

# Frege, contextuality and compositionality

Theo M.V. Janssen

Computer Science, University of Amsterdam

Plantage Muidergracht 24, 1018TV Amsterdam, The Netherlands

email: theo@wins.uva.nl

## 1 Introduction

The name ‘Frege’s principle’ is well known: it is discussed, obeyed or rejected in philosophy, linguistics, logic and computer science (see e.g. [33] and [45]). Surprisingly, two completely different principles which are denoted by that name. In most fields it refers to ‘the principle of compositionality of meaning’:

The meaning of a compound expression is a function of the meaning of its parts and of the syntactic rule by which they are combined.

But many philosophers understand it as the ‘context principle’ or ‘the principle of contextuality’. That is a principle that Frege mentions in the beginnings of ‘Grundlagen der Arithmetik’:

Never ask for the meaning of a word in isolation, but only in the context of a sentence

Authors who interpret the principle in the one way, often are not aware of the fact that others do so in the other way. An impressive description of the intellectual landscape in which the two interpretations bloom is given by Pelletier [45].

In this article I will investigate whether the name ‘Frege’s principle’ is justified in view of Frege’s writings. Did he accept both principles at the same time, did he hold the one principle but not the other, or did he at some moment change his opinion?

## 2 On interpretations of the principles

The principles are formulated in colloquial language, and that leaves room for interpretation. There is a whole gamut: the interpretations vary from ones in which the principles are contradictory, to ones in which they supplement each other. Below an impression of the possibilities is given.

### \* Contextuality

The literal interpretation of contextuality says that the notion ‘meaning of a word in a given sentence’ might be considered, but the ‘meaning of a word in isolation’ is not a sensible notion and should play no role in the semantic

theory. One might have the position that meanings of words in isolation do not exist at all, or, alternatively, that no meanings are assigned to such words. An extreme interpretation of contextuality is that only meanings of sentences are defined. A modern theory in which this is –by and large– the case, is Hintikka’s game theoretical semantics [29].

If one accepts the literal version and allows for meanings of subexpressions of sentences, then one might use the meanings of sentences to find these. The meaning of a word in a given sentence is the contribution it makes to the meaning of that sentence. This method will in the sequel be called the *contextual method*. One might have the position that this method finds meanings among already defined ones, or, alternatively, introduces such meanings. An author who says that Frege used (a version of) the contextual method is e.g. Currie [9]. The contextual method is not a necessary consequence of contextuality (e.g. there might be other sources for determining word meanings).

\* Compositionality

The standard interpretation of compositionality is that for basic expressions a meaning is given, and that operations are defined on these meanings which yield meanings for compound expressions. Almost all modern linguistic theories which give serious attention to semantics follow this idea, for instance MG (Montague grammar), GPSG (Generalized phrase structure grammar), CG (Categorical grammar) and LTAG (Lexicalized tree adjoining grammar). In these theories a meaning is assigned to words in isolation; what the meanings are differs considerably (a logical expression, a feature bundle, or something else).

A technical description of the standard interpretation is that syntax and semantics are algebras, and meaning assignment is a homomorphism from syntax to semantics. This definition of compositionality is found with authors as Montague [43], Janssen [34], and Hodges [31]. Hodges also mentions some variants. For the present discussion the relevant aspect is that in these approaches the ‘meaning of an expression’ always is defined.

If one accepts compositionality, one might use this as a method for finding the meanings of compound expressions. Start with (representations of) the meanings of words, perform on these the semantic operations which correspond to the formation of the sentence, and eventually one obtains the meaning of the compound. This will be called *the compositional method*. One might have the position that this method finds meanings among already defined ones, or, alternatively, introduces such meanings.

\* Reconciling interpretations

Note that with the interpretations considered above, it is not possible to accept both principles at the same time: there is a conflict between them. This is underscored by the fact that the only modern theory which obeys contextuality [29] is proud of being non-compositional and uses this feature to defeat other theories.

There are several proposals for a reconciliation of the principles, and these

are more widespread than the literal interpretation. The most influential proposal is by Dummett in his *Frege's philosophy of language* ([10] pp.192–196). In his interpretation contextuality expresses that it makes no sense to consider the meaning of word in isolation and next go to some unrelated other issue: speaking about the meaning of word only makes significance as preparation for considering the meaning of a sentence. In his *The interpretation of Frege's philosophy* [11] other arguments are mentioned. Recently he presented a new interpretation of the principle [12]; this one is, as he states, in conflict with his earlier publications. But it falls in the same category: Frege did not take the principle literally.

Also in modern semantic theories the two principles are reconciled. Words do have a meaning in isolation, but which meaning that is, is determined by the role a word plays in sentences. Let us consider as an example the determiner *every*. When considered in isolation, it may be assumed to have as meaning the universal quantifier ( $\forall x$ ). Motivated by its contribution to sentence meanings, it gets as meaning the generalized quantifier  $\lambda P \lambda Q \forall x [P(x) \rightarrow Q(x)]$ . Another example is the subjunctive connective *whether*. Asking someone what this word means, makes no sense, for no one will have an opinion about it. But about the sentence *John knows whether Mary comes* one has intuitions: e.g. in case Mary does not come, the sentence expresses that John knows that Mary does not come. Based upon sentence meanings the connective can get a meaning (see [23]).

This approach does justice to both principles. To contextuality, because what a word means, is determined in the context of sentences, viz. by investigating its contribution to the meanings of the sentences. And to compositionality, because the meaning of a compound expression is a function of the (thus determined) meanings of its parts. Contextuality and compositionality are in this interpretation complementary principles which cooperate in the process of designing a semantic theory. A philosophic motivation for this approach is given by Rott [49].

In all these reconciling interpretations contextuality is not taken literally: it is allowed that words in isolation do have a meaning. In the sequel we will see that the literal interpretation also might be combined with some non standard interpretation of compositionality (sections 4 and 6.6).

The aim of this article is to find out whether the name ‘Frege’s principle’ has a basis in his writings, and for which interpretation. Therefore we have to find out where Frege stood in the gamut. The main feature of our method is a careful reading of what Frege wrote about subjects related with contextuality and compositionality. A difference with most of the literature is that we will strictly follow the chronological order. That enables us to discover developments in Frege ideas.

In this paper several writings of Frege are quoted in translation. Then ‘Bedeutung’ will be translated by ‘meaning’, and ‘Satz’ by ‘sentence’. Thus I follow the decision of Long and White, the translators of Frege’s posthumous writings [27], and other authors. They argue that these translations express what the

words mean, any other translation would load them with an interpretation.

### 3 1884: The foundations of arithmetic

#### 3.1 The situation in mathematics

In the 19th century there were important developments within calculus, and remarkable results were obtained. For instance, a curve was discovered that has nowhere a tangent; imaginary numbers, as the name says, strange numbers, turned out to be very useful, and the infinity was counted by Cantor. Many notions that previously seemed obvious, now required fundamental investigations. Frege's publication *Grundlagen der Arithmetik* [13] was preceded by a many other publications on foundations. To illustrate this, I will give a *chronological* list of 19th century authors which are discussed by Frege - he often gives incomplete information (partial title, no year of publication).

In *Grundlagen* (1884) Frege discusses: M. Cantor (1855) [7], H. Grassmann (1860) [22], H. Hankel (1867) ([24] and [25]), G. Köpp (1867) [37], J. J. Baumann (1868) [2], E. Kossak (1872) [38], O. Hesse (1872) [28], E. Schröder (1873) [51], J.S. Mill (1877) [41] - Frege used this German translation, the original is from 1843 [40] -, Lipschitz (1877) [39], W.S. Jevons (1879) [36] - Frege used this 3rd edition, original is from 1873 -, J. Thomae (1880) [55], O. Schloemilch (1881) [50], and G. Cantor (1883) [6]. Furthermore Frege discusses several authors from the 17th and 18th century: Descartes, Hobbes, Leibniz, Kant, Locke, Hume, Berkeley. For them he used a collection from 1868 [2], and the collected writings of Kant and Leibniz.

Frege mentions several notions of which the foundations have to be investigated: function, continuity, limit, infinity, negative numbers, and imaginary numbers [13, §1]. Remarkably enough, in his book on foundations most of these notions are *not* considered at all; the discussion mainly concerns natural numbers like 0, 1 and 2. As will be shown below, his main motivation for *Grundlagen* is not to solve all these fundamental problems, but to warn for the dangerous direction taken by most of his predecessors.

#### 3.2 Frege's aims

As an example of Frege's argumentation ([13] §7) we paraphrase his discussion of J.S. Mill [41], (the quotations are mainly taken from the translation by Austin [17]). Mill was in those days the most discussed philosopher and politician, both in England and Germany, and was the authority not only for economists and politicians, but also for philosophically interested scientists (witness [57] p.VII).

Frege reports the following. According to John Stuart Mill all knowledge is empirical. His definitions of numbers are not definitions in the logical sense, because they do not only state the meaning of an expression, but also express empirical facts. For instance the number 3 consists, according to Mill, 'in this

that collections of objects exists, which while they impress the senses thus  $\star \star \star$ , may be separated into two parts, thus  $\star \star \quad \star$ .' ([13] p.9<sup>e</sup>).

Frege comments as follows. 'What a mercy that not everything in the world is nailed down; for if it were, we should not be able to bring off this separation, and  $2+1$  would not be  $3!$  What a pity that Mill did not also illustrate the physical facts underlying the numbers 0 and 1!' ([17] p.9<sup>e</sup>). There are several other reasons why this approach is ridiculous, for instance 'On Mill's view we could actually not put  $1,000,000 = 999,999 + 1$  unless we observed a collection of things split up in precisely this particular way [...]' ([17] p.11<sup>e</sup>). Such an approach can be called 'gingerbread and pebble arithmetic [...]' ([17] p.vii<sup>e</sup>). It remains only to ascribe to the flavor of the bread some special meaning for the concept of number' ([17] p.vii<sup>e</sup>).

The essential aspect of Frege's objection is that if we call a proposition empirical on the ground that we must have made observations in order to become conscious of its content, we are making a psychological statement, which concerns solely the content of the proposition; the question of its truth is not touched ([13] §8).

### 3.3 The role of contextuality

The discussion of Mill is one of the sharpest discussions in *Grundlagen*. It expresses Frege's motivations: his aim is the objective truth of mathematics, and the greatest danger is the subjectivity introduced by psychology. This danger is the main theme in the introduction of his book. In his introduction he discusses psychologism (the movement in those days that tried to base the foundations of logical and mathematical knowledge to psychology). The first remark is somewhat positive: 'It may serve some purpose to investigate the ideas and changes of ideas which occur during the course of mathematical thinking; but psychology should not imagine that it can contribute anything whatever to the foundation of arithmetic' ([17] p.vi<sup>e</sup>). But then he becomes much sharper: 'But this account makes everything subjective. And if we follow it through to the end, it does away with truth.' ([17] p.vii<sup>e</sup>). Therefore mathematics should refuse all assistance from the direction of psychology. ([17] p.ix<sup>e</sup>).

So Frege's aim was: to defend the objectivity of mathematics against the influence of psychologism. In order to reach this aim, he keeps three principles for his investigations of the foundations of arithmetic: ([17] p.x<sup>e</sup>)

1. always to separate sharply the psychological from the logical, the subjective from the objective
2. never ask for the meaning of a word in isolation, but only in the context of a sentence
3. never lose sight of the distinction between concept and object.

From the previous discussion we understand why the first point is mentioned: it is his main theme. And 'If the second point is not observed, one is almost forced to take as the meanings of words mental pictures or acts of the individual mind, and so to offend against the first principle as well. As to the third point,

it is a mere illusion to suppose that a concept can be made an object without altering it' [17, p. x<sup>e</sup>].

It is remarkable that Frege does not argue for the correctness of the principle. The only arguments Frege presents in favor of the principle are that its consequences suit him. This can be explained from the historic context [35].

In 1891, so seven years after *Grundlagen*, Husserl published his *Philosophie der Arithmetik. Logische and psychologische Untersuchungen* [32]. Seeing the subtitle, it is not surprising that there was a discussion between Husserl and Frege, and later a negative review article by Frege [15]. Frege scholars (e.g. [3] appendix 4) say that the review had the effect of converting Husserl to anti-psychologism, but Husserl scholars informed me that from Husserl's diary it is clear that his standpoint had changed already before the review.

### 3.4 Applications of contextuality

Frege gives several examples where we cannot associate a concept with a word, so where a word has no meaning in isolation.

#### Infinitely small numbers (§60)

Frege argues as follows. 'Even so concrete a thing as the Earth we are unable to imagine as we know it to be.' It is too large, there is no way to have a conception of it. Accordingly, any word, 'for which we can find no mental picture appears to have no content.' But 'That we can form no idea of its content is [...] not a reason for excluding it from our vocabulary'. 'It is enough if the sentence as a whole has a sense; by this also its parts get content'. This approach may clarify difficult concepts, such as infinitesimals (infinitely small units), which arise in calculus e.g. in  $df(x) = g(x)dx$ . 'The problem is not, as might be thought, to produce a segment bounded by two distinct points whose length is  $dx$ , but rather to define the sense of an identity of the type  $df(x) = g(x)dx$ ' [...] we ought always to keep before our eyes a complete sentence. Only therein do the words really have a meaning.'

#### Finite numbers (§60, §61, §62)

The discussion concerning natural numbers is introduced by the last sentence of §60, which gives a repetition of the contextuality principle. 'The self-subsistence I claim for number, is *not* taken to mean that a number word signifies something when removed from the context of a sentence.' [my emphasis TJ] Next Frege again argues that we cannot have any idea or intuitions about numbers, and he asks how numbers then can be given to us. And in §62 he says 'It is only in the context of a sentence that words mean something. Therefore the point is to explain the sense of a sentence in which a numeral occurs.' There is a large supply of such sentences, but only some require special attention: 'In our case we have to define the sense of the sentence "the number which belongs to the concept F is the same as that which belongs to the concept G".'

Thereafter, Frege considers identity notions in geometry, and then arrives at his definition of the identity statement asked for above. This definition says that there is a one to one correspondence between the objects falling under concept F with the objects falling under the concept G. ([13] p.85). Next he presents the sentences which define 0 (§74), 1 (§77), and the other finite numbers.

### Infinitely large numbers (§84)

‘Contrasted with the finite numbers are the infinite numbers.’ However, ‘About the infinite number  $\infty_1$  [...] there is nothing mysterious or wonderful’. ‘We may use sentences like “the number which belongs to the concept F is  $\infty_1$ ” and this has ‘a perfectly clear and unambiguous sense’, viz. ‘that there exists a relation which correlates one to one the objects falling under the concept F with the finite numbers.’ ‘This suffices to justify the use of the symbol  $\infty_1$  and to assure it of a meaning’. ‘That we cannot form any idea of an infinite number of objects is absolutely of no importance; the same is equally true of finite numbers.’

### 3.5 Conclusion

We have seen that Frege discusses cases where one cannot have an idea associated with a number ( $dx$  and  $\infty$ ), and that he argues that the same holds for natural numbers. About searching for the meaning of such words he says ‘that is not the problem’, ‘that is of absolutely no importance’, and that a number in isolation signifies nothing. Throughout the whole book he attacks authors who assign meanings to words in isolation.

There is no glimpse of an attempt to find the meaning of a word as a preparation for its use in a sentence (Dummetts interpretation, see 2), nor any of the other reconciling interpretations. Although Frege leaves open the possibility that words within a sentence have a meaning, in his proposal this is not the case. So in fact he obeyed the extreme interpretation of contextuality. This makes clear that Frege really literally meant what he wrote when he formulated the context principle. Confronted with the principle of compositionality, Frege would (in the period of Grundlagen) have been vehemently against it.

## 4 1892: On sense and meaning

In his paper *Über Sinn und Bedeutung* [14] Frege introduced the distinction between sense and meaning. If one is interested in Frege’s attitude to compositionality, then, so many authors say, one has to distinguish between compositionality of sense and compositionality of meaning. This will be investigated below.

Frege’s aim in this paper is to explain the difference between the informative sentence  $b = a$  and the *a priori* true sentence  $a = a$ . The names  $a$  and  $b$  may designate the same object, but they do so in a different way. For this way

of presentation he introduced the name ‘sense’. The meanings of  $a$  and  $b$  in the two sentences are the same, but their senses are different. Therefore the first equation is informative. The situation is like this only in ordinary use; in reported speech words mean what in normal speech is their sense (‘In ungerade Rede ... das bedeuten was gewöhnlich ihr Sinn ist’ [14, p.28]).

Note that Frege’s position in this paper is more liberal than in *Grundlagen*. He acknowledges that at least some expressions within a sentence may have a meaning, whereas in *Grundlagen* only whole sentences get a meaning.

Frege investigates the assumption that the meaning of a sentence is its truth value and uses as argument a substitution property of meanings of subexpressions [14, p.35]:

If our supposition that the meaning of a sentence is its truth value is correct, the latter must remain unchanged when a part of the sentence is replaced by an expression with the same meaning. And this is indeed the case.

He also investigates the substitution property for sentences [14, p.35]:

But we have not yet considered the case that the expression for which we substitute is a sentence itself. If our insight is correct, then the truth value of a sentence which has another one as part must remain unchanged if we replace the subsentence by another one which has the same truth value.

These quotes are interesting because there is a relation between the principle of compositionality of meaning and the substitution property of meaning: some authors consider these properties to be equivalent. A general formulation of the substitution property is: ‘If in an expression a subexpression is replaced by one with the same meaning, then the meaning of the complete expression is unchanged.’ Below we will investigate whether, starting from the substitution property, a compositional reconstruction of Frege’s ideas would be possible.

The substitution property follows immediately from compositionality (see e.g. [33]). One expects a relation in the other direction as well, and the proof proceeds as follows. All expressions which can be substituted for each other in a given sentence make the same contribution to the meaning of the sentence. Hence they form an equivalence class of meanings, and one might introduce a common meaning for them (which can then be used in a compositional system). A proof along the lines just sketched is given by Hodges [31]. It requires some additional tidiness conditions to be fulfilled, and then the for us relevant result is that properties 1. and 2. are equivalent (Theorems 4 and 6, section 5):

1. The language has a compositional semantics
2. The synonymy relation of the language is a congruence relation.

The second clause is a technical formulation of the following property: synonymous expressions can be substituted for each other in all contexts, and the resulting expressions are synonymous as well.

The meanings resulting from the theorem are not without complications. It might not be obvious how to select a representative for such an equivalence class. And the construction might end up with a notion of meaning that one



does not like. A classical example consists of the expressions ‘a creature with a heart’ and ‘a creature with kidneys’; probably they are intersubstitutable in all contexts, so would get the same meaning. This does, however, not conform to our intuitions. According to Hodges [30], the selection of a representative is a problem of linguistics, not of compositionality.

Frege warns repeatedly that there are exceptions to the substitution property. For instance when he discusses the examples ‘Copernicus believed that the orbits of the planets are circles’ and ‘Copernicus believed that the apparent motion of the sun is produced by the real motion of the earth’. These embedded sentences have the same truth value, so one might expect them to be substitutable for each other without changing the truth of the whole. But Frege says ([14, p.37], my emphasis T.J.):

In these cases it is not allowed to replace in the subordinate clause an expression by another one which has the same customary meaning, but only by one which has the same indirect meaning, i.e. its customary sense. If someone would conclude that the meaning of a sentence is not its truth value ‘since if that were the case one could replace a sentence in all cases by another one with the same truth value’ would conclude too much: with the same argumentation one could state that the meaning of the word ‘morning star’ would not be Venus. One only has the right to conclude that the meaning of a sentence is *not always* its truth value and that ‘the morning star’ does not always mean the planet Venus, viz. when the word has its indirect meaning.

As for the meaning of a sentence the situation seems comparable with proper names: in ordinary use its meaning is a truth value, and in indirect speech its sense. However, Frege mentions many more possibilities. The meaning of the sentence can be an order or a request [14, p.39] if the main verb expresses this. Another meaning arises in [14, p.39]:

Whoever discovered the elliptic form of the planetary orbits died in misery.

Here the meaning of the embedded sentence (Whoever . . . orbits) is not a truth value, but an individual. Another intriguing example is [14, p. 47]:

Bebel fancies that the return of Alsace-Lorraine would appease France’s desire for revenge.

This sentence expresses *two* thoughts:

1. Bebel believes that the return of Alsace-Lorraine would appease France’s desire for revenge;
2. the return of Alsace-Lorraine would not appease France’s desire for revenge.

This discussion shows the context dependency of meaning in Frege’s approach. Therefore it is an consequence of Hodges’ theorem that Frege’s ideas concerning meaning can *not* be reconstructed in a compositional way. For Frege synonymy was not a congruence relation: two expressions which have the same meaning in direct speech cannot always be inter-substituted in indirect speech. So the second condition of Hodges’ theorem is not satisfied, and therefore not the

first either. As a consequence, the quotes concerning substitution given above cannot be used to argue for compositionality: from the substitution property compositionality follows only under conditions which are *not* fulfilled in Frege's theory.

So far for substitution of meanings. As for sense, there is not much ground to claim that Frege assumed a substitution property. The above quote concerning Copernicus contains a sentence which has been used to argue for it:

In these cases it is not allowed to replace in the subordinate clause an expression by another one which has the same customary meaning, but only by one which has the same indirect meaning, i.e. its customary sense.

Another indication arises when Frege discusses the meaning of 'Odysseus' in the sentence 'Odysseus was set ashore in Ithaca while deeply asleep' [14, p.34]:

If only the sense of the sentence, the thought were relevant, than it was not necessary to bother with the meaning of the name; as it concerns the sense of the sentence, only the sense, and not the meaning of this part can be relevant.

These quotes say that the senses of the parts are relevant to determine the sense of the sentence, but not more. As a description of a substitution property of sentences they are not convincing. And even if we would accept them as expressing a substitution property, the same problem arises as with the substitution property for meanings. Meanings of sentences are context dependent, and therefore their senses are, too.

Two alternatives have been proposed. Dummett [11, pp. 266–267] proposes to let the sense of an expression be the same in all contexts (see also section 6.6). Then the direct relation between meaning and sense has to be given up. But it was *not* Frege's choice to have the same sense in all contexts. Hodges[31] proposes that two expressions are synonymous if they have the same sense *and* the same meaning. This would work if the sense and the meaning of an expression were context independent - again not Fregean.

Carnap [8] discusses extensively the role of the substitution properties in Frege's paper and argues that they are heuristic principles. He gives the following characterization of the differences with Frege's method [8, p. 125]:

A decisive difference between our method and Frege's consists in the fact that our concepts, in distinction to Frege's, are independent of the context. An expression in a well-constructed language system always has the same extension and the same intension: but [in Frege's theory] in some contexts it has its ordinary nominatum and its ordinary sense, in other contexts its oblique nominatum and its oblique sense.

Carnap presents several arguments why his method is preferable over Frege's: that theory gives an unnecessary multiplicity of entities and names (with a word is associated its meaning, its sense, the sense of its sense, etc.), and in certain contexts a name may have simultaneously several nominata. And indeed, Montague's successful work [42] was based upon Carnap's intension-extension distinction, and not on Frege's Sinn-Bedeutung distinction.

The conclusion is that Frege's statements concerning substitution properties give no basis at all for a compositional reconstruction of Frege's ideas, neither for meaning, nor for sense. On the contrary, Frege would attack compositionality because that goes against his conviction that the meaning of an expression depends on the context, and that conviction is fundamental to his approach.

## 5 1893-1913: Contextuality repeated

It has been claimed that contextuality makes no appearance in Frege's writings after *Grundlagen* and that he gave it up (e.g. Dummett [10, p.192] and Resnik [46, p.46] [47] [48]), whereas other authors deny this (e.g. Currie [9] and Sluga [52], [53] and [54]). In this section we will investigate several papers that play a role in this discussion.

### 5.1 1893(1880):

#### Boole's calculating logic and the concept-script

In 1969 Frege's *Posthumous writings* appeared [27], and Sluga wrote a review [52]. He remarks that contextuality is repeated several times therein, and points to two papers. The first one is a fragment from *Booles rechnende Logik und die Begriffsschrift* which is considered here. It is a very interesting one, because it goes along with a remarkable story. The second paper which Sluga mentions will be considered section 6.3.

Frege writes [26, p.204], [27, p.17]:

So, instead of putting the judgement together out of an individual entity as subject, and an already previously formed concept as predicate, we let, in the opposite direction, the judgeable content fall into parts and obtain in this way the concept.

Here Frege describes the contextual method. In a footnote he mentions one of the dangers of the opposite method: it would lead to unpleasant discussions about negative concepts, such as 'non-triangle'. A few lines later Frege warns us that the above method is not a way to obtain isolated properties or relations, and paraphrases the contextuality principle:

But it does not follow from this that the ideas of these properties and relations are formed apart from entities. Hence in the *Begriffsschrift* their designations never occur on their own, but only in combinations which express the judgeable contents. I could compare this with the behavior of the atoms, of which it is supposed that one atom never occurs on its own, but only in a combination with others, moving out of one combination only in order to enter immediately into another one.

After Sluga's review Resnik published a second paper [47] in which he argues that Frege rejected contextuality. He seems to accept the above quote as a formulation of contextuality, but says 'that it is clearly not relevant because

it was written *before* the *Grundlagen*' [47, p.47]. Indeed *Grundlagen* is from 1884, and *Booles rechnende Logik* from 1880 (Frege submitted it several times in vain). But we will show in a minute that the passage is highly relevant for our investigations.

Frege has a footnote on the last sentence of the quotation: 'As I have seen since, Wundt uses in his *Logik* the same image in a similar way'. The authors of the *German* edition of Frege's *Posthumous Writings* inform us that this picture of parts as atoms does *not* occur in the first edition from 1880 of Wundt's *Logik*, but in the 3th edition from 1906. This is not quite correct: the picture already occurs in the 2nd edition from 1893. It means that Frege cannot have added the footnote before 1893 and proves that he accepted contextuality also at that moment. Although the *English* version of the *Posthumous writings* does not have such an editorial footnote, this cannot explain Resnik's ignorance of the reference to Wundt: Resnik's paper appeared three years earlier.

Due to the added footnote *Booles rechnende Logik und die Begriffsschrift* certainly is relevant, and illustrates that Frege held the same view on contextuality both before and after *Grundlagen*.

## 5.2 1896: Letter to Peano

The quote below played a role in the discussion whether Frege at a certain moment abandoned the principle of contextuality. It is taken from a letter Frege wrote to Peano as reaction on his positive review of *Grundgesetze der Arithmetik*. Frege makes some remarks about using concepts in proofs, and mentions the inscrutability of vernacular languages. Then he says [18, p.183], [19, p.115]:

The case is different when inferences have to be drawn: then it is essential that in two sentences the same expression occurs, and that it has exactly the same meaning in both. It must therefore have a meaning for its own that is independent of the other parts of the sentence.

Note that Frege acknowledges again that expressions within a sentence (may) have a meaning. The point Frege makes in the above quote would nowadays be made by requiring the expression to be 'unambiguous', or, without that term, that the expression has just one meaning. Frege, however, uses a more complex formulation. He speaks about the meaning which the expression has within the two sentences ('it has *in both* the same meaning' [my emphasis T.J]), and not just about its meaning. In this way he obeys contextuality.

The last sentence of the quote deserves special attention because of its role in the discussion in the literature (see below). Its last phrase is: 'the other parts of the sentence'. Let us call the sentence of which the parts are supposed to be taken 'S'. The phrase 'other parts' assumes a dichotomy: the 'it' in 'it must have a meaning' concerns the remainder of S. So the sentence says that the meaning of that remainder must be independent of other parts of S. Hence the meaning of that remainder is conceived of as the meaning of an expression within the context of a sentence; thus also this sentence is not in conflict with contextuality.

Resnik ([46] and [47]) used a variant of the last sentence of the above quote as an example of an explicit rejection by Frege of the context principle. He does not tell us why he considers it as a rejection of contextuality, but I will try to give an explanation.

The original German text reads ‘Er muss also für sich eine Bedeutung haben, die unabhängig ist von den anderen Theilen des Satzes.’ Resnik translates this as [46, p.362]: ‘Therefore, it must have a reference by itself which is independent of the parts of the sentence.’ On two points I disagree with Resnik’s translation. The first is that ‘für’ is not translated into ‘for’ (or ‘of’ as in [19]) but into ‘by’, the second that ‘anderen’ (‘other’) is not translated. These cannot be printing errors because nine years later he uses almost the same translation [47, p.46]. The translation ‘by itself’ suggests that the meaning originates from the expression itself, and excludes that it comes from somewhere else (e.g. derived from the sentence meaning). Since ‘other’ is lacking, the translation does not exhibit the dichotomy created by the original text, and the expression is no longer seen in its coherence with other parts of a sentence. So Resnik’s translation lacks all indications of contextuality. With these indications the text is, as we have seen, in accordance with contextuality. Maybe he understood the sentence under discussion as stating that the expression has a meaning, and that the sentence in which it occurred and its parts are irrelevant.

Also Dummett [11, p.543] does not consider the quote as an explicit rejection of contextuality (although the formulation conforms to his opinion - based on other grounds - that Frege has rejected it). Dummett interprets it as stating that each word should have the same sense and reference in every context (cf. section 4). That is in my opinion too strong, Frege’s remark only concerns the expression involved in drawing conclusions.

### 5.3 1902: Letter to Huntington

In a letter to E. V. Huntington, probably dating from 1902, Frege writes [18, p.90], [19, p.58]:

Such sign combinations as ‘ $a + b$ ’, ‘ $f(a, b)$ ’ do therefore not mean anything, and have no sense for their own, but can help to express a sense in the coherence of a sentence, as in ‘ $a + b = b + a$ ’ or ‘if  $a + b = c$ , then  $a = c - b$ ’ and similar ones.

The quote repeats the idea of contextuality, eighteen years after *Grundlagen*. In the quote  $a$  and  $b$  are a kind of variables. If they were replaced by specific numbers, Frege would not have used this formulation, because earlier in the paper he says: ‘The signs “2”, “3” I call proper names; each of them designates or means a definite number’. This is different from *Grundlagen* where in his own proposal he did not speak about meanings for number signs, only for sentences containing such signs.

## 5.4 1906: Introduction to Logic

In *Einleitung in die Logik*, an unpublished manuscript (a kind of diary notes) dating from 1906, Frege says [26, p.204], [27, p.185]:

By splitting up a singular thought we obtain components of the complete and of the unsaturated kind, which of course cannot occur in isolation; but each component of the one kind together with any component of the other kind will form a thought.

So twenty-two years after *Grundlagen*, Frege repeats the principle of contextuality: he denies that parts of thoughts can exist in isolation. Although this quote seems relevant, it is not considered in the discussion in the literature.

Note that Frege acknowledges here that within a sentence the parts are associated with parts of the thought. This does not only hold for the parts of the complete kind (as in the 1902 letter), but also for parts of the unsaturated kind.

## 6 1914-1925: Creativity of language

From 1914 on, Frege used several times the argument from creativity of language: how is possible that we understand sentences we have never heard before? Several authors base their opinion concerning the relation between Frege and compositionality on his statements on this subject.

### 6.1 1914: Letter to Jourdain (concept)

In a letter to Frege, Jourdain had reminded him of the fact that Russell had shown that propositions can be analyzed to a form which only assumes that a name has a meaning and not a sense. Jourdain asks Frege some questions, and the third is whether he ‘would hold that sense was merely a psychological property of a name’. The answer Frege wrote (January 1914) is well known for its example concerning mounts Ateb and Aphla. Nevertheless, one might hesitate to let it play a role in the discussion: it is a letter that was never sent. The answer that was sent has no corresponding passage.

Frege gives the following answer on Jourdain’s question [18, p.127] [19, p.79]:

As concerns your third question, I do not believe that we can dispense with the sense of a name in logic; for a sentence must have a sense if it is to be useful. But a sentence consists of parts, which must somehow contribute to the expression of the sense of the sentence, so they themselves must somehow have a sense. [...] The possibility for us to understand sentences which we have never heard before, is evidently based on this, that we construct the sense of a sentence from parts, which correspond to the words.

This last sentence expresses compositionality. However, the passage occurs in the middle of an argumentation answering Jourdain’s question. It is an argument why word senses have a role at all, and has *not* the aim to describe how

the sense of a sentence is formed. This becomes clear from the sequel of the letter, where Frege speaks several times about the sense of a word, and always as part of the sense of a sentence. For instance, immediately after the above quote he says [my emphasis TJ]:

If we find in two sentences the same word, e.g. ‘Etna’, then we also recognize something common in the corresponding thoughts. Without this, language in the proper sense would be impossible [...]. Now this *part of the thought* which corresponds to the name ‘Etna’, cannot be mount Etna itself [...].

It would be simpler to say that the sense of ‘Etna’ cannot be mount Etna itself. But Frege describes the situation as originating from the thought (as contextuality requires), and does *not* start from the senses of the words.

Later in the same letter he considers the situation of two explorers who both have seen a mountain, one thinks it is mount ‘Aphla’, and the other, approaching from another direction, that it is mount ‘Ateb’. When later on it is discovered that it was in fact the same mountain, this is reported by the sentence *Ateb is Aphla*. This thought is different from the thought expressed by *Ateb is Ateb*. This explains why Frege says [my emphasis TJ]:

What corresponds to the name ‘Ateb’ *as part of the thought* must therefore be different from what corresponds to the name ‘Aphla’ *as part of the thought*.

It would be much shorter and clearer to state that ‘Ateb’ and ‘Aphla’ have different senses. Frege carefully avoids this, and only speaks about these senses as parts of the thought of sentences in which these words occur. He does so throughout the whole letter, except for the very last part where he summarizes his conclusions. Then he does not consider specific sentences, and speaks about the sense of a name without further qualifications.

So in this letter, Frege indeed used an argument that was provided by the compositional perspective: the creativity of language. In the actual discussion, he carefully obeys contextuality in its strict form, except when that becomes impractical. Then he uses formulations that are closer to the compositional perspective.

## 6.2 1914: Logic in mathematics

*Logik in der Mathematik* is an unpublished paper, probably written in 1914 as lecture notes. Many themes from Frege’s previous work return, e.g. his explanation of the difference between ‘ $2+3=5$ ’ and ‘ $5=5$ ’. Concerning the relation between thoughts and sentences he says [26, p.212], [27, p.225]:

It is wonderful what language can achieve. With a few sounds and combinations of sounds it is capable of expressing a huge number of thoughts, and in particular also thoughts which have never before been grasped or expressed by any man. What makes these achievements possible? The fact that thoughts are built up from building blocks of thoughts. And these building blocks correspond to groups of sounds,

out of which the sentence expressing the thought is built up, so that the construction of the sentence out of the parts of the sentence corresponds to the construction of a thought out of parts of thoughts. And we may call the part of the thought the sense of that part of the sentence which corresponds to it, in the same way as a thought can be conceived of as the sense of the sentence.

This quote describes the relations between parts of thoughts and parts of sentences which one gets when working in accordance with compositionality. Frege uses a compositional terminology: he speaks about building thoughts from building blocks. One might expect that on a later occasion Frege would make the minor step to the compositional version and start with the parts of sentences, because that would make the explanation of the power of language more convincing: now it seems rather to be about the power of thoughts. But the opposite will happen: starting with building blocks will be rejected (see the sections 6.3 and 6.5 below). Note furthermore that in the last sentence the situation is described again in a contextual way: starting with the thought, from there to the parts of thought, and finally to parts of the sentence. The statement about building blocks is such a remarkable statement that the editors of the German edition added a footnote. It tells us that on all other occasions Frege warns that such building blocks should not be conceived of as being independent of the thought in which they arise.

### 6.3 1919: Notes for Ludwig Darmstaedter

Ludwig Darmstadter, an historian of science, asked Frege for an autograph for his collection. Frege wrote a kind of overview of his scientific enterprise: *Aufzeichnungen für Ludwig Darmstaedter*. It gives the following characterization of his method [26, p.272], [27, p.253]:

So I do not begin with the concepts and put them together to form the thought or the judgement, but I obtain the parts of thought by letting the thought fall into parts. This is the difference between my *Begriffsschrift* and similar creations by Leibniz and his followers, despite of the name I gave it, which perhaps was not a happy choice.

Frege considered the name *Begriffsschrift* ('Concept-script') as not well chosen because the name suggests that he started from 'Begriffe' ('Concepts'), and that was not the case. The above quote describes again the contextual method, and underscores the distinction with the compositional method.

Resnik [47] has another opinion. He claims that nothing Frege says is incompatible with senses of words being associated directly with words. Maybe that is true, strictly speaking, but then it remains to be explained why Frege did not describe his approach in that way. The main question, however, is whether the connection between words and senses is established in the context of a sentence, or in isolation. The following fragment from the paper under discussion describes Frege's view [26, p.274], [27, p.255]:

We can regard a sentence as a mapping of a thought: corresponding



to the part-whole relation between a thought and its parts we have, by and large, the same relation between a sentence and its parts.

As one sees, Frege's point of departure remains the complete thought, not the sentence or the senses of words. Thus he obeys contextuality. But one also finds the flavor of compositionality: within a thought a structure can be distinguished that reflects the structure of the sentence.

## 6.4 1923-I: Compound thoughts

The most well known fragment of Frege which reminds of compositionality is the first sentence of *Gedankegefüge* [16] in the translation by Geach & Stoothoff [21]:

It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even a thought grasped by a terrestrial being for the very first time can be put into a form of words which will be understood by someone to whom the thought is entirely new. This would be impossible, were we not able to distinguish parts in the thoughts corresponding to the parts of a sentence, so that the structure of the sentence serves as the image of the structure of the thoughts.

This fragment has been used to show that Frege abandoned contextuality and adopted compositionality (e.g. Resnik [47, p.47]). There are several arguments why that is not correct.

First about the translation of the second sentence. It contains the phrase 'a thought [...] can be put into a form of words which will be understood by someone ...'. If one parses this as 'words which will be understood' one might see here a glimpse of the idea that thoughts are built. The original version does not have such a possibility because it reads: 'eine Einkleidung findet, in der ihn ein anderer erkennen kann ...'. A more literal translation is: 'finds a form in which someone else can recognize it'. So Frege said that the thought is recognized, and *not* that (as a first step) words are understood.

The second reason is that the last sentence of the quote ('this would be impossible') does not speak about the formation of the sense of a sentence from the senses of its parts. The sentence obeys contextuality because it starts from the sense of the sentence, the thought, and distinguishes therein parts of the thought.

Thirdly, immediately after the quote follows: 'to be sure, we really talk figuratively when we transfer the relation of whole and part to thoughts; yet the analogy is so ready to hand and so generally appropriate that we are hardly ever bothered by the hitches which occur from time to time.' So the quote is an analogy, used to express an idea about thoughts, not to convey any principle on natural language in general.

Finally, the analogy concerns only compound *thoughts*, i.e. thoughts which have a thought as part. Frege warns us that not all sentences composed from a sentence provide a serviceable example, e.g. relative clauses are not an example

‘because they do not express a thought (we cannot tell what the relative pronoun is supposed to refer to)’. So Frege acknowledges here only meanings for those parts which had a meaning anyhow: sentences which express thoughts.

The conclusion is that Frege in the first and second sentence of the quote does not say anything about the formation of sentence meaning from smaller units. The quote is not an expression of what we understand by the principle of compositionality.

## 6.5 1923-II: Compound thoughts

In *Gedankengefüge* Frege makes a remark which is most relevant for the investigations concerning the relation between Frege and the principles; for this reason it is discussed in a section of its own.

Frege investigates compound thoughts like ‘A and B’ (called compound thoughts of the first kind), ‘not [A and B]’ (the second kind), ‘not A and not B’ (the third kind) etc.. He gives a complete inventory of binary connectives made up from negation and conjunction. When considering compound thoughts of the form ‘Not [... and ...]’ Frege says [16, in original p.40],

By filling the gaps with expressions of thoughts, we form the expression of a compound thought of the second kind. But we really should not talk of the compound thought as originating in this way, for it is a thought, and a thought does not originate.

In the line of his argumentation Frege is almost forced to say that a compound thought of the form ‘Not [... and ...]’ is formed from two other thoughts, thus following the compositional approach. But that is a way of speaking which he cannot accept. Thoughts are objective (recall *Grundlagen*), we can only grasp or recognize them, but thoughts do not originate. Frege explicitly denies in this published paper that thoughts are formed, and therefore he would deny compositionality as well.

## 6.6 Discussion

From 1914 on Frege begins to use formulations which are compatible with compositionality, and several of the arguments he uses could also be seen as arguments in favor of compositionality. A practical reason for this shift is that the arguments he uses are more easily expressed from the compositional perspective. A more fundamental reason is his interest in the creativity of language. He used that argument several times: twice in a formulation that says that the sense of a sentence is built, on all other occasions the thought is the initial source, thus obeying contextuality. The picture he gives of the relation between a thought and the corresponding sentence resembles the picture which emerges when compositionality is obeyed. So in the last decade of his career Frege’s position moves to a weak form of compositionality: the thought expressed by a sentence is a function of the parts of thought associated within that sentence with its parts. Or in a *modern* terminology: the meaning of the sentence is a function of the meanings its parts have within that sentence.

The just mentioned weak version of compositionality is a version which is not in conflict with contextuality. It is not included in the gamut in section 2 because it differs in a principled way from what is understood in the literature by compositionality. In linguistic theories meanings are associated with lexical items, independent of context. Authors who argue against compositionality give examples where this approach is in their opinion not acceptable (see e.g. [44]). This criticism would not apply to the weak version, illustrating that it is not what is understood by the principle of compositionality. Furthermore, with the above weak version, the discussion about compositionality would be senseless: because trivially true. For instance, assume sentences to be built from two parts: let the first part mean the same as the given sentence, and let the second part be a nothing-changing modifier. So acceptance of the above weak version, does not mean acceptance of the principle of compositionality.

There are proposals which restrict the weak version to one which is equivalent with standard compositionality. Dummett ([10, p. 268] and [10, pp. 363-364]) proposes to require that an expression has the same meaning in all contexts. This proposal is isomorphic with standard compositionality: the meaning of an expression can then be defined as the one it has in all contexts. Also Hodges [30, section 8.2] considers compositionality without meanings for isolated expressions as a serious option (although in his mathematical work he follows the standard interpretation). Since it was not Frege's opinion that expressions would have the same meaning in all contexts, he would reject these versions.

## 7 Conclusion

In *Grundlagen* Frege was very strict: he did not use meanings for words in isolation, not even for numerals like 1 and 2. He literally meant what he expressed with the context principle. In that period he evidently was against compositionality. Thereafter, the context principle was not repeated in his *published* writings, but he did not give it up as has been claimed. It appears on several occasions in correspondence and in unpublished writings (also in one intended for publication). In the period 1883-1913 there is a gradual development of the way in which Frege speaks about contextuality. First he accepts for number signs a meaning in isolation, but not for unsaturated expressions. Thereafter he accepts that a sense is associated with every word in a sentence. But he repeatedly warns that such senses do not exist in isolation. So he gradually moved from extreme compositionality towards literal contextuality. From 1914 on, Frege's position, pushed by the needs of research, comes close to compositionality. But he could never make the final step for principled reasons. The main reason is that he did not accept the notion 'meaning of an expression': for him meanings were contextually dependent, and he always considered the meaning of an expression within a given sentence. A second reason is that he cannot accept that thoughts are formed. So Frege never gave up contextuality, and would always be against compositionality.

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