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Frequency Response Diagram for Pipeline Leak Detection: Comparing the Odd and the Even Harmonics

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Abstract

Pipeline leak detection using hydraulic transient analysis is a relatively new detection technique. For single pipeline systems, recent work has led to two different approaches for determining leak parameters based on leak-induced patterns displayed in a pipeline's frequency response diagram (FRD). The major difference between the two techniques is that one uses the leak-induced pattern within the odd harmonics of an FRD, while the other one uses the leak-induced pattern at the even harmonics. In order to compare and contrast the two approaches, the current research analyses the relationship between the characteristics of the leak-induced patterns and the parameters of the pipeline system. A dimensionless analysis, based on hydraulic impedance, is adopted to simplify the equations. The amplitudes of leak-induced patterns at both the odd and the even harmonics in the FRD are found to be dependent on a critical parameter: the dimensionless steady-state valve impedance, Z_V^*. The value of Z_V^* is dependent on the steady-state valve opening. As a result, amplitudes of the leak-induced patterns within the FRD for any specific pipeline system can be controlled by the initial valve opening. This

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research also derives the equations for calculating the dimensionless leak size based on the value of Z_V^* and the amplitude of either leak-induced pattern. Finally, the two existing FRD-based leak detection methods are compared, and the approach using the odd harmonics is found to be superior.

Keywords: pipelines; transient; water hammer; water distribution systems; leak detection; frequency response diagram; steady oscillatory flow; harmonic analysis

Introduction

The problem of leakage in water distribution systems not only imposes a large economic cost on users and authorities, but also poses significant potential risks to public health due to possible contamination of the potable water supply. Attention to this problem has increased over the last decade due to a growing awareness of the need for water security. During this time, a number of transient based methods for leak detection have been developed (Colombo et al. 2009).

Transient based leak detection methods utilize transient pressure waves that travel at a high speed inside pipelines. Leaks, or any other physical changes in a pipe, can induce reflections on a travelling wave. In the time domain, these reflections are observed as discontinuities in the pressure signal measured along the pipe (Lee et al. 2007). In the frequency domain, the frequency response diagram (FRD) of the pipeline system may be distorted due to the reflected signals (Lee et al. 2005a). Accordingly, numerous transient leak detection methods have been developed either in the time domain or the frequency domain.

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Time domain transient leak detection methods mainly include time domain reflectometry techniques (Silva et al. 1996; Brunone 1999), impulse response techniques (Vítkovský et al. 2003; Lee et al. 2007) and inverse transient analysis (ITA) methods (Liggett and Chen 1994; Vítkovský et al. 2007; Jung and Karney 2008). Frequency domain techniques are usually based on the analysis of a pipeline system's FRD, which describes the amplitude of the pressure response fluctuation corresponding to each frequency component in an input signal. Compared with time domain methods, frequency domain methods require less computational time because the head and flow responses are determined directly through analytic relationships (Colombo et al. 2009).

Jönsson and Larson (1992) first proposed that the spectral analysis of a measured pressure trace could be used for leak detection. Several years later, Mpesha et al. (2001) proposed a leak detection method based on the analysis of an experimental FRD of a pipeline system; Ferrante and Brunone (2003) presented a leak detection method based on the analysis of hydraulic impedances of a pipeline; Covas et al. (2005) proposed a standing wave difference method (SWDM) which used the spectral analysis of an FRD to determine the leak-resonance frequency, thereby determining the leak location. However two solutions existed for a single leak in the SWDM.

In the same year, Lee et al. (2005a) introduced a method that could both determine the location and the size of a leak by analyzing the leak-induced pattern (pressure oscillations) at the resonant frequency components (the odd harmonics) in the FRD. In a later paper, Lee et al. (2006) presented the first experimental validation of the FRD based leak detection method. Sattar and Chaudhry (2008) then suggested a similar leak detection

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method but using the leak-induced pattern at the anti-resonant frequencies (the even harmonics).

According to the analyses presented in Lee et al. (2005a) and Sattar and Chaudhry (2008), a leak-induced pattern appears as sinusoidal oscillations at the odd or the even harmonic magnitudes in the FRD for a leaking pipe. The period of the leak-induced pattern can be used to determine the leak location. The amplitude of the pattern is indicative of the leak size. In Lee et al. (2005a) the leak-induced pattern at the odd harmonics is evident, while only slight perturbations can be observed at the even harmonics. In contrast, the leak-induced pattern at the even harmonics possesses a larger amplitude in Sattar and Chaudhry (2008). As a larger leak-induced pattern amplitude (which has a higher signal to noise ratio) is desirable in real applications, two questions naturally arise from this discrepancy between these two FRD based leak detection methods: Firstly, *what controls the amplitudes of the leak-induced patterns in the FRD for a single pipe with a leak?* Secondly, *if the amplitudes of the leak-induced patterns in the FRD for a specific pipe configuration can be controlled, which method would lead to a more accurate solution?*

The research reported here answers these questions by analyzing and comparing the leak-induced patterns at the odd and the even harmonics in the FRD. This study initially reviews the fundamental equations involved in the analysis of steady oscillatory flow in a single pipe using the transfer matrix method (Chaudhry 1987). Analytic expressions for the magnitude of pressure response at the odd and even harmonics are adapted from Lee et al. (2005a) and Sattar and Chaudhry (2008).

Following this, a non-dimensionalization approach for the governing equations is performed, during which the amplitudes of the leak-induced patterns are observed to depend only on two dimensionless impedances: the dimensionless steady-state valve impedance (Z_V^*) and the dimensionless leak impedance (Z_L^*). For any specific pipeline system, the value of Z_L^* is constant, so that the relative sizes of the two amplitudes of the leak-induced patterns are controlled only by the value of Z_V^*. When Z_V^* is equal to unity, the amplitudes of the two leak-induced patterns at the odd and the even harmonics are equivalent. When it is larger than unity, the leak-induced pattern at the odd harmonics has a larger amplitude, while the oscillatory pattern at the even harmonics is more evident if Z_V^* is less than unity. As the value of Z_V^* is dependent on the steady-state valve opening, this finding indicates that the amplitudes of the leak-induced patterns in the FRD can be controlled by the initial steady-state valve opening.

Equations for determining the dimensionless leak size are also derived in this research. The dimensionless leak size can be determined from the amplitude of the leak-induced pattern at either the odd or the even harmonics. In order to illustrate the effects of the two dimensionless impedances (Z_V^* and Z_L^*) on the amplitudes of the leak-induced patterns, numerical simulations are conducted for the dimensionless system neglecting friction. A case study for a specific pipeline system with steady friction is also performed to determine the effect of Z_V^* on the accuracy of the two leak detection methods. Finally, the two leak detection methods are compared and contrasted. Recommendations for implementing the FRD based leak detection methods in the real world are outlined in the concluding section of this paper.

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Fundamental Equations

Unsteady flow in pipes can be described by simplified one-dimensional momentum and continuity equations, as shown by Chaudhry (1987):

$$\frac{\partial H}{\partial x}+\frac{1}{gA}\frac{\partial Q}{\partial t}+\frac{fQ^2}{2gDA^2}=0 \tag{1}$$

$$\frac{\partial Q}{\partial x}+\frac{gA}{a^2}\frac{\partial H}{\partial t}=0 \tag{2}$$

where H and Q represent the piezometric head and the flow rate; g is the gravitational acceleration; A and D are the cross sectional area and the inside diameter of the pipeline; f is the Darcy-Weisbach friction factor; a is the pressure wave speed; x and t are the spatial coordinate and time, respectively. Instantaneous H and Q values can be considered as the sum of the mean values, H_0 and Q_0, and the sinusoidal oscillations around the mean, $h^O=\mathrm{Re}[h(x)e^{j\omega t}]$ and $q^O=\mathrm{Re}[q(x)e^{j\omega t}]$, where h^O and q^O denote the head and flow sinusoidal oscillations; ω is the angular frequency, in radians per second; $j=\sqrt{-1}$; $h(x)$ and $q(x)$ are complex valued functions of x; $\mathrm{Re}[\;]$ stands for the real part of the variable inside the brackets.

Provided the flow and head oscillation at the upstream end (entrance) of the ith pipe are known, the expressions for the amplitudes of head and flow fluctuation at the downstream end of the ith pipe can be written in the matrix notation as (Chaudhry 1987)

$$\begin{Bmatrix} q \\ h \end{Bmatrix}^{n+1}=\begin{bmatrix} \cosh(\mu_i L_i) & \dfrac{-1}{Z_{Pi}}\sinh(\mu_i L_i) \\ -Z_{Pi}\sinh(\mu_i L_i) & \cosh(\mu_i L_i) \end{bmatrix}\begin{Bmatrix} q \\ h \end{Bmatrix}^{n} \tag{3}$$

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where the superscripts n and $n+1$ represent the upstream and downstream positions respectively; L_i is the length of the ith pipe; $Z_{Pi} = \mu_i a_i^2 /(j\omega g A_i)$ is the characteristic impedance for the ith pipe; and μ_i is the propagation operator given by $\mu = \sqrt{-\omega^2 / a^2 + j\omega gAR / a^2}$, in which R is a linearised resistance term. For turbulent flow and steady friction $R = fQ_0 /(gDA^2)$.

Similar matrices can be derived for other components such as inline valves and leaks. The point transfer matrix for a leak presented in Lee et al. (2005a) is

$$\mathbf{P_L} = \begin{bmatrix} 1 & -\dfrac{Q_{L0}}{2H_{L0}} \\ 0 & 1 \end{bmatrix} \tag{4}$$

in which Q_{L0} and H_{L0} are the steady-state flow through the leak and the head at the leak, respectively. The terms Q_{L0} and H_{L0} are related by the orifice equation $Q_{L0} = C_d A_L \sqrt{2gH_{L0}}$, where A_L is the area of the leak orifice; C_d is the coefficient of discharge; and $C_d A_L$ is called the lumped leak parameter. The transfer matrix for a sinusoidal oscillating valve can be written as (Lee et al. 2005a)

$$\begin{Bmatrix} q \\ h \end{Bmatrix}^{n+1} = \begin{bmatrix} 1 & 0 \\ -\dfrac{2\Delta H_{V0}}{Q_{V0}} & 1 \end{bmatrix} \begin{Bmatrix} q \\ h \end{Bmatrix}^{n} + \begin{Bmatrix} 0 \\ \dfrac{2\Delta H_{V0}\Delta\tau}{\tau_0} \end{Bmatrix} \tag{5}$$

where ΔH_{V0} and Q_{V0} represents the steady-state head loss across and flow through the valve, respectively; τ_0 is the mean dimensionless valve-opening coefficient; and $\Delta\tau$ stands for the magnitude of the dimensionless valve-opening oscillation generating the transients; the active input is given by the second vector term on the right side of the equation.

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The overall transfer matrix $\mathbf{U}$ for a pipeline is obtained by an ordered multiplication of the individual field and point matrices starting at the downstream end. For a single pipeline with a leak (see Fig. 1), $\mathbf{U} = \mathbf{F_2 P_L F_1}$, in which $\mathbf{F}_1$ and $\mathbf{F_2}$ are the field transfer matrices for the two pipe sections separated by the leak.

Fig. 1 goes here

Frequency Response Equations for a Leaking Pipe

Similar to the systems in Lee et al. (2005a) and Sattar and Chaudhry (2008), a reservoir-pipeline-valve system is adopted for the research described in this paper, as shown in Fig. 1. For a frictionless intact pipeline (that is, without any anomalies such as leaks), the FRD has a uniform value of $2\Delta H_{V0}\Delta\tau/\tau_0$ at the odd harmonics and a zero value at the even harmonics (Lee et al. 2005a). Neglecting friction, where a leak exists in the pipe, the magnitude of the pressure fluctuation at the upstream side of the valve can be derived as (Lee et al. 2005a)

$$h = \frac{-2\Delta H_{V0}\Delta\tau/\tau_0}{1-\dfrac{2\Delta H_{V0}\left[\cos\left(\dfrac{(L_1+L_2)\omega}{a}\right)+\dfrac{jQ_{L0}a}{2gAH_{L0}}\sin\left(\dfrac{L_1\omega}{a}\right)\cos\left(\dfrac{L_2\omega}{a}\right)\right]}{Q_{V0}\left[\dfrac{Q_{L0}a^2}{2H_{L0}g^2A^2}\sin\left(\dfrac{L_1\omega}{a}\right)\sin\left(\dfrac{L_2\omega}{a}\right)-\dfrac{ja}{gA}\sin\left(\dfrac{(L_1+L_2)\omega}{a}\right)\right]}} \tag{6}$$

where L_1 and L_2 are the lengths of the pipe sections upstream and downstream of the leak, as shown in Fig. 1. The odd harmonics are defined as $\omega = \omega_r^{odd}\omega_{th}$, in which ω_{th} is the fundamental angular frequency of the reservoir-pipeline-valve system [$\omega_{th} = \pi a/(2L)$, where L is the total pipeline length] (Chaudhry 1987); ω_r^{odd} is the relative angular frequency for the odd harmonics ($\omega_r^{odd} = 1, 3, 5, \ldots$). By simplification and elimination of

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a small coefficient $(Q_{L0}a)/(4H_{L0}gA)$ [which is usually much smaller than unity under realistic combinations of leak size and head at the leak (Lee et al. 2005a)], the magnitude of the pressure fluctuation at the odd harmonics can be derived as

$$|h_{odd}| = \frac{2\Delta H_{V0}\Delta\tau/\tau_0}{\frac{\Delta H_{V0}Q_{L0}}{2Q_{V0}H_{L0}}\left[\cos\left(\omega_r^{odd}\pi x_L^* - \pi\right)+1\right]+1} \tag{7}$$

where $x_L^* = L_1/(L_1+L_2)$ is the dimensionless leak location.

Similarly, Sattar and Chaudhry (2008) derived the expression for the pressure fluctuation magnitude at the even harmonics:

$$|h_{even}| = \frac{2\Delta H_{V0}\Delta\tau/\tau_0}{\dfrac{1}{\left(\dfrac{a}{gA}\right)^2\dfrac{Q_{V0}Q_{L0}}{8\Delta H_{V0}H_{L0}}\left[\cos\left(\omega_r^{even}\pi x_L^*\right)-1\right]}-1} \tag{8}$$

where ω_r^{even} is the relative angular frequency for the even harmonics ($\omega_r^{even} = 2, 4, 6, \ldots$).

Dimensionless Analysis of the Leak-induced Patterns

As part of conducting the research described in this paper, it is necessary to simplify the governing equations using a dimensionless analysis on the leak-induced patterns using hydraulic impedances (Wylie and Streeter 1993). Based on the characteristic impedance for a frictionless pipeline, $Z_C = a/(gA)$, the hydraulic impedance for a steady-state valve, $Z_V = 2\Delta H_{V0}/Q_{V0}$, and the hydraulic impedance for a leak (or a fixed orifice), $Z_L = 2H_{L0}/Q_{L0}$, dimensionless impedances for the valve and leak can be defined as

$$Z_V^* = Z_V/Z_C \tag{9}$$

$$Z_L^* = Z_L/Z_C \tag{10}$$

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The magnitude of the pressure fluctuation h is non-dimensionalised by dividing it by the active input $2\Delta H_{V0}\Delta\tau/\tau_0$, i.e.

$$h^* = h/(2\Delta H_{V0}\Delta\tau/\tau_0) \tag{11}$$

Dimensionless Analysis of Frictionless Leaking Pipes

For a frictionless intact pipe, the values of h^* at the odd harmonics are all equal to unity, while those at the even harmonics are all zero. For a frictionless pipe with a leak, the expression of h^* can be obtained by substituting Eqs (9) to (11) into Eq. (6), which leads to

$$h^* = \frac{-1}{1 - Z_V^* \dfrac{\cos\left(\omega_r \dfrac{\pi}{2}\right) + j(Z_L^*)^{-1}\sin\left(\omega_r \dfrac{\pi}{2} x_L^*\right)\cos\left(\omega_r \dfrac{\pi}{2}(1-x_L^*)\right)}{(Z_L^*)^{-1}\sin\left(\omega_r \dfrac{\pi}{2} x_L^*\right)\sin\left(\omega_r \dfrac{\pi}{2}(1-x_L^*)\right) - j\sin\left(\omega_r \dfrac{\pi}{2}\right)}} \tag{12}$$

in which $\omega_r = \omega/\omega_{th}$ is the dimensionless relative angular frequency. For the head fluctuation at the odd harmonics, Eq. (7) can be written as

$$\left|h_{odd}^*\right| = \frac{1}{\dfrac{Z_V^*}{2Z_L^*}\left[\cos\left(\omega_r^{odd}\pi x_L^* - \pi\right) + 1\right] + 1} \tag{13}$$

Similarly, for the head fluctuation at the even harmonics, Eq. (8) yields

$$\left|h_{even}^*\right| = \frac{1}{\dfrac{2Z_V^* Z_L^*}{\cos\left(\omega_r^{even}\pi x_L^*\right) - 1} - 1} \tag{14}$$

Eqs (13) and (14) show that, in addition to the relative angular frequency, the dimensionless leak-induced patterns are dependent only on three dimensionless parameters: Z_V^*, Z_L^* and x_L^*. The periods of the sinusoidal patterns are both equal to

$2/x_L^*$. The dimensionless amplitude of a leak-induced pattern can be defined as the maximum head value minus the minimum head value within the oscillatory pattern. From Eq. (13), the theoretical minimum head value at the odd harmonics is $1/\left(Z_V^*/Z_L^*+1\right)$ when the cosine function $\cos(\omega_r^{odd}\pi x_L-\pi)$ equals 1. When the cosine function equals -1, the maximum head value is achieved as 1. As a result, the theoretical dimensionless amplitude of the leak-induced pattern at the odd harmonics is given as

$$\left|h_{odd}^*\right|_{amp}=1-\frac{1}{Z_V^*/Z_L^*+1}=\frac{1}{Z_L^*/Z_V^*+1} \tag{15}$$

The theoretical dimensionless amplitude for the leak-induced pattern at the even harmonics can be derived by a similar process, and written as

$$\left|h_{even}^*\right|_{amp}=\frac{1}{Z_L^*Z_V^*+1} \tag{16}$$

Eqs (15) and (16) demonstrate that the amplitudes of the leak-induced pattern at the odd and the even harmonics are equal only if $Z_V^*=1$. The expression of $\left|h_{odd}^*\right|_{amp}$ is a monotonically increasing function of Z_V^*, while $\left|h_{even}^*\right|_{amp}$ is a monotonically decreasing function of Z_V^*. When the value of Z_V^* is greater than 1, the amplitude of the leak-induced pattern at the odd harmonics is larger than that at the even harmonics; however, when Z_V^* is less than 1, the amplitude of the leak-induced pattern at the even harmonics is larger. These results indicate that, physically, the amplitudes of leak-induced patterns at the odd and even harmonics are controlled by the steady state valve opening, because the value of Z_V^* depends on the steady state head loss across the valve and the flow through the valve, which are in turn dependent on the steady state valve opening [see Eq. (17) in the next section].

Impedance Parameter Ranges

An analysis is now performed to determine the physical ranges for the above dimensionless impedances. The possible range of a parameter is presented as $O\{10^n, 10^m\}$, which indicates that the order of magnitude of the parameter is between 10^n and 10^m. Indicative values for n and m are discussed subsequently.

Considering the valve as an orifice and using the orifice equation, the expression of the dimensionless steady-state valve impedance can be written as

$$Z_V^* = \frac{\sqrt{2g\Delta H_{V0}}A}{aC_d A_{V0}} \tag{17}$$

where A_{V0} is the opening area of the steady-state valve; C_d is the coefficient of discharge for the valve opening, which is usually less than 1. It can be seen from Eq. (17) that the value of Z_V^* can be controlled by setting the valve opening A_{V0}. Similarly, the dimensionless leak impedance can be written as

$$Z_L^* = \frac{\sqrt{2gH_{L0}}A}{aC_{Ld} A_L} \tag{18}$$

In practice, the reasonable physical range for wave speed a is $O\{10^2, 10^3\}$; for valve head loss ΔH_{V0} it is $O\{10^{-2}, 10^2\}$; for head at a leak H_{L0} it is $O\{10^{-2}, 10^2\}$. The size of $C_d A_{V0}$ can be either very small or comparable to A, so a reasonable estimate for the range of $A/(C_d A_{V0})$ is $O\{10^0, 10^4\}$. In contrast, $C_{Ld} A_L$ should be much smaller than A, so the range of $A/(C_{Ld} A_L)$ is assumed to be $O\{10^2, 10^4\}$. In addition, note that when simplifying Eq. (6) to obtain Eq. (7) and Eq. (8), the eliminated small coefficient $(Q_{L0}a)/(4H_{L0}gA)$ is equivalent to $1/(2Z_L^*)$. It must be small enough for the elimination to be satisfied, so the value of Z_L^* is assumed to be larger than 10^0. By substituting all these elements into Eq. (17) and Eq. (18), the ranges for the dimensionless steady-state valve and leak impedance are derived as

$$Z_V^* \in O\{10^{-4}, 10^4\} \tag{19}$$

$$Z_L^* \in O\{10^0, 10^4\} \tag{20}$$

Leak Size Derivation

The dimensionless leak size $C_{Ld}A_L/A$ is a critical parameter that needs to be estimated during the leak detection procedure. Lee et al. (2005a) offered a procedure and equations for determining $C_{Ld}A_L/A$ using the leak-induced pattern at the odd harmonics for dimensional systems. Sattar and Chaudhry (2008) published a way to estimate the leak size through the use of the leak-induced pattern at the even harmonics, but by using a semi-empirical look-up curve rather than an analytical approach.

In contrast, the current research derives the equations for determining the dimensionless leak size $C_{Ld}A_L/A$ by utilizing the dimensionless impedances and either of the two dimensionless amplitudes of the leak-induced patterns. Rearranging the expression for the dimensionless leak impedance [Eq. (10)] yields $Q_{L0} = 2H_{L0}gA/(aZ_L^*)$. By substituting it into the orifice equation for the leak, $C_{Ld}A_L/A$ can be written as

$$C_{Ld}A_L/A = \sqrt{2H_{L0}g} \big/ (aZ_L^*) \tag{21}$$

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It is possible to estimate the value of H_{L0} once the leak location has been confirmed. The value of Z_L^* can then be obtained from the dimensionless amplitude of the leak-induced pattern either at the odd harmonics or at the even harmonics. Two new equations are developed using Eq. (15) and Eq. (16):

$$Z_L^* = Z_V^*(1 - |h_{odd}^*|_{amp}) \Big/ |h_{odd}^*|_{amp} \tag{22}$$

$$Z_L^* = (1 - |h_{even}^*|_{amp}) \Big/ (|h_{even}^*|_{amp} Z_V^*) \tag{23}$$

Values of the leak-induced pattern amplitudes can be read from the experimental FRD, and the value of Z_V^* can be calculated by the steady-state head loss across, and flow through, the inline valve. As a result, the dimensionless leak size $C_{Ld}A_L / A$ can be determined mathematically either from the leak-induced pattern at the odd harmonics or that at the even harmonics. The above procedure is equivalent to the leak size determination procedure in Lee et al. (2005a), but with dimensionless parameters.

Dimensionless Modeling of Frictionless Leaking Pipes

In order to demonstrate the effects of the two dimensionless impedances (Z_V^* and Z_L^*) on the leak induced patterns, a frictionless reservoir-pipeline-valve system is described here. A specific set of impedance values is described first, followed by an explanation of the value of the dimensionless impedances one by one.

A leak with a dimensionless impedance of 12.12 (which comes from the pipeline system in the later *case study* section) is located at the dimensionless leak location $x_L^* = 0.1$. According to Eqs (15) and (16), the location of the leak does not impact on the amplitudes of the leak-induced patterns. The value of Z_V^* is set at 1.0. The dimensionless FRD for this system is obtained from Eq. (12) and shown in Fig. 2.

Fig. 2 goes here

The leak-induced patterns at the odd and even harmonics are illustrated in Fig. 3. The circle points are values at the harmonics, and the solid lines are the sinusoidal fitted lines of the form $Y_{data} = x_1 \sin(x_2\omega_r + x_3) + x_4$, where $x_1, \ldots, x_4$ are coefficients determined from a least squares fit to the data; ω_r is equal to ω_r^{odd} for the pattern at the odd harmonics and ω_r^{even} is the value for the even harmonics; and Y_{data} is the magnitude of head oscillation at the harmonics.

Fig. 3 goes here

The amplitudes of the leak-induced patterns at the odd and even harmonics are both 0.076 (from the sinusoidal fitting line). This result validates the finding that when Z_V^* equals unity, the amplitudes of the leak-induced patterns are the same.

When Z_L^* is assigned the value of 12.12 and Z_V^* is increased from 10^{-4} to 10^4, the behavior of the leak-induced patterns is determined and shown in Fig. 4. The theoretical amplitude of the leak-induced pattern at the odd harmonics (the thick solid line) rises, while that at the even harmonics (the thin solid line) drops, both monotonically. When Z_L^* is decreased to 6.06, the amplitudes of leak-induced patterns (dashed lines) show the same pattern but with larger values. The intersections occur only when $Z_V^* = 1.0$ (Fig. 4). These results confirm the finding that when Z_V^* is greater than unity, the leak-induced pattern at the odd harmonics is more evident, while when Z_V^* is less than unity, the pattern at the even harmonics has a larger amplitude. As mentioned in previous sections,

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the value of Z_V^* is controlled by the initial valve opening. The results shown in Fig. 4 also verify the finding that the amplitudes of the leak-induced patterns can be adjusted by the initial steady-state valve opening.

Fig. 4 goes here

When the value of Z_V^* remains constant and the value of Z_L^* is changed, the effect of Z_L^* on the amplitudes of the leak-induced patterns can be determined and is demonstrated in Fig. 5. The amplitudes of the two leak-induced patterns alter with varying Z_L^* values, but are equal for the entire range of Z_L^* as long as Z_V^* equals unity (see the solid lines in Fig. 5). The amplitudes decrease monotonically with the increase of Z_L^* (equivalent to a decrease in leak size). When Z_V^* is equal to 2.0, the amplitude of the leak-induced pattern at the odd harmonics is always larger than the pattern amplitude at the even harmonics (see the dashed lines in Fig. 5), as expected.

Fig. 5 goes here

The plots in Fig. 5 indicate that a smaller value of Z_L^* (a larger leak size) leads to greater values for the leak-induced pattern amplitudes, for both patterns at the odd and the even harmonics. Note that in frictionless systems, ΔH_{V0} and H_{L0} are both constant and equal to the head of the reservoir, so that Z_V^* and Z_L^* are independent variables. For a given system, Z_L^* is always constant [Eq. (18)], while Z_V^* is a function of the valve opening [Eq. (17)].

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Case Study of a Specific Pipeline with Steady Friction

The previous dimensionless analysis is for frictionless pipes. In order to investigate the effect of Z_V^* on the leak-induced patterns for real pipelines, a case study is now presented for a leaking pipeline with steady friction. The steady-state valve opening is changed to obtain various steady-state flow rates as well as various values of Z_V^*. Leak detection procedures are performed based on the leak-induced pattern at the odd harmonics and the pattern at the even harmonics, respectively. The accuracies of the derived leak sizes are contrasted. The results are presented with dimensionless parameters to be consistent with the previous dimensionless analysis.

A reservoir-pipeline-valve system similar to that in Sattar and Chaudhry (2008) is adopted. The pipeline system layout is the same as that in Fig. 1. The parameters of this system are summarized in the following table, where H_r is the reservoir head.

Table 1 goes here

When the steady-state valve discharge Q_{V0} is set to be 0.2995 L/s by adjusting the steady-state valve opening, the value of Z_V^* is almost 1.0 and the value of Z_L^* is 12.07 (it is 12.12 if friction is neglected). The dimensionless FRD is determined numerically by using the original transfer matrices shown in Eqs (3) to (5) and the result is depicted in Fig. 6.

Fig. 6 goes here

The values of $\left|h_{odd}^*\right|_{amp}$ and $\left|h_{even}^*\right|_{amp}$ determined by the fitting functions are 0.0701 and 0.0702, respectively. The equivalence indicates that, for a specific pipeline system with steady friction, the amplitudes of the leak-induced patterns are still very close to being the same when Z_V^* equals unity. The dimensionless leak location is determined to be 0.1 or 0.9 (a symmetric location due to the symmetric nature of the cosine function) by the periods of the leak-induced patterns. The aliased leak location ($x_L^* = 0.9$) can be eliminated using the leak-induced pattern at the odd harmonics and the phase-based technique presented in Lee et al. (2005a). Fig. 7 shows the dimensionless FRD for the same pipeline system but the leak is now located at $x_L^* = 0.9$. The leak-induced pattern at the odd harmonics has a different phase compared with the pattern at the odd harmonics shown in Fig. 6, while the leak-induced patterns at the even harmonics are identical.

Fig. 7 goes here

Once the leak location has been determined, the head at the leak H_{L0} can be estimated by linear interpolation, which yields a value of 29.73 m. The value of Z_L^* derived from the dimensionless amplitudes at the odd and the even harmonics are 13.27 and 13.25 by using Eq. (22) and Eq. (23), respectively. Finally, $C_{Ld} A_L / A$ comes to 0.0018 with both Z_L^* values using Eq. (21), which indicates the leak size can be determined relatively accurately from both leak-induced patterns when Z_V^* equals unity.

To determine the behavior of the dimensionless amplitudes of the leak-induced patterns for a specific pipeline system, various values of Z_V^* are now adopted (obtained by changing the steady-state opening of the valve). The range of Z_V^* is selected to be around unity in order to give a clear view about the behavior of the dimensionless amplitudes of the leak-induced patterns around this critical Z_V^* value, and the results are demonstrated in Fig. 8.

Fig. 8 goes here

The dimensionless amplitude of the leak-induced pattern at the odd harmonics, $\left|h^*_{odd}\right|_{amp}$ (derived by using fitted functions), shows a monotonic increasing trend as Z^*_V increases, which is similar to the trend shown in Fig. 4. In contrast, the changing curve of $\left|h^*_{even}\right|_{amp}$ is not monotonic and different compared with the corresponding trend line for frictionless systems shown in Fig. 4. With an increasing value of Z^*_V, an increasing trend is observed at the beginning of the $\left|h^*_{even}\right|_{amp}$ curve. A decreasing trend starts when the value of Z^*_V reaches around 0.25. The discrepancy between the curve of $\left|h^*_{even}\right|_{amp}$ for friction systems (Fig. 8) and that for frictionless systems (Fig. 4) is caused only by the effect of friction. When friction is included, the values of ΔH_{V0} and H_{L0} change along with the varying of the steady-state valve opening, while they are constant and equal to the reservoir head in the frictionless analysis. A larger valve opening corresponds to a larger steady-state discharge and a smaller Z^*_V value, in which the effect of steady friction is also enlarged.

When the effect of steady friction is severe, or when the value of Z^*_V is less than 0.25 as shown in Fig. 8, the governing equations derived in the frictionless analysis are no longer applicable for depicting the behavior of real pipelines with friction. This discrepancy is mainly exhibited in the distortion of the leak-induced pattern at the even harmonics in the FRD. In contrast, the leak-induced pattern at the odd harmonics is not affected to any significant degree. The above hypothesis is verified by calculating the dimensionless leak sizes within the whole range of Z^*_V and determining the accuracy. The results are presented in Fig. 9.

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Fig. 9 goes here

When Z_V^* equals 1, the values of $C_{Ld} A_L / A$ determined from the leak-induced patterns at the odd and even harmonics are equivalent and close to the theoretical leak size (0.002). When Z_V^* is increased from 1, a slight decreasing trend that deviates from the theoretical leak size is observed for the dimensionless leak size determined from the odd harmonics, which indicates that values of Z_V^* that are too large are not desirable. Meanwhile, the leak size derived from the leak-induced pattern at the even harmonics seems to be more accurate. However, as can be seen in Fig. 8, the value of $\left|h_{even}^*\right|_{amp}$ is small and keeps on decreasing as the value of Z_V^* increases from 1.0. Considering that the head values at the even harmonics in the FRD are small values and close to zero [refer to the FRD showed in Lee et al. (2005a)], the amplitude of the leak-induced pattern at the even harmonics is hard to observe accurately from experimental data due to a poor signal to noise ratio (SNR).

When Z_V^* decreases from 1, the value of $C_{Ld} A_L / A$ derived from the odd harmonics remains steady but fluctuates when $\left|h_{odd}^*\right|_{amp}$ is too small. In contrast, the value of $C_{Ld} A_L / A$ derived from the even harmonics drops dramatically, even though the value of $\left|h_{even}^*\right|_{amp}$ is larger than $\left|h_{odd}^*\right|_{amp}$ when Z_V^* is smaller than 1. The relative error of the leak size derived from the even harmonics when compared to the real dimensionless leak size is around 30% when Z_V^* is 0.5. The deviation from the real leak size is more obvious for smaller Z_V^* values, which corresponds to greater friction effects.

The above case study on a specific pipeline system with steady friction verifies that the amplitudes of the leak-induced patterns can be adjusted in real applications by adjusting the steady-state valve opening. The steady friction has more effect on the leak-induced pattern at the even harmonics for values of Z_V^* less than 1. The fluctuation and large error in the derived leak sizes presented in Fig. 9 demonstrate that, for small values of Z_V^* (less than 0.5), the influence of the steady friction is so great that the expression for the leak-induced pattern at the even harmonics derived by frictionless analysis is insufficient to depict the behavior of the head response or to be used to determine the leak size accurately.

Comparison of the Two Existing Leak Detection Methods

The leak detection method proposed by Lee et al. (2005a) and that presented in Sattar and Chaudhry (2008) are now compared to illustrate which harmonics (odd or even) possess the greater utility for leak detection. Leak detection consists of two main goals: (1) to detect the leak location, and (2) to estimate the leak size. The dimensionless leak location can be determined from the period of the leak-induced pattern. The periods of the leak induced patterns at the odd and the even harmonics are the same regardless of whether steady friction is included or not. However, this approach leads to two possible symmetric locations for a leak corresponding to a single oscillation period due to the symmetric nature of the cosine function. In the method proposed by Lee et al. (2005a), the phase of the inversed leak-induced pattern at the odd harmonics can be used to determine in which half of the pipeline the leak is located. For the method presented in Sattar and Chaudhry (2008), however, the aliased position cannot be eliminated.

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Regarding the estimation for the leak size, there are two different strategies. In Lee et al. (2005a), Eq. (7) together with the orifice equation were used to calculate the lumped leak parameter $C_{Ld}A_L$, which is equivalent to the procedure proposed in this paper but in the dimensional domain. The distortion in the FRD caused by unsteady friction can be corrected numerically by using a least squares regression of a scale- and trend-corrected sinusoid to the inverted peak data points (Lee et al. 2006). The procedure for determining the leak size has a strong mathematical foundation and the results are observed to be relatively accurate for a wide range of Z_V^* values in the case study described in this research. The mathematics and the results of the case study indicate that the method based on odd harmonics is robust and reliable in estimating the leak size.

The leak size estimation technique in Sattar and Chaudhry (2008) is not based on rigorous mathematical equations. Firstly, the amplitudes of the leak-induced pattern at the even harmonics are derived numerically with various leak sizes. Then, a semi-empirical relationship between the dimensionless amplitude (non-dimensionalised by dividing the head of the reservoir) and the dimensionless damage [$(C_{Ld}A_L/A)^{0.5}$] requires derivation by curve fitting. In Sattar and Chaudhry (2008), when an experimental FRD was obtained from a real pipeline system, the leak size was read from a look-up curve after the amplitude of the leak-induced pattern at the even harmonics had been determined. The effect of unsteady friction was neglected, which led to an average error of 7%, as mentioned in their work. Moreover, as illustrated in this research, the amplitude of the leak-induced pattern (either at the odd or the even harmonics) is not only dependent on the leak size but also influenced by the steady-state valve opening or the impedance of the valve. A simple semi-empirical curve is not sufficient to predict the leak size for arbitrary pipeline configurations.

The above analysis indicates that the leak detection method proposed by Lee et al. (2005a) using the leak-induced pattern at the odd harmonics has more advantages. It can eliminate the aliased leak location, and provide a leak size with acceptable accuracy with appropriate valve opening settings.

Challenges to current FRD-based leak detection techniques

This research indicates that the leak detection technique based on the analysis of the leak-induced pattern of resonant responses is promising, and extraction of the leak-induced pattern can be improved by using the appropriate valve impedance. However, it should be noted that challenges exist for the application to real pipelines. These challenges result from two major aspects: the assumptions made in the development of the mathematical leak detection algorithms, and practical limitations due to the complexities of real pipeline systems in the field. Details about these two aspects are discussed below.

Summary of the assumptions

A number of assumptions have been made during the development of the FRD-based leak detection algorithms. The major assumptions are summarized and presented below.

- **Main assumptions associated with the momentum and continuity equations [Eqs (1) and (2)]:** (i) The flow is one dimensional and pressure is uniform at cross sections; (ii) The fluid is slightly compressible and the conduit walls are linearly elastic, but variations of the mass density ρ and the flow area A due to variations of the inside pressure are negligible during the mathematical deviation; (iii) The head losses during the transient state for a given flow velocity are the same as in steady flows at that velocity.

- **Main assumptions associated with the transfer matrix method [Eqs (3) to (5)]:** (i) Sinusoidal steady-oscillatory flow can be established in pipelines, in which the instantaneous pressure head $H = H_0 + h^O$ and the instantaneous discharge $Q = Q_0 + q^O$, where H_0 and Q_0 are the mean values of head and discharge (i.e. the steady-state head and discharge), and h^O and q^O are the sinusoidal oscillations around the mean; (ii) The friction term and the non-linear boundary conditions are linearized, which requires that the amplitude of oscillations (h^O and q^O) to be small; (iii) The pipeline system is a linear system, so that any periodic forcing function (input signals) can be decomposed into various harmonics by Fourier analysis and analyzed separately. The overall system response is determined by superposition of individual responses; (iv) The pipeline section described by the field matrix Eq. (3) is constant and uniform in cross-sectional area, wall thickness, wave speed and wall material.
- **The orifice equation:** It is assumed that the head at the leak and the flow through the leak is governed by the orifice equation $Q_{L0} = C_d A_L \sqrt{2gH_{L0}}$. This equation is developed for steady-state conditions, and can be accurate if the lumped leak parameter $C_d A_L$ is calibrated appropriately. This equation is assumed to be still valid in the FRD-based leak detection using transient waves, where the value of $C_d A_L$ may vary during transient event. To make this assumption applicable, the transient waves

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used to excite the pipeline must be low in amplitude compared with the steady-state head. The IRS-based persistent signals proposed in this paper are suitable options.

- **The FRD-based leak detection techniques [Eqs (6) to 8]:** (i) The effects of friction are negligible, i.e, the pipeline is lossless; (ii) The size of the leak is small compared with the cross-sectional area of the pipeline; (iii) The pipeline under test is a single pipeline with simple boundary conditions (reservoir-pipeline-valve or reservoir-pipeline-reservoir).

Challenges in practical applications

As presented in the previous sub-section, current FRD-based leak detection techniques are developed based on a number of assumptions. These assumptions facilitate the deviation of the mathematical equations; however, they also impose challenges on the application of the FRD-based leak detection techniques on real pipelines. Major challenges and possible solutions to date are summarized below.

- **Effects of friction:** The effects of friction are neglected in the FRD-based leak detection techniques. However, the effects of friction exist in every pipeline and affect the shape of the measured FRD. Lee et al. (2005a) found that steady friction can cause a uniform decrease of the amplitude of a FRD, but that it has negligible effects on the leak-induced pattern at the odd harmonics, while unsteady friction is frequency-dependent and cause a non-uniform distortion of the amplitude of resonant responses. To deal with the non-uniform distortion, a least squares regression fitting algorithm was proposed by Lee et al. (2006) through an experimental study. An equation with ten unknown parameters was calibrated to fit the measured data, so that the frequency of the leak-induced pattern, which is indicative of the location of the

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leak, could be estimated. This technique was successful in the laboratory. This technique leads to another practical challenge: it requires the input signal to have a wide bandwidth, as a number of resonant responses need to be measured for the calibration of the ten unknowns.

- **Bandwidth in the input signal:** The bandwidth required depends on the fundamental frequency of the pipeline under test. A shorter pipeline has a higher fundamental frequency, thus requires a higher bandwidth in the input signal. Traditional input transient signals, such as discrete step or pulse pressure waves generated by an abrupt valve movement, cannot fulfill this requirement for short pipelines. The discrete step or pulse pressure waves are not sharp enough to gain a wide enough bandwidth due to the limitation in the maneuverability of the valve. In addition, the energy distribution of these two types of signal is uneven in the frequency domain. The energy drops rapidly as the frequency increases, yielding low signal-to-noise-ratio (SNR) for high frequency components. A potential solution of this problem is to use persistent signals that have wide bandwidth and lower amplitude to replace the traditional discrete signals. Lee et al. (2008) have used pseudo-random binary signal (PRBS)-based input transients to extract pipeline FRD in the laboratory. A laboratory-sized customized side-discharge valve-based transient generator was developed to produce the persistent and periodical signal. The experiments were successful as the estimated FRD was consistent with the theoretical FRD. However, some practical issues still exist, such as the effects of nonlinearities in real pipelines and the potential need to rapidly maneuver the side-discharge valve.

- **Pipeline parameter variations:** The FRD-based leak detection techniques developed by Lee et al. (2005a) and Sattar and Chaudhry (2008) both assume that the pipeline is uniform in cross-sectional area, wall thickness, wave speed and wall material. However, these properties may vary along a pipeline in the field, for example, due to wall deterioration. Duan et al. (2011) have studied the effects of complex series pipelines on the FRD-based leak detection, which shows that the reflections resulting from internal series junctions modify the system resonant frequencies but have a small effect on the leak-induced information contained within the system frequency responses. Numerical simulations performed by Duan et al. (2011) indicate that single or multiple leaks in complex series pipelines can still be detected by the FRD-based technique developed by Lee et al. (2005a), provided that the location and size of the resonant peaks of the system frequency responses are accurately determined.
- **Pipeline networks:** The current FRD-based leak detection techniques are developed for single pipelines lying between well-defined boundary conditions (reservoir or valve). However, in practice, few systems exist within this narrow category. In contrast, pipelines often contain multiple sections or form a complex network. Due to the complexities in boundary conditions, until now, no field tests have been conducted for the FRD-based leak detection techniques.

Another challenge in pipeline networks is the demand-induced operational noise. Transients can be introduced into a pipeline system because of the use of water by an unknown consumer. The demand-induced transient noise is usually small in large scale systems, but need to take into account where necessary.

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The extension of the FRD-based leak detection techniques to pipeline networks is important. Lee et al. (2005b) have studied this problem and outlined a FRD extraction procedure for pipelines in a network. An individual pipeline can be partially separated from the network by closing the valve at one end of the pipe section. A side-discharge valve located adjacent to the closed valve is then used to generate a transient pulse. The transient trace is measured by a transducer located at the same location as the generator from the beginning of the excitation (time 0) up to a time of $2L/a$, where L is the length of the single pipe section (from the closed valve to the junction in the other end) and a is the wave speed in this section. This initial length of data contains only the reflections within the pipe section but excludes reflections from the rest of the network. The FRD of the specified pipe section can be obtained by assuming that a reservoir exists at the open boundary, and extending the original data to a length of $4L/a$ through inverting the measured data and attaching it to the end of the original data (Lee et al. 2005b). Although further studies and testing are necessary to validate this technique, it shows a potential of applying the FRD-based leak detection techniques to pipelines in the real world.

Conclusions

Leak-induced patterns at the odd and the even harmonics in the frequency response diagram (FRD) of a leaking pipe are analyzed and compared in this paper. A non-dimensionalization approach is performed to simplify the governing equations of the FRD, and to analyze the leak-induced patterns with dimensionless parameters. Equations for determining the dimensionless leak size $C_{Ld}A_L/A$ are derived utilizing the dimensionless impedances and the dimensionless amplitudes of the leak-induced patterns.

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The dimensionless analysis conducted as part of this research illustrates that the amplitudes of the leak-induced patterns can be controlled by the steady-state valve opening. The amplitudes of the leak-induced patterns at the odd and the even harmonics are equivalent when the dimensionless steady-state valve impedance, Z_V^*, equals unity. When Z_V^* is greater than unity, the leak-induced pattern amplitude at the odd harmonics is larger than that at the even harmonics; whereas when Z_V^* is smaller than unity, the leak-induced pattern at the even harmonics is more evident. The value of Z_V^* can be adjusted by changing the steady-state opening of the inline valve. This finding is applicable to both frictionless pipeline systems and systems with steady friction.

A case study of a specific pipeline system is analyzed to investigate the effect of steady friction on the amplitudes of the leak-induced patterns. The results indicate that when the effect of steady friction is severe, where the value of Z_V^* is much less than unity, the leak sizes determined from the leak-induced pattern at the odd harmonics have some fluctuations, and those determined from the leak-induced pattern at the even harmonics have huge errors.

Two existing leak detection methods are compared. The method proposed by Lee et al. (2005a), which uses the leak-induced patterns evident at the odd harmonics, is found to be more robust. It can provide a unique leak location and a relatively accurate leak size through mathematical calculations. Challenges to the existing FRD-based leak detection techniques are summarized, including physical assumptions made during mathematical deviation and typical practical issues in real pipeline systems.

This research suggests that, in real applications, the leak detection method proposed by Lee et al. (2005a) with a value of Z_V^* larger than unity should be employed. The opening of the inline valve needs to be small, ensuring a large steady-state valve head loss, a small valve discharge, and the slight effect of steady friction. However, in cases where Z_V^* cannot be over 1.0, i.e. the steady-state discharge is required to be large, the method presented in Sattar and Chaudhry (2008) can be employed to determine whether there is a leak and to estimate the location.

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Notations

The following symbols are used in this paper:

A	=	inside pipe cross sectional area;
a	=	wave speed;
A_L	=	area of a leak orifice;
A_{V0}	=	opening area of a steady-state valve;
C_d, C_{Ld}	=	coefficient of discharge for a valve opening and for a leak orifice;
D	=	internal pipe diameter;
$\mathbf{F}_1$, $\mathbf{F}_2$	=	field matrices of the two pipe sections divided by a leak;

f = Darcy-Weisbach friction factor;

g = gravitational acceleration;

H = instantaneous head;

H_0 = steady-state head;

H_r = reservoir head;

H_{L0} = steady-state head at a leak orifice;

h, h^* = amplitude and dimensionless amplitude of head fluctuation;

h^O = head oscillation around the mean;

$h(x)$ = complex head amplitude;

$|h_{odd}|$, $|h^*_{odd}|$ = magnitude and dimensionless magnitude of head fluctuation at the odd harmonics;

$|h_{even}|$, $|h^*_{even}|$ = magnitude and dimensionless magnitude of head fluctuation at the even harmonics;

$|h^*_{odd}|_{amp}$, $|h^*_{even}|_{amp}$ = dimensionless amplitude of leak-induced pattern at the odd harmonics and the even harmonics;

j = imaginary unit, $\sqrt{-1}$;

L = total length of pipe;

L_1, L_2 = lengths of the two pipe sections divided by a leak;

$O\{10^n, 10^m\}$ = an order of magnitude range from 10^n to 10^m;

$\mathbf{P_L}$ = point matrix for a leak;

Q = instantaneous flow rate;

Q_0 = steady-state discharge;

Q_{L0} = steady-state flow out of a leak;

Q_{V0} = steady-state flow through a valve;

q^{O} = discharge oscillation around the mean;

$q(x)$ = complex discharge amplitude;

R = linearised resistance term;

Re[] = real part of the variable inside the brackets;

t = time;

$\mathbf{U}$ = overall transfer matrix for a pipeline;

x = distance along pipe;

$x_1, \ldots x_4$ = coefficients to be determined in fitting function;

x_L^* = dimensionless position of a leak;

Y_{data} = oscillatory head values at harmonics;

Z_C = characteristic impedance of a frictionless pipe;

Z_L, Z_L^* = hydraulic impedance and dimensionless hydraulic impedance for a leak orifice;

Z_{Pi} = the characteristic impedance for the i th pipe;

Z_V, Z_V^* = hydraulic impedance and dimensionless hydraulic impedance for a steady-state valve;

Greek symbols:

ΔH_{V0} = steady-state head loss across a valve;

$\Delta\tau$ = magnitude of the dimensionless valve-opening oscillation;

μ_i = propagation operator for the i th pipe;

τ_0 = mean dimensionless valve-opening coefficient, centre of oscillation;

ω, ω_r = angular frequency and dimensionless relative angular frequency;

ω_r^{odd}, ω_r^{even} = relative angular frequency for odd harmonics and for even harmonics;

ω_{th} = fundamental angular frequency for a reservoir-pipeline-valve system;

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Figures:

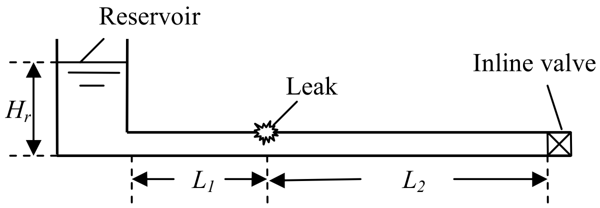

Fig. 1. A reservoir-pipeline-valve system with a leak.

Figure2

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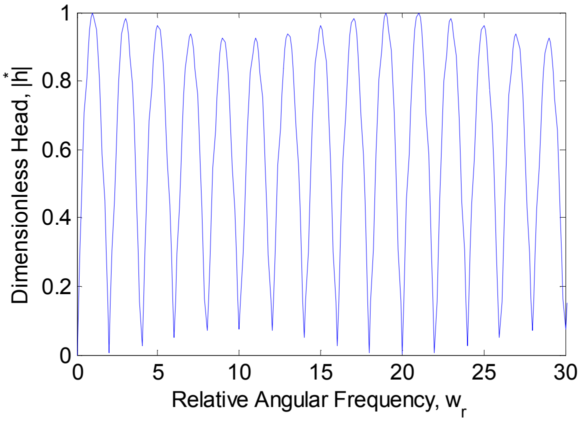

Fig. 2. A dimensionless FRD with same leak-induced pattern amplitudes at the odd and even harmonics for a frictionless pipe simulation.

Figure3

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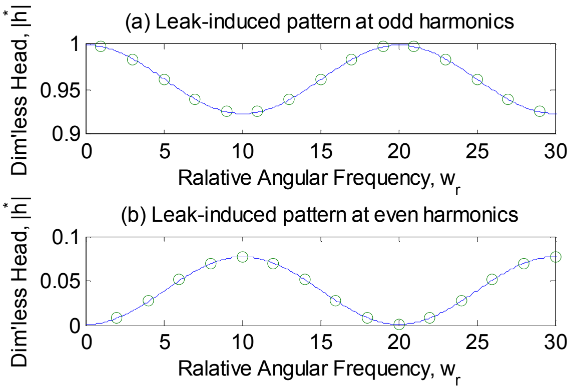

Fig. 3. Dimensionless leak-induced patterns at the odd and even harmonics (frictionless pipe). The circles are values at harmonics, and the solid lines are the sinusoidal fitted lines.

Figure4

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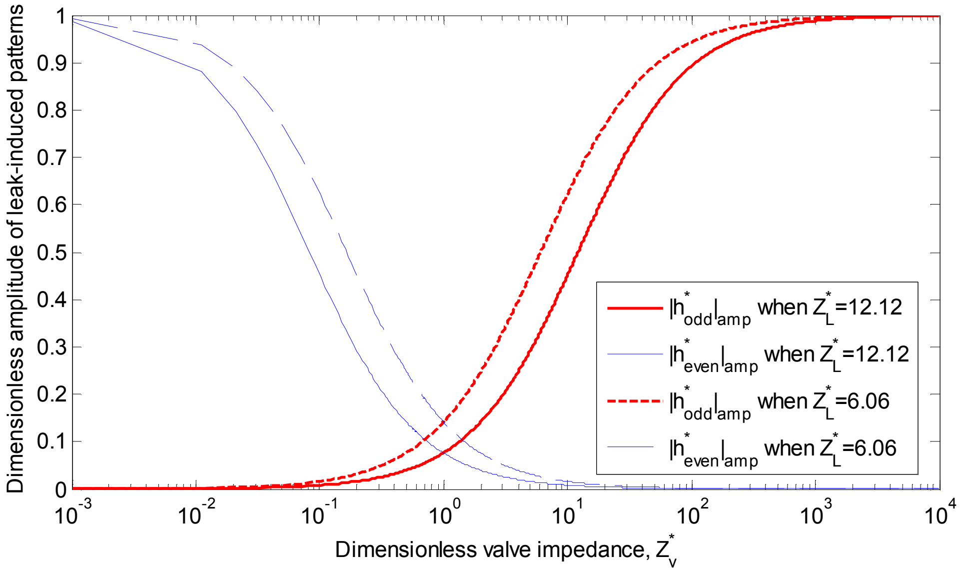

Fig. 4. Variation of the leak-induced pattern amplitudes as the dimensionless valve impedance (Z_V^*) varies for a frictionless pipe simulation.

Figure5

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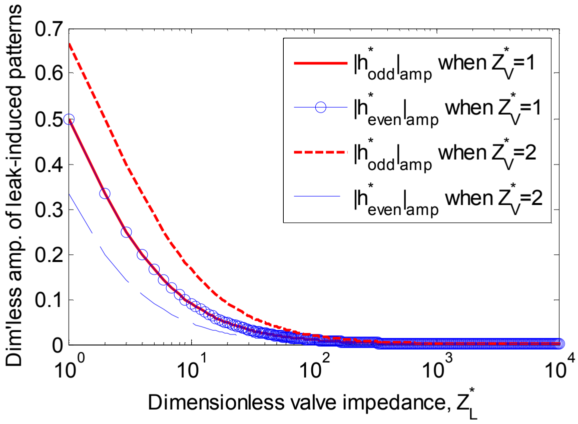

Fig. 5. Variation of the leak-induced pattern amplitudes according to changes in the dimensionless leak impedance (Z_L^*) for a frictionless pipe simulation.

Figure6

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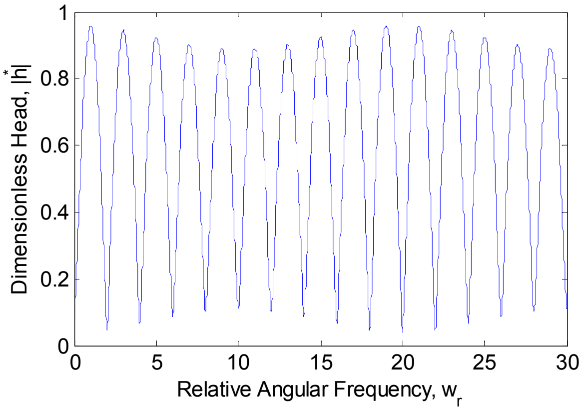

Fig. 6. A dimensionless FRD for the pipeline system in the case study, $x_L^* = 0.1$.

Figure7

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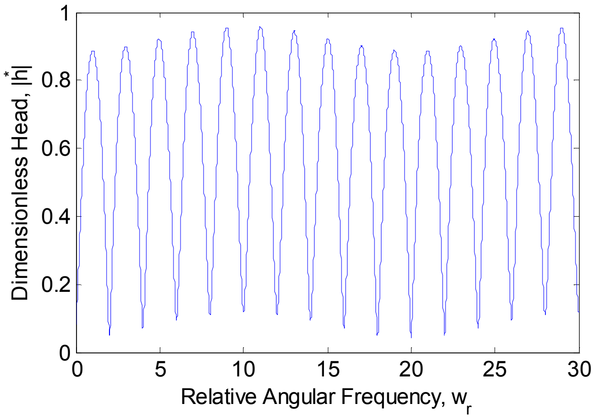

Fig. 7. A dimensionless FRD for the pipeline system in the case study, $x_L^* = 0.9$.

Figure8

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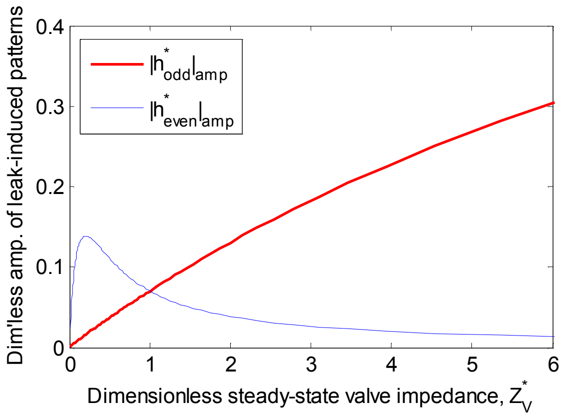

Fig. 8. Variation of the dimensionless leak-induced pattern amplitudes according to changes in the dimensionless valve impedance (Z_V^*), for case study with steady friction.

Figure9

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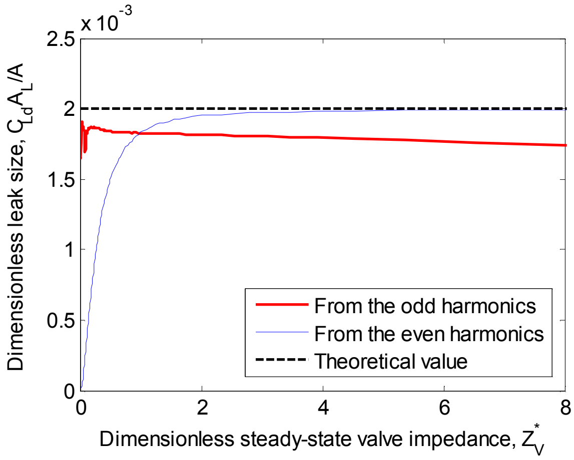

Fig. 9. Variation of the dimensionless leak size ($C_{Ld} A_L / A$) (derived from the odd and the even harmonics, respectively) for changes in the dimensionless steady-state valve impedance (Z_V^*), for the case study with steady friction.

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Tables:

Table 1. System Parameters for the Case Study

Parameter	Value
L	160 m
L_1	16 m
L_2	144 m
D	25.4 mm
H_r	30 m
a	1000 m/s
f	0.024
$\Delta\tau/\tau_0$	0.05
$C_{Ld}A_L/A$	0.002