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Frequency Domain versus Time Domain Modal Identifications for Ambient Excited Structures

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Abstract

Ambient vibration test is one of the most applicable and reliable methods among vibration tests due to recently fast evolvement of measurement, sensor technologies and data processing, and economical benefit as well. Modal identification methods of the ambient vibration structures are output-only identification techniques which have commonly been branched into either the frequency domain or the time domain. Each the frequency domain-based or the time domain-based modal identification methods has its own advantage and limitation. The frequency domain-based modal identification methods or non-parametric identification ones have been widely used for the output-only system identification due to their advantages in estimating natural frequencies high frequency resolution and mode shapes, but they have uncertainty and troublesome in the damping estimation. Inversely, the time domain-based modal identification methods or parametric ones benefit in the damping estimation, but difficulty in parameters. This paper will present theoretical backgrounds of the most modern and recent modal identification methods in the frequency domain and in the time domain such as Frequency Domain Decomposition (FDD) and Stochastic Subspace Identification (SSI). Full-scale ambient measurements of 5-storey steel building have been carried out for the modal parameter estimation. Applicable evaluation of the frequency domain-based and the time domain-based modal identification methods will be also discussed.

Key Words: System identification; Ambient vibration; Output-only identification; Frequency domain; Time domain; Frequency domain decomposition; Stochastic subspace identification

1. Introduction

Full-scale vibration testing is the most reliable method to determine modal parameters of structures (e.g. natural frequencies, damping ratios and mode shapes) which are served as multipurpose uses as model updating and validation, response prediction, structural control and damage detection of both experimental and operation structures. Generally, the vibration testing can be subdivided depending on types of excitation by two main branches: forced vibration testing including free vibration testing and ambient vibration testing. In the first branch, the structures are excited by shaker or impulse hammer. In case of the free vibration testing, exciting force is suddenly lifted. The forced testing usually involve with measuring both input excitations and output responses. Accordingly, input-output identification methods are used to estimate the modal parameters of structures through indentifying either frequency response function (FRF) or impulse response function (IRF). This testing advantages over deterministic input excitation which can be controlled and optimized to the response of interested vibration modes. However, the forced vibration test requires additional equipments like shaker and accompanying instrumentations, traffic shut-down. Thus it is suitable to conduct this test with small-scaled or medium-scaled structures, however, in cases of structures with flexible, large-scaled and low-ranged, closed natural frequencies, the controlling of input excitation for optimized level of response is often difficult and costly. It is noted that output-only identification with measured responses only also can be applied for the forced vibration testing, but effects of input excitation and harmonics must be eliminated from the output response before the modal parameters extracted. The ambient vibration testing using natural and environmental excitations such as traffic vehicles, human activities, wind and wave… does not require the traffic disturbance, shaker and additional instrumentations, but require highly sensitive sensors and data processing techniques. However, with development of sophisticated sensor technology and identification techniques, the ambient testing has become the most practical and reliable method for the modal parameter identification from multipurpose laboratory experiments and full-scale measurements. Because only the output response can be measured without the input excitations in the ambient testing, thus the output-only identification is applied to estimate the modal parameters (Cunha et al., 2006).

Modal parameter identification or output-only identification of the ambient excited structures is key issue and the most attractive topic for both experimental modal analysis and operational modal analysis. So far, a number of the output-only identification techniques have been developed for MDOF systems, which is often classified into three branches (1) Frequency domain-based methods; and (2) Time domain-based methods. The frequency domain-based modal identification methods, also known non-parametric identification ones have been widely used for the output-only system identification of the ambient vibration structures due to their reliability, effectiveness and advantages in estimating high-resolved natural frequencies and mode shapes. The modern modal identification methods for the ambient testing in the frequency domain are Frequency domain decomposition (FDD) and Enhanced frequency domain decomposition (EFDD). The ambient testing does not require to identify either FRFs or IRFs because the input force cannot be measured. Ambient testing has been successfully applied for many types of civil engineering structures such as bridges (Abdel_Ghaffar and Scanlan, 1985; Siringoringo and Fujino, 2008), buildings and engineering towers (Tamura et al., 2005). However, the ambient testing and the output-only identification contain its shortcomings and uncertainties due to unknown input excitations, input influence, noise disturbances and leakage effect due to the Fourier transform and so on. Moreover, the frequency-domain identification methods are often based on strict hypotheses such as uncorrelated white noise inputs, light damping and effectiveness of decomposition techniques for simplification. Especially, there is troublesome damping estimation in the frequency domain due to difficulty and unreliability in estimating auto power spectral function and free decay function for each structural mode (Brincker et al., 2001a; Tamura et al., 2005). Just so-called operational deflection shapes or unscaled mode shapes can be obtained from the ambient testing because scaling factor not to be determined due to unknown input forces (Brincker et al., 2001b).

The Stochastic Subspace Identification (SSI) is the powerful and most applicable technique for the output-only identification methods in the time domain, originally presented by Van Overschee and De Moor, 1993. SSI has been branched by either COV-SSI or DATA-SSI due to dealing with covariance matrix (Toeplitz matrix) of the output data or data matrix (Hankel matrix) directly. It is noted that number of parameters such as the number of block rows, the number of system orders and so on must be required for the SSI procedure and carefully selected. Similar to FDD, the theory of SSI is also assumed that excitation forces are Gaussian distributed stochastic broad-band white noises. SSI has applied for estimating modal parameters of variety of structures by some authors such as Peeters and De Reock, 1999; Weng et al., 2008; Reynders and De Roeck, 2008.

This paper presents fundamental theories of the most practical modern identification methods, concretely the Frequency Domain Decomposition and its enhanced version in the frequency domain, and Stochastic Subspace Identification in the time domain. Comparative results will be discussed. Full-scale ambient measurements of 5-storey steel structure have been carried out for the modal parameter identification with emphasis on natural frequencies and damping ratios.

2. Frequency domain-based methods

2.1. Frequency Domain Decomposition

Analysis of the output-only response data is carried out in the frequency domain using well-known FDD technique. The key point of the FDD will be briefly summarized hereafter. Relationship between excitation inputs and output response can be expressed in the frequency domain through the FRF matrix as follows:

$$S_{XX}(\omega) = H(\omega) S_{FF}(\omega) H(\omega)^{*T} \tag{1}$$

where *, T denote conjugate and transpose operations; $S_{FF}(\omega), S_{XX}(\omega)$: power spectral matrices of inputs and outputs; $H(\omega)$: FRF matrix.

The FRF matrix can be expressed commonly under a form of residues/poles (He and Fu, 2001; Brincker et al., 2001a):

$$H(\omega) = \sum_{i=1}^{N} \left(\frac{a_i}{j\omega - \lambda_i} + \frac{a_i^*}{j\omega - \lambda_i^*} \right) \tag{2a}$$

$$H(\omega) = \sum_{i=1}^{N} \left(\frac{\gamma_i \phi_i \phi_i^T}{j\omega - \lambda_i} + \frac{\gamma_i^* \phi_i^* \phi_i^{*T}}{j\omega - \lambda_i^*} \right) \tag{2b}$$

where i, N: index, number of modes; a_i, λ_i: i-th complex residue and complex pole in which $a_i = \gamma_i \phi_i \phi_i^T$ with light damping and $\lambda_i = -\zeta_i \omega_i + j\omega_i \sqrt{1-\zeta_i^2}$; ω_i, ζ_i: i-th natural frequency and damping ratio; ϕ_i, γ_i: i-th mode shape vector and scaling factor.

If the inputs are uncorrelated white noise inputs, the input power spectral matrix of the inputs is diagonal constant one $S_{FF}(\omega) = diag(c_1, c_2, ...c_N)$ and damping is light, one can obtains the output power spectral matrix at evaluated frequency ω_i decomposed modally as follows:

$$S_{XX}(\omega) = \sum_{i=1}^{N}\left(\frac{d_i\phi_i\phi_i^T}{j\omega-\lambda_i} + \frac{d_i^*\phi_i^*\phi_i^{*T}}{j\omega-\lambda_i^*}\right) \quad (3)$$

where d_i: i-th scalar constant. Expression of the output power spectral matrix in Eq.(3) is similar one of some matrix decompositions in the complex domain, thus these can be used to decompose the output power spectral matrix.

The output power spectral matrix has been orthogonally decomposed using the eigenvalue decomposition to obtain spectral eigenvalues and spectral eigenvectors:

$$S_{XX}(\omega) = \Phi(\omega)\Theta(\omega)\Phi(\omega)^{*T} \quad (4a)$$

$$S_{XX}(\omega) = \sum_{k=1}^{M}\varphi_i(\omega)\theta_1(\omega)\varphi_i(\omega)^{*T} \quad (4b)$$

where $\Phi(\omega), \Theta(\omega)$: spectral eigenvectors and spectral eigenvalues matrices; k, M: index and number of spectral eigenvectors. Because the eigenvalue decomposition is fast-decaying, thus the output power spectral matrix can be approximated by using the lowest-order spectral eigenvalue and eigenvector as follows:

$$S_{XX}(\omega) \approx \varphi_1(\omega)\theta_1(\omega)\varphi_1(\omega)^{*T} \quad (5)$$

where $\varphi_1(\omega), \theta_1(\omega)$: the first spectral eigenvector and first spectral eigenvalue.

Due to the first-order spectral eigenvalue and eigenvector are dominant in a term of energy contribution, thus the first spectral eigenvalue contains full information of dominant frequencies to be used for extracting the natural frequencies, whereas the first spectral eigenvector brings information of mode shapes at each dominant frequency. The i-th mode shape can be estimated from the first spectral eigenvector at certain dominant frequencies (ω_i: i-th natural frequency) as follows:

$$\phi_i = \varphi_1(\omega_i) \quad (6)$$

where ϕ_i: i-th mode shape

2.2. Enhanced Frequency Domain Decomposition

Originally, the damping ratios estimation cannot be carried out by the FDD, but its enhanced version has been developed for this purpose. Enhanced frequency domain decomposition has been developed basing on the frequency domain decomposition for estimating the damping ratios only (Brincker et al., 2001b). As can be seen from Eq.(6) that FDD extracts the mode shape from the first spectral eigenvector at selected natural frequency, thus prior knowledge of the natural frequencies must be required for this identification technique. Accuracy of estimated mode shapes can be evaluated via correlation criteria between estimated mode shapes and analytical mode shapes, moreover, among these criteria Modal Assurance Criterion (MAC) is preferably used:

$$MAC(\%) = \frac{|\phi_E\phi_A^T|^2}{\{\phi_E^T\phi_E\}\{\phi_A^T\phi_A\}} \quad (7)$$

where ϕ_E, ϕ_A: estimated mode shapes and analytical mode shapes, respectively.

For the damping estimation, the key point here is to identify the auto power spectral density function of the single-DOF generalized coordinate of certain mode from the spectral eigenvalues. Because the first spectral eigenvalue contributes dominantly almost energy of system, thus it is often used to extract the auto power spectral density functions. Searching the auto power spectral function of certain mode from the first eigenvalue is carried out on both sides of value of the natural frequency, and it is terminated until the desirable limit of MAC reached. The remaining values of the auto spectral density function in the calculated frequency range are set to zero. From identified auto power spectral density functions, the damping ratios are obtained via logarithmic decrement technique of free decay functions, of which these free decay functions obtained by converting the auto spectral density function in the frequency domain back to the time domain by inverse fast Fourier transform technique (Brincker et al. 2001b). It is also noted that validation of the natural frequencies can be checked through these free decay functions.

3. Time domain-based methods

3.1. Stochastic Subspace Identification

Consider a discrete-time response stochastic state-space model (Van Overschee and De Moor, 1993)

$$Y_{k+1} = AY_k + w_k \tag{8a}$$

$$X_k = CY_k + \upsilon_k \tag{8b}$$

where A, C: system matrices; Y : state vector; X : measurement response; k: index of discrete state; and w, υ : process noise and measurement noise. System matrices A, C containing the system modal parameters will be identified by SSI technique.

All response data are normally represented by the data matrix of the block Hankel matrix, which this is convenient to divide original data into two parts: the past worked as reference basis and the future as processing data. For the reference-based stochastic subspace identification, the Hankel matrix plays a critically important role in the SSI algorithm. Hankel matrix of single response data $X = \{x_0, x_1, ...x_j, ...x_N\}^T$ can be formulated here for a brevity:

$$H = \left[\begin{array}{cccc} x_0 & x_1 & \dots & x_{j-1} \\ x_1 & x_2 & \dots & x_j \\ \dots & & & \\ x_{i-1} & x_i & \dots & x_{i+j-2} \\ \hline x_i & x_{i+1} & \dots & x_{i+j-1} \\ x_{i+1} & x_{i+2} & \dots & x_{i+j} \\ \dots & & & \\ x_{2i-1} & x_{2i} & \dots & x_{2i+j-1} \end{array}\right] = \left[\begin{array}{c} X_p^{ref} \\ \hline X_f \end{array}\right] \tag{9}$$

where i: a user-defined index; N: number of samples in response data; j: number of columns; and X_p^{ref}, X_f : the past and future parts. To ensure all samples of the output vector X_k populate in the Hankel matrix, the number of j can be equal to N-2i+1 or N=2i+j-1. Thus, dimensions of the matrices are $H \in R^{2Nixj}$, $X_p^{ref} \in R^{Nixj}, X_f \in R^{Nixj}$. If there are M response outputs, do similar way for the Hankel matrix of each output and organize in column-based format to build up the entire Hankel matrix of all outputs.

The orthogonal projection of the future part on the past one is carried out. This projection is introduced as geometrical tool or conditional mean operation under following formula (Van Overschee and De Moor, 1993):

$$O = \left(X_f / X_p^{ref}\right) = X_f X_p^{ref} \left(X_p^{ref} X_p^{ref\,T}\right)^{-1} X_p^{ref} \tag{10}$$

where /: projection operator; O : orthogonal projection of Hankel matrix.

Because so-called observability matrix is used to identify initial conditions, then the system matrices, but the observability matrix itself can be estimated by factoring the orthogonal projection due to such following steps. Firstly, singular value decomposition (SVD) is applied for the orthogonal projection, the singular values and singular vectors estimated through SVD:

$$O = USV^T = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} \tag{11}$$

where U, V: singular vectors; S: singular values. Here, we organize these singular values, vectors into two groups with indices 1 and 2. Dominant low-order singular values and their associated singular vectors group in U_1, S_1, V_1, whereas the smallest high-order singular values and vectors in U_2, S_2, V_2. Actually, this is approximation, reduced-order model or noise effect reduction, in which the dominant singular values and vectors are used to estimate the system parameters, while the small singular values are neglected, we have:

$$O \approx U_1 S_1 V_1 \quad (12)$$

where $U_1 \in R^{Nixk}; S_1 \in R^{kxk}; V_1 \in R^{jxk}$; k: number of system order or reduced order.

Secondly, the observability matrix can be estimated from reduced-order model:

$$\Gamma = U_1 S_1^{0.5} \quad (13)$$

where Γ: observability matrix $\Gamma \in R^{Nixk}$

Finally, the system matrices A, C can be determined from this observability matrix as follows:

$$A = \underline{\Gamma}^{-1}\overline{\Gamma} \quad (14a)$$

$$C = \hat{\Gamma} \quad (14b)$$

where $\underline{\Gamma} \in R^{N(i-1)xk}$ denotes Γ without last N rows; $\overline{\Gamma} \in R^{N(i-1)xk}$ denotes Γ without first N rows; and $\hat{\Gamma} \in R^{Nxk}$: first N rows of Γ.

3.2. Modal parameter estimation

The modal parameters then are estimated from the system matrices. Using either SVD or eigenvalue decomposition to decompose the system matrix A as follows:

$$A = \Psi \Lambda \Psi^{*T} \quad (15)$$

where Λ : diagonal eigenvalue matrix containing complex poles and natural frequencies; and Ψ: eigenvector matrix, containing information of mode shapes.

Concretely, i-th natural frequencies and damping ratios can be determined as following formulae (Weng et al., 2008):

$$\omega_i = \frac{a_i}{2\pi\Delta T} \text{ and } \zeta_i = \frac{b_i}{\sqrt{a_i^2 + b_i^2}} \quad (16)$$

where $a_i = |\arctan(\mathrm{Im}(\lambda_i)/\mathrm{Re}(\lambda_i))|; b_i = \ln(\lambda_i)$

The i-th mode shape can be estimated from the system matrix C and eigenvectors:

$$\phi_i = C\psi_i \quad (17)$$

In the SSI method, the all response outputs are arranged firstly to form the Hankel matrix. Second, the orthogonal projection of the Hankel matrix is carried out with reduced-order approximation using the SVD, before to estimate the observability matrix. Third, the system matrices are determined from the observability matrix. Final, the eigenvalue decomposition or SVD is used to decompose the system matrix, and the modal parameters can be identified.

4. Full-scale measurements

Full-scale ambient measurements have been carried out on a 5-storey steel frame at the test site of the Disaster Prevention Research Institute (DPRI), Kyoto University (see Figure 1). Ambient data were recorded at all 5 floor levels and ground as reference, by tri-axial velocity sensors with output velocity signals (VCT Corp., Models UP255S/UP252) with A/D converter, amplifier and laptop computer. All data were sampled for period of 30 minutes per floor (5 minutes per a set-up) with sampling rate of 100Hz. Sensors have been located to capture ambient motions in lateral X-direction and horizontal Y-direction from ground level to 5th floor, see Figure 1 (Kuroiwa and Iemura, 2007).

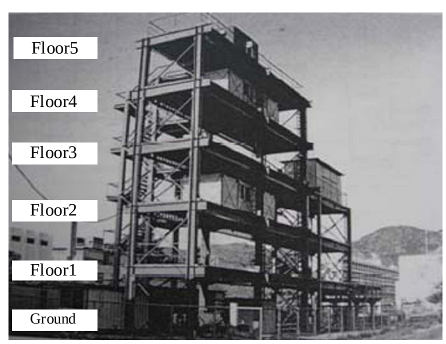

Figure 1. Five-storey steel structure

Only outputs sensors and modal parameters in the X direction have been discussed in this paper. It is noted that all outputs were velocity time series, thus a single integration in the time domain using a trapezoic integration approach has been

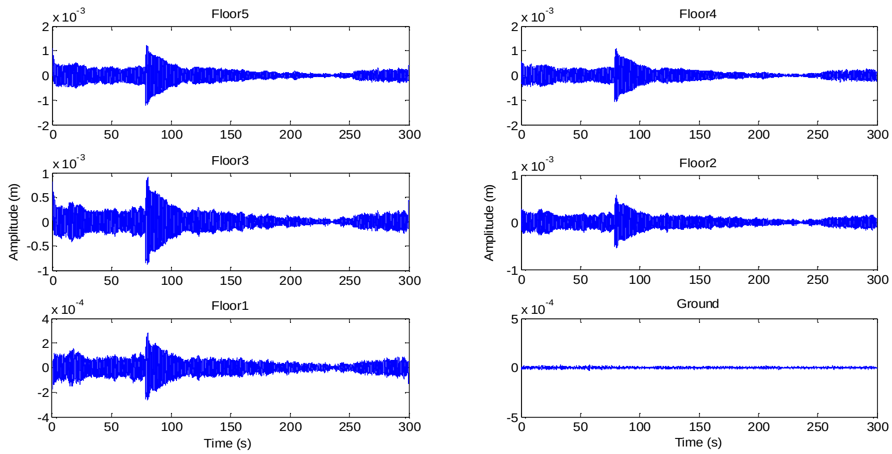

Figure 2. Integrated output displacements

required to obtain output displacements which are necessary for estimating mode shapes in next steps. A integration drift due to unknown initial condition of the displacements during the time integration have been treated through compensation to be zero-mean output displacements. Integrated output displacements at all floors are shown in Figure 2.

5. Modal parameter estimation

In the traditional way using Peak Picking (PP) technique, the peak frequencies can be extracted from the output series using power spectral analysis, then the mode shapes can be estimated based on values and directions of the power spectral matrix and phase matrix at each frequency peaks. However, this method cannot extract the damping.

The output power spectral density (PSD) matrix has been established from the output displacements $X(t)=\{X_1(t), X_2(t),...X_M(t)\}^T$ (M: number of sensors). Spectral eigenvalues and eigenvectors have been determined via Proper Orthogonal Decomposition of the output PSD matrix. All six normalized spectral eigenvalues and first four eigenvectors are shown in Figure 4 and Figure 5. Energy contributions of each spectral eigenvector and its corresponding eigenvalue can be estimated roughly based on its summed eigenvalues on analyzed frequency band (here 0-30Hz bandwidth). Concretely, first four eigenvalues and associated eigenvectors roughly contribute 99.9%, 0.07%, 0.01%, 0.00% respective to the dynamically structural system. As a results, the first spectral eigenvalues and

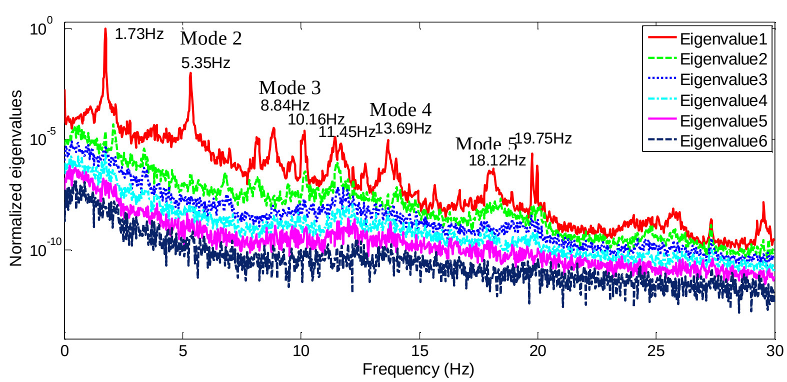

Figure 3. Normalized spectral eigenvalues

Mode 1

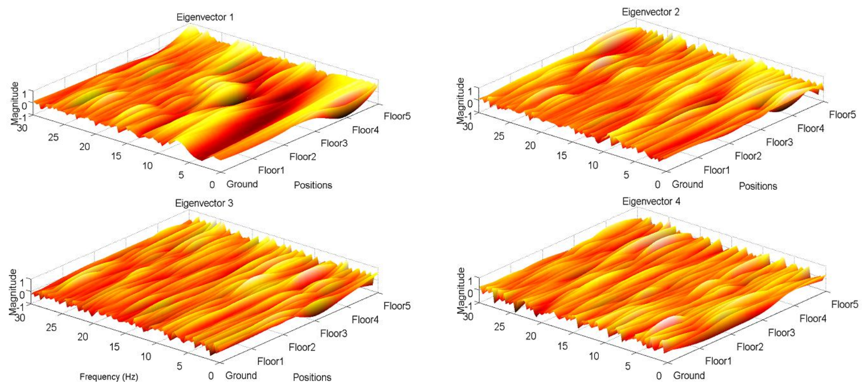

Figure 4. First four spectral eigenvectors

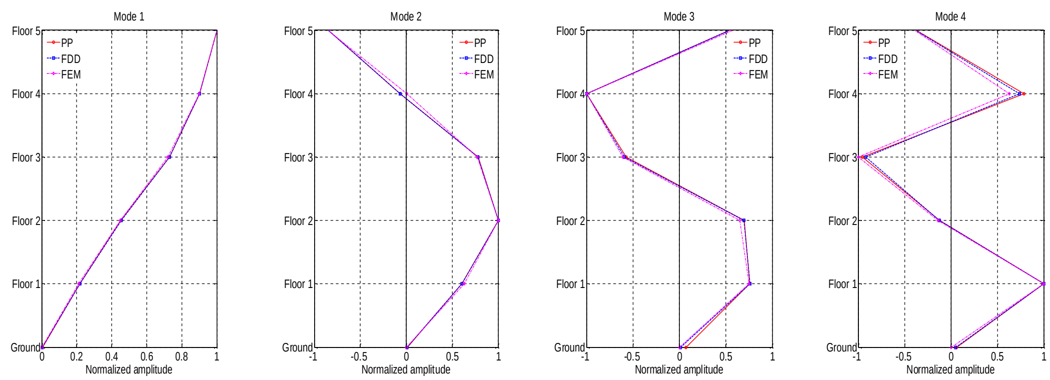

Figure 5. First four mode shapes

eigenvectors characterize modal parameters of the structure.

Estimated peak frequencies in Figure 4 are compared with analytical finite element model (FEM) results to identify nutural frequencies associated with order of mode shapes. Bending modes in the X direction corresponding the estimated natural frequencies can be determined via the first spectral eigenvector at these frequencies following the Eq.(6). MAC values of the first five mode shapes evaluated by Eq.(7) are respectively 100%, 99.76%, 99.8%, 98.95% and 99.3%. Thus, there is a good agreement between the FDD-based mode shapes and FE-based ones. First four mode shapes are

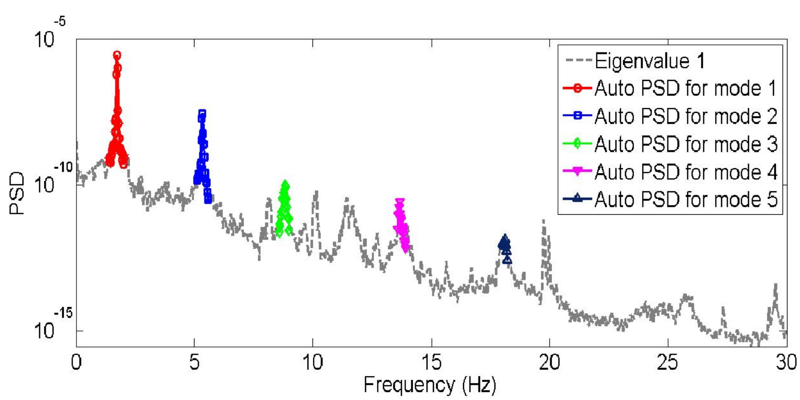

Figure 6. Estimated auto-spectral functions at each natural frequencies (MAC=98%)

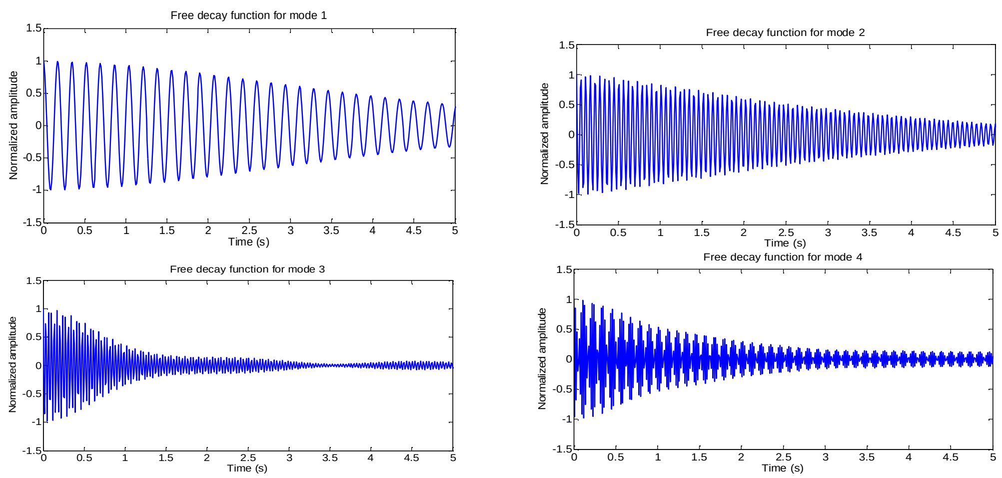

Figure 7. Estimated free decay functions of first four modes

shown in Figure 5 in comparison between FEM, PP and FDD. There are good agreements in estimated mode shapes between analytical and identification methods.

For the damping extraction, firstly auto power spectral density functions of each mode have been identified in the Enhanced Frequency Domain Decomposition (EFDD) via the MAC value, secondly, the free decay functions corresponding to mode shapes have been identified to be convenient for the logarithmic decrement estimation and damping ratios of each mode. Figure 6 shows the estimated auto spectral density functions of the first five structural modes at MAC=98%, while Figure 7 indicates the estimated free decay functions of these mode shapes.

Next, modal parameters have been identified using the SSI algorithm in the time domain as presented in the part 4. Number of measured outputs are 6, with N=30000 samples each. In order to build up the stability diagrams, the number of block rows in Hankel matrix and number of system orders or number of singular values used in reduced-order model have been varies, concretely i=20÷125 with step 10, k=5÷60 step 5. Stability diagram for the frequency at varied k=5÷60 step 5 and i=50 is shown in Figure 8. First five natural frequencies can be observed in this frequency stability diagram. Apparently, the natural frequencies extracted

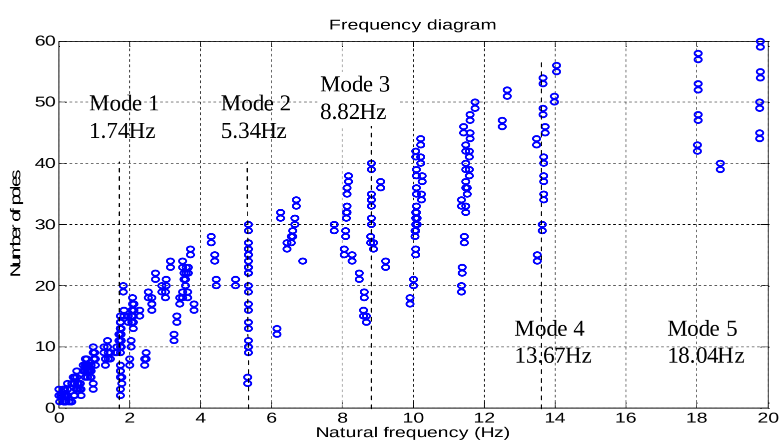

Figure 8. Frequency stability diagram

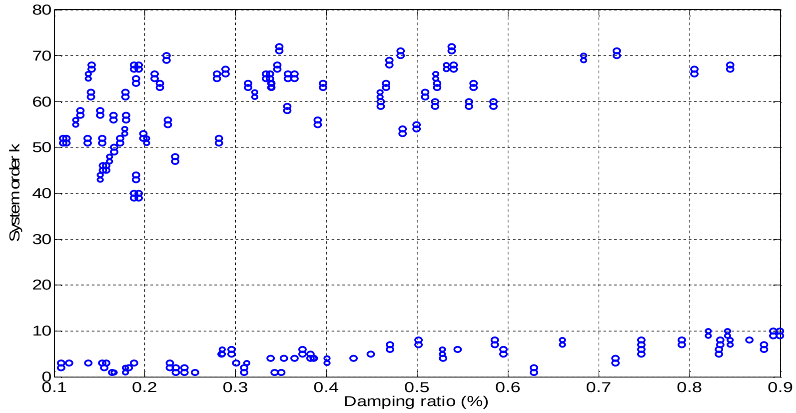

Figure 9. Damping ratio stability diagram

by SSI are similar to those from FDD. Damping stability diagram is indicated in Figure 9.

6. Conclusion

Modal parameters identifications of the 5-storey steel structure using FDD and SSI-DATA have been investigated. Identified natural frequencies are well agreement between them and with FEM results. Identified mode shapes also are good fitted between FDD, PP and FEM methods. Damping ratios can be identified from FDD and SSI, however, refined technique should be further investigated for more reliability of the damping estimation.

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