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ABSTRACT

Two different terms are used in literature for the frictional force between moving ion and neutral gases. It is shown how the corresponding collision frequencies have to be defined in order to keep both terms valid. An expression is derived for the momentum transfer collision frequency ν_{in} between ions and neutral particles as a function of temperature and the relative flow velocity $|\underline{v}_i - \underline{v}_n|$. The result indicates that for most practical applications ν_{in} may be considered as independent of $|\underline{v}_i - \underline{v}_n|$. However, for very high velocities, e.g. a convection flow from the magnetosphere into the ionosphere, this dependence may be significant. Numerical values for ν_{in} are presented. In the case of collisions between ions and their parent neutral gases, laboratory values are used for the resonant charge exchange cross section obtained for high energies, but a correction is made for the much smaller thermal energies in the temperature range typical for ionospheric conditions. This gives rise to an enhancement of the collision frequencies, compared with the values obtained from the uncorrected charge exchange cross sections, the amount of which depends on the particle mass.

FRICTIONAL FORCES AND COLLISION FREQUENCIES BETWEEN MOVING ION AND NEUTRAL GASES

1. INTRODUCTION

The dynamic behaviour of the ionospheric plasma and atmospheric neutral gas can be described by the Navier-Stokes equation of hydrodynamics. This equation expresses the acceleration of a fluid in terms of a pressure gradient force, a viscous force and the resultant of all external forces. One of the most important external forces is the frictional force between an ion gas and a neutral gas moving with different velocities. The frictional force can be expressed in terms of the relative velocity between the ion and neutral gas and the number of collisions between ions and neutral particles.

Two different expressions are used in literature for the frictional force, both relating the frictional force to the collision frequency between ions and neutral particles. It is necessary, therefore, to define different collision frequencies in order to keep both expressions valid. One of the goals of the present work shall be to give the proper definitions for these collision frequencies.

Collision frequencies between ions and neutral particles are usually assumed to be independent of the relative flow velocity. It shall be investigated in this paper to which extent this assumption is correct.

The frequency of collisions between ions and their parent neutral particles is greatly influenced by charge exchange processes. Charge exchange cross sections for all processes of atmospheric interest have been measured for energies far above the thermal energy range, and they have been extrapolated to thermal energies without any correction. In the present paper an attempt shall be made to correct the charge exchange cross sections for small energies which must lead to an enhancement of the collision frequencies.

2. EQUATION OF MOTION FOR AN ION GAS

The Navier-Stokes equation, applied to an ion gas in the earth's atmosphere, reads as follows:

$$\rho_i \frac{d\underline{v}_i}{dt} = - \operatorname{grad} p_i + \eta_i \left(\frac{1}{3} \operatorname{grad} \operatorname{div} \underline{v}_i + \Delta \underline{v}_i\right) + \underline{k} \qquad (1)$$

ρ_i = mass density of the ion gas

p_i = partial pressure of the ion gas

$\underline{v}_i$ = ion velocity

η_i = coefficient of viscosity

$\underline{k}$ = resultant external force per unit volume.

The external force $\underline{k}$ may be written as the sum of a gravity force, an electromagnetic force, a Coriolis force, a centrifugal force and a frictional force (ion drag force):

$$\underline{k} = \rho_i \, \underline{g} + N_i \, e\left(\underline{E} + \underline{v}_i \times \underline{B}\right) + 2\rho_i\left(\underline{v}_i \times \underline{\Omega}\right) + \rho_i \, \underline{\Omega} \times (\underline{r} \times \underline{\Omega}) + \underline{k}_f \qquad (2)$$

$\underline{g}$ = acceleration due to gravity

N_i = ion number density

e = ionic charge

$\underline{E}$ = electric field strength

$\underline{B}$ = magnetic induction

$\underline{\Omega}$ = angular velocity of the earth

$\underline{r}$ = position vector, measured from the earth's center

$\underline{k}_f$ = external frictional force or ion drag force.

For a practical application of Equations (1) and (2) to the earth's ionosphere the acceleration force, the internal friction force, the Coriolis force, and the centrifugal force may be omitted.

3. FRICTIONAL FORCE AND COLLISION FREQUENCY

Two different terms for $\underline{k}_f$ are in use in literature. Some authors (e.g. Dougherty, 1961; Kendall and Pickering, 1967; Kohl and King, 1967; Priester, Roemer and Volland, 1967) use the expression

$$\underline{k}_f = -N_i \nu_{in} m_i (\underline{v}_i - \underline{v}_n) = -\rho_i \nu_{in} (\underline{v}_i - \underline{v}_n) \tag{3}$$

while others (e.g. Schlüter, 1950; Chandra, 1964) use the expression

$$\underline{k}_f = -N_i \nu_{in} \mu_{in} (\underline{v}_i - \underline{v}_n) \tag{4}$$

$\mu_{in} = m_i m_n / m_i + m_n$ = reduced mass

m_i = ion mass

m_n = neutral particle mass

ν_{in} = number of collisions per second of one ion with the neutral particles

$\underline{v}_n$ = neutral gas velocity.

Since ν_{in} is a frictitious quantity which merely has the dimension of a frequency, it can be defined so that either (3) or (4) are valid.

Two different definitions for ν_{in} are given in literature. According to Banks (1966a), the momentum transfer collision frequency is defined as

$$\nu_{in} = \frac{4\pi}{3} N_n \left(\frac{\mu_{in}}{kT}\right) \left(\frac{\mu_{in}}{2\pi kT}\right)^{3/2} \int_0^\infty Q(g)\, e^{-\mu_{in}/2kT\, g^2}\, g^5 \, dg \tag{5}$$

$Q(g) = 2\pi \int_0^\infty (1 - \cos\chi) b \, db$ = momentum transfer cross section

g = relative thermal velocity

χ = angle of deflection (see Figures 12 and 13)

b = collision parameter (see Figure 14)

In order to find out how the frictional force term reads when we use ν_{in} as defined by Equation (5), we refer to the momentum transfer equation provided by statistical plasma mechanics. After Burgers (1960), the frictional force in the terminology of this paper is given by

$$\underline{k}_f = -(\underline{v}_i - \underline{v}_n) \cdot \frac{4\pi}{3} \mu_{in} N_i N_n \left(\frac{\mu_{in}}{kT}\right) \left(\frac{\mu_{in}}{2\pi kT}\right)^{3/2} \cdot \int_0^\infty Q(g) \cdot e^{-\mu_{in}/2kT\, g^2} g^5 \, dg \qquad (6)$$

or, inserting (5):

$$\underline{k}_f = -N_i \nu_{in} \mu_{in} (\underline{v}_i - \underline{v}_n) \qquad (7)$$

Equation (7) is identical with (4).

Dalgarno (1961), on the other hand, defines the collision frequency as

$$\nu_{in} = \frac{kT}{m_i D_i} \qquad (8)$$

or, when we insert for the diffusion coefficient D_i the expression given by Chapman and Cowling (1961):

$$\nu_{in} = \frac{4\pi}{3} \cdot N_n \cdot \frac{\mu_{in}}{m_i} \cdot \left(\frac{\mu_{in}}{kT}\right) \left(\frac{\mu_{in}}{2\pi kT}\right)^{3/2} \int_0^\infty Q(g) \, e^{-\mu_{in}/2kT\, g^2} g^5 \, dg \qquad (8a)$$

By comparison of (8a) with (6) we obtain

$$\underline{k}_f = -N_i \nu_{in} m_i (\underline{v}_i - \underline{v}_n) \qquad (9)$$

Equation (9) is identical with (3).

The numerical values for ν_{in}, given by Cowling (1945) and Dalgarno (1961 and 1964), are based on the defining Equations (8) or (8a). Hence, when these collision frequencies are used, Equation (3) must be applied for $\underline{k}_f$. On the other hand, the collision frequencies presented by Banks (1966b) and later on in this paper (paragraphs 5 and 6), are calculated by means of Equation (5). Therefore, these values have to be taken in conjunction with Equation (4). Thereby, the apparent discrepancy between the collision frequencies of Dalgarno (1964) on the one side and of Banks (1966b) on the other side is removed.

The frictional force as given by Equation (6) leads, when decomposed into the constituents of Equation (4), to an expression for the collision frequency which does not depend on the relative flow velocity $|\underline{v}_i - \underline{v}_n|$. Such a dependency, however, should be expected, since the velocity distribution functions of the ion and neutral gas cannot only be influenced by a change in T, but also by a change in $|\underline{v}_i - \underline{v}_n|$.

We will derive here an expression for $\underline{k}_f$ leading to a collision frequency which shows a dependence on $|\underline{v}_i - \underline{v}_n|$. Our basic assumption is that the velocity distributions of the ion and neutral gas are Maxwellian distributions, displaced by the drift velocities $\underline{v}_i$ and $\underline{v}_n$, respectively (Smith, 1964):

$$\frac{dN_i}{N_i} = \left(\frac{m_i}{2\pi kT_i}\right)^{3/2} e^{-m_i/2kT\,(\underline{c}_i-\underline{v}_i)^2}\ d^3\underline{c}_i$$

$$\frac{dN_n}{N_n} = \left(\frac{m_n}{2\pi kT_n}\right)^{3/2} e^{-m_n/2kT\,(\underline{c}_n-\underline{v}_n)^2}\ d^3\underline{c}_n$$

$\underline{c}_i, \underline{c}_n$ = thermal velocities.

The result of our calculations, which are presented in the appendix, is:

$$\underline{k}_f = -\left(\underline{v}_i - \underline{v}_n\right)\cdot\frac{4\pi}{3}\cdot N_i\cdot N_n\cdot\mu_{in}\cdot\left(\frac{\mu_{in}}{kT_R}\right)\cdot\left(\frac{\mu_{in}}{2\pi kT_R}\right)^{3/2}\cdot e^{-\mu_{in}/2kT_R\,|\underline{v}_i-\underline{v}_n|^2}$$

$$\cdot\int_0^\infty Q(g)\cdot\left[1+\frac{3}{5}\cdot\frac{1}{3!}X^2g^2+\frac{3}{7}\cdot\frac{1}{5!}X^4g^4+\frac{3}{9}\cdot\frac{1}{7!}X^6g^6+\cdots\right]e^{-\mu_{in}/2kT_R\,g^2}\,g^5\,dg \qquad (10)$$

$$X = \frac{\mu_{in}}{kT_R} \left| \underline{v}_i - \underline{v}_n \right|$$

$$T_R = \mu_{in} \left(\frac{T_i}{m_i} + \frac{T_n}{m_n} \right)$$

By comparison with (4), the momentum transfer collision frequency is given by:

$$\nu_{in} = \frac{4\pi}{3} N_n \left(\frac{\mu_{in}}{kT_R} \right) \left(\frac{\mu_{in}}{2\pi kT_R} \right)^{3/2} e^{-\mu_{in}/2kT_R \left| \underline{v}_i - \underline{v}_n \right|^2}$$

$$\cdot \int_0^\infty Q(g) \left[1 + \frac{3}{5} \cdot \frac{1}{3!} X^2 g^2 + \frac{3}{7} \cdot \frac{1}{5!} X^4 g^4 + \frac{3}{9} \cdot \frac{1}{7!} X^6 g^6 + \cdots \right]$$

$$\cdot \, e^{-\mu_{in}/2kT_R \, g^2} g^5 \, dg \qquad (11)$$

For $\underline{v}_i = \underline{v}_n$ and $T_i = T_n$ Equation (11) is identical to (5). We will discuss this formula later (paragraph 5). In the following we must distinguish between ions in unlike gases and ions in their parent gases. In the first case, the momentum transfer cross section is determined by electrostatic interactions, while in the second case charge exchange plays a predominant role.

4. MOMENTUM TRANSFER CROSS SECTION FOR IONS MOVING IN THEIR PARENT NEUTRAL GAS

The momentum transfer section Q was defined as

$$Q(g) = 2\pi \int_0^\infty (1 - \cos \chi) \, b \, db = 4\pi \int_0^\infty \cos^2 \vartheta \, b \, db \qquad (12)$$

χ, ϑ and b are defined in Figures 12, 13 and 14.

Due to electrostatic interaction forces, which are described later, the integrand is greater than zero for all finite values of b and g, but, for given g and increasing b, it converges fast enough to zero to guarantee the convergence of the above integrals.

When the ion gas is embedded in its parent neutral gas, the value of Q is much less determined by electrostatic interactions than by charge exchange processes of the type

$$X^+ + Y \rightleftharpoons X + Y^+$$

where X and Y are particles of the same neutral gas species. After Holstein (1952), the effect of charge exchange processes on the momentum transfer cross section can be described as follows:

When the particle trajectories are linear, a critical collision parameter b_c with the following features can be defined: For $b < b_c$ the charge exchange probability P_{ex} is a rapidly changing function of b, oscillating between 0 and 1 and having an average of 1/2. For $b > b_c$, P_{ex} decreases rapidly with increasing b. Therefore, on the average, every second collision with $b < b_c$ leads to a charge exchange process. Let us consider a particular impact, in the first instance without charge exchange (Figure 1a), then with charge exchange (Figure 1b). In the first case the ion is deflected by χ_1,

Figures 1a and 1b

in the second case (since X and Y are of the same species) by $\chi_2 = \pi - \chi_1$ (i.e. $\cos \chi_2 = -\cos \chi_1$). Hence, on the average, for $b < b_c$ the integrand $(1 - \cos\chi) \cdot b$ is equal to b. Therefore, if we neglect the very small contribution of P_{ex} for $b > b_c$, we can describe the net result of charge exchange processes in the following manner: Charge exchange processes between ions and their parent neutral particles influence the momentum transfer so as if all collisions with an impact parameter b smaller than the critical impact parameter b_c occur with a deflection angle of 90°. Thus, the momentum transfer cross section can be written as

$$Q(g) = \pi b_c^2 + 4\pi \int_{b_c}^{\infty} \cos^2 \vartheta \, b \, db \tag{13}$$

The charge exchange cross section S_{ex}, which has been measured for many processes of atmospheric interest, is defined as the integral

$$S_{ex} = \int_0^{\infty} P_{ex}\, dq \;, \tag{14}$$

where dq is a differential cross section and P_{ex} is the charge exchange probability related to dq. Since P_{ex} is a function of b only, S_{ex} can be written as

$$S_{ex} = 2\pi \int_0^{\infty} P_{ex}\, b\, db \tag{14a}$$

Omitting the small contributions of the integrand for $b > b_c$, we get

$$S_{ex} = \frac{\pi b_c^{\,2}}{2} \qquad \text{or} \qquad b_c = \sqrt{\frac{2S_{ex}}{\pi}} \tag{15}$$

Therefore, Q(g) and S_{ex} are related by

$$Q(g) = 2S_{ex} + 4\pi \int_{\sqrt{2S_{ex}/\pi}}^{\infty} \cos^2 \vartheta\, b\, db \tag{16}$$

Laboratory measurements (e.g. Stebbings, Smith and Ehrhardt, 1964) as well as theoretical studies (Knof, Mason and Vanderslice, 1964) showed that for energies above 1 eV, when the trajectories are approximately linear, S_{ex} and the relative kinetic energy $E = (\mu_{in}/2) g^2$ are connected by

$$S_{ex} = (C_1 - C_2 \cdot \log_{10} E)^2 \;, \tag{17}$$

where C_1 and C_2 are constants. In order to evaluate collision frequencies or diffusion constants for ions (e.g. Banks, 1966b and Knof et al., 1964), the authors used the unmodified Equation (17). However, at thermal energies characteristic of atmospheric conditions, the curvature of the particle trajectories due to long range attractive forces should have a marked influence on S_{ex} and Q. We will therefore make an attempt to correct the charge exchange cross sections for small energies. The corrected values of the charge exchange cross section and the critical collision parameter shall be denoted by S_{ex}' and b_c', respectively, to distinguish them from the uncorrected values, given by (15) and (17).

While the task to relate b_c' to b_c for a given relative velocity g can be exactly solved, an exact relationship between S_{ex}' and b_c' can be found only when all branches of the charge exchange interaction potential are known, but even then it is a very difficult problem. We will assume that S_{ex}' and b_c' are related in the same manner as S_{ex} and b_c:

$$S_{ex}' = \frac{\pi b_c'^2}{2} \quad \text{or} \quad b_c' = \sqrt{\frac{2S_{ex}'}{\pi}} \tag{15a}$$

This assumption is in agreement with a proposal by Holstein (1952, p. 835). Equation (15a) would be correct to the same extent as (15) if the transformation from b' to b were strictly linear. The meanings of b' and b are explained in Figure 2.

Figure 2

When b_c' is known, Q is given by

$$Q(g) = \pi b_c'^2 + 4\pi \int_{b_c'}^{\infty} \cos^2 \vartheta \, b' \, db' , \tag{13a}$$

or, because of (15a), by

$$Q(g) = 2S_{ex}' + 4\pi \int_{\sqrt{2S_{ex}'/\pi}}^{\infty} \cos^2 \vartheta \, b' \, db' \tag{16a}$$

We are left now with the problem to calculate $\cos^2 \vartheta$ as a function of b' and g and b_c' as a function of b_c and g.

We assume a long range interaction potential between an ion and a neutral particle of the form (e.g. McDaniel, 1964)

$$\left.\begin{array}{llll} U(r) & = -\left(\frac{A}{r}\right)^4 \left[1 + \left(\frac{B}{r}\right)^2\right] & \text{for} & r \geq \sigma \\ U(r) & = +\infty & \text{for} & r < \sigma \end{array}\right\} \tag{18}$$

The fourth order term is due to dipole interactions, the sixth order term to quadrupole interactions. σ is the sum of the gas kinetic radii of the particles. A can be expressed in terms of the polarizability η of the neutral particle.

$$A = \left(\frac{e^2 \eta}{2}\right)^{1/4} \tag{19}$$

It would be more realistic to describe the repulsive potential by a term of the form C/r^{12}, but since no values for C are available, this would only impede the calculations without yielding a higher accuracy. The potential given by Equation (18) is illustrated in Figure 3.

Figure 3

Using the symbols defined by Figure 4 and applying the conservation laws

Figure 4

for energy and angular momentum, we obtain the relations

$$\frac{\mu_{in}}{2} w^2 + U(r) = \frac{\mu_{in}}{2} g^2 \tag{20a}$$

$$\mu_{in} r^2 \dot{\phi} = \mu_{in} g b' \tag{20b}$$

w can be expressed as

$$w^2 = \dot{r}^2 + r^2 \dot{\phi}^2$$

$\dot{r}$ and $\dot{\phi}$ stand for the time derivatives dr/dt and $d\phi/dt$, respectively. Considering the identity

$$\dot{\phi} = \dot{r} \frac{d\phi}{dr} ,$$

$\dot{r}$ and $d\phi/dr$ are given by

$$\dot{r}^2 = g^2 \left(1 - \frac{b'^2}{r^2}\right) - \frac{2U(r)}{\mu_{in}} \tag{21}$$

$$\frac{d\phi}{dr} = \frac{gb'}{r^2} \left[g^2 \left(1 - \frac{b'^2}{r^2}\right) - \frac{2U(r)}{\mu_{in}}\right]^{-1/2} \tag{22}$$

By means of (22) we can immediately obtain an expression for ϑ:

$$\vartheta\left(b', g\right) = b' \int_b^{\infty} \left(1 - \frac{b'^2}{r^2} - \frac{2U(r)}{\mu g^2}\right)^{-1/2} \frac{dr}{r^2} \tag{23}$$

b is the minimum distance between the two particles for given b' and g. If r_{PCA} (PCA = point of closest approach) is defined as the greatest nullpoint of $\dot{r}^2(r)$, b is given by (see Figure 5)

$$b = \begin{cases} r_{PCA} & \text{for} \quad r_{PCA} \geq \sigma \\ \sigma & \text{for} \quad r_{PCA} < \sigma \end{cases} \tag{24a}$$

For sufficiently small values of b', however, there exists no nullpoint of

Figure 5

$\dot{r}^2(r)$. In this case, b is simply given by

$$b = \sigma \tag{24b}$$

If b is greater than σ, i.e. $b = r_{PCA}$, the integrand in Equation (34) has a singularity for $r = b$. In order to remove this singularity and to make the range of integration finite, we change the variables by using the transformation (Mason and Schamp, 1958)

$$\sin \delta = \frac{b}{r} \tag{25}$$

We thus obtain for ϑ:

$$\vartheta(b', g) = \frac{b'}{b} \int_0^{\pi/2} \left(1 - \frac{b'^2 \sin^2 \delta}{b^2} - \frac{2U\left(\frac{b}{\sin \delta}\right)}{\mu g^2}\right)^{-1/2} \cos \delta \, d\delta \, . \tag{26}$$

The critical collision parameter b_0' which has the property that for $b' < b_0'$ $\dot{r}^2(r)$ has no nullpoint, is of significance for the determination of b_c'. As we read off from Figure 5, b_c' is given by the condition

$$r_{PCA}(b_c') = b_c \tag{27a}$$

if $b_c \geq r_{PCA}(b_0')$ and by

$$b_c' = b_0' \tag{27b}$$

if $b_c < r_{PCA}(b_0')$. Now all formulae necessary for a calculation of $Q(g)$ and $S'_{ex}(g)$ have been provided. Numerical results are presented in the next paragraph.

5. NUMERICAL VALUES FOR MOMENTUM TRANSFER CROSS SECTIONS AND COLLISION FREQUENCIES FOR IONS IN THEIR PARENT NEUTRAL GASES

Experimental values of $S_{ex}(g)$ are available for all processes of atmospheric interest. We will base our calculations on the values for S_{ex} and η given in Table 1. The values for η have been taken from Banks (1966b).

Table 1

Charge Exchange Cross Sections and Polarizabilities of Neutral Particles. Sources: (1) Stebbings et al. (1964); (2) Fite et al. (1962); (3) Cramer and Simons (1957); (4) Knof et al. (1964); (5) Amme and Utterback (1964).

Process	η (10^{-24} cm^3)	$S_{ex}^{1/2}$ (10^{-8} cm)	Source for S_{ex}
$O^+ + O \rightarrow O + O^+$	0.89	$5.95 - 0.63 \log_{10} E(eV)$	(1)
$H^+ + H \rightarrow H + H^+$	0.67	$7.60 - 1.06 \log_{10} E(eV)$	(2)
$He^+ + He \rightarrow He + He^+$	0.21	$5.25 - 0.74 \log_{10} E(eV)$	(3)
$N^+ + N \rightarrow N + N^+$	1.13	$5.53 - 0.46 \log_{10} E(eV)$	(4)
$O_2^+ + O_2 \rightarrow O_2 + O_2^+$	1.60	$5.37 - 0.54 \log_{10} E(eV)$	(5)
$N_2^+ + N_2 \rightarrow N_2 + N_2^+$	1.76	$7.36 - 0.68 \log_{10} E(eV)$	(5)

Using the formulae derived in paragraph 4, $Q(g)$ and $S_{ex}'(g)$ have been calculated numerically with the help of an electronic computer. Since B (defined by Equation (18)) is practically unknown, a set of values for B, ranging from 0.5 to 2.0 10^{-8} cm has been used. The result was that the influence of the sixth order potential term is completely negligible, since in no case was the deviation in $Q(g)$ caused by this term more than 0.5%. Therefore, all further calculations have been carried out with $B = 0$, i.e. for a pure fourth order potential law.

Results for $Q(g)$ and $S_{ex}'(g)$, compared with $S_{ex}(g)$, for the processes $O^+ + O \rightarrow O + O^+$ and $H^+ + H \rightarrow H + H^+$ are shown in Figures 6 and 7.

Figure 6

Figure 7

The numerical values for Q(g) have been used to claculate the momentum transfer collision frequency ν_{in} after Equation (11) for the processes listed in Table 1. Figure 8 shows ν_{in} for the most important charge exchange collision process in the ionosphere, namely $O^+ + O \rightarrow O + O^+$, as a function of the reduced temperature T_R for $|\underline{v}_i - \underline{v}_n| = 0$. In order to indicate the effect of electrostatic interactions, ν_{in} has also been calculated for the uncorrected momentum transfer cross section $Q = 2S_{ex}$, which would be the case if no electrostatic interactions would be present.

Figure 8

Since the difference between Q and $2S_{ex}$ increases with decreasing velocity, the influence of electrostatic interactions is conspicuous only for low temperatures (below 1000°K), and is more noticeable for heavier particles because the mean thermal velocity is proportional to $m^{-1/2}$. For the process $O_2^+ + O_2 \rightarrow O_2 + O_2^+$, for instance, ν_{in}, calculated with the exact Q(g), is greater than ν_{in} for $Q = 2S_{ex}$ by 39% for 400°K and 17% for 1000°K. On the other hand, the corresponding numbers for the process $H^+ + H \rightarrow H + H^+$ are 2.3% and 0.4%.

In the temperature range $500°K \leq T_R \leq 3000°K$ the collision frequency ν_{in} for $|\underline{v}_i - \underline{v}_n| = 0$, as a function of T_R, can be approximated by the following expressions, when the neutral particle number density is measured in the unit cm^{-3}:

$$\nu(O^+, O) = 1.86 \cdot 10^{-9} (T_R/1000)^{0.37} \, n(O) \quad sec^{-1}$$

$$\nu(H^+, H) = 12.03 \cdot 10^{-9} (T_R/1000)^{0.38} \, n(H) \quad sec^{-1}$$

$$\nu(He^+, He) = 2.92 \cdot 10^{-9} (T_R/1000)^{0.37} \, n(He) \quad sec^{-1}$$

$$\nu(N^+, N) = 1.75 \cdot 10^{-9} (T_R/1000)^{0.34} \, n(N) \quad sec^{-1}$$

$$\nu(O_2^+, O_2) = 1.17 \cdot 10^{-9} (T_R/1000)^{0.28} \, n(O_2) \quad sec^{-1}$$

$$\nu(N_2^+, N_2) = 2.11 \cdot 10^{-9} (T_R/1000)^{0.38} \, n(N_2) \quad sec^{-1}$$

These values can be used to calculate the ambipolar diffusion constant D_a for an electrically neutral electron-ion gas in the ionosphere, which is given by

$$D_a = \frac{k(T_i + T_e)}{\nu_{in}\mu_{in}} \sin^2 I$$

In the F2 region, where O^+ and O are the major ionic and neutral constituents, D_a can be written as

$$D_a = 0.55 \cdot 10^{16} \left(\frac{1000}{T_R}\right)^{0.37} \frac{(T_i + T_e) \sin^2 I}{n(O)} \qquad \text{cm}^2 \text{ sec}^{-1}$$

In the exosphere, where H^+ and H are predominant, D_a is

$$D_a = 1.38 \cdot 10^{16} \left(\frac{1000}{T_R}\right)^{0.38} \frac{(T_i + T_e) \sin^2 I}{n(H)} \qquad \text{cm}^2 \text{ sec}^{-1}$$

ν_{in} as a function of $|\underline{v}_i - \underline{v}_n|$ for the processes $O^+ + O \rightarrow O + O^+$ and $H^+ + H \rightarrow H + H^+$ is shown in Figures 9 and 10. When we consider diffusional flows, electromagnetic drifts or neutral gas winds in the upper atmosphere and ionosphere, $|\underline{v}_i - \underline{v}_n|$ should not exceed about 300 m/sec.

Figure 9

Figure 10

Therefore, when applied to these mechanisms, ν_{in} may be considered as independent of $|\underline{v}_i - \underline{v}_n|$. However, for particle flows from the magnetosphere into the ionosphere, as proposed by Axford and Hines (1961), this dependence may be of significance, since the plasma convection velocity is expected to be of the order of kilometers per second.

6. NUMERICAL VALUES OF COLLISION FREQUENCIES FOR IONS IN UNLIKE NEUTRAL GASES

For reasons of completeness, collision frequencies for ions in unlike neutral gases are also presented, although the basic studies in this field have been carried out more than six decades ago by Langevin (1905) and later on by Hassé (1926). Langevin and Hassé give an expression for the ion mobility K which, using the terminology of this paper, can be written as

$$K = \frac{A(\lambda)}{2N_n \sqrt{\pi \mu_{in} \eta}} \tag{28}$$

where $A(\lambda)$, λ being defined as

$$\lambda = \frac{\sigma^2}{e}\left(\frac{2kT}{\eta}\right)^{1/2} \tag{29}$$

is given by Hassé (1926, Table III). A is not identical with the quantity defined by Equation (18). Using Equations (1), (2), (4) and taking $\underline{v}_n = 0$, K and ν_{in} are found to be related by

$$\nu_{in} = 2 \cdot F(\lambda)\left(\frac{\eta e^2}{\mu_{in}}\right)^{1/2} N_n \tag{30}$$

$F(\lambda) = \sqrt{\pi}/A(\lambda)$ is shown in Figure 11.

<u>Figure 11</u>

Since σ, the sum of the gas kinetic particle radii, is not known well enough, the usual method is to take the limiting value $F(0)$, i.e. for vanishing σ or infinite η, instead of the correct value $F(\lambda)$. This leads to the approximation

$$\nu_{in} = 6.94\left(\frac{\eta e^2}{\mu_{in}}\right)^{1/2} N_n \tag{31}$$

We must realize, however, that the values obtained from Equation (31) can easily be wrong by about 20% for temperatures below 2000°K and by more than this for higher temperatures. Especially the temperature dependence of ν_{in} is not correctly described by the approximation (31). For $\lambda < 0.6$ the collision frequencies decrease with increasing temperature, while for $\lambda > 0.6$ the opposite is true. For $\lambda > 1.2$ the collision frequencies approximately follow a $T^{1/2}$-law. Assuming $T = 500$°K and $\eta = 10^{-24}$ cm^3, a value of $\lambda \geq 1.2$ is adopted for $\sigma \geq 4\ 10^{-8}$ cm. ν_{in} after Equation (31) is based on the defining Equation (5). The defining Equation (8a) would yield

$$\nu_{in} = 6.94 \cdot \frac{\mu_{in}}{m_i}\left(\frac{\eta e^2}{\mu_{in}}\right)^{1/2} \cdot N_n \tag{31a}$$

Although momentum transfer collision frequencies can simply be evaluated from Equation (31), we will give some numerical values for the most important collision processes in the ionosphere:

$$\nu(O^+, O_2) = 1.00\ 10^{-9}\ n(O_2) \quad sec^{-1}$$

$$\nu(O^+, N_2) = 1.08\ 10^{-9}\ n(N_2) \quad sec^{-1}$$

$$\nu(O^+, H) = 2.19\ 10^{-9}\ n(H) \quad sec^{-1}$$

$$\nu(H^+, O) = 2.52\ 10^{-9}\ n(O) \quad sec^{-1}$$

$$\nu(NO^+, O_2) = 0.83\ 10^{-9}\ n(O_2) \quad sec^{-1}$$

$$\nu(NO^+, N_2) = 0.90\ 10^{-9}\ n(N_2) \quad sec^{-1}$$

$$\nu(NO^+, O) = 0.76\ 10^{-9}\ n(O) \quad sec^{-1}$$

$$\nu(O_2^+, N_2) = 0.89\ 10^{-9}\ n(N_2) \quad sec^{-1}$$

$$\nu(O_2^+, O) = 0.75\ 10^{-9}\ n(O) \quad sec^{-1}$$

As mentioned before, the collision frequencies defined by (5) and (8a) are fictitious quantities, that is, they are quantities having the dimension of a frequency and were introduced merely for reasons of convenience. When two particles approach each other so closely that they are separated only by short range repulsive forces or, in other words, when two particles have a direct contact, we will call this a "real" collision. According to this definition, the number of real collisions per second, ν_R, is given by (Stubbe, 1966)

$$\nu_R = 2\pi \left(\frac{\eta e^2}{\mu_{in}}\right)^{1/2} N_n \left[\frac{2}{\sqrt{\pi}} \int_0^{u_0} e^{-u^2} du + \frac{1}{2v_0} \left(\frac{8kT}{\pi\mu_{in}}\right)^{1/2} \cdot e^{-\mu v_0^2/2kT}\right], \tag{32}$$

where u_0 and v_0 are abbreviations standing for the expressions

$$u_0 = \left(\frac{\mu_{in} v_0^2}{2kT}\right)^{1/2}$$

$$v_0 = \frac{e}{\sigma^2}\left(\frac{\eta}{\mu_{in}}\right)^{1/2}$$

In terms of λ, defined by (29), ν_R can be written as

$$\nu_R = 2\pi\left(\frac{\eta e^2}{\mu_{in}}\right)^{1/2} N_n \left[\frac{2}{\sqrt{\pi}}\int_0^{1/\lambda} e^{-u^2}\,du + \frac{\lambda}{\sqrt{\pi}}\, e^{-1/\lambda^2}\right] \tag{32a}$$

For temperatures below about 2000°K, (32) and (32a) can be approximated by

$$\nu_R = 2\pi\left(\frac{\eta e^2}{\mu_{in}}\right)^{1/2} N_n \tag{32b}$$

By comparison of (31), (31a) and (32b) we see that ν_R lies between ν_{in} after (31a) and ν_{in} after (31).

8. SUMMARY

a) The relationship between the frictional force and the collision frequency between ion and neutral gases is studied. Proper definitions are given for the collision frequency to fit the expressions used for the frictional force.

b) An expression is derived for the momentum transfer collision frequency ν_{in} as a function of the relative flow velocity $|\underline{v}_i - \underline{v}_n|$. It is shown that for most practical applications ν_{in} may be considered as independent of $|\underline{v}_i - \underline{v}_n|$, but that for velocities in the km/sec range, as they may be expected in convection flows from the magnetosphere, this dependence is significant.

c) Numerical values of ν_{in} for the most important collision processes in the ionosphere are presented. In the case of ions in their parent neutral gases, resonant charge exchange is of great influence on the collision frequencies. Laboratory values are used for the charge exchange cross sections, although obtained for higher energies, but an approximate correction is made for thermal energies in order to take into account the effect of curved particle trajectories on the charge exchange cross section.

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APPENDIX

DERIVATION OF EQUATION (10)

The frictional force between two gases is the result of the momentum transfer between the colliding particles. We introduce the following quantities:

$\underline{c}_i$, $\underline{c}_n$ = ion and neutral particle velocity before the collision

$\underline{c}_i'$, $\underline{c}_n'$ = ion and neutral particle velocity after the collision

$\underline{g} = \underline{c}_i - \underline{c}_n$ relative velocity before the collision

$\underline{g}' = \underline{c}_i' - \underline{c}_n'$ relative velocity after the collision

$\underline{I}_i = m_i \underline{c}_i$ ion momentum before the collision

$\underline{I}_i' = m_i \underline{c}_i'$ ion momentum after the collision

$\Delta\underline{I}_{in} = \underline{I}_i' - \underline{I}_i$ momentum transfer

Using the conservation law for momentum, it can easily be derived that $\Delta\underline{I}_{in}$ is given by

$$\Delta\underline{I}_{in} = \mu_{in}\,(\underline{g}' - \underline{g}) \tag{37}$$

The conservation law of energy which holds for elastic collisions gives

$$|\underline{g}| = |\underline{g}'| = g \tag{38}$$

In order to determine $\underline{g}' - \underline{g}$ we will distinguish between direct collisions (direct contact of the colliding particles) and indirect collisions (no direct contact of the particles, but curved orbits due to long range electrostatic forces).

a) Direct Collisions: Under the assumption that no angular momentum is transferred during the collision, the orbit is symmetrical to the radial unit

Figure 12

vector $\underline{e}_r$. According to Figure 12, we therefore obtain:

$$\underline{g}' - \underline{g} = |\underline{g}' - \underline{g}| \cdot \underline{e}_r$$

$$|\underline{g}' - \underline{g}| = 2g \cos \vartheta$$

b) Indirect Collisions: According to Figure 13:

$$\underline{g}' - \underline{g} = - |\underline{g}' - \underline{g}| \cdot \underline{e}_r$$

$$|\underline{g}' - \underline{g}| = - 2g \cos \vartheta$$

Figure 13

In both cases we get the same result, namely

$$\underline{g}' - \underline{g} = 2g \cos \vartheta \, \underline{e}_r \tag{39}$$

or

$$\Delta \underline{I}_{in} = 2\mu_{in} \, g \cos \vartheta \, \underline{e}_r \tag{40}$$

We now introduce a Cartesian coordinate system (Figure 14) and we especially assume that the relative macroscopic velocity $\underline{v}_i - \underline{v}_n$ has the direction of the negative z-axis. In this case we are interested only in the z-component

Figure 14

of $\Delta \underline{I}_{in}$ since, on the average, all other components cancel out because of the complete symmetry about the z-axis.

$$\left(\Delta I_{in}\right)_z = 2\mu_{in} \, g \cos \vartheta \, e_{rz} \tag{41}$$

It can easily be shown that

$$e_{rz} = \cos\vartheta\cos\theta + \cos\alpha\sin\vartheta\sin\theta$$

Hence, the z-component of the momentum transferred from an ion to a neutral particle for any particular collision is given by

$$\left(\Delta I_{in}\right)_z = 2\mu_{in}\, g\left(\cos^2\vartheta\cos\theta + \cos\alpha\sin\vartheta\cos\vartheta\sin\theta\right) \quad (42)$$

$\left(\Delta I_{in}\right)_z$ and the frictional force $\underline{k}_f$ are connected by

$$d\underline{k}_f = \left(\Delta I_{in}\right)_z N_i\, d\nu_i\, \underline{e}_z \quad (43)$$

$d\nu_i$ is the number of collisions of one ion per second impinging on the differential cross section $dq = b \cdot db \cdot d\alpha$ (see Figure 14) and occurring in the velocity range $g \cdots g + dg$ and the angular range $\theta \cdots \theta + d\theta$. According to this definition, $d\nu_i$ is given as the product of the relative velocity g, the cross section dq, and the number of neutral particles $\overline{dN_n}$ per unit volume having a velocity between g and g + dg and being in the angular range between θ and $\theta + d\theta$, related to the particular ion under consideration.

In order to determine $\overline{dN_n}$, we assume that the ion gas and the neutral gas have Maxwellian velocity distributions, displaced by the velocities $\underline{v}_i$ and $\underline{v}_n$, respectively. Furthermore we assume, in the first instance, that the ion temperature and the neutral gas temperature are equal.

$$\frac{dN_i}{N_i} = \left(\frac{m_i}{2\pi kT}\right)^{3/2} e^{-m_i/2kT\,(\underline{c}_i - \underline{v}_i)^2}\, d^3\underline{c}_i \quad (44a)$$

$$\frac{dN_n}{N_n} = \left(\frac{m_n}{2\pi kT}\right)^{3/2} e^{-m_n/2kT\,(\underline{c}_n - \underline{v}_n)^2}\, d^3\underline{c}_n \quad (44b)$$

We multiply the distribution functions with each other and introduce a new velocity

$$\underline{U} = \frac{m_i \underline{c}_i + m_n \underline{c}_n}{m_i + m_n}$$

After some simple manipulations we get:

$$dN_i\, dN_n = N_i N_n \left(\frac{m_i m_n}{4\pi^2 k^2 T^2}\right)^{3/2} e^{-1/2kT\left[(m_i+m_n)(\underline{U}-m_i\underline{v}_i+m_n\underline{v}_n/m_i+m_n)^2+\mu_{in}(\underline{v}_i-\underline{v}_n-\underline{g})^2\right]}\, d^3\underline{U} d^3\underline{g}$$

The integration over $\underline{U}$ can simply be carried out yielding

$$\overline{dN_i\, dN_n} = N_i N_n \left(\frac{\mu_{in}}{2\pi kT}\right)^{3/2} e^{-\mu_{in}/2kT(\underline{v}_i-\underline{v}_n-\underline{g})^2}\, d^3\underline{g} \tag{45}$$

When we drop the restriction $T = T_i = T_n$ we obtain the same result provided that we replace T with the reduced temperature T_R defined by

$$T_R = \mu_{in}\left(\frac{T_i}{m_i} + \frac{T_n}{m_n}\right) \tag{46}$$

Using Figure 14 and defining

$$X = \frac{\mu_{in}}{kT_R} |\underline{v}_i - \underline{v}_n|$$

we obtain

$$e^{-\mu_{in}/2kT_R(\underline{v}_i - \underline{v}_n - \underline{g})^2} = e^{-\mu_{in}/2kT_R|\underline{v}_i - \underline{v}_n|^2} \cdot e^{-\mu_{in}/2kT_R g^2} \cdot e^{\mu_{in}/kT_R g|\underline{v}_i - \underline{v}_n|\cos\theta}$$

$$e^{\mu_{in}/kT_R g|\underline{v}_i - \underline{v}_n|\cos\theta} = 1 + X g \cos\theta + \frac{1}{2!} X^2 g^2 \cos^2\theta + \frac{1}{3!} X^3 g^3 \cos^3\theta + \cdots$$

$$d^3 g = g^2 \sin\theta \, d\theta \, d\phi \, dg$$

Inserting these expressions into (45), carrying out the integration over $\phi (0 \leq \phi \leq 2\pi)$ and dividing by N_i, we get for $\overline{dN}_n$:

$$\overline{dN}_n = N_n \cdot 2\pi \left(\frac{\mu_{in}}{2\pi k T_R}\right)^{3/2} \cdot e^{-\mu_{in}/2kT_R|\underline{v}_i - \underline{v}_n|^2} \cdot g^2 \cdot e^{-\mu_{in}/2kT_R g^2}$$

$$\cdot \left(\sin\theta + X g \sin\theta \cos\theta + \frac{1}{2!} X^2 g^2 \sin\theta \cos^2\theta + \cdots\right) dg \, d\theta \quad (47)$$

$d\nu_i$ is given by

$$d\nu_i = g \cdot b \cdot \overline{dN}_n \cdot db \cdot d\alpha \quad (48)$$

Hence, after integrating over $\alpha (0 \leq \alpha \leq 2\pi)$:

$$d\underline{k}_f = \underline{e}_z \, 4\pi \mu_{in} N_i \, g^2 \, b \cos^2\vartheta \cos\theta \, \overline{dN}_n \, db \quad (49)$$

ϑ is a function of the collision parameter b and the relative velocity g. We introduce a velocity dependent collision cross section Q(g) defined by

$$Q(g) = 4\pi \int_0^\infty \cos^2 \vartheta \, b \, db \qquad (50)$$

where Q(g) is identical to the momentum transfer cross section

$$Q(g) = 2\pi \int_0^\infty (1 - \cos \chi) \, b \, db \qquad (50a)$$

commonly used in literature since, according to Figures 12 and 13, the deflection angle χ is related to ϑ by means of

$$2 \cos^2 \vartheta = 1 - \cos \chi$$

We integrate over $\theta (0 \leq \theta \leq \pi)$ and over $b (0 \leq b < \infty)$ in Equation (49), insert Q(g) given by (50), consider the relationship

$$\underline{e}_z = - \frac{\underline{v}_i - \underline{v}_n}{|\underline{v}_i - \underline{v}_n|}$$

and integrate over $g (0 \leq g < \infty)$ to get the final result

$$\underline{k}_f = - (\underline{v}_i - \underline{v}_n) \cdot \frac{4\pi}{3} \cdot N_i N_n \mu_{in} \left(\frac{\mu_{in}}{kT_R}\right) \left(\frac{\mu_{in}}{2\pi k T_R}\right)^{3/2} e^{-\mu_{in}/2kT_R |\underline{v}_i - \underline{v}_n|^2}$$

$$\cdot \int_0^\infty Q(g) \left[1 + \frac{3}{5} \cdot \frac{1}{3!} X^2 g^2 + \frac{3}{7} \cdot \frac{1}{5!} X^4 g^4 + \frac{3}{9} \cdot \frac{1}{7!} X^6 g^6 + \dots \right] \qquad (10)$$

$$e^{-\mu_{in}/2kT_R \, g^2} \, g^5 \, dg$$

FIGURE CAPTIONS

Figure 1a, b. Left side (1a): Collision without charge exchange
Right side (1b): Collision with charge exchange

Figure 2. Illustration of some quantities used in the text

Figure 3. Interaction potential U as a function of the particle distance r

Figure 4. Illustration of some quantities used in the text

Figure 5. $\dot{r}^2$ ($\dot{r}$ = radial velocity) as a function of the particle distance r for three different collision parameters b'

Figure 6. Q, $2S_{ex}'$ and $2S_{ex}$ as a function of the relative velocity g for the process $O^+ + O \rightarrow O + O^+$

Figure 7. Q, $2S_{ex}'$ and $2S_{ex}$ as a function of the relative velocity g for the process $H^+ + H \rightarrow H + H^+$

Figure 8. $\nu(O^+, O)/n(O)$ as a function of the reduced temperature T_R for $|\underline{v}_i - \underline{v}_n| = 0$. Upper curve for the momentum transfer cross section Q after Equation (16a), lower curve for the uncorrected momentum transfer cross section $2S_{ex}$

Figure 9. $\nu(O^+, O)/n(O)$ as a function of $|\underline{v}_i - \underline{v}_n|$ for three different temperatures

Figure 10. $\nu(H^+, H)/n(H)$ as a function of $|\underline{v}_i - \underline{v}_n|$ for four different temperatures

Figure 11. F as a function of λ

Figure 12. Relation between $g' - g$ and ϑ for direct collisions

Figure 13. Relation between $g' - g$ and ϑ for indirect collisions

Figure 14. Illustration of some quantities used in the text

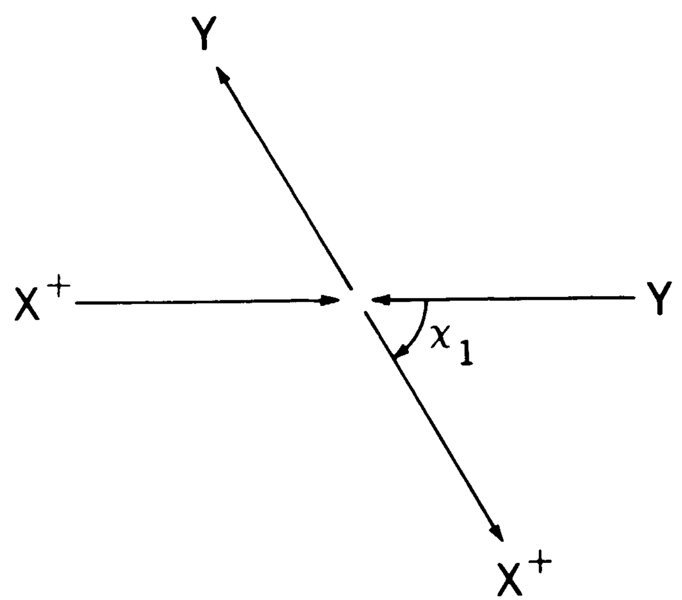

Figure 1a. Collision Without Charge Exchange

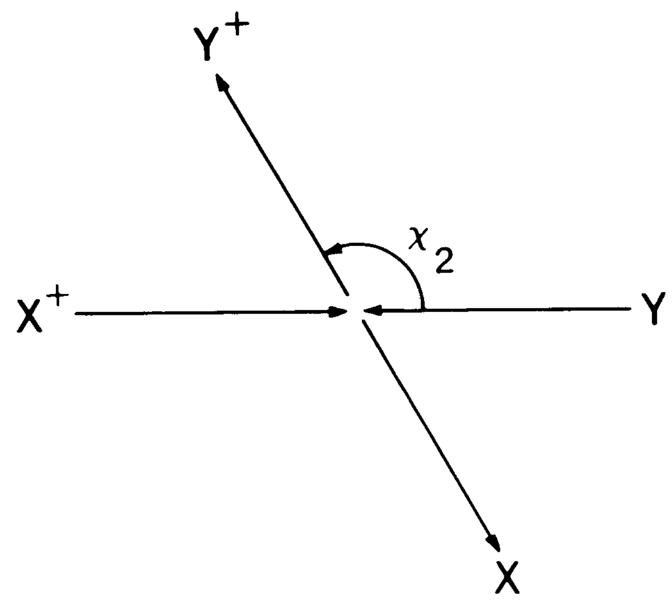

Figure 1b. Collision With Charge Exchange

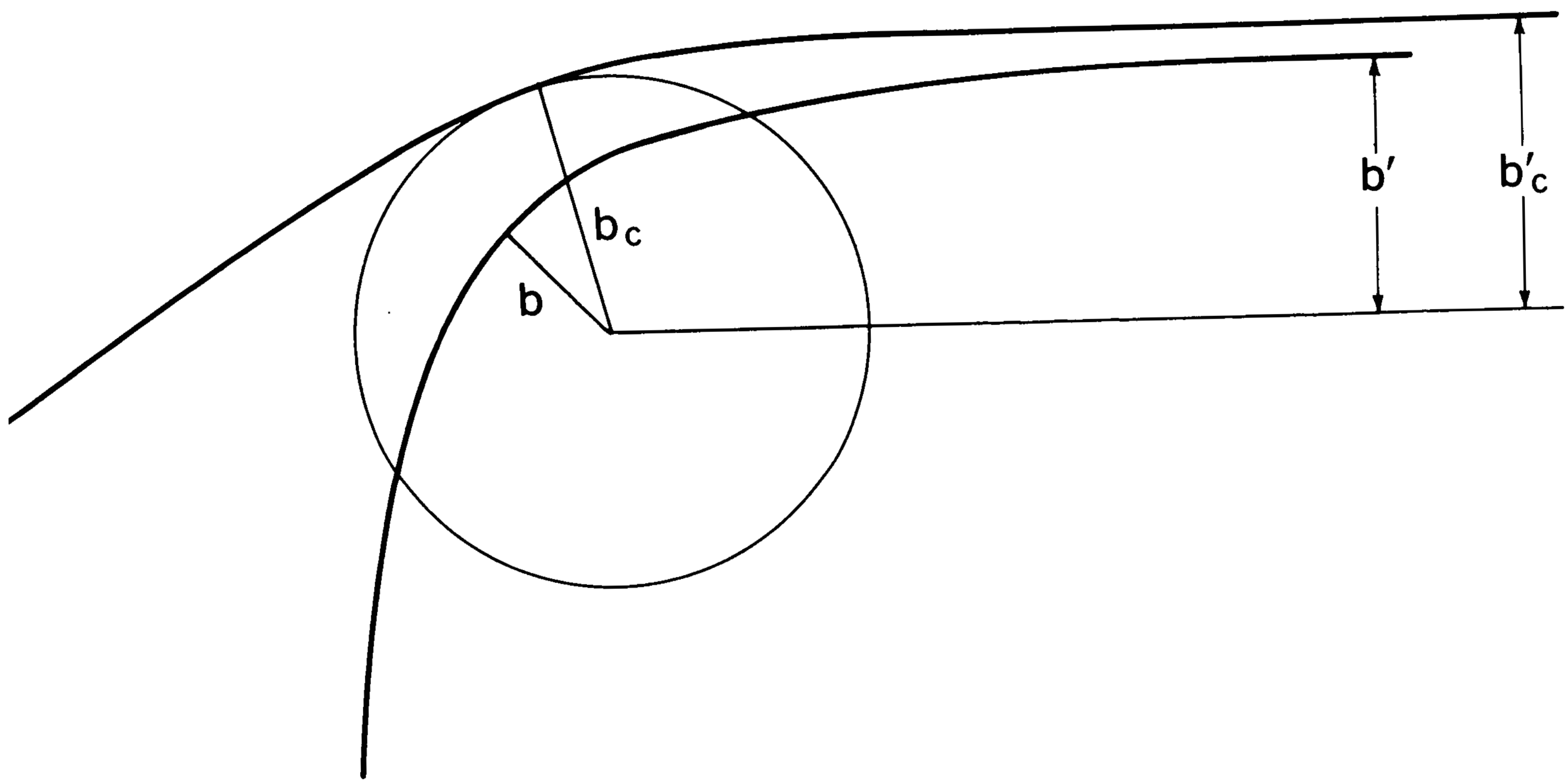

Figure 2. Illustration of Some Quantities Used in the Text

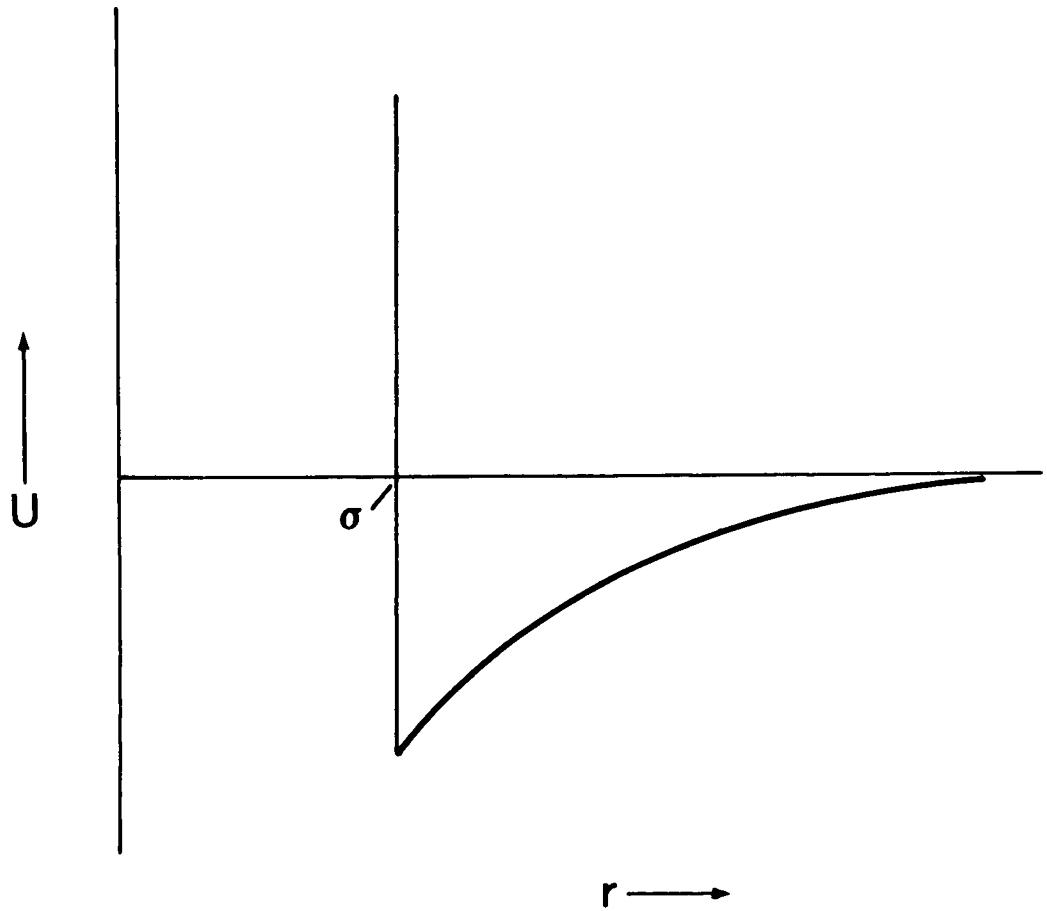

Figure 3. Interaction Potential U as a Function of the Particle Distance r

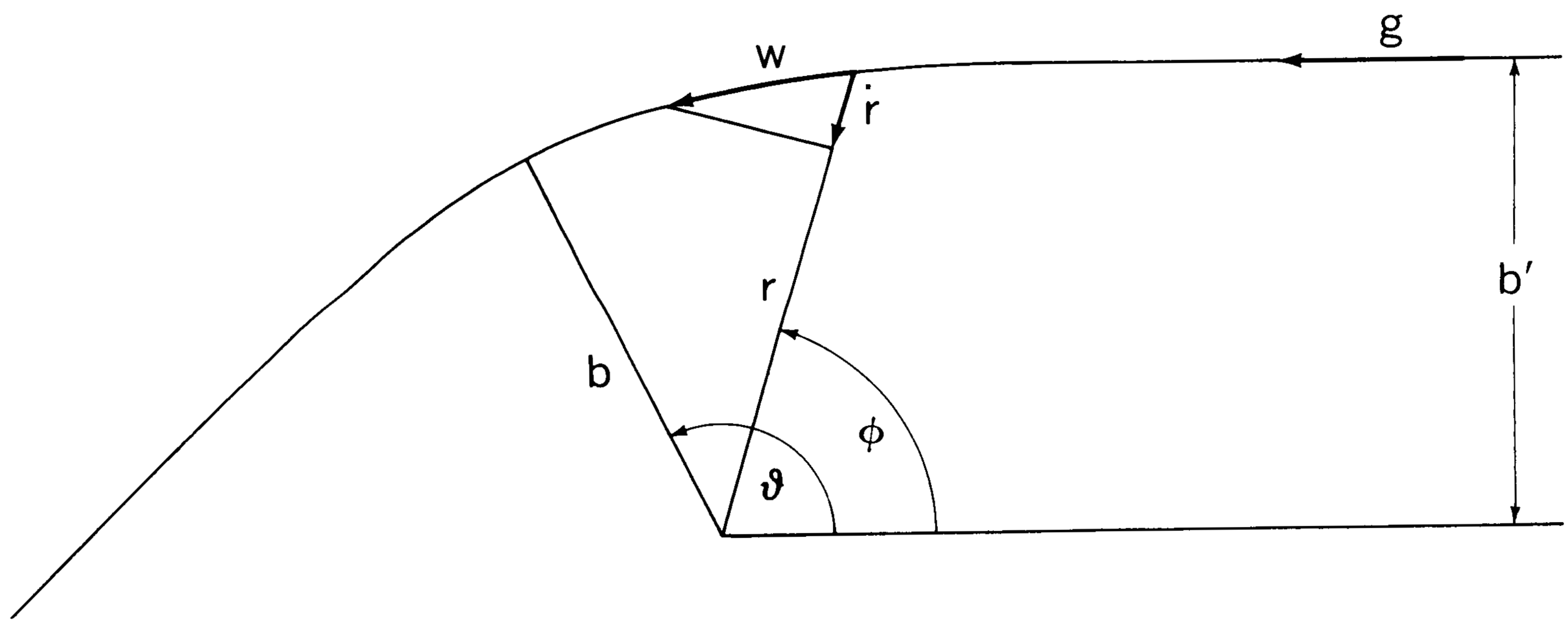

Figure 4. Illustration of Some Quantities Used in the Text

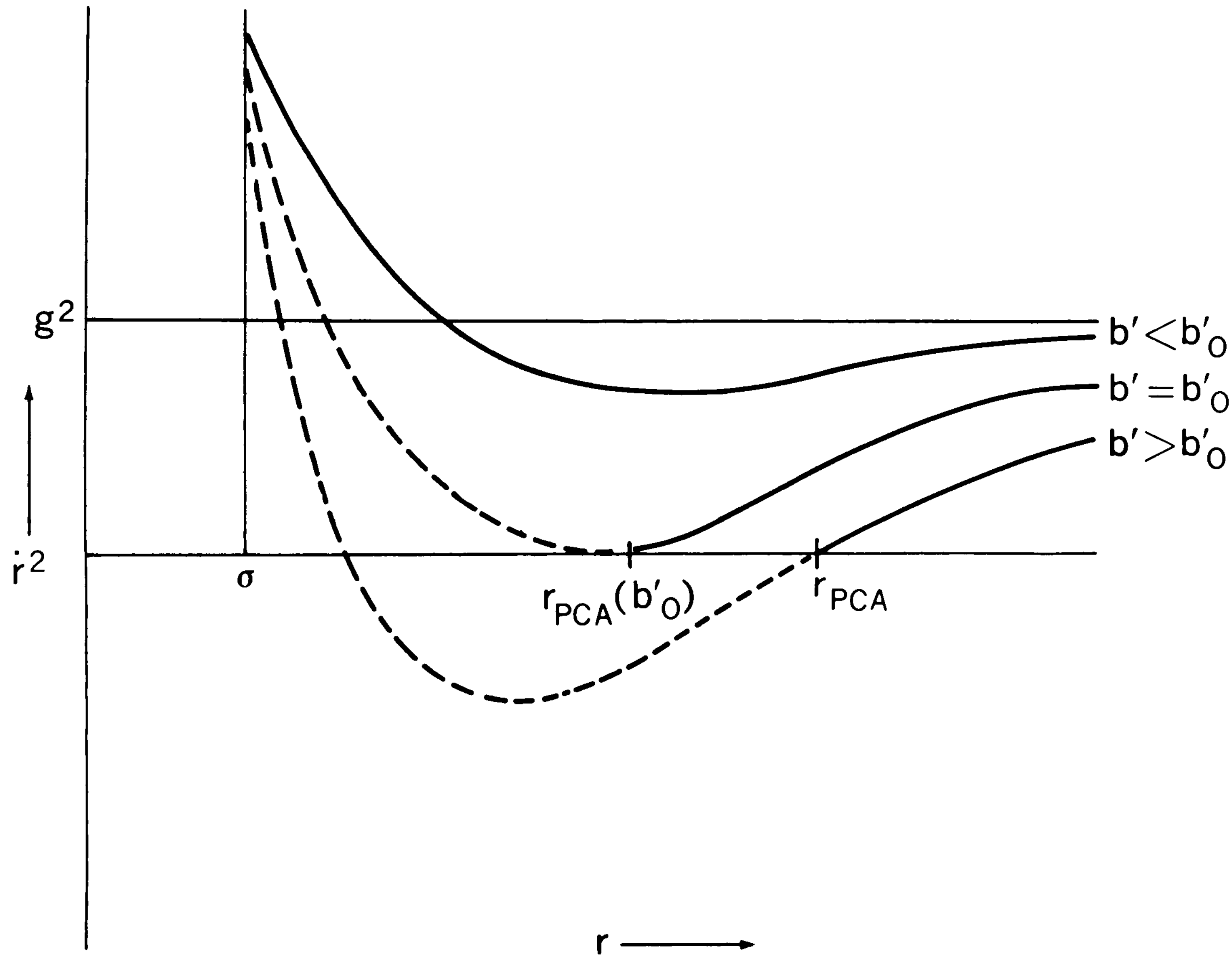

Figure 5. $\dot{r}^2$ ($\dot{r}$ = radial velocity) as a Function of the Particle Distance r for Three Different Collision Parameters b'

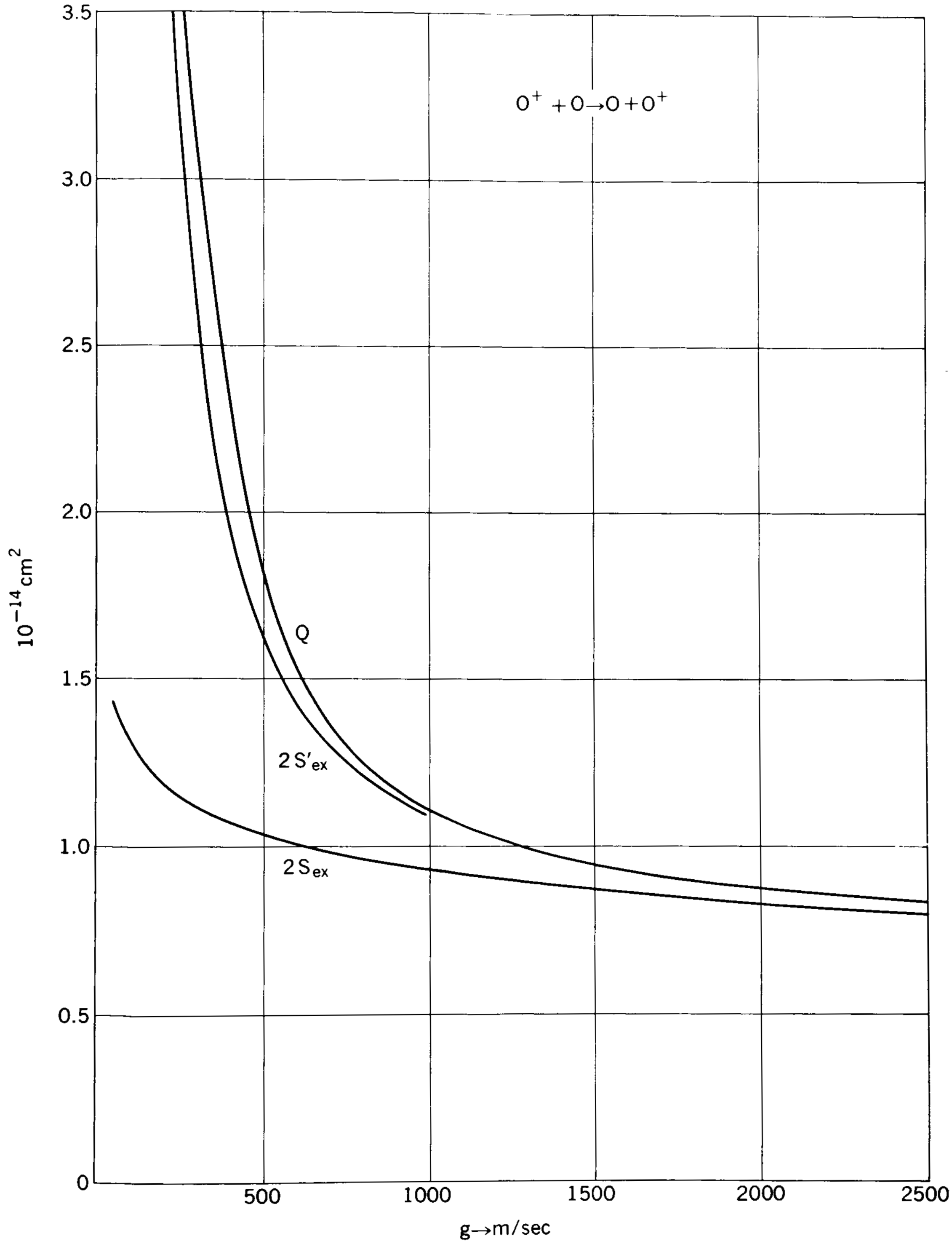

Figure 6. Q, $2S'_{ex}$ and $2S_{ex}$ as a Function of the Relative Velocity g for the Process $O^+ + O \rightarrow O + O^+$

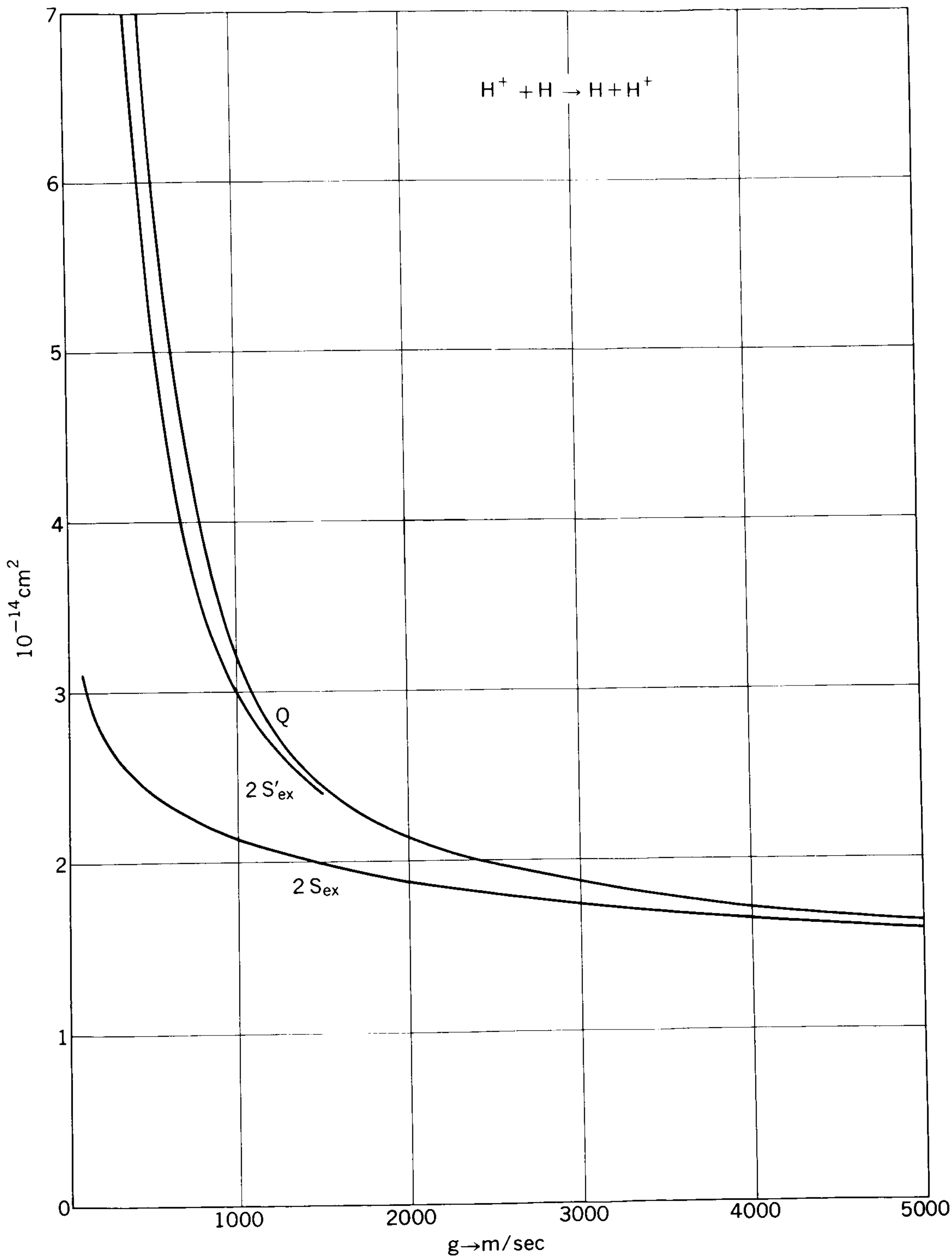

Figure 7. Q, $2S'_{ex}$ and $2S_{ex}$ as a Function of the Relative Velocity g for the Process $H^+ + H \rightarrow H + H^+$

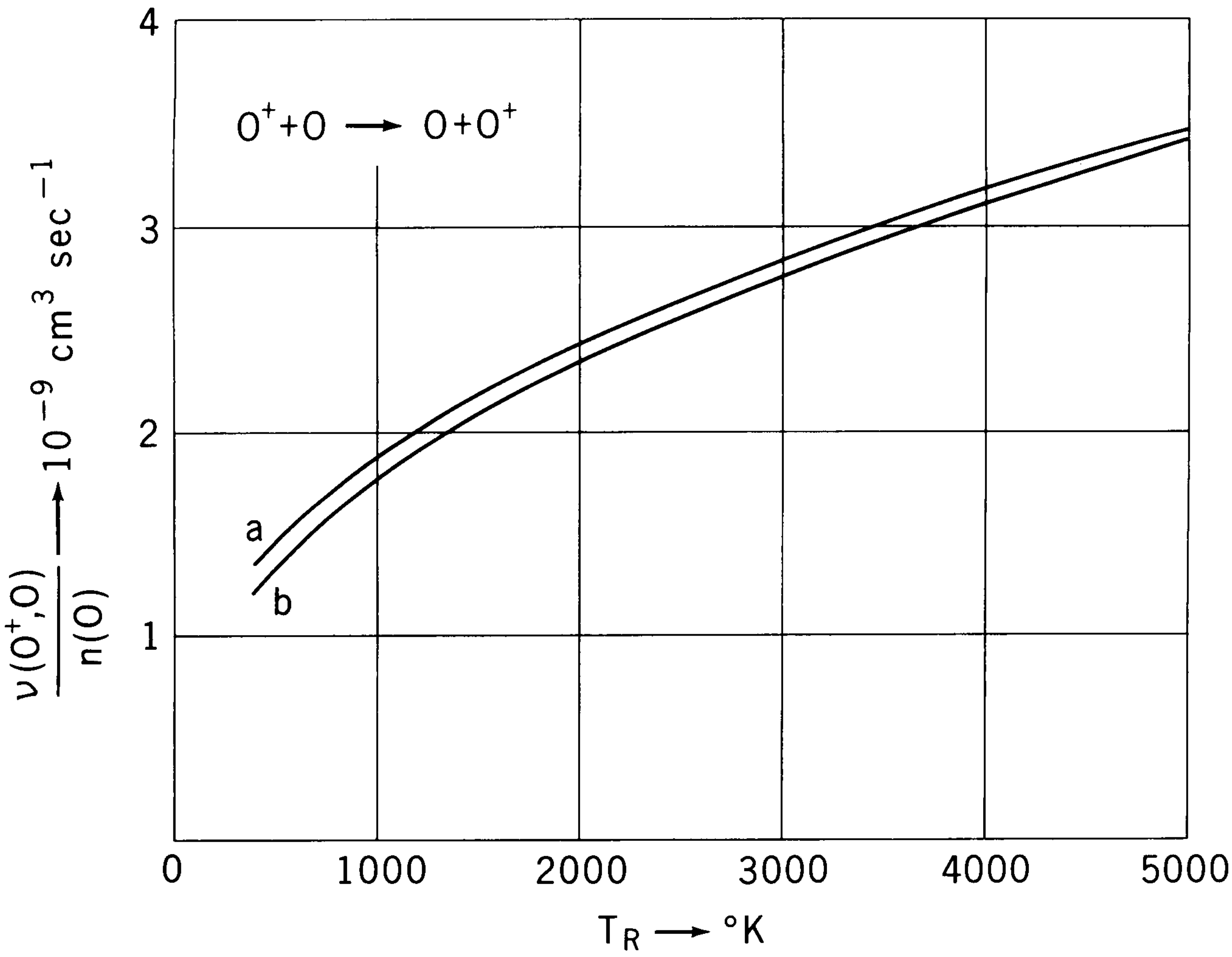

Figure 8. $\nu(O^{+}, O)/n(O)$ as a Function of the Reduced Temperature T_R for $|\underline{v}_i - \underline{v}_n| = 0$. Upper Curve for the Momentum Transfer Cross Section Q After Equation (16a), Lower Curve for the Uncorrected Momentum Transfer Cross Section $2S_{ex}$

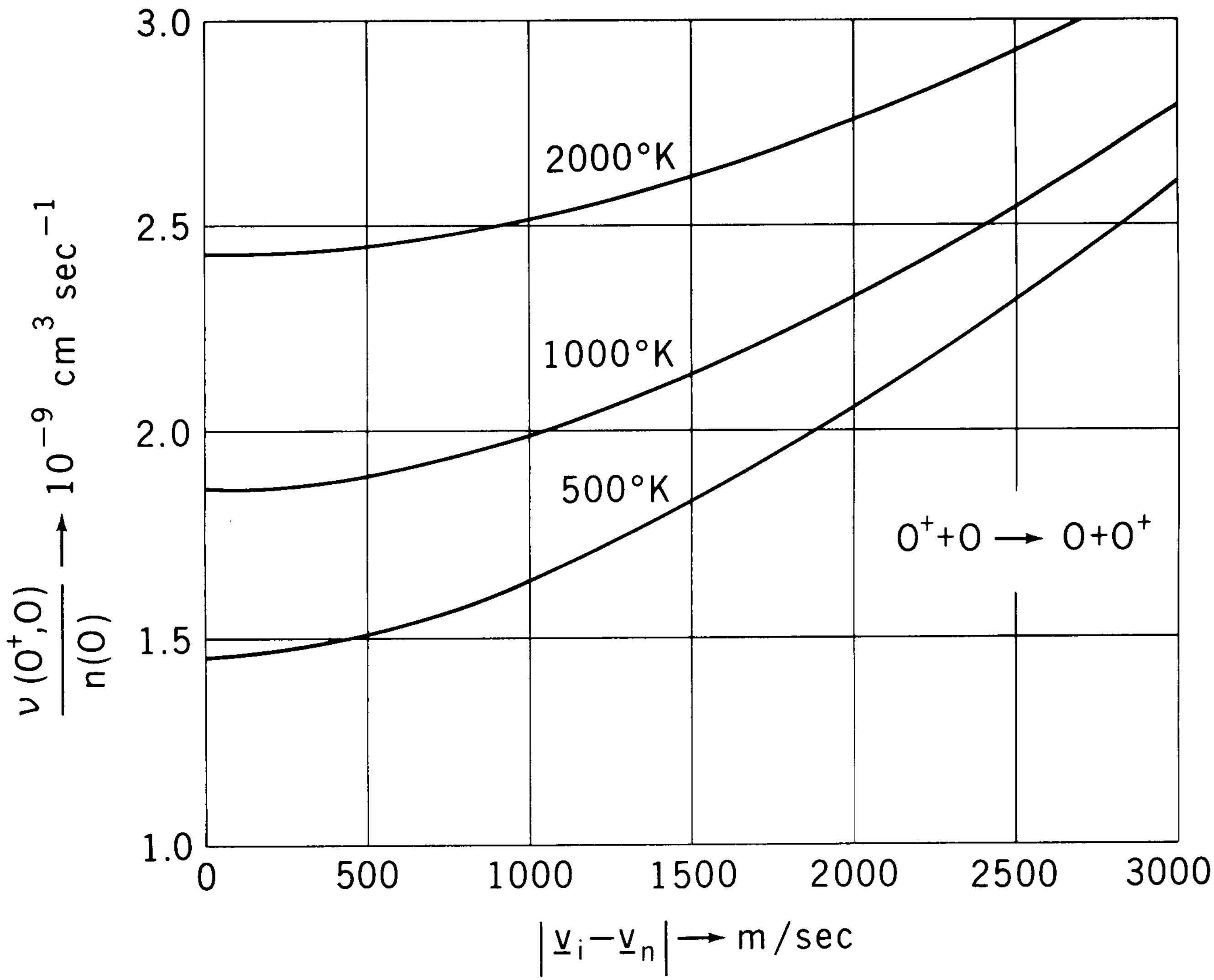

Figure 9. $\nu(O^+, O)/n(O)$ as a Function of $|\underline{v}_i - \underline{v}_n|$ for Three Different Temperatures

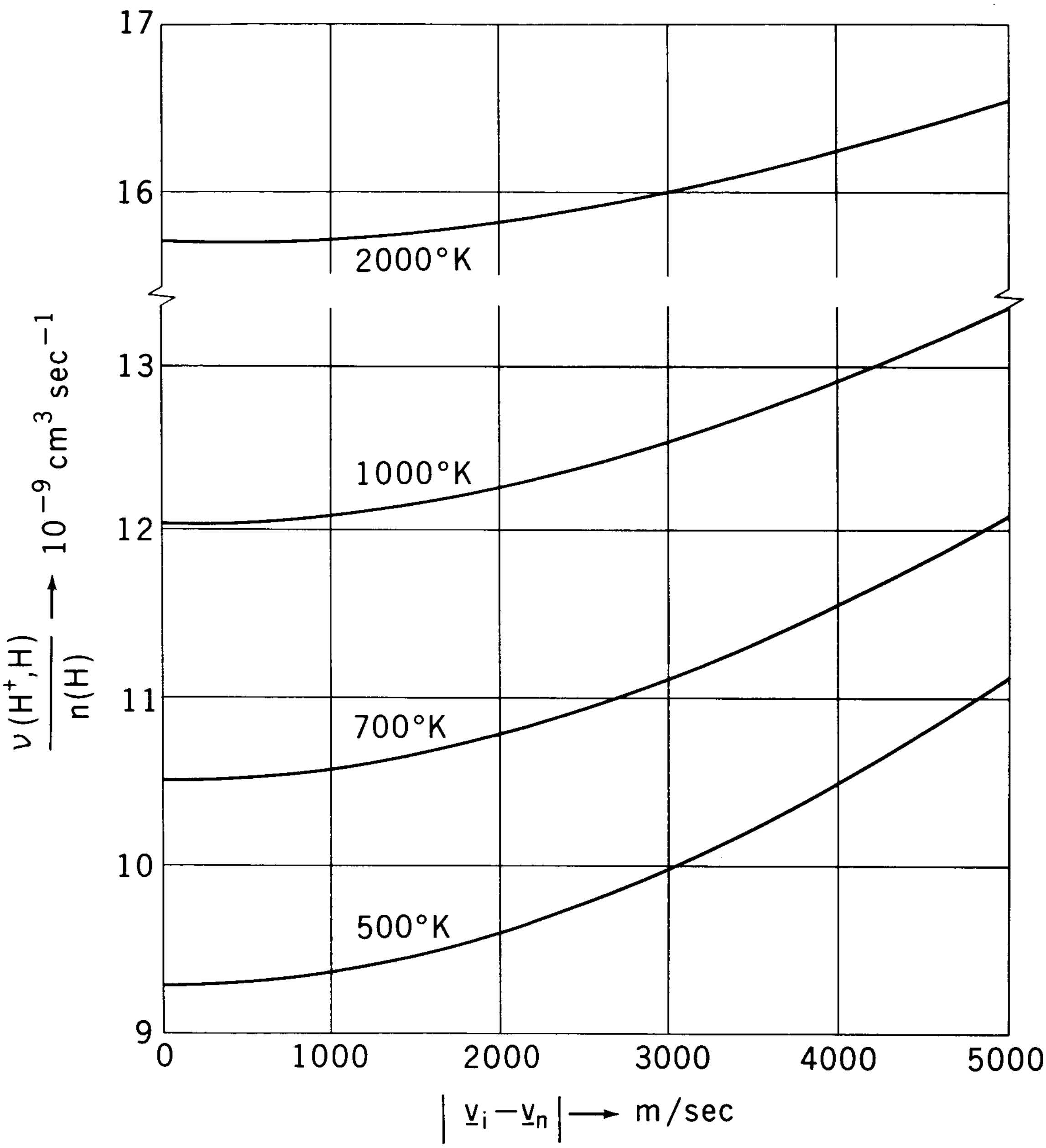

Figure 10. $\nu(H^+, H)/n(H)$ as a Function of $|\underline{v}_i - \underline{v}_n|$ for Four Different Temperatures

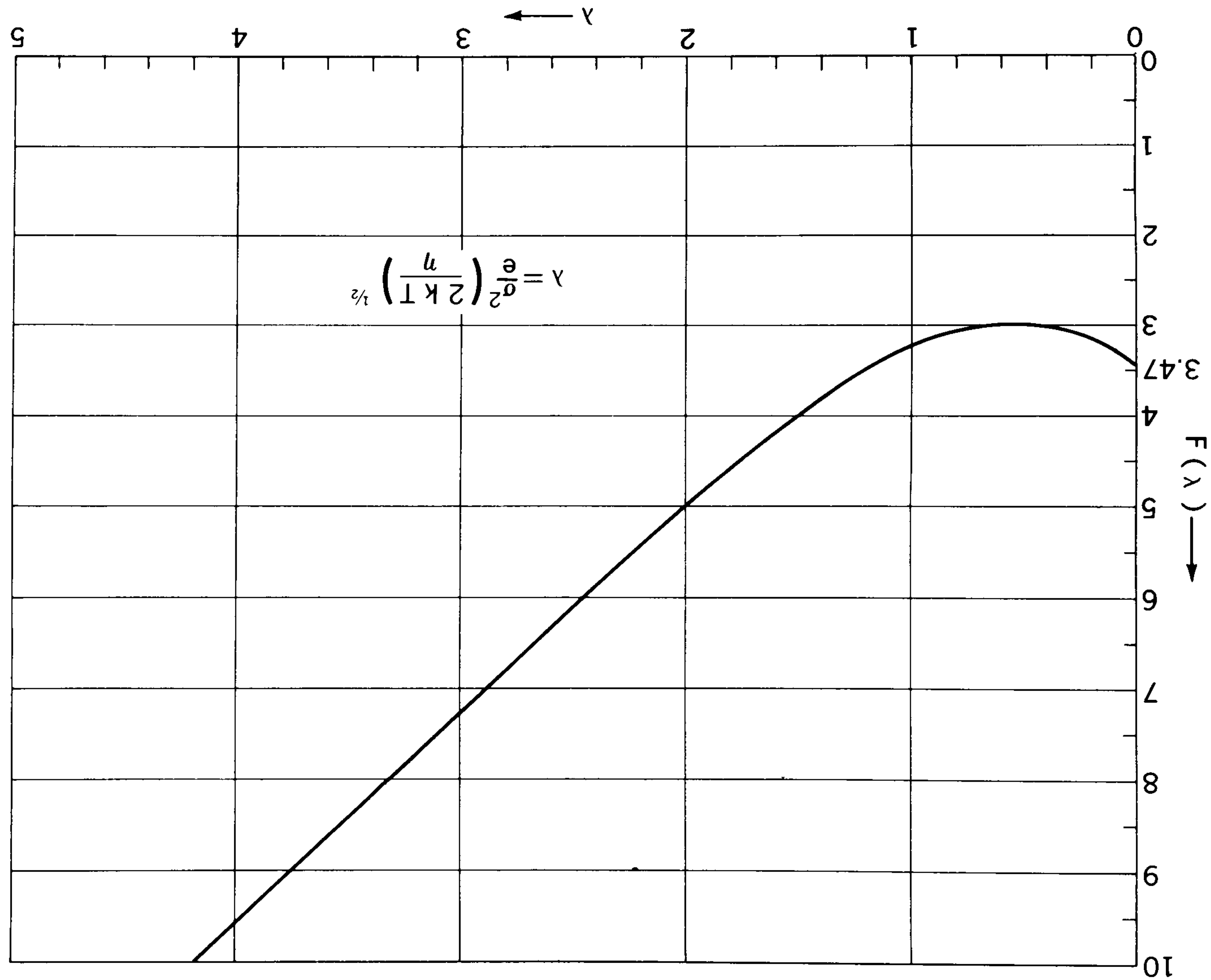

Figure 11. F as a Function of λ

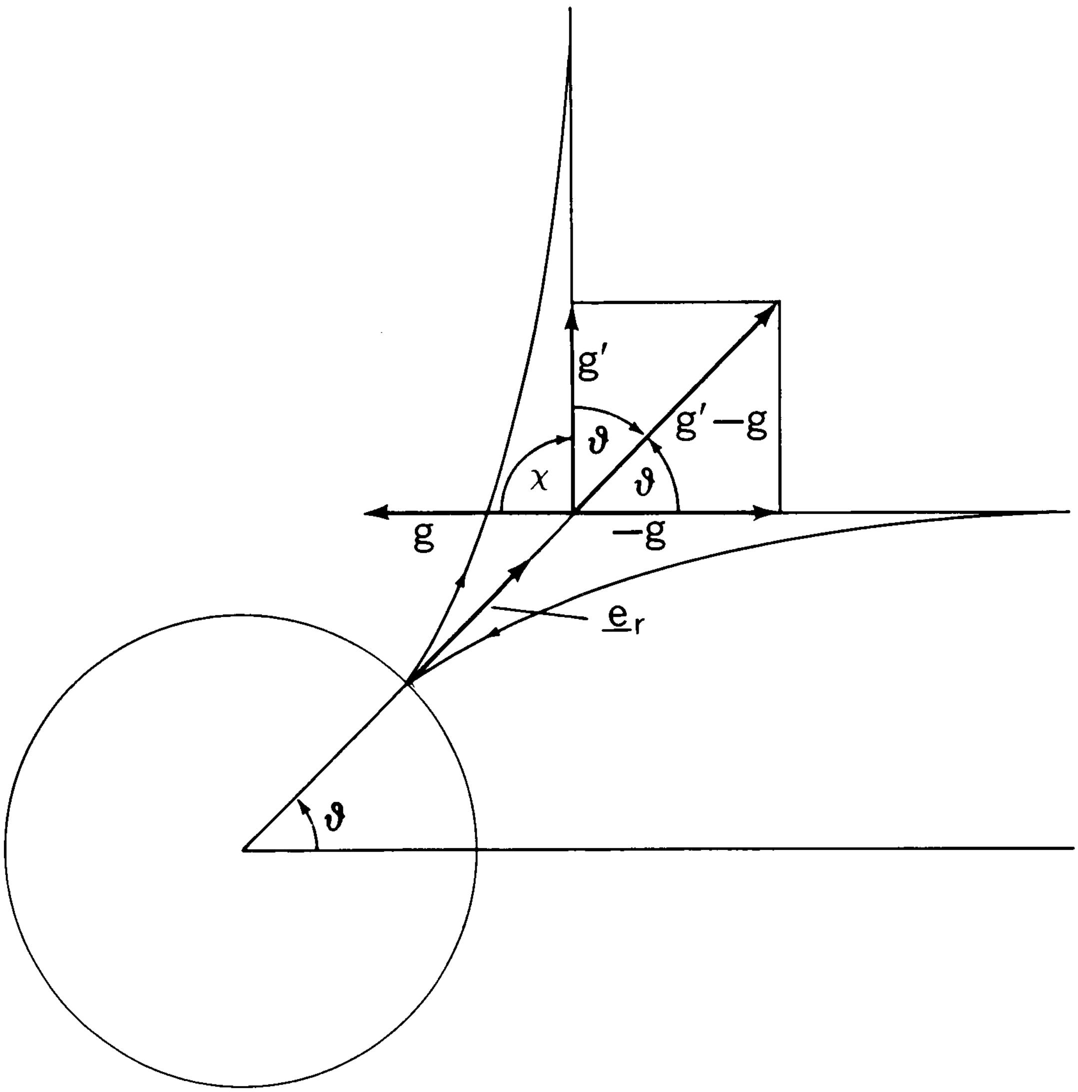

Figure 12. Relation Between $\underline{g}'-\underline{g}$ and ϑ for Direct Collisions

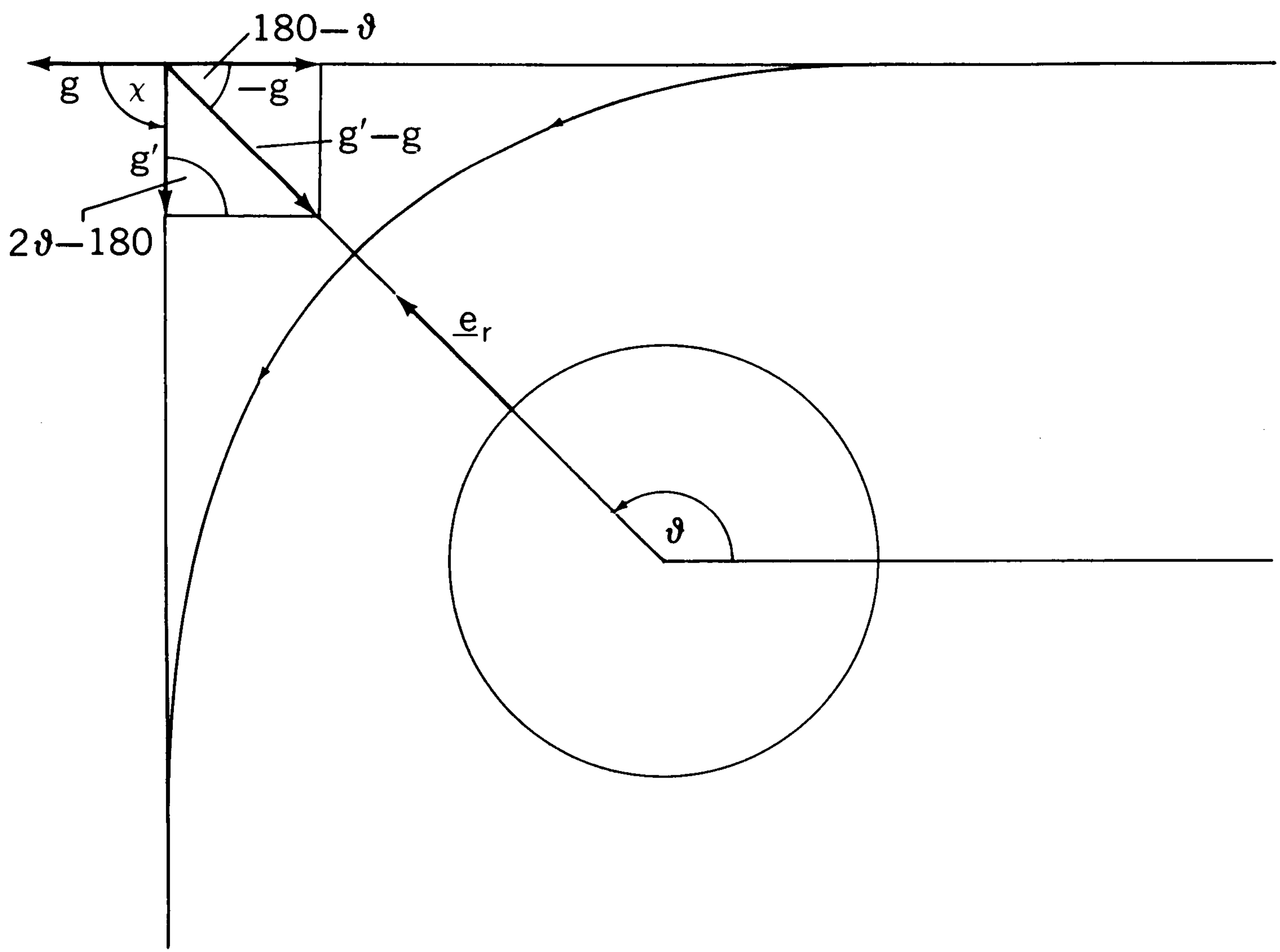

Figure 13. Relation Between $\underline{g}' - \underline{g}$ and ϑ for Indirect Collisions

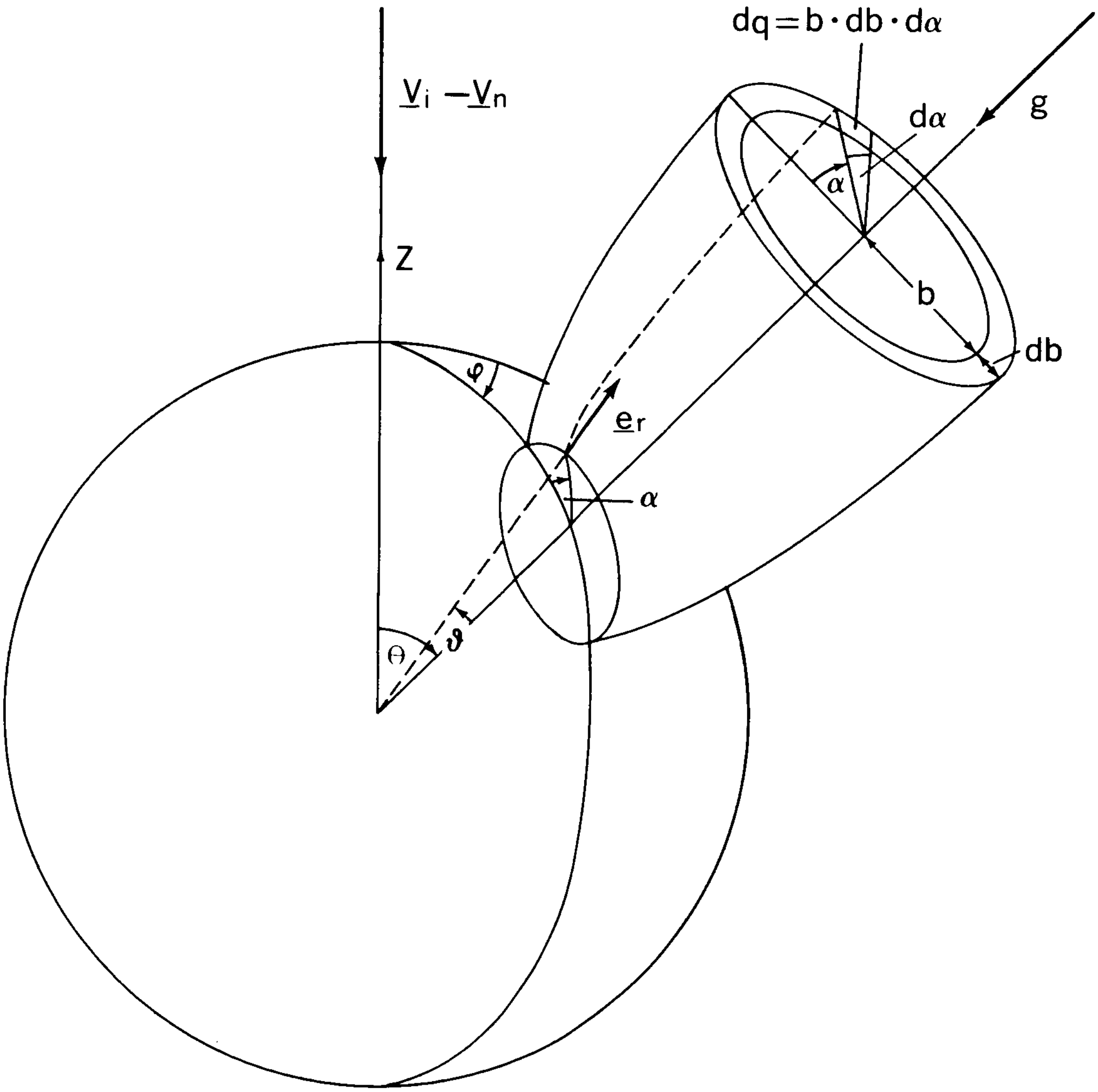

Figure 14. Illustration of Some Quantities Used in the Text