A Frequency-Domain Description of a Lock-in Amplifier

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The basic principles behind the operation of a lock-in amplifier are described. Particular emphasis is placed on looking at the frequency components of the signal present at the various stages of the lock-in during a typical measurement. The description presented here has been used successfully at Oberlin College to explain lock-in operation to students.

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1. Introduction

While not as important as the oscilloscope, the lock-in amplifier is a laboratory instrument whose operation should be familiar to undergraduate physics majors, especially those planning to pursue graduate studies in experimental physics. Several loci-in experiments have been proposed for inclusion in intermediate-level laboratory courses.[1,2] At Oberlin College, students are introduced to the operation of a lock-in amplifier in the mandatory junior course in laboratory techniques. The following year, in the also mandatory senior lab, students are expected to be able to readily use a lock-in to perform measurements associated with several solid-state experiments; these include experiments to measure the temperature-dependence of the resistance of a pure metal and a YBCO superconductor, and the Hall effect in a metallic thin film.

Here, I describe the operation of a lock-in amplifier with emphasis on the frequency-, rather than time-domain. This treatment has been successful in introducing Oberlin students to lock-ins.

2. Purpose of a Lock-in amplifier

In a nut shell, what a lock-in amplifier does is measure the amplitude V_o of a sinusoidal voltage,

$$V_{in}(t) = V_o\cos(\omega_o t), \tag{1}$$

where $\omega_0 = 2\pi f_0$ and f_0 are the angular- and natural-frequencies of the signal respectively. You supply this voltage to the *signal input* of the lock-in, and its meter tells you the amplitude V_o, typically calibrated in V-rms. What makes a lock-in different from a simple AC voltmeter, which is what I just described, is that you must also supply the lock-in with a reference input, that is, a decent size, say 1 volt p-p, sinusoidal voltage that is synchronized with the signal whose amplitude you are trying to measure.[3] The lock-in uses this signal (like an external trigger for an oscilloscope) to "find" the signal to be measured, while ignoring anything that is not synchronized with the reference. In practice, the lock-in can measure voltage amplitudes as small as a few nanovolts, while ignoring signals even thousands of times larger. In contrast, an AC voltmeter would measure the sum of all of the voltages at its input.

3. Block Diagram

While a lock-in amplifier is an extremely important and powerful measuring tool, it is also quite simple. The block diagram of a lock-in amplifier is shown in Figure 1. The lock-in consists of five stages: 1) an AC amplifier, called the signal amplifier; 2) a voltage controlled oscillator (VCO); 3) a multiplier, called the phase sensitive detector (PSD); 4) a low-pass filter; and 5) a DC amplifier. The signal to be measured is fed into the input of the AC amplifier. The output of the DC amplifier is a DC voltage proportional to V_0. This voltage is displayed on the lock-in's own meter, and is also available at the output connector. I now elaborate on the functions of the five stages.

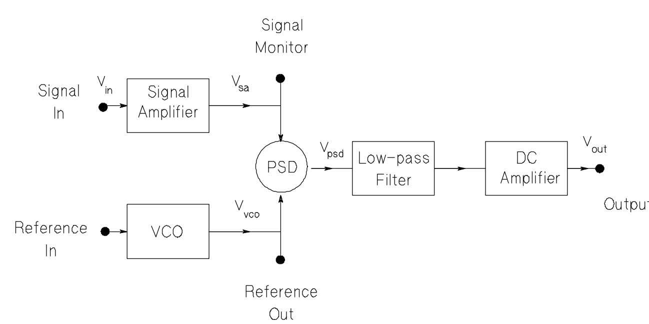

Figure 1. Block diagram of a lock-in amplifier.

The *AC amplifier* is simply a voltage amplifier combined with variable filters. Some lock-in amplifiers let you change the filters as you wish, others do not. Some lock-in amplifiers have the output of the AC amplifier stage available at the signal monitor output. Many do not.

The *voltage controlled oscillator* is just an oscillator, except that it can synchronize with an external reference signal (i.e., trigger) both in phase and frequency. Some lock-in amplifiers contain a complete oscillator and need no external reference. In this case they operate at the frequency and amplitude that you set, and you must use their oscillator output in your experiment to derive the signal that you ultimately wish to measure. Virtually all lock-in amplifiers

1 Paul A. Temple, "An introduction to phase-sensitive amplifiers: an inexpensive student instrument," *American Journal of Physics*, **43**, 801-807 (1975).
2 Richard Wolfson, "The lock-in amplifier: a student experiment," *American Journal of Physics*, **59**, 569-572 (1991).
3 Actually, the reference signal does not have to be sinusoidal, the only requirement is that it be periodic, synchronous with the desired signal.

are able to synchronize with an external reference signal. The VCO also contains a phase-shifting circuit that allows the user to shift its signal from 0-360 degrees with respect to the reference.

The *phase sensitive detector* is a circuit which takes in two voltages as inputs V_1 and V_2 and produces an output which is the product V_1*V_2. That is, the PSD is just a multiplier circuit. (This is not quite true, since the PSD is a very specialized multiplier, but more on that later.)

The *low pass filter* is an RC filter whose time constant may be selected. In many cases you may choose to have one RC filter stage (single pole filter) or two RC filter stages in series (2-pole filter). In newer lock-in amplifiers, this might be a digital filter with the attenuation of a "many-pole" filter.

The *DC amplifier* is just a low-frequency amplifier similar to those frequently assembled with op-amps. It differs from the AC amplifier in that it works all the way down to zero frequency (DC) and is not intended to work well at very high frequencies, say above 10 KHz.

4. Time-Domain Description

Briefly, a lock-in works like this.[4] A signal $V_0\cos(2\pi f_0 t)$ is fed into the signal channel input and amplified by the AC amplifier stage. Since this signal is at a frequency f_0, it is not necessary to amplify other frequencies. In fact, a lot of unwanted "stuff" is eliminated by placing filters in the AC amplifier to pass only a narrow band of frequencies around f_0. The AC amplifier has a voltage gain G_{ac}, that is determined by the sensitivity setting of the lock-in.[5] The output of the AC amplifier becomes one of the two inputs to the multiplier stage, namely

$$V_{ac}(t) = G_{ac}V_0\cos(\omega_o t). \tag{2}$$

The multiplier stage is the heart of the lock-in amplifier. A multiplier produces an output voltage that is the product of the voltages at its two inputs, $V_1(t)$ and $V_2(t)$. As mentioned above, the output of the AC amplifier is one of the multiplicands. The other is a voltage

$$V_{vco}(t) = E_0\cos(\omega_o t+\phi), \tag{3}$$

furnished by the voltage controlled oscillator. The multiplier output is then

$$V_{psd}(t) = G_{ac}V_0E_0\cos(\omega_o t)\cos(\omega_o t+\phi), \tag{4}$$

For simplicity, assume $\phi = 0$, i.e. that the VCO output is in-phase with the signal that is being measured. Recall that when two sinusoidal signals at frequencies f_1 and f_2 are multiplied together, their product may be represented by the sum of two new sinusoids, one having a frequency equal to the sum f_1+f_2 and the other at the difference frequency, f_2-f_1.[6] For this application the two frequencies f_1 and f_2 are identical, so the output of the multiplier has components at the second harmonic (i.e., $2f_0$) and at DC (i.e., at $0 = f_0-f_0$). Using the appropriate trigonometric identify, the above equation may be rewritten

$$V_{psd}(t) = \tfrac{1}{2}G_{ac}V_0E_0[1 + \cos(2\omega_o t)], \tag{5}$$

The amplitudes of the second harmonic and the DC voltage are both proportional to V_o, the signal we are trying to measure. We can throw away the redundant information at $2f_0$ and concentrate only on the DC component. This is accomplished by feeding the multiplier output into a low-pass filter whose time-constant is chosen so that the signal at $2f_0$ is strongly attenuated. The output of the low-pass filter may not yet be large enough, so we amplify it further with a DC amplifier. The final output voltage is a DC voltage directly proportional to the amplitude V_o we were trying to measure, namely

$$V_{out} = \tfrac{1}{2}G_{dc}G_{ac}V_0E_0, \tag{6}$$

where G_{dc} is the voltage gain of the DC amplifier stage.[7] This voltage is both displayed on the meter, calibrated in rms-voltage, and also made available at the output connector.

The low-pass filter serves several functions. First, as already mentioned, it eliminates the second harmonic signal produced by the multiplier. Secondly, it is an integrator. Much of the random noise that ends up in the DC output of the multiplier will integrate to zero, and thus contribute negligibly to the measurement.

5. Frequency-Domain Description

To understand better what is going on it is useful to look at the frequency components present in the signal at various stages of the lock-in amplifier. Figure 2 shows the circuit that was set-up to make these measurements. The internal oscillator of a PAR-124A lock-in amplifier furnished a very small current to resistors series resistors R = 100 kΩ and r = 1000 Ω at a frequency f_0 = 200 Hz. The voltage drop across the smaller resistor, r, was connected to the signal input of the lock-in amplifier. A Hewlett-Packard model 3562A dynamic signal analyzer was used to measure the power spectral density of the voltages present at the various stages of the lock-in amplifier.[8]

4 A variety of useful technical notes are available from *EG&G Princeton Applied Research*. Two of these are Technical Note 115, "Explore the lock-in amplifier," and Technical Note 116, "Specifying lock-in amplifiers."

5 More precisely, the AC amplifier is characterized by a complex transfer function, $G_{ac}(\omega)$, where $\omega = 2\pi f$.

6 This, of course, is easily shown using trigonometric identity $\cos(\alpha)\cos(\beta) = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$.

7 Actually, it makes more sense to consider the low-pass filter and DC amplifier as a single, low-frequency amplifier having a transfer function $G_{dc}(\omega)$.

8 The power spectral density or power spectrum, $S_V(f)$, of a voltage V(t) is defined to be the mean square voltage per unit frequency. Just as the power contained between frequencies f_1 and f_2 for black-body radiation is

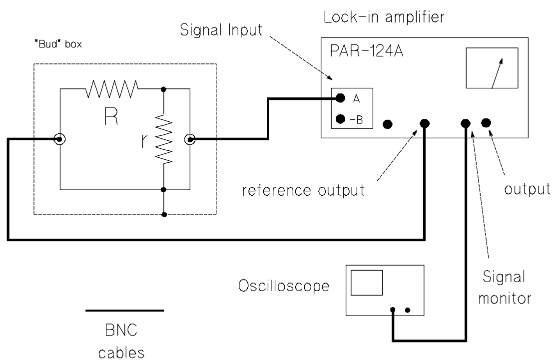

Figure 2. Measurement circuit.

Figure 3 contains a semilog graph of the power spectral density (S_V) versus frequency (f) of the signal present at the input of the lock-in.[9] (Note that the power in a certain bandwidth is not just the area under the curve because of the logarithmic vertical scale.) In addition to the small desired signal at frequency f_0, there were also present large interfering signals at the power line frequency, 60 Hz, and its (mainly odd) harmonics. In addition to these, the input signal contains broad-band noise which tends to be frequency-independent at high frequencies but increases at low-frequencies as 1/f.[10]

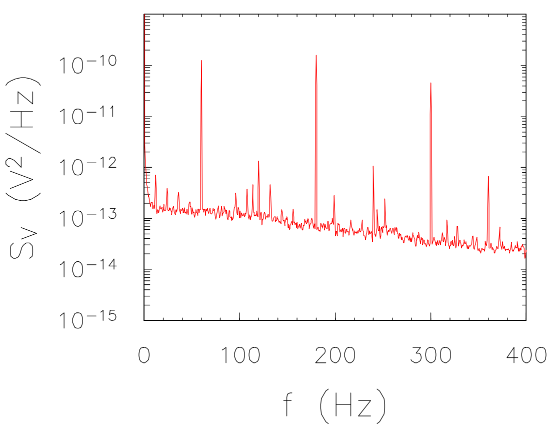

Figure 3. Graph of the power spectral density of the input signal. The desired signal is the "spike" at 200 Hz. Other spikes are due to unwanted pickup from the 110 VAC line at 60 Hz and its harmonics.

The broad spike at zero frequency is 1/f noise. The flat background noise is the unavoidable thermal noise.

Figure 4 shows the output of the AC amplifier. The AC amplifier was set to operate as a bandpass filter centered at f_0 = 200 Hz with a "quality factor," Q = 5. Note how the desired signal is amplified while unwanted frequencies, notably those associated with 60 Hz and its harmonics, are significantly attenuated. Despite this attenuation the third harmonic of the line frequency remains the largest signal present.

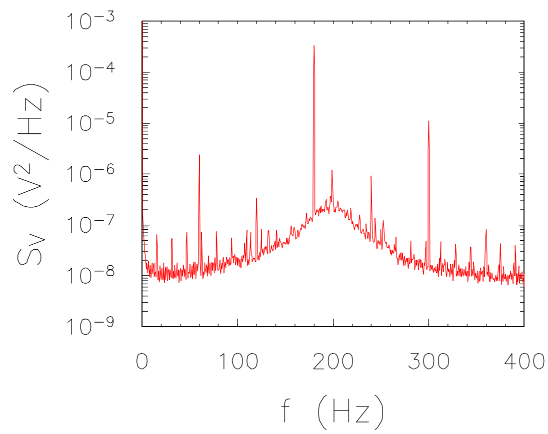

Figure 4. Power spectral density of the output of the AC amplifier stage. The bandpass character allows the desired frequency of 200 Hz to be amplified while frequencies well away from 200Hz are attenuated. In particular we have reduced the effects of line frequency pickup and the 1/f noise near DC.

The output of the multiplier stage is shown in Figure 5. All of the "spikes" in the spectrum have now been shifted from their original position f_j to f_0+f_j and f_0-f_j. This phenomenon is called "beating." The desired signal is now contained in both the "spikes" at zero frequency (DC) and $2f_0$ = 400 Hz. The interference from the third harmonic of the line frequency now appears at 20 Hz and 380 Hz.

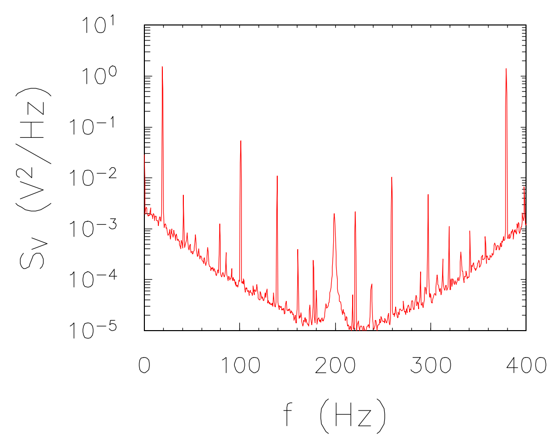

Figure 5. Power spectral density of the output of the multiplier stage. The effect of beating has shifted each spike at frequency f_j in Figure 4 to two new positions, 200 Hz + f_j and 200 Hz - f_j. The desired signal is now located at zero frequency and at 400 Hz. The broad spike now located at 200 Hz is mostly associated with the "1/f

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obtained by integrating the black-body spectral distribution over this bandwidth, the mean-square-voltage between frequencies f_1 and f_2 is given by $\langle v^2\rangle = \int_{f_1}^{f_2} S_V(f)df$.

9 This was actually measured by connecting the spectrum analyzer to the signal monitor connector and setting the frequency response of the signal amplifier to be flat. The measured spectrum was then divided by the square of the known voltage gain of the signal amplifier to obtain the spectrum for the signal at the input.

10 This, so-called, "1/f noise," is actually added by the AC amplifier stage and is present in virtually all electrical components.

noise" of the signal amplifier.(a) This work was supported, in part, by an AWU-DOE faculty sabbatical fellowship.

Figure 6 shows the output of the DC amplifier with the low-pass filter time constant set to 1 s. Note the different frequency scale than for Figures 3 through 6. All of the frequency components present in the output in the range 0-1 Hz are present in the input signal at frequencies 199-201 Hz! Thus the low-pass filter has, in some sense, allowed the lock-in to act like a narrow-band amplifier with a high Q ≈ (200 Hz)/(2 Hz) = 100. If we are willing to settle for a slower response time, a time constant of 100 s gives an effective Q of 10^5!

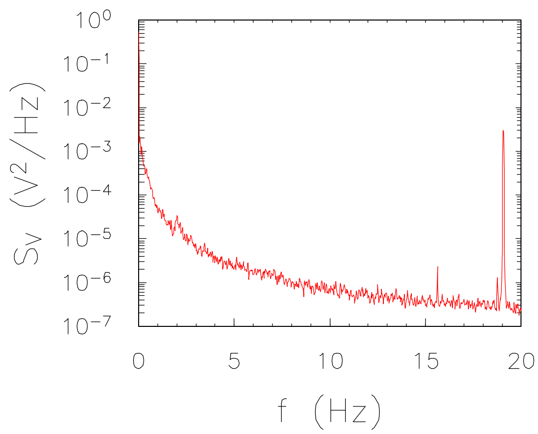

Figure 6. Power spectral density at the output of the DC amplifier. Note the change in frequency scale from the earlier plots. The desired signal at DC is now the only significant signal since the low-pass filter has removed everything above 1 Hz. Though difficult to see on the graph, $S_V(0) = 0.55\ V^2$/Hz.

6. Important Complications

Real lock-in amplifiers differ from that described above in two important ways. First, the reference may be shifted in phase from the desired signal so that an accurate phase-shifting circuit is included in the voltage controlled oscillator stage. This is important since, in practice, we may not know the phase of the input signal. More precisely, the input signal of Eq. (1) is more accurately written as $V_{in}(t) = V_o\cos(\omega_o t+\delta)$, where δ is unknown. With this change, the final output represented by Eq. (6) should be written as

$$V_{out} = \tfrac{1}{2}G_{dc}G_{ac}V_0E_0\cos(\delta+\phi). \qquad (7)$$

In this case it is not desirable to set the VCO phase angle ϕ to be zero. Instead, one adjusts ϕ so as to maximize V_{out}, i.e., so that $\delta+\phi$ is zero.

Secondly, for many commercial lock-in amplifiers, the multiplier doesn't multiply by a sinusoid at all, but rather by a square-wave

$$V_{vco}(t) = \sum_{n=0}^{\infty} \frac{4E_0}{(2n+1)} \sin[(2n+1)2pf_0 t + f_n] \qquad (8)$$

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at frequency f_0.[11] Since a square wave can be represented by a Fourier series, whose dominant component is the fundamental, this doesn't alter the result much provided that the AC amplifier is operated in a narrow-band mode, stripping away any input signal at the higher harmonics. However, a square-wave multiplier certainly complicates the analysis.[12]

The reason for multiplying by a square-wave is a practical one. Until recently, it was not possible to build sufficiently stable multipliers. It has been possible, however, to make very stable transistor switches that turn "on" and "off" like a square wave. In practice, most lock-in amplifiers use these in place of the multiplier stage. The input of the PSD is just the output of the AC amplifier stage as before. When the PSD is "switched on" its output is equal to its input. When it is "switched off" its output is zero. The net result is that the output of the PSD is equal to its input multiplied by a square-wave. Thus most real lock-in amplifiers multiply by square-waves rather than sine-waves, and the PSD is intimately connected with the VCO.

7. Summary

In summary, I have described the operation of a lock-in amplifier. The lock-in amplifier consists of five stages: the AC amplifier, the VCO, the PSD, the low-pass filter, and the DC amplifier. The functions of the various stages has been illustrated by looking at power spectral densities of signals present at each them during a typical measurement.

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[11] One vendor, *Stanford Research Instruments*, sells a "digital" lock-in that multiplies by a pure sinusoid.

[12] For a more detailed analysis see John H. Scofield, "AC method for measuring low-frequency resistance fluctuation spectra,"*Review of Scientific Instruments*, **58**, 985-993(1987).