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Research Article

Frequency Analysis of Functionally Graded Curved Pipes Conveying Fluid

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The curved pipe made of functionally graded material conveying fluid is considered and the in-plane free vibration frequency of the resulting composite pipe is investigated. The material properties are assumed to distribute continuously along the pipe wall thickness according to a power law and the effective mass, flexural rigidity, and mass ratio are used in the governing equations. The natural frequencies are derived numerically by applying the modified inextensible theory. The lowest four natural frequencies are studied via the complex mode method, the validity of which is demonstrated by comparing the results with those in available literatures. A parametric sensitivity study is conducted by numerical examples and the results obtained reveal the significant effects of material distribution gradient index, flow velocity, fluid density, and opening angle on the natural frequencies of the FGM curved pipes conveying fluid.

1. Introduction

Over the past decades, a great deal of effort has been devoted to the examination of vibrational properties of macro and micro/nanopipes and cylindrical shells conveying fluid due to their significance of academic research and engineering application. Although the straight and planar curved pipes are generally deemed as the most basic structures for conveying fluid, up to now, most of the existing literatures are still concerned with the dynamical study of straight pipes. The main reason is that the simplicity of a straight configuration results in convenience of modeling and solution, and the dynamical behaviors in a certain direction may reflect the mechanical nature of a straight pipe due to the perfect symmetry in geometry morphology. In contrast, the curved pipes conveying fluid have not achieved much attention as straight ones, reasonably because the complicated configuration of a curved pipe requires more variables to describe the state of motion. Further, both in-plane and out-of-plane oscillations should be taken into account as they are completely distinct dynamical behaviors, unlike a straight pipe. In the sense of engineering practice, a curved pipe has a more flexible geometric configuration to meet the demands of surrounding structures, which leads to a broader application of fluid-conveying curved pipes than straight ones.

Research work on dynamics of a curved pipe conveying fluid was originated in the 1970s, and it has just experienced a brief history of less than 50 years so far. In the early studies, attentions were mainly concentrated on the linear aspect of this dynamical problem. In 1972, Chen [1, 2] proposed a linear model for analysis of in-plane vibration of cantilevered, supported curved pipes conveying fluid with inextensible axis theory. One year later, Chen [3] extended his model to the analysis of out-of-plane vibration. However the results obtained in his work did not reveal the dynamical distinction of curved pipes from straight ones. To improve the model of Chen, Misra et al. [4, 5] developed a modified inextensible theory, in which the combined axial force was considered, to examine the linear characteristics of curved pipes conveying fluid. It was found that the curved pipes with supported ends may exhibit different instability feature from the straight pipes. Païdoussis [6] later summarized the dynamical analysis of a curved pipe conveying fluid, involving linear and nonlinear investigations, in his academic work. Ni et al. [7]

and Wang and Ni [8] focused on nonlinear behaviors, especially the bifurcations and chaotic motions of a fluid-conveying curved pipe subjected to nonlinear constraints and harmonic excitation. Their research not only demonstrated the existence of chaotic motions, but also pointed out that the route to chaos for the curved pipe is through period-doubling bifurcations. Recently, Ni et al. [9] presented a nonlinear study on the parametric resonance of a curved pipe conveying pulsating fluid, and the results obtained were in good agreement with the experimental results by literature. Meanwhile, Ni et al. [10] performed the analysis on the forced vibration of fluid-conveying curved pipes resting on a nonlinear elastic foundation under external excitation. It showed that the excitation amplitude, material damping, and foundation stiffness all can greatly affect the steady-state responses of the system. In addition, Zhai et al. [11] provided a means of random vibration analysis for the curved pipes conveying fluid with a Timoshenko curved beam model. Tang et al. [12], Xia and Wang [13], and Ghavanloo et al. [14] attempted to apply the theories associated with curved macro pipes and shells into vibrational analysis of curved micro/nanopipes conveying fluid, and the significant effects of small scale on the dynamical properties of the curved pipe were revealed in their studies.

Even though there have been some theoretical and experimental achievements of study on the dynamics of curved pipes conveying fluid, the models heretofore available are confined to the homogeneous materials, which may result in a serious limit of theoretical contributions on the engineering practice. Actually, a new conception of employment of composites has been advanced towards fluid-conveying pipes. A representative example is concerned with the laminated structures. An extensive research on dynamical behaviors of laminated pipes or cylindrical shells conveying fluid was performed by Chang and Chiou [15], Xi et al. [16], Sharma et al. [17], Toorani and Lakis [18], Kochupillai et al. [19], Kadoli and Ganesan [20], Alijani and Amabili [21], and Ray and Reddy [22]. However as is known, there always exists a fatal flaw in laminated structures that the interface between layers may easily be subjected to cracks as the material property of a certain layer is different from that of adjacent ones. In order to deal with this problem, in the 1980s, the conception of functionally graded materials (FGMs) was introduced. Subsequently, comprehensive attention was paid to the FGMs due to their superior performance in resisting high temperature gradients and eliminating the stress concentration of a common composite [23–26]. Particularly, in comparison with laminated composites, the FG structures possess a continuous variation of the material properties in the thickness direction and thus give rise to a so-called thickness stretching effect [27–31], which reveals that the transverse displacement of the FG structures is variational through the thickness, especially for thick structures, instead of a constant as in the conventional composite theories. In view of this, in several reports, a FGM beam model was established to predict the dynamical behaviors of fluid-conveying pipes or cylindrical shells. Sheng and Wang [32, 33] first studied the thermomechanical vibration of a fluid-conveying shell based on a FGM model. It was found that the material composition and distribution had great effect on the free vibration and dynamic response of the system. Shen et al. [34] conducted an analysis of beam-mode stability of fluid-conveying shells with a periodic FGM model. The results showed that the critical flow velocity can be raised and the stress concentration can be effectively reduced by introducing this model. Setoodeh and Afrahim [35] and Ansari et al. [36] extended the FGM model to micropipes and microshells conveying fluid and found that the material property distribution can also make huge impacts on the natural frequency and critical flow velocity of micro structures conveying fluid, whereas, in the aforementioned researches, the models considered were confined to straight pipes, which were the simplest system conveying fluid and generally inappropriate for a complex structure with special geometrical configuration. To the best of the authors' knowledge, there is no investigation available on the dynamics of a fluid-conveying curved pipe based on FGM models, and it remains unclear how the material properties influence the vibrations of fluid-conveying curved structures.

This study aims to explore the in-plane free vibration frequency of a FGM curved pipe conveying fluid by the complex mode method. Under the assumption that the material properties vary continuously along the pipe wall thickness according to power law distribution, the equation governing in-plane vibration is obtained by introducing the effective mass, flexural rigidity, and mass ratio into the conventional equation by Misra et al. [4]. A typical clamped-clamped end condition is taken into account and the natural frequencies are calculated via the complex mode method. Numerical examples are employed to carry out a parametric sensitivity study and the effects of material distribution gradient index and some key parameters on the free vibration of the FGM curved pipes conveying fluid are discussed in detail. The present work may extend the application of FGMs in complicated fluid-conveying structures and provide theoretical supports and technical reserves for the solution to the vibration-induced failure of composite pipes conveying fluid in chemical engineering and energy systems.

2. Mechanical Model and Material Property

The system under consideration is depicted in Figure 1. It comprises a planar FGM curved pipe of centerline radius R, opening angle α, a uniform annular cross section with inner and outer radii r_i and r_o, respectively, effective mass per unit length m_{eff} and effective flexural rigidity $(EI)_{eff}$, and an incompressible plug flow conveyed with a mass per unit length M, constant velocity U, and pressure p. Variable ϕ is the angular coordinate of the pipe centerline and r and θ are the radial and angular coordinates of the cross section, respectively. In the present study, two types of materials, namely, ceramic and metal, are considered to distribute, respectively, in the inner and outer surfaces of the FGM pipe. Thus, the effective material property of the pipe, Γ_{eff}, can be expressed as

$$\Gamma_{eff} = \Gamma_{in} V_{in} + \Gamma_{out} V_{out},$$
$$V_{in} + V_{out} = 1, \tag{1}$$

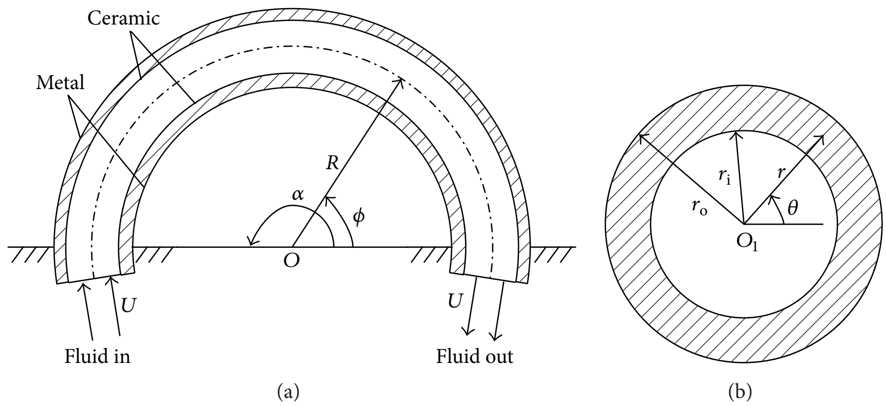

FIGURE 1: Mechanical model of a FGM curved pipe conveying fluid: (a) longitudinal-sectional view and (b) enlarged view of cross section.

where Γ_{in} and Γ_{out} denote the properties of the inner and outer materials, respectively, and V_{in} and V_{out} are the corresponding volume fractions. For the FGM curved pipe considered, V_{in} can be expressed according to the power law as

$$V_{in}(r,k) = \left(\frac{r_o - r}{r_o - r_i}\right)^k, \tag{2}$$

where k is the power gradient index of the volume fraction of the inner material. The effective mass density ρ_{eff} and Young's modulus E_{eff} are then given by

$$\begin{aligned} \rho_{eff}(r,k) &= \rho_{in}V_{in}(r,k) + \rho_{out}V_{out}(r,k), \\ E_{eff}(r,k) &= E_{in}V_{in}(r,k) + E_{out}V_{out}(r,k). \end{aligned} \tag{3}$$

The effective mass, flexural rigidity, and mass ratio can be, respectively, calculated by

$$m_{eff}(k) = 2\pi \int_{r_i}^{r_o} \rho_{eff}(r,k)\, r\, dr, \tag{4}$$

$$(EI)_{eff}(k) = \int_0^{2\pi}\int_{r_i}^{r_o} E_{eff}(r,k)\, r^3 \sin^2\theta\, dr\, d\theta, \tag{5}$$

$$(M_r)_{eff}(k) = \left(\frac{M}{M + m_{eff}(k)}\right)^{1/2}. \tag{6}$$

3. Governing Equation and Solution

By applying the modified inextensible theory, the equation governing in-plane vibration of a fluid-conveying curved pipe made of a single homogeneous material (without external liquid) can be derived as [4]

$$\begin{aligned} &EI\frac{\partial^5 v}{\partial s^5} + \left(\frac{EI}{R^2} + MU^2 + pA_f - T\right)\frac{\partial^3 v}{\partial s^3} \\ &\quad + \left(\frac{MU^2 + pA_f - T}{R^2}\right)\frac{\partial v}{\partial s} + 2MU\frac{\partial^3 v}{\partial s^2 \partial t} \\ &\quad + (M+m)\frac{\partial^3 v}{\partial s \partial t^2} + \frac{MU}{R^2}\frac{\partial v}{\partial t} + \frac{EI}{R}\frac{\partial^4 w}{\partial s^4} \\ &\quad + \frac{EI + \left(MU^2 + pA_f - T\right)R^2}{R^3}\frac{\partial^2 w}{\partial s^2} + \frac{MU}{R}\frac{\partial^2 w}{\partial s \partial t} \\ &\quad - \frac{M+m}{R}\frac{\partial^2 w}{\partial t^2} + \frac{MU^2 + pA_f - T}{R^3}w = 0, \end{aligned} \tag{7}$$

where EI, m, and T stand for the flexural rigidity, mass per unit length, and axial tension of the curved pipe, respectively, A_f stands for the flow cross-sectional area, and w, v, s, and t stand for the in-plane tangential displacement, radial displacement, curvilinear coordinate, and time variable, respectively. Furthermore, by inextensible theory [1], the radial and tangential displacements possess the following relation:

$$v = R\frac{\partial w}{\partial s}. \tag{8}$$

The relation between angular and curvilinear coordinates can be given as

$$\phi = \frac{s}{R}. \tag{9}$$

By considering (8) and (9) in conjunction with the effective mass per unit length and effective flexural rigidity of FGM pipes determined by (4) and (5), we can obtain the following sixth-order partial differential equation of w for a FGM curved pipe conveying fluid:

$$\begin{aligned} &\frac{\partial^6 w}{\partial \phi^6} + \left(2 + \frac{MU^2R^2}{(EI)_{eff}}\right)\frac{\partial^4 w}{\partial \phi^4} + \left(1 + \frac{2MU^2R^2}{(EI)_{eff}}\right)\frac{\partial^2 w}{\partial \phi^2} \\ &\quad + \frac{2MUR^3}{(EI)_{eff}}\frac{\partial^3 w}{\partial \phi^3 \partial t} + \frac{(M + m_{eff})R^4}{(EI)_{eff}}\frac{\partial^4 w}{\partial \phi^2 \partial t^2} \\ &\quad + \frac{2MUR^3}{(EI)_{eff}}\frac{\partial w}{\partial \phi \partial t} - \frac{(M + m_{eff})R^4}{(EI)_{eff}}\frac{\partial^2 w}{\partial t^2} \\ &\quad + \frac{MU^2R^2}{(EI)_{eff}}w \\ &\quad + \frac{(pA - T)R^2}{(EI)_{eff}}\left(\frac{\partial^4 w}{\partial \phi^4} + 2\frac{\partial^2 w}{\partial \phi^2} + w\right) = 0. \end{aligned} \tag{10}$$

The following dimensionless variables and parameters are introduced:

$$\eta = \frac{w}{R},\quad \zeta = \frac{\phi}{\alpha},\quad \tau = \left[\frac{(EI)_{\mathrm{eff}}}{M + m_{\mathrm{eff}}}\right]^{1/2}\frac{t}{R^2},\quad u = \left[\frac{M}{(EI)_{\mathrm{eff}}}\right]^{1/2} RU,\quad \Pi^{\circ} = \frac{(pA - T)R^2}{(EI)_{\mathrm{eff}}}. \tag{11}$$

Substituting (6) and (11) into (10), we can yield the following dimensionless governing equation:

$$\begin{aligned}&\frac{\partial^6\eta}{\partial\zeta^6} + \alpha^2\left(2 + u^2\right)\frac{\partial^4\eta}{\partial\zeta^4} + \alpha^4\left(1 + 2u^2\right)\frac{\partial^2\eta}{\partial\zeta^2}\\ &+ 2\alpha^3 u\left(M_r\right)_{\mathrm{eff}}\frac{\partial^4\eta}{\partial\zeta^3\partial\tau} + \alpha^4\frac{\partial^4\eta}{\partial\zeta^2\partial\tau^2}\\ &+ 2\alpha^5 u\left(M_r\right)_{\mathrm{eff}}\frac{\partial^2\eta}{\partial\zeta\partial\tau} - \alpha^6\frac{\partial^2\eta}{\partial\tau^2} + \alpha^6 u^2\eta\\ &+ \Pi^{\circ}\left(\alpha^2\frac{\partial^4\eta}{\partial\zeta^4} + 2\alpha^4\frac{\partial^2\eta}{\partial\zeta^2} + \alpha^6\eta\right) = 0.\end{aligned} \tag{12}$$

The clamped-clamped end condition demands

$$\left.\frac{\partial\eta(\zeta,\tau)}{\partial\zeta}\right|_{\zeta=0} = 0,\quad \left.\frac{\partial^2\eta(\zeta,\tau)}{\partial\zeta^2}\right|_{\zeta=0} = 0,\quad \eta(0,\tau) = 0,\quad \left.\frac{\partial\eta(\zeta,\tau)}{\partial\zeta}\right|_{\zeta=1} = 0,\quad \left.\frac{\partial^2\eta(\zeta,\tau)}{\partial\zeta^2}\right|_{\zeta=1} = 0,\quad \eta(1,\tau) = 0. \tag{13}$$

It should be noted that, in (12), Π° is the steady-state combined axial force associated with the modified inextensible theory, which consists of fluid pressure and axial tension of the pipe and depends on dimensionless fluid flow velocity, in addition to the gravity loading and the orientation of the pipe. In the conventional inextensible theory, Π° is usually neglected. The effect of internal flow on the eigenfrequencies manifests itself via the centrifugal and Coriolis forces. In the modified inextensible theory, where Π° is taken into account, the internal flow exerts only a Coriolis force [14]. If both ends of the pipe are supported and the gravity effect is neglected, $\Pi^{\circ} = -u^2$ [4] and so the terms associated with the initial forces cancel out those arising from the centrifugal force.

In this paper, we employ a convenient but efficient approach, named complex mode method, to solve (12). A complex mode is adopted instead of the real one to generate a phase difference of the vibration nodes. This is particularly suitable for the analysis of vibrations with moving nodes, which always occurred in axially moving continua and fluid-conveying pipes.

In order to implement the complex mode method, firstly, we assume that the nth-order solution of (12) has a complex form as

$$\eta_n(\zeta,\tau) = B_n(\zeta)\,\mathrm{e}^{\mathrm{i}\omega_n\tau}, \tag{14}$$

where ω_n and $B_n(\zeta)$ are the nth dimensionless circular natural frequency and the corresponding mode function, respectively. If the nth natural frequency of the system is denoted by f_n, ω_n can be given by

$$\omega_n = 2\pi f_n R^2\left[\frac{M + m_{\mathrm{eff}}}{(EI)_{\mathrm{eff}}}\right]^{1/2}. \tag{15}$$

Substitutions of (14) into (12) and (13) yield

$$\begin{aligned}&\frac{\mathrm{d}^6 B_n}{\mathrm{d}\zeta^6} + \alpha^2\left(2 + u^2 + \Pi\right)\frac{\mathrm{d}^4 B_n}{\mathrm{d}\zeta^4}\\ &+ 2\mathrm{i}\omega_n\alpha^3 u\left(M_r\right)_{\mathrm{eff}}\frac{\mathrm{d}^3 B_n}{\mathrm{d}\zeta^3}\\ &+ \alpha^4\left(1 + 2u^2 - \omega_n^2 + 2\Pi\right)\frac{\mathrm{d}^2 B_n}{\mathrm{d}\zeta^2}\\ &+ 2\mathrm{i}\omega_n\alpha^5 u\left(M_r\right)_{\mathrm{eff}}\frac{\mathrm{d}B_n}{\mathrm{d}\zeta}\\ &+ \alpha^6\left(\omega_n^2 + u^2 + \Pi\right)B_n = 0,\end{aligned} \tag{16}$$

$$\left.\frac{\mathrm{d}B_n(\zeta)}{\mathrm{d}\zeta}\right|_{\zeta=0} = 0,\quad \left.\frac{\mathrm{d}^2 B_n(\zeta)}{\mathrm{d}\zeta^2}\right|_{\zeta=0} = 0,\quad B_n(0) = 0,\quad \left.\frac{\mathrm{d}B_n(\zeta)}{\mathrm{d}\zeta}\right|_{\zeta=1} = 0,\quad \left.\frac{\mathrm{d}^2 B_n(\zeta)}{\mathrm{d}\zeta^2}\right|_{\zeta=1} = 0,\quad B_n(1) = 0. \tag{17}$$

Equation (16) is a homogeneous ordinary differential equation with sixth order, the solution of which can be written as

$$B_n(\zeta) = C_{1n}\left(e^{id_{1n}\zeta} + C_{2n}e^{id_{2n}\zeta} + C_{3n}e^{id_{3n}\zeta} + C_{4n}e^{id_{4n}\zeta} + C_{5n}e^{id_{5n}\zeta} + C_{6n}e^{id_{6n}\zeta}\right), \tag{18}$$

where C_{jn}, $j = 1\sim6$, are six unknown constants depending on the boundary conditions and d_{jn}, $j = 1\sim6$, satisfy the following characteristic equation:

$$\begin{aligned} & d_{jn}^6 - \alpha^2\left(2 + u^2 + \Pi\right)d_{jn}^4 - 2\omega_n\alpha^3 u\left(M_r\right)_{\text{eff}} d_{jn}^3 \\ & + \alpha^4\left(1 + 2u^2 - \omega_n^2 + 2\Pi\right)d_{jn}^2 \\ & + 2\omega_n\alpha^5 u\left(M_r\right)_{\text{eff}} d_{jn} - \alpha^6\left(\omega_n^2 + u^2 + \Pi\right) = 0. \end{aligned} \tag{19}$$

Substitute (18) into (17); we get a series of equations with respect to $C_{1n}\sim C_{6n}$ as follows:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ d_{1n} & d_{2n} & d_{3n} & d_{4n} & d_{5n} & d_{6n} \\ d_{1n}^2 & d_{2n}^2 & d_{3n}^2 & d_{4n}^2 & d_{5n}^2 & d_{6n}^2 \\ e^{id_{1n}} & e^{id_{2n}} & e^{id_{3n}} & e^{id_{4n}} & e^{id_{5n}} & e^{id_{6n}} \\ d_{1n}e^{id_{1n}} & d_{2n}e^{id_{2n}} & d_{3n}e^{id_{3n}} & d_{4n}e^{id_{4n}} & d_{5n}e^{id_{5n}} & d_{6n}e^{id_{6n}} \\ d_{1n}^2e^{id_{1n}} & d_{2n}^2e^{id_{2n}} & d_{3n}^2e^{id_{3n}} & d_{4n}^2e^{id_{4n}} & d_{5n}^2e^{id_{5n}} & d_{6n}^2e^{id_{6n}} \end{pmatrix} \begin{pmatrix} 1 \\ C_{2n} \\ C_{3n} \\ C_{4n} \\ C_{5n} \\ C_{6n} \end{pmatrix} C_{1n} = 0. \tag{20}$$

To obtain a nontrivial solution of the above equations, the determinant of the coefficient matrix must be zero. Based on this condition and (18)~(20), we can calculate the circular natural frequency ω_n and mode function $B_n(\zeta)$ numerically. The nth-order solution of (12) is then expressed as

$$\eta_n(\zeta,\tau) = \operatorname{Re}\left[C_{1n}e^{i\omega_n\tau}\left(e^{id_{1n}\zeta} + C_{2n}e^{id_{2n}\zeta} + C_{3n}e^{id_{3n}\zeta} + C_{4n}e^{id_{4n}\zeta} + C_{5n}e^{id_{5n}\zeta} + C_{6n}e^{id_{6n}\zeta}\right)\right]. \tag{21}$$

4. Numerical Results and Discussion

4.1. Validation of the Present Method. Before applying to solve the present problem, in this section, the validity of the complex mode method will be first confirmed through comparing some results with that in the literatures. Owing to the absence of an available report regarding FGM curved pipes conveying fluid, the present FGM model is herein simplified to a model made of a single homogeneous material (by setting the power gradient index k to zero or infinity) to perform the validation. Figure 2 shows the comparison of the lowest four dimensionless natural frequencies calculated by the complex mode method with those by the finite element method (FEM) of [4], in which the values of opening angle and mass ratio are $\alpha = \pi$ and $M_r = 0.71$, respectively. It can be seen that the results obtained by the two techniques are in very good agreement, which demonstrates the effectiveness of the complex mode method. Besides, we can find that the complex mode method has a relatively simple process of solution for higher-order partial differential equations and the calculation is not too much. This also indicates the convenience of the complex mode method in dealing with the dynamic problem of axially moving systems.

4.2. Numerical Example. The geometric size of the curved pipe is taken as $R = 0.5$ m, $\alpha = \pi$, $r_i = 6.4$ mm, and $r_o = 7.4$ mm. Referring to [32], Zirconia and Aluminum are, respectively, taken as the ceramic and metallic materials distributed in the inner and outer surfaces of the FGM curved pipe and the values of the related physical parameters are given as $\rho_{Zir} = 3000\ \text{kg/m}^3$ and $E_{Zir} = 151$ GPa for Zirconia and $\rho_{Al} = 2707\ \text{kg/m}^3$ and $E_{Al} = 70$ GPa for Aluminum. The fluid conveyed inside the pipe is Water with $\rho_w = 1000\ \text{kg/m}^3$.

Figures 3(a)–3(d) show the lowest four natural frequencies of the FGM curved pipes conveying fluid as functions of flow velocity for different values of power gradient index k. It can be concluded that the effect of power gradient index on the free vibration of the FGM pipes is significant. The four natural frequencies all decrease as k increases, especially for small values of k, and the natural frequencies of higher-mode vibrations have a greater change than those of lower-mode ones. Actually, when the value of k varies from 0 to ∞, the curved pipe may gradually change from a fully ceramic pipe to a fully metallic one, which leads to a more flexible structure. In order to better elucidate the impact of material distribution gradient index, a further detailed analysis is illustrated in Figure 4, which describes the fundamental frequency (the first natural frequency) of the FGM curved pipes as a function of k for a specified flow velocity $U = 120$ m/s. The results of other natural frequencies have the similar tendency with that of the fundamental frequency. As can be seen in Figure 4, the variation of the fundamental frequency has grown slower with k increasing. The frequency is subjected to a sharp decrease as k increases for $k < 4$. When $k > 4$, the frequency has little change as k increases, and it is close to the value of a metallic curved pipe. This indicates that the material property has obvious effect on the free vibration of FGM curved pipes conveying fluid only as k is small.

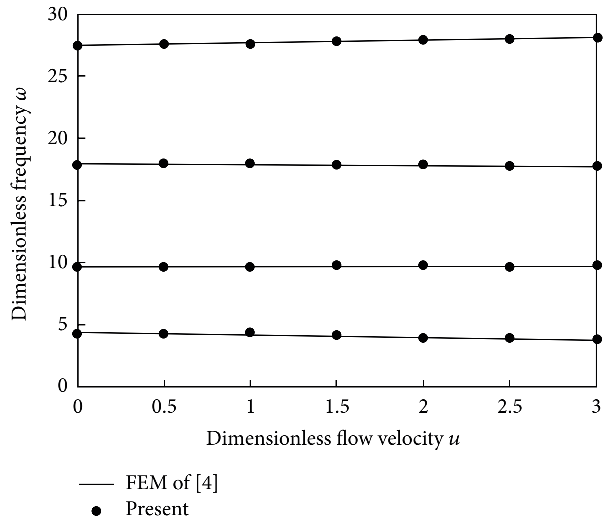

Figure 2: The lowest four dimensionless natural frequencies of a single homogeneous curved pipe conveying fluid as functions of dimensionless flow velocity for $\alpha = \pi$ and $M_r = 0.71$.

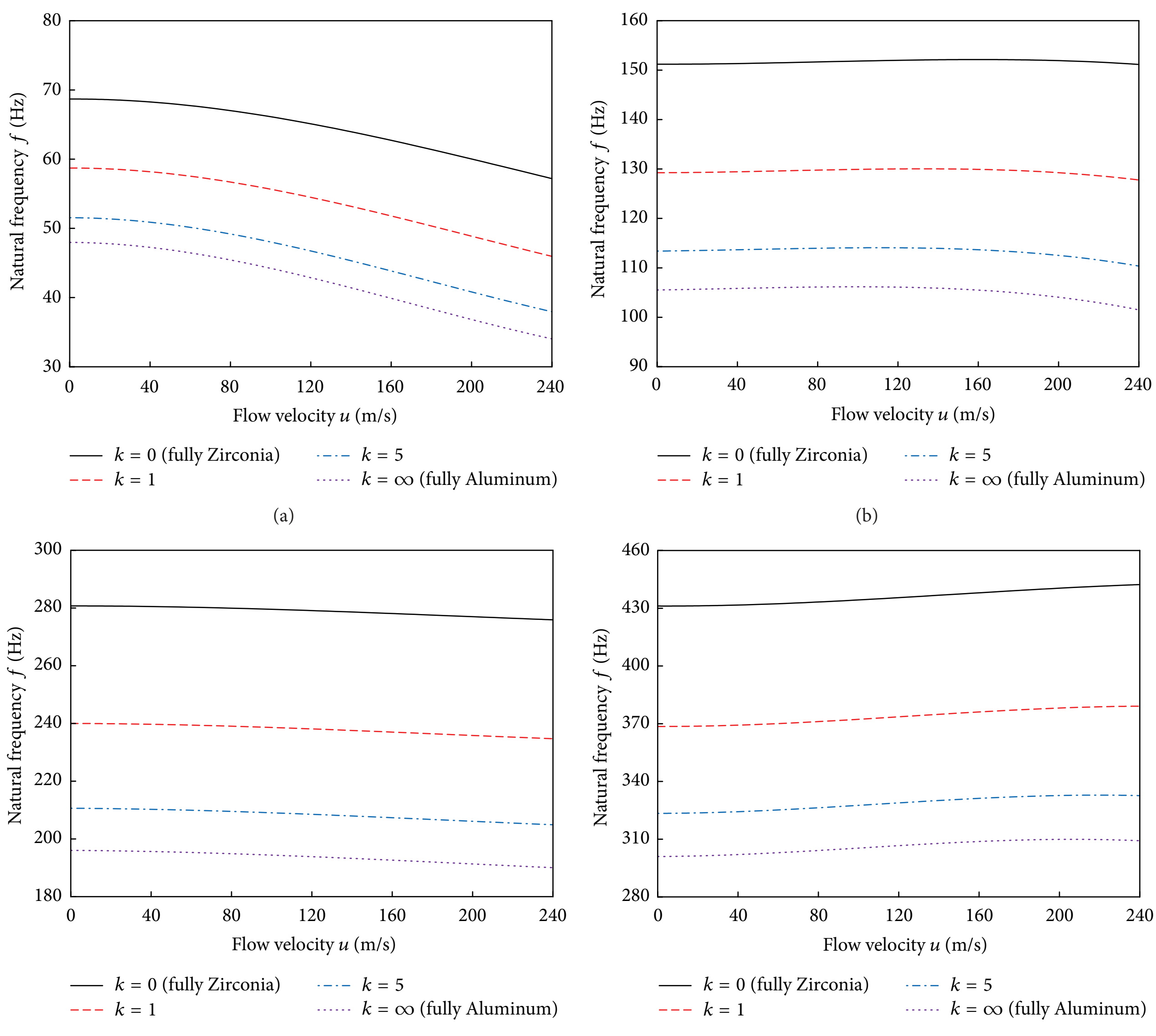

Figure 3: The lowest four natural frequencies of the FGM curved pipes conveying fluid as functions of flow velocity for different values of power gradient index: (a) 1st mode; (b) 2nd mode; (c) 3rd mode; (d) 4th mode.

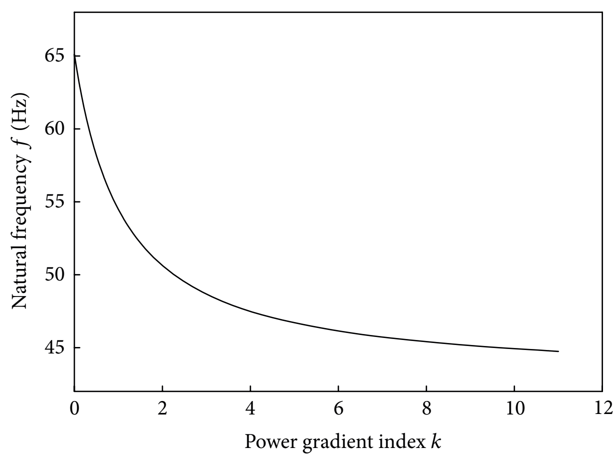

FIGURE 4: Fundamental frequency of the FGM curved pipes conveying fluid as a function of power gradient index for $U = 120$ m/s.

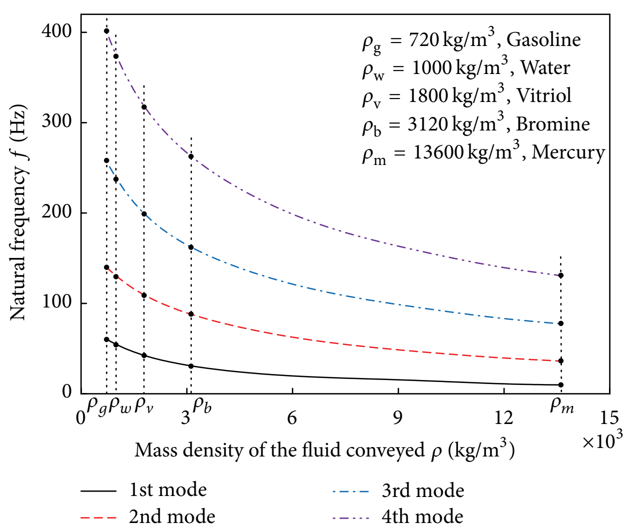

FIGURE 5: The lowest four natural frequencies of the FGM curved pipes conveying fluid as functions of fluid density for $U = 120$ m/s and $k = 1$.

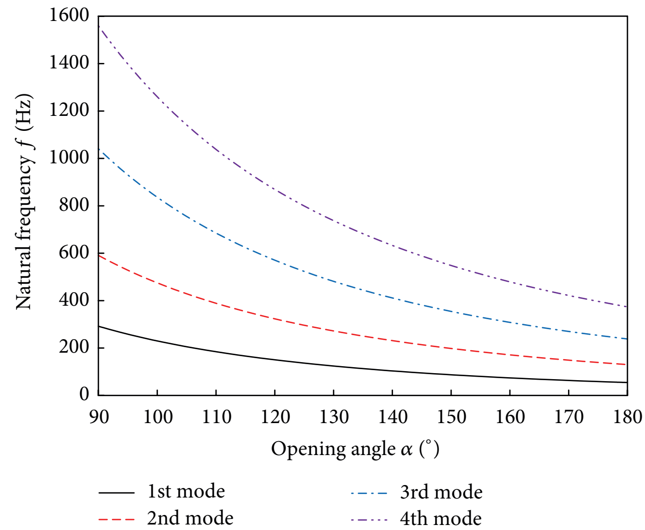

FIGURE 6: The lowest four natural frequencies of the FGM curved pipes conveying fluid as functions of opening angle for $U = 120$ m/s and $k = 1$.

Another important phenomenon should be mentioned here that in both of Figures 2 and 3, unlike the conventional supported curved pipe, in which the combined axial force is neglected, the effect of flow velocity on the natural frequencies is not evident, particularly for those from the second to the fourth mode, and the natural frequencies sometimes increase with the flow velocity increasing (under critical flow velocity), as displayed in Figures 2, 3(b), and 3(d). It is reasonably because the combined axial force appears in the present model, which results in a stiffer structure of the curved pipes.

As the inner surface of FGM pipes is generally made of excellent materials, the FGM pipes can be used to transport various fluids. Figure 5 reveals the influence of fluid density on the lowest four natural frequencies of FGM curved pipes conveying fluid for specified values of $U = 120$ m/s and $k = 1$. The results of several common liquids, namely, Gasoline, Water, Vitriol, Bromine, and Mercury, are marked in the figure. It is clear that the natural frequencies decrease as the fluid density increases. The reason is that increasing fluid density means to increase the inertial and Coriolis forces, which leads to a more flexible and unstable structure. Additionally, the variation of natural frequencies becomes slow with the fluid density increasing, and further the natural frequencies of higher-order modes have a stronger change than those of lower-order modes.

Figure 6 exhibits the lowest four natural frequencies as functions of the opening angle of the FGM curved pipes. It can be observed that the natural frequencies decrease as the opening angle increasing, especially for small values of opening angle and higher-order modes. In fact, for a specified value of centerline radius R, increasing opening angle leads to increasing the span length of the curved pipe, which consequently results in a more flexible and unstable structure. In this sense, the fluid density and opening angle have a similar effect on the free vibration of the FGM curved pipes conveying fluid.

5. Conclusions

In this investigation, a FGM model consisting of ceramic and metallic materials is considered to apply in curved pipes conveying fluid and the in-plane free vibration frequency of the resulting composite pipe is explored by the complex mode method. According to the analysis, some significant and interesting conclusions have been drawn as follows:

(i) If the material properties are distributed along the pipe wall thickness according to a power law, then the power gradient index may be a key parameter affecting the natural frequencies of the FGM curved pipe. The contribution of the power gradient index is prominent as it has small value.

(ii) In the presence of the combined axial force, the impact of internal fluid on the natural frequencies is not so strong as compared with the traditional models without the combined axial force.

(iii) The mass density of the fluid conveyed and the opening angle of the FGM curved pipes have similar effects on the natural frequencies of the system; that is, increase of the parameter values may lead to decrease of the natural frequencies. Higher values of mass density weaken the stability of the system during vibrations, especially for small parameter values and higher-order modes.

Competing Interests

The authors declare that they have no competing interests.

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