \int_0^1 \left[ (1 - \bar{\Theta}_1 \cdot \xi)^2 + \Lambda_1 \Omega_G \xi^2 \right. \\ &\left. \left. + j(\Omega_G - \Lambda_1) \cdot \xi (1 - \bar{\Theta}_1 \cdot \xi) \right]^{\alpha/2-1} d\xi \right\} \quad (8) \end{aligned}$$

where  $\xi = 1 - z/L$  ( $z$  is the propagation distance),  $\Omega_G = 2L/kW_G^2$  is a non-dimensional parameter characterizing the spot radius of the Gaussian collecting lens,  $W_G^2 = D_G^2/8$  and  $\bar{\Theta}_1 = 1 - \Theta_1$ .

At this point we plot  $\alpha_1^2(1, D_G)$  as a function of alpha for a particular horizontal case. We take

$$\begin{aligned} L &= 1000 \text{ m}; \tilde{C}_n^2 = 7 \cdot 10^{-14} \text{ m}^{-\alpha+3} \\ \lambda &= 1.55 \text{ } \mu\text{m}; D_G = 0.1 \text{ m} \\ W_0 &= 0.01 \text{ m}, \Theta_0 = 1. \end{aligned}$$

The results are shown in Fig. 3.

We deduce from Fig. 3 that for alpha values lower than Kolmogorov value  $\alpha = 11/3$ , excluding alpha values close to 3, there is a increase of scintillation. Consequently, scintillation in this case leads to a larger penalty on system performance. We deduce also that there is a maximum value of scintillation where the curve changes its slopes because the term  $A(\alpha)$  begins to decrease to zero. In addition for alpha values higher than  $\alpha = 11/3$ , scintillation slightly decreases and consequently it will lead to a slight gain in system performance. The physical interpretation of alpha approaching 4 is that the power spectrum contains fewer eddies of high wave numbers; therefore scintillation effect are reduced.

#### V. PROBABILITY OF FADE

Given a PDF model for irradiance fluctuations  $p_I(I)$ , the *probability of fade* describes the percentage of time the irradiance of the received signal is below some prescribed threshold

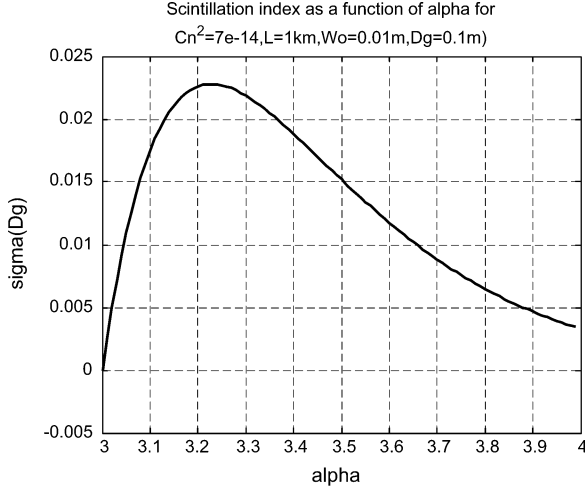


Fig. 3. Scintillation index as a function of alpha for horizontal path.

value  $I_T$ . Hence, the *probability of fade* as a function of threshold level is defined by the cumulative probability [1]

$$p_I(I < I_T) = \int_0^{I_T} p_I(I) dI. \quad (9)$$

The PDF most often used under weak irradiance fluctuations is the lognormal model and the resulting *probability of fade* leads to

$$p_I(I < I_T) = \frac{1}{2} \left\{ 1 + \operatorname{erf} \left[ \frac{\frac{1}{2} \cdot \sigma_I^2(\alpha, D_G) - 0.23 \cdot F_T}{\sqrt{2} \cdot \sigma_I(\alpha, D_G)} \right] \right\} \quad (10)$$

where  $\operatorname{erf}(x)$  is the error function. In arriving at this expression we have introduced the fade threshold parameter

$$F_T = 10 \cdot \log_{10} \left( \frac{\langle I \rangle}{I_T} \right) [dB]. \quad (11)$$

The fade parameter  $F_T$ , given in decibels [dB], represents the dB level below the on-axis mean irradiance that the threshold  $I_T$  is set.

Using scintillation index (8) into (10), we calculate the *probability of fade* as a function of alpha for a fixed fade threshold parameter for a particular horizontal case.

We take

$$\begin{aligned} L &= 1 \text{ km}; \quad \tilde{C}_n^2 = 7 \cdot 10^{-14} m^{-\alpha+3} \\ \lambda &= 1.55 \text{ } \mu\text{m}; \quad D_G = 0.1 \text{ m}, \quad F_T = 3 \text{ dB} \\ W_0 &= 0.01 \text{ m}, \quad \Theta_0 = 1. \end{aligned}$$

The plot is shown in Fig. 4.

## VI. MEAN SIGNAL TO NOISE RATIO

In this paragraph is shown the *mean signal to noise ratio* in presence of atmospheric turbulence using non Kolmogorov power spectrum. The received irradiance over long measurement intervals must be treated like random variable because of

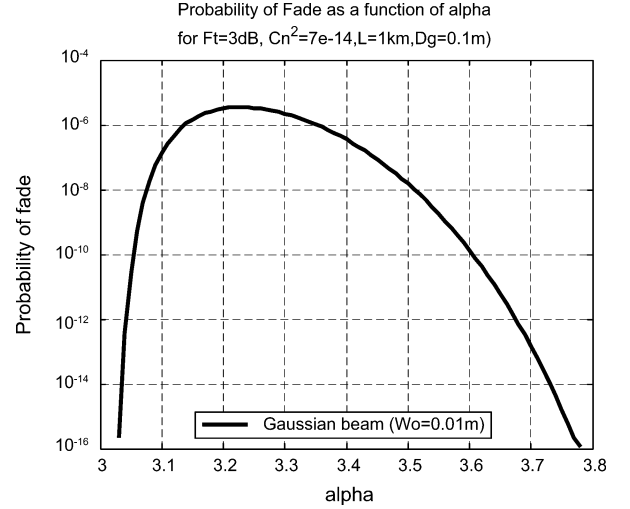


Fig. 4. *Probability of fade* as a function of alpha using the log-normal PDF.

the turbulence. Based on [1], [2], the *mean signal to noise ratio*  $\langle \text{SNR} \rangle$  at the output of the detector in the case of a shot-noise limited system assumes the form

$$\langle \text{SNR} \rangle = \frac{\text{SNR}_0}{\sqrt{1 + \sigma_I^2(\alpha, D_G) \cdot \text{SNR}_0^2}} \quad (12)$$

where  $\sigma_I^2(\alpha, D_G)$  has been defined before,  $\text{SNR}_0$  is the signal to noise ratio in absence of turbulence.

We plot in dB units *mean signal to noise ratio*  $\langle \text{SNR} \rangle$  as a function of *signal to noise ratio without turbulence*  $\text{SNR}_0$  for several alpha values, using Gaussian beam model for scintillation. We take the following parameters:

$$\begin{aligned} L &= 1 \text{ km}; \quad \tilde{C}_n^2 = 7 \cdot 10^{-14} m^{-\alpha+3} \\ \lambda &= 1.55 \text{ } \mu\text{m}; \quad D_G = 0.1 \text{ m} \\ W_0 &= 0.01 \text{ m}, \quad \Theta_0 = 1. \end{aligned}$$

The plot, shown in Fig. 5, illustrates the impact of the alpha variation on the  $\langle \text{SNR} \rangle$  performance. For alpha values lower than  $\alpha = 11/3$ , excluding alpha values close to 3, there is a penalty on the system performance with respect to the case of Kolmogorov  $\alpha = 11/3$ . For alpha values higher than  $\alpha = 11/3$  there is a gain on the system performance with respect to the case of Kolmogorov  $\alpha = 11/3$ . Finally also there is a gain on system performance with respect to Kolmogorov  $\alpha = 11/3$  when alpha assumes values very close to  $\alpha = 3$  because the amplitude factor  $A(\alpha)$  assumes very low values and consequently the *scintillation* reported before in Fig. 2 approaches zero.

## VII. MEAN BIT ERROR RATE

In the presence of optical turbulence, the probability of error is considered a conditional probability that must be averaged

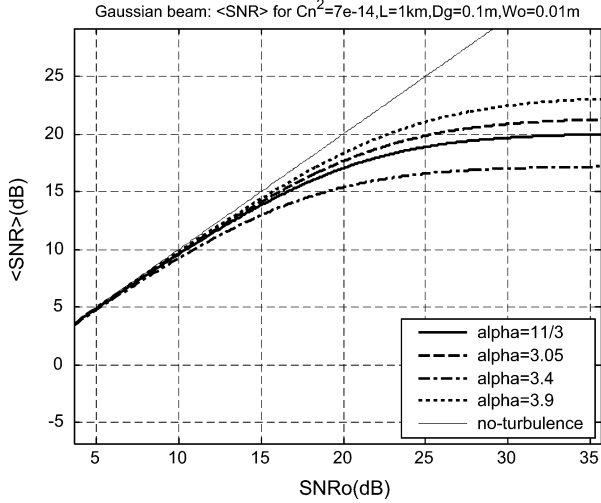


Fig. 5. Mean signal to noise ratio as a function of signal to noise ratio without turbulence for several alpha values, using Gaussian beam model for scintillation.

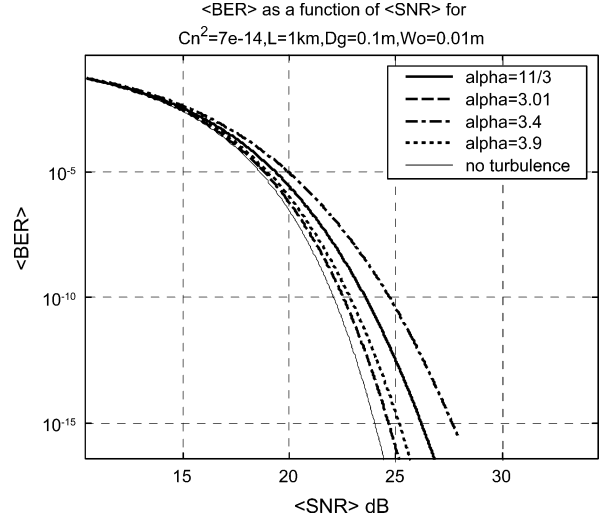


Fig. 6. Mean bit error rate as a function of mean signal to noise ratio for several alpha values.

over the PDF of the random signal to determine the unconditional mean BER. In terms of a normalized signal with unit mean, this leads to the expression [1], [2]

$$\Pr(E) = \langle \text{BER} \rangle = \frac{1}{2} \cdot \int_0^{\infty} p_I(u) \cdot \text{erfc} \left( \frac{\langle \text{SNR} \rangle \cdot u}{2 \cdot \sqrt{2}} \right) du \quad (13)$$

where  $p_I(u)$  is taken to be the log normal distribution with unit mean

$$p_I(u) = \frac{1}{u \cdot \sigma_I(D_G, \alpha) \cdot \sqrt{2 \cdot \pi}} \cdot \exp \left\{ -\frac{[\ln(u) + \frac{1}{2} \cdot \sigma_I^2(D_G, \alpha)]^2}{2 \cdot \sigma_I^2(D_G, \alpha)} \right\}, u > 0. \quad (14)$$

We plot the mean bit error rate  $\langle \text{BER} \rangle$  as a function of mean signal to noise ratio  $\langle \text{SNR} \rangle$  (dB) for several alpha values using Gaussian beam model for scintillation. We take the same parameters

$$\begin{aligned} L &= 1 \text{ km}; \quad \tilde{C}_n^2 = 7 \cdot 10^{-14} m^{-\alpha+3} \\ \lambda &= 1.55 \mu\text{m}; \quad D_G = 0.1 \text{ m} \\ W_0 &= 0.01 \text{ m}, \quad \Theta_0 = 1. \end{aligned}$$

The plot is shown in Fig. 6.

It shows the impact of the alpha variation on  $\langle \text{BER} \rangle$  performance. Also in this analysis, when alpha is lower than  $\alpha = 11/3$ , excluding alpha values close to 3, there is a penalty, but for alpha higher than  $\alpha = 11/3$ , there is an improvement on the system performance. However, when alpha assumes values close to  $\alpha = 3$ , there is a gain on the  $\langle \text{BER} \rangle$  performance with respect to  $\langle \text{BER} \rangle$  value corresponding to  $\alpha = 11/3$ , because the scintillation approaches zero.

## VIII. CONCLUSION

In this paper we introduced a non Kolmogorov power spectrum which uses both a generalized exponent and a generalized amplitude factor instead of a constant standard exponent value  $\alpha = 11/3$  and a constant amplitude factor 0.033 associated with the conventional Kolmogorov spectrum. This non-Kolmogorov spectrum has been developed from a generalized structure function. It has been shown, for a Gaussian beam wave propagating along horizontal link, the long term beam spread, scintillation, probability of fade, mean SNR and mean BER as variations depending on the alpha exponent lead to results somewhat different than those obtained with the standard value of Kolmogorov  $\alpha = 11/3$ .

It is shown that for alpha values lower than  $\alpha = 11/3$ , but not for alpha close to  $\alpha = 3$ , there is a remarkable increase of scintillation and consequently a major penalty on the system performance. However, when alpha assumes a value close to  $\alpha = 3$ , the amplitude factor  $A(\alpha)$  assumes a very low value and consequently the long term beam spread and scintillation decrease, leading to an improvement on the system performance. Finally, also for higher alpha values than  $\alpha = 11/3$  the scintillation decreases and consequently it improves system performance.

## REFERENCES

- [1] L. C. Andrews and R. L. Phillips, *Laser Beam Propagation through Random Media*, 2nd ed. Philadelphia, PA: SPIE Press, 2005.
- [2] L. C. Andrews, R. L. Phillips, and C. Y. Hopen, *Laser Beam Scintillation with Applications*. Philadelphia, PA: SPIE Press, 2001.
- [3] D. Dayton, B. Pierson, and B. Spielbusch, "Atmospheric structure function measurements with a Shack-Hartmann wave front sensor," *Opt. Lett.*, vol. 17, no. 24, pp. 1737–1739, 1992.
- [4] J. McWilliam, "A demonstration of the suppression of turbulent cascades by coherent vortices in two-dimensional turbulence," *Phys. Fluids, A 2*, vol. 547, 1990.
- [5] M. S. BeLenkii, S. J. Karis, J. M. Brown, II, and R. Q. Fugate, "Experimental study of the effect of non-Kolmogorov stratospheric turbulence on star image motion," *SPIE*, vol. 3126, pp. 113–123.
- [6] E. Golbraikh and N. S. Kopeika, "Behavior of structure function of refraction coefficients in different turbulent fields," *Appl. Opt.*, vol. 43, no. 33, pp. 5151–5156, 2004.
- [7] G. Falkovich and K. R. Sreenivasan, "Lessons from hydrodynamic turbulence," *Phy. Today*, pp. 43–49, Apr. 2006.

- [8] A. I. Kon, "Qualitative theory of amplitude and phase fluctuations in a medium with anisotropic turbulent irregularities," *Wave in Random Media*, vol. 4, pp. 297–305, 1994.
- [9] B. E. Ströbling, B. M. Welsh, and M. C. Roggemann, "Optical propagation in non-Kolmogorov atmospheric turbulence," *SPIE*, vol. 2471, pp. 181–196.
- [10] D. T. Kyrzasis, J. Wissler, D. D. B. Keating, A. J. Preble, and K. P. Bishop, "Measurement of optical turbulence in the upper troposphere and lower stratosphere," *SPIE*, vol. 2110, pp. 43–55, 1994.
- [11] M. S. Belen'kii, S. J. Karis, and C. L. Osmon, "Experimental evidence of the effects of non-Kolmogorov turbulence and anisotropy of turbulence," *SPIE*, vol. 3749, pp. 50–51, 1999.
- [12] R. R. Beland, "Some aspects of propagation through weak isotropic non-Kolmogorov turbulence," *SPIE*, vol. 2375, pp. 6–16.
- [13] I. Toselli, L. C. Andrews, and R. L. Phillips, "Free space optical system performance for laser beam propagation through non Kolmogorov turbulence," *SPIE*, vol. 6457, 2007.
- [14] I. Toselli, L. C. Andrews, R. L. Phillips, and V. Ferrero, "Angle of arrival fluctuations for free space laser beam propagation in non Kolmogorov turbulence," *SPIE*, vol. 6551, 2007.
- [15] A. Zilberman, E. Golbraikh, and N. S. Kopeika, "Lidar studies of aerosol and non Kolmogorov turbulence in the mediterranean troposphere," *SPIE*, vol. 5987, 2005.
- [16] I. Toselli, L. C. Andrews, R. L. Phillips, and V. Ferrero, "Free space optical system performance for laser beam propagation through non Kolmogorov turbulence for uplink and downlink paths," *SPIE*, vol. 6708, 2007.
- [17] I. Toselli, L. C. Andrews, R. L. Phillips, and V. Ferrero, "Scintillation index of optical plane wave propagating through non Kolmogorov moderate-strong turbulence," *SPIE*, vol. 6747, 2007.
- [18] A. Tunick, "Analysis of free-space laser signal intensity over a 2.33 km optical path," *SPIE*, vol. 6708, 2007.
- [19] J. Oz and E. Heyman, "Modal theory for the two-frequency mutual coherence function in random media. General solution and plane wave solution," *Waves in Random Media*, vol. 7, pp. 79–93, 1997.
- [20] J. Oz and E. Heyman, "Modal theory for the two-frequency mutual coherence function in random media," *Plane Wave Solution (Part II)*, *Waves in Random Media*, vol. 7, pp. 95–106, 1997.
- [21] J. Oz and E. Heyman, "Modal theory for the two-frequency mutual coherence function in random media. Beam waves," *Waves in Random Media*, vol. 8, pp. 159–174, 1998.



**Italo Toselli** was born in 1975 in Nettuno, Rome, Italy. He received the M.Sc. degree in electrical engineering from the "Università La Sapienza" Rome, Italy, and the postgraduate "XIV Master in Information Technology" from the "Cefriel-Politecnico di Milano," Milan, Italy, both in 2002 and the Ph.D. degree in electrical engineering from the Politecnico di Torino, Italy, in 2008.

He was an Officer in the Italian Navy from October 2002 to December 2003. From January 2006 to December 2007, he was a Visiting Student at the

University of Central Florida (UCF), Orlando, where he researched free space optical propagation through atmospheric turbulence. He is currently a Visiting Researcher at the Florida Space Institute, UCF.



**Larry C. Andrews** received the Ph.D. degree in theoretical mechanics from Michigan State University, East Lansing, in 1970.

Currently, he is a Professor of mathematics at the University of Central Florida, Orlando, and an Associate Member of the College of Optics/CREOL. He is also an Associate Member of the Florida Space Institute. Previously, he held a faculty position at Tri-State University and was a staff mathematician with Magnavox Company, antisubmarine warfare operation. He has been an active researcher in optical wave propagation through random media for more than 25 years and is the author or coauthor of ten textbooks on topics of differential equations, boundary value problems, special functions, integral transforms, wave propagation through random media, and mathematical techniques for engineers. Along with wave propagation through random media, his research interests include special functions, random variables, atmospheric turbulence, and signal processing.



**Ronald L. Phillips** received the Ph.D. degree in electrical engineering from Arizona State University, Tempe, in 1970.

Currently, he is Director of the Florida Space Institute and a Professor in the Department of Electrical and Computer Engineering at the University of Central Florida. He is also a member of the Department of Mathematics and an Associate Member of the College of Optics/CREOL. He has held positions on the faculties at Arizona State University and the University of California, San Diego. He has been an active researcher in wave propagation through random media for more than 28 years. He has co-authored two textbooks on wave propagation through random media and mathematical techniques for engineers. In addition to optical wave propagation, his research interests include optical communications and imaging through atmospheric turbulence.

Dr. Phillips was awarded a Senior NATO Postdoctoral Fellowship in 1977 and the American Society for Engineering Education 1983 Medal for outstanding contributions in research.



**Valter Ferrero** received the Laurea in Ingegneria Elettronica (*summa cum laude*) from the Politecnico di Torino, Turin, Italy, in 1994.

In 1994, he collaborated with the Politecnico di Torino, working on optical coherent systems. From 1995 to 1996, he was with GEC Marconi, Genova, Italy, where he worked on the design and implementation of Erbium doped fibre optical amplifier. He also worked on the design of optical interfaces for SDH systems STM4 and STM16. In 1997, he was in charge of the Optical Communication Group, Electrical Department, Politecnico di Torino and, in February 2001, he became Assistant Professor. He supervises lab conduction and is involved on several research topics related to optical communications. Currently he is primarily involved in optical coherent communications and free space communications.