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RESEARCH ARTICLE

Frictionless contact problem between a rigid moving punch and a homogeneous layer resting on a Winkler foundation

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Abstract

In this paper, the moving contact problem between the punch and a layer supported by a Winkler type foundation is considered under plane strain conditions. The cylindrical punch moves smoothly with constant velocity over the homogeneous layer. With the use of Galilean transformation and Fourier integral transform, the plane contact problem is converted into a Cauchy type singular integral equation. The singular integral equation is solved numerically by performing the appropriate collocation method, and the contact width, the contact and in-plane stresses are calculated. The results show that the stiffness of the Winkler foundation and the moving velocity have a significant effect on the contact width and the behavior of the stress field.

Keywords

1. Introduction

The contact mechanics of deformable components are among the most difficult problems in mechanics. Since mechanical and structural components are usually in contact with each other, contact problems have gained importance over the years. Particularly, the contact problem of a layer resting on an elastic foundation lies at the hearth of soil mechanics.

The studies on contact mechanics have focused on functionally graded materials (FGMs) in recent years. The load transfer problems in the presence of friction (sliding contact) are the potential applications of FGMs [1]. The sliding contact problems of the coated half plane are examined by Guler and Erdogan [1] Guler and Erdogan [2] and Alinia et al. [3] using Fourier integral transform technique. In the mentioned studies, the shear modulus of the coating is assumed to vary exponentially in the depth direction and the system is loaded by rigid cylindrical, flat or triangular stamps. Ke and Wang [4] used the multi-layered model to solve the sliding contact problem of FG coated half plane. With the multi-layered model they divided the FG layer into several sub-layers assuming the elastic properties is a linear function. The sliding contact problem of a half plane coated with a linear graded layer is examined by Chen and Chen [5]. The sliding contact problem of an FG half plane loaded by a flat punch is investigated by Choi [6] and Chen et al. [7]. They also assumed that the shear modulus varies exponentially in an arbitrary direction [7]. The sliding contact analysis of

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laterally graded medium is investigated by Dag et al. [8] and Dag [9]

In the sliding contact problems, the frictional heating may occur since the presence of friction between the contacting components. The frictional heating may influence the performance of the system and leads to contact damage. Liu et al. [10] examined the thermoelastic contact problem of a FGM coated half plane with multilayered model. Choi and Paulino [11] studied thermoelastic contact of coating-graded interlayer-substrate system. The thermoelastic contact problem of FG coated half plane is studied with finite element method (FEM) by Balci et al. [12]. Barik et al. [13] investigated the effect of thermal conductivity on the thermoelastic contact problem. Mao et al. [14] analyzed the thermoelastic instability of an FGM layer and a homogeneous layer by the perturbation method.

In the absence of body forces the contacting bodies get separated from each other with the loading and the contact area reduces to a finite size. This kind of contact referred to as receding. Rhimi et al. [15] and Rhimi et al. [16] considered the axisymmetric receding contact problem between an FG layer and a homogeneous layer using Hankel transform. The frictional receding contact problems of an FG layer resting on a homogeneous substrate are investigated by El-Borgi et al. [17] and El-Borgi and Çömez [18] using the Fourier transformation technique. The frictionless receding contact problem of a homogeneous layer or bilayer resting on a homogeneous substrate or a layer is examined by Çömez et al. [19], Kahya et al. [20] Adibelli et al. [21]. The layer and substrate are assumed to be anisotropic in [20]. Çömez [22] and Parel and Hills [23] studied the frictional receding contact problem of a homogeneous layer resting on a homogeneous substrate. The studies of Çömez et al. [19] and Adibelli et al. [21] are extended to the FGM case by Jan and Li [24] and Jan and Mi [25].

The contact problem involving a layer supported by a Winkler foundation have many applications particularly in soil mechanics and ice engineering fields. In the Winkler foundation model, it is assumed that the foundation behaves like an infinite number of individual elastic springs

and the foundation reaction is proportional to its displacement [26]. Although there are considerable number of studies related to the beam or the plate problem resting on a Winkler foundation, the contact problem of a layer resting on a Winkler foundation is quite limited in the literature. The plane contact problem of an elastic homogeneous layer lying on a Winkler foundation is first investigated by Dempsey et al. [27]. The layer is pressed by line load, uniform load, rigid cylinder or flat punch. This study is extended to the axisymmetric case by Dempsey et al. [28]. Wozniak et al. [29] studied the axisymmetric contact problem of a layer lying on a rigid support with a Winkler type excavation using the Hankel transform. The contact problem of a homogeneous bilayer supported by a Winkler foundation is analyzed by Birinci et al. [30]. Çömez [26] investigated the contact problem of FG layer supported by Winkler type foundation. Birinci et al. [31] studied the continuous and discontinuous contact problem of the bilayer resting on a Winkler foundation using the FEM. Adıyaman and Birinci [32] investigated the contact problem of an FG layer pressed to a Winkler foundation with the arbitrary rigid punches.

There are problems that arise in practice in which the speed of one body relative to other is quite large, and we therefore need to investigate whether it is necessary to take the dynamic character of the problem into account [33]. In this respect, Georgiadis [34] studied the rigid flat moving punch problem over the highly orthotropic strip and examined the effect of the velocity of the punch as well as of the orthotropy. De and Patra [35] investigated the contact problem of a single punch or a row of equally spaced punches moving over the surface of the orthotropic medium. The contact problem between the moving flat or cylindrical punch and the magneto-electro-elastic half plane is analyzed by Zhou and Lee [36], [37]. Zhou et al. [38] and Zhou et al. [39] considered the frictional and frictionless moving contact problem of a rigid punch over the orthotropic layer using Galilean transform and Fourier integral transform technique. It is assumed that the flat or cylindrical

rigid punch moving at a constant velocity on the surface of orthotropic medium. The first study of the frictionless moving contact problem of an FG layer indented by a rigid cylindrical punch is examined by Çömez [40]. Çömez [41] considered the contact problem between the moving punch and a homogeneous layer in the presence of friction. Balcı and Dag [42] solved the moving contact problem between the rigid punch and a coated half plane.

It can be seen from the literature that the moving contact problem of a layer resting on a Winkler foundation has not been considered, yet. To fill this gap in the literature, the contact problem of a rigid cylindrical punch moving at a constant velocity on the surface of a homogeneous layer resting on a Winkler foundation is investigated in this study. The plane contact problem was reduced to a singular integral equation of the second kind using the Galilean transformation and Fourier integral transform technique. The integral equation is solved numerically utilizing the Gauss- Chebyshev integration formula. The main objective of this paper is to study effect of the moving velocity and stiffness of the Winkler foundation on the contact area and the distribution of the stress components.

2. Formulation of the problem

Consider the plane moving contact problem in Fig 1. A homogeneous layer of a thickness h is resting on a Winkler type foundation. The layer is pressed by a rigid cylindrical punch with radius R subjected to an external force P. The punch moves over the surface of the layer in x_1 direction at a constant velocity V.

The governing equations of motion can be written as follows

$$\frac{\partial \sigma_x}{\partial x_1} + \frac{\partial \tau_{xy}}{\partial y_1} = \rho \frac{\partial^2 u}{\partial t^2}$$
$$\frac{\partial \tau_{yx}}{\partial x_1} + \frac{\partial \sigma_y}{\partial y_1} = \rho \frac{\partial^2 v}{\partial t^2} \tag{1}$$

where u, v are the $x_1 -$ and $y_1 -$ components of the displacement vector, respectively and t denotes the time variable.

The stress and displacement relations (Hooke's law) in a state of plane strain can be expressed as follows

$$\sigma_x = \frac{\mu}{\kappa-1}\left[(\kappa+1)\frac{\partial u}{\partial x_1} + (3-\kappa)\frac{\partial v}{\partial y_1}\right] \tag{2a}$$

$$\sigma_{y1} = \frac{\mu}{\kappa-1}\left[(3-\kappa)\frac{\partial u}{\partial x_1} + (\kappa+1)\frac{\partial v}{\partial y_1}\right] \tag{2b}$$

$$\tau_{xy} = \mu\left[\frac{\partial u}{\partial y_1} + \frac{\partial v}{\partial x_1}\right] \tag{2c}$$

where μ represents the shear modulus and $\kappa = 3-4\nu$ for the plane strain case, ν being the Poisson's ratio.
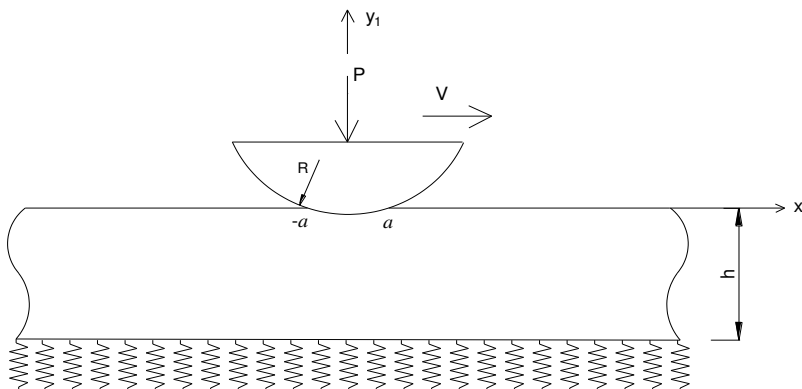

Fig 1. Geometry of the moving contact problem

Introducing the following Galilean transformation [33], the dynamic contact problem may be turn into the steady contact problem

$$x = x_1 - Vt \qquad\qquad y = y_1 \qquad\qquad (3)$$

Substituting Eqs. (3) into Eqs. (1) and using Hooke's law (2), the equations of motion can be written in terms of the derivatives of $u(x, y)$ and $v(x, y)$ as follows

$$[(\kappa+1) + (1-\kappa)\frac{V^2}{c_T^2}]\frac{\partial^2 u}{\partial x^2} + (\kappa-1)\frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 v}{\partial x \partial y} = 0$$

$$-[(1-\kappa) + (1+\kappa)\frac{V^2}{c_L^2}]\frac{\partial^2 v}{\partial x^2} + (\kappa+1)\frac{\partial^2 v}{\partial y^2} + 2\frac{\partial^2 u}{\partial x \partial y} = 0$$

$$(4)$$

where c_L and c_T are the speed of propagation of longitudinal and shear waves in the layer, respectively.

$$c_T = \sqrt{\frac{\mu}{\rho}} \qquad\qquad c_L = \sqrt{\frac{2\mu+\lambda}{\rho}} \qquad (5)$$

Note that Eqs. (4) contain partial differential equation system. Using the Fourier integral transform technique, Eqs (4) can be converted to the ordinary differential equation system. Due to the plane symmetry, it is adequate to consider the problem in the field $0 \le x < \infty$ only. Utilizing the symmetry of the problem and Fourier transform technique, the following transforms may be written

$$u(x, y) = \frac{2}{\pi}\int_0^\infty \tilde{u}(\alpha, y)\sin(\alpha x)d\alpha \qquad (6a)$$

$$v(x, y) = \frac{2}{\pi}\int_0^\infty \tilde{v}(\alpha, y)\cos(\alpha x)d\alpha \qquad (6b)$$

where $\tilde{u}(\alpha, y)$ and $\tilde{v}(\alpha, y)$ are the Fourier transforms of the displacement vectors $u(x, y)$ and $v(x, y)$, respectively.

Using the Fourier transforms defined (6) to Eqs. (4) with respect to x, yields following ordinary differential equation system

$$(\kappa-1)\frac{d^2\tilde{u}}{dy^2} - 2\alpha\frac{d\tilde{v}}{dy} - \alpha^2[(\kappa+1) + (1-\kappa)\frac{V^2}{c_T^2}]\tilde{u} = 0$$

$$(\kappa+1)\frac{d^2\tilde{v}}{dy^2} + 2\alpha\frac{d\tilde{u}}{dy} + \alpha^2[(1-\kappa) + (1+\kappa)\frac{V^2}{c_L^2}]\tilde{v} = 0$$

$$(7)$$

Substituting the necessary derivatives of Eqs. (6) into Eqs. (7) and after some routine manipulations, the following characteristic equation may be obtained

$$n^4 + \left(-2 + \frac{V^2}{c_T^2} + \frac{V^2}{c_L^2}\right)n^2$$

$$+ \frac{1}{-1+\kappa^2}\left(-1 + \frac{V^2}{c_T^2}(-1+\kappa) - \kappa\right) \qquad (8)$$

$$\times\left(1 - \kappa + (1+\kappa)\frac{V^2}{c_L^2}\right) = 0$$

The roots of the characteristic equation can be found as

$$n_{1,2} = \pm\sqrt{1 - \frac{V^2}{c_T^2}}, \qquad n_{3,4} = \pm\sqrt{1 - \frac{V^2}{c_L^2}} \qquad (9)$$

Hence, the solution of the Eqs. (7) may be expressed as

$$u = \frac{2}{\pi}\int_0^\infty \Big[A_1 e^{n_1\alpha y} + A_2 e^{n_2\alpha y} + A_3 e^{-n_1\alpha y}$$

$$+ A_4 e^{-n_2\alpha y}\Big]\sin(\alpha x)d\alpha \qquad (10a)$$

$$v = \frac{2}{\pi}\int_0^\infty\Big[-\frac{1}{n_1}A_1 e^{n_1\alpha y} - n_2 A_2 e^{n_2\alpha y}$$

$$+ \frac{1}{n_1}A_3 e^{-n_1\alpha y} + n_2 A_4 e^{-n_2\alpha y}\Big]\cos(\alpha x)d\alpha \qquad (10b)$$

Using Hooke's law (2) and Eqs. (10) the stress field can be obtained as follows

$$\frac{\sigma_x}{\mu} = \frac{2}{\pi}\int_0^\infty \alpha \left[2A_1 e^{n_1\alpha y} + \frac{(1+n_2^2(-3+\kappa)+\kappa)}{-1+\kappa} A_2 e^{n_2\alpha y} \right.$$
$$\left. +2A_3 e^{-n_1\alpha y} + \frac{(1+n_2^2(-3+\kappa)+\kappa)}{-1+\kappa} A_4 e^{-n_2\alpha y} \right]$$
$$\times \cos(\alpha x) d\alpha \tag{11a}$$

$$\frac{\sigma_y}{\mu} = \frac{2}{\pi}\int_0^\infty -\alpha \left[2A_1 e^{n_1\alpha y} + (1+n_1^2) A_2 e^{n_2\alpha y} \right.$$
$$\left. +2A_3 e^{-n_1\alpha y} + (1+n_1^2) A_4 e^{-n_2\alpha y} \right] \cos(\alpha x) d\alpha \tag{11b}$$

$$\frac{\tau_{xy}}{\mu} = \frac{2}{\pi}\int_0^\infty \alpha \left[\frac{(1+n_1^2)}{n_1} A_1 e^{n_1\alpha y} + 2n_2 A_2 e^{n_2\alpha y} \right.$$
$$\left. -\frac{(1+n_1^2)}{n_1} A_3 e^{-n_1\alpha y} - 2n_2 A_4 e^{-n_2\alpha y} \right] \sin(\alpha x) d\alpha \tag{11c}$$

where A_j $(j=1,...4)$ are the unknown functions to be determined from the boundary conditions of the problem.

3. The boundary conditions and the singular integral equation

The moving contact problem under consideration must be solved using the following boundary conditions

$$\sigma_y(x,0) = \begin{cases} -p(x) & 0 \le x < a \\ 0 & a \le x < \infty \end{cases} \tag{12a}$$

$$\tau_{xy}(x,0) = 0 \tag{12b}$$

$$\sigma_x(x,-h) - k_w v(x,-h) = 0 \tag{12c}$$

$$\tau_{xy}(x,-h) = 0 \tag{12d}$$

where $p(x)$ is the contact stress between the rigid punch and the homogeneous layer on the contact area $(-a,a)$ and k_w is the stiffness of the Winkler foundation.

Upon taking the Fourier transforms of the boundary conditions given by (12), the unknowns A_j $(j=1,...4)$ and consequently the displacement

and the stress components may be determined in terms of the unknown contact stress $p(x)$. The vertical displacement is known based on the profile of the cylindrical stamp which is cylindrical. Additional condition may be written in the form of a derivative of the rigid punch profile to eliminate rigid-body displacement and ensures continuity of normal displacements as follows

$$v(x,0) \cong \frac{x^2}{2R} + \delta, \qquad \frac{\partial v(x,0)}{\partial x} = \frac{x}{R} \tag{13}$$

where δ is the maximum indentation depth. Substituting the unknowns A_j into the mixed condition (13) and determining the singular behavior of the solution following singular integral equation can be obtained as

$$\frac{1}{\pi}\int_{-a}^a p(\xi) d\xi \left[\frac{1}{\xi - x} + k_1(x,\xi) \right] = \frac{\mu}{\varphi_1}\frac{x}{R} \quad (0 \le x < a) \tag{14}$$

The expressions for $k_1(x,\xi)$ and φ_1 from Eq. (14) are given in Appendix A.

In the singular integral equation (14) the contact width a is also unknown, a priori. To complete the solution, $p(x)$ must satisfy the following equilibrium condition

$$\int_0^\infty \sigma_y(x,0) dx = -\int_0^a p(\xi) d\xi = -\frac{P}{2} \tag{15}$$

4. Numerical solution of the singular integral equation

Since the smooth contact at the ends of the contact area, the index of the integral equation (14) is -1 [43].

The following transformations are defined to solve the integral equation numerically

$$s = x/a, \qquad \xi = r/a \tag{16a}$$

$$\varphi(r) = \frac{p(r)}{P/h} \tag{16b}$$

Using the transformations, the integral equation (14) and the equilibrium condition (15) becomes

$$\frac{1}{\pi}\int_{-1}^{1}\phi(r)\left[\frac{1}{r-s}+\frac{a}{h}k_1(s,r)\right]dr$$

$$=\frac{1}{\varphi_1}\frac{1}{R/h}\frac{\mu}{P/h}\frac{a}{h}s \tag{17}$$

$$\frac{a}{h}\int_{-1}^{1}\phi(r)dr=1 \tag{18}$$

The solution of the integral equation can be expressed as

$$\phi(r)=g(r)\sqrt{(1-r^2)} \tag{19}$$

where $g(r)$ is a continuous and bonded function in the interval [-1,1]. Applying the conventional collocation technique [43], [44] the integral equation (17) can be transformed into the equivalent system of algebraic equations as follows

$$\sum_{i=1}^{N}W_i^N\left[\frac{1}{r_i-s_k}+\frac{a}{h}k(s_k,r_i)\right]g(r_i)$$

$$=\frac{1}{\varphi_1}\frac{1}{R/h}\frac{\mu}{P/h}\frac{a}{h}s_k \quad k=1,...N+1 \tag{20}$$

and the equilibrium condition becomes

$$\frac{a}{h}\sum_{i=1}^{N}W_i^N g(r_i)=\frac{1}{\pi} \tag{21}$$

where r_i and s_k are the roots of the related Chebyshev polynomials and W_i^N is the weighting constant:

$$r_i=\cos(\frac{i\pi}{N+1}) \; i=1,...N \tag{22a}$$

$$s_k=\cos(\frac{\pi}{2}\frac{2k-1}{N+1}) \quad k=1,...N+1 \tag{22b}$$

$$W_i^N=\frac{1-r_i^2}{N+1} \tag{22c}$$

Note that there are $N+1$ equations to determine the N unknowns $g(r_i)$ in Eq. 20. It can be shown that the (N/2+1)-th equation in (20) is automatically

satisfied [44]. Thus, Eq (20) and Eq (21) give $N+1$ equations to determine the $N+1$ unknowns, which are $g(r_i)$ and a. The system of equations (20) is linear in terms of the $g(r_i)$ but the contact area a is unknown in priory and makes the problem highly nonlinear. Therefore, an iterative method is used to obtain unknowns.

After solving the contact stress and contact areas, the in-plane stress at the surface of the layer can be expressed as

$$\frac{\sigma_x(x,0)}{P/h}=\begin{cases}\varphi_2\dfrac{p(x)}{P/h}+H(x), & -a<x<a \\ H(x), & x\le-a, \; x\ge a\end{cases} \tag{23}$$

where

$$H(x)=\frac{1}{\pi}\int_{-a}^{a}p(\xi)k_2(x,\xi)d\xi \tag{24a}$$

$$k_2(x,\xi)=\int_{0}^{\infty}(M_2(\alpha)-\varphi_2)\cos\alpha\xi\cos\alpha x d\alpha \tag{24b}$$

$$M_2(\alpha)=\alpha\left[2A_1+\frac{(1+n_2^2(-3+\kappa)+\kappa)}{-1+\kappa}A_2+2A_3\right.$$

$$\left.+\frac{(1+n_2^2(-3+\kappa)+\kappa)}{-1+\kappa}A_4\right] \tag{24c}$$

$$\varphi_2=\lim_{\alpha\to\infty}M_2(\alpha) \tag{24d}$$

5. Numerical results

The contact width a/h, the contact stress $p(x)/(P/h)$, the in-plane stress component on the surface of the layer $\sigma_x(x,0)/(P/h)$ and along the y axis $\sigma_x(0,y)/(P/h)$ is computed numerically for various material and geometrical parameters such as the moving velocity $V^2\rho/\mu$, the stiffness of the Winkler foundation k_w/μ, the punch radius R/h and the external load $P/(\mu h)$. During the computation, the Poisson's ratio is taken as $\nu=0.2$ The results obtained from the study are presented in table and showed graphically in figures.

This study can be validated with those of Çömez [26] by letting the problem static, i. e. $V^2\rho/\mu \to 0$ For the comparison the contact widths obtained from the two studies are presented in Table 1. It can be observed from the table that the numerical results for the contact width under the punch is quite close to the each other.

Fig 2. shows the influences of the moving velocity $V^2\rho/\mu$ and punch radius R/h on the contact width. It may be seen from the figure that an increase in the moving velocity and punch radius tends to widen the contact width. The effect of the moving velocity on the contact width becomes certain when the punch radius gets bigger. Note that when the punch radius very small the contact width does not change with moving velocity.

The effect of the stiffness of the Winkler foundation k_w/μ and the load parameter $P/(\mu h)$ on the contact width is depicted in Fig 3. Note that the foundation becomes flexible when the stiffness of the Winkler foundation is getting smaller. As a result, the contact width tends to widen with decreasing values of the stiffness of the Winkler foundation. When the external load increases, the punch penetrates the layer more, so the contact width increases. It can be observed from the figure that the contact width close to zero when the external load decreases.

Due to the absence of the friction the contact stress and the in-plane stress distributions on the upper surface of the layer are symmetric (Figs. 4-6). The

peak values of the contact stress and the in-plane stress $\sigma_x(x,0)/(P/h)$ occurs at the symmetry plane $x = 0$. It is observed from the Figs. (4-6) the tensile peaks occur both on the upper and bottom surface of the layer. The in-plane stress gives some implications for the crack propagation problem. The axial stress $\sigma_x(0,y)/(P/h)$ is compressive in the upper part of the layer and tensile in the lower part of the layer.

The influences of the moving velocity on the contact stress and in-plane stresses are plotted in Fig 4. As the punch moves faster the peak values of the contact stress decreases but the tensile peak increase at the bottom and top surface of the layer.
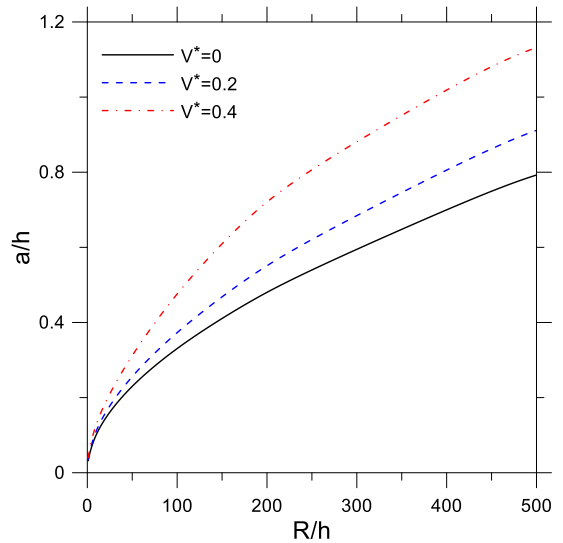

Fig 2. Variations of the contact width under the punch a/h versus punch radius R/h for some selected moving velocity $V^* = V^2\rho/\mu$ with fixed parameters $k_w/\mu = 1$, $\mu/(P/h) = 500$.

Table 1. Comparison of the contact width under the punch with that of Çömez (2013) $(R/h = 100, \mu/(P/h) = 500)$

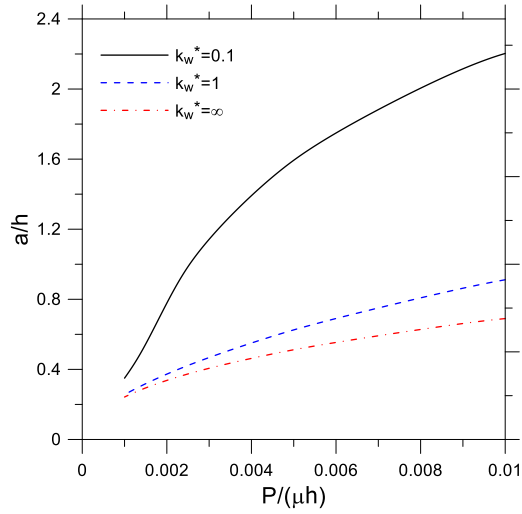| k_w/μ	a/h		
	Comez, 2013 $(\gamma h \cong 0)$	This study $(V^2\rho/\mu \cong 0)$	**Error %**
0.1	0.3725	0.3729	**0.1074**
1	0.3304	0.3306	**0.0605**
10	0.3151	0.3152	**0.03173**
∞	0.3116	0.3117	**0.03209**

Fig 3. Variations of the contact width under the punch a/h versus load parameter $P/(\mu h)$ for some selected the stiffness of the Winkler foundation $k_w^* = k_w/\mu$ with fixed parameters $V^2\rho/\mu = 0.2, R/h = 100$.
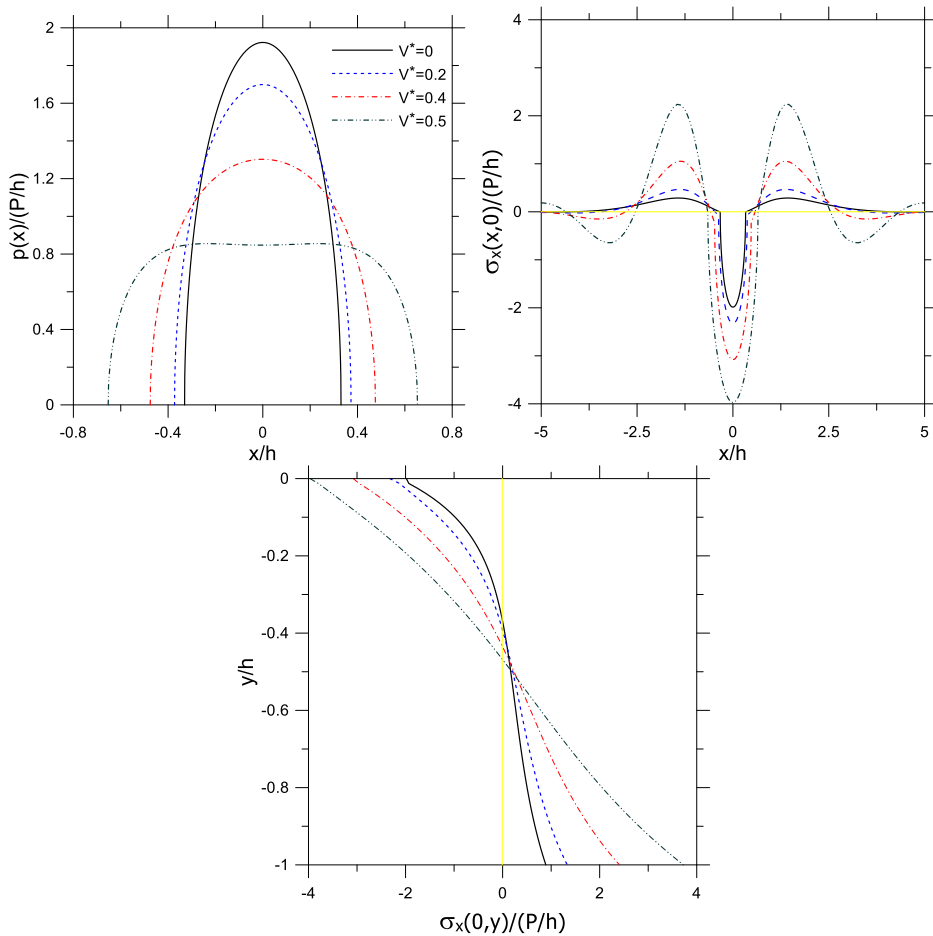

Fig 4. Contact stress and in-plane stress distributions for different values of moving velocity $V^* = V^2\rho/\mu$. $(R/h = 100, \mu/(P/h) = 500, k_w/\mu = 1)$

Fig 5. shows the effect of the stiffness of the Winkler foundation on the contact stress and the in-plane stresses. An increasing value of the stiffness of the elastic foundation tend to increase the peak values of the contact stress. As the elastic foundation is more flexible ($k_w = 0.1$) the axial stress $\sigma_x(0,y)/(P/h)$ distribution becomes linear and it matches the simple supported beam stress distribution. Also, the bigger tensile peaks occur

when the foundation is softer, and the layer become susceptible to a possible cracking problem.

The effect of punch radius on the contact stress and the in-plane stresses is depicted in Fig 6. With the increasing values of the punch radius the peak values of contact stress and in-plane stresses decrease. The peak values occur when the punch radius $R/h = 10$. This is because when the punch radius smaller the loading approaches to the point load case.
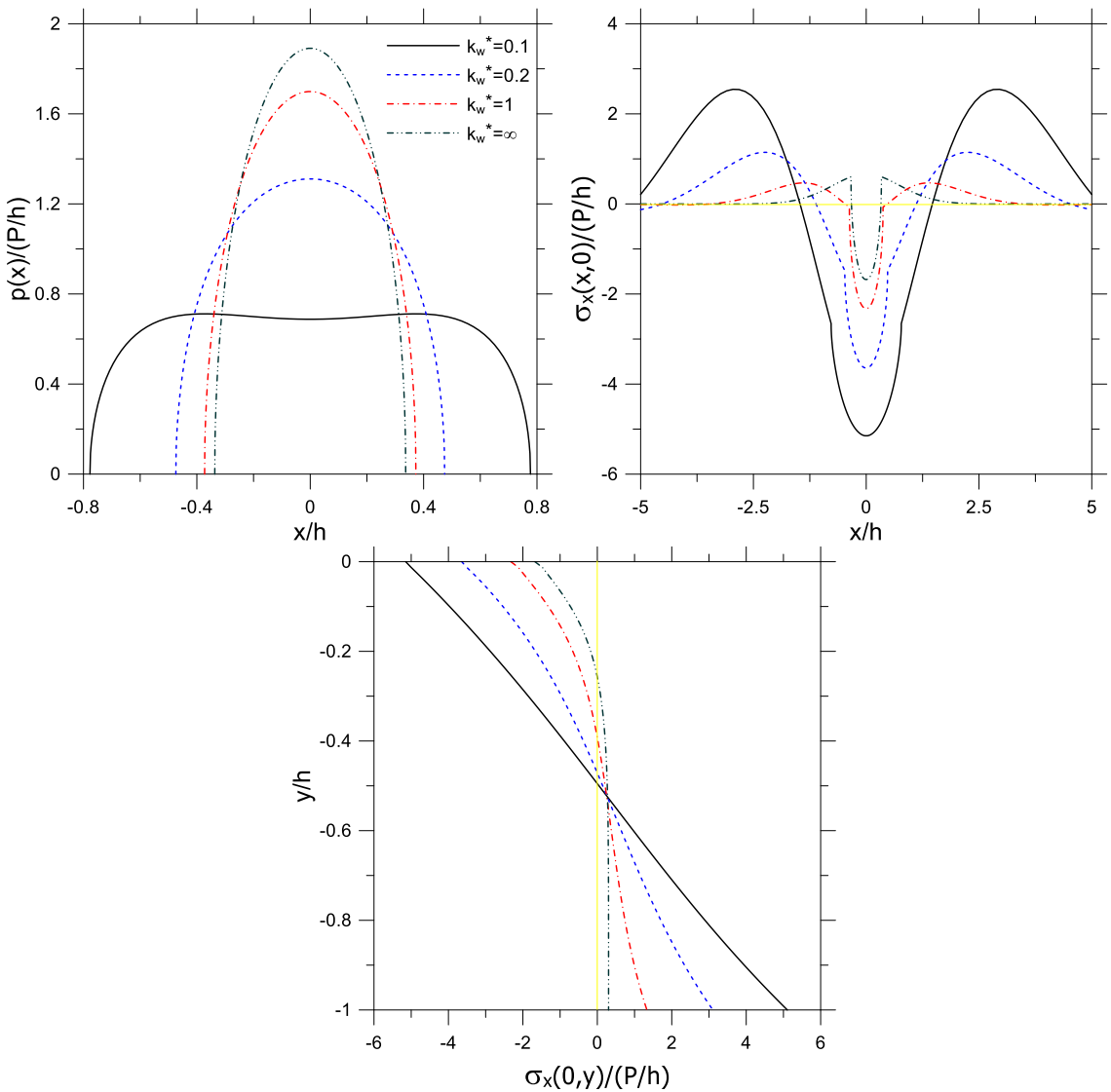

Fig 5. Contact stress and in-plane stress distributions for different values of the stiffness of Winkler foundation $k_w^* = k_w/\mu$. ($V^2\rho/\mu = 0.2, R/h = 100, \mu/(P/h) = 500$)
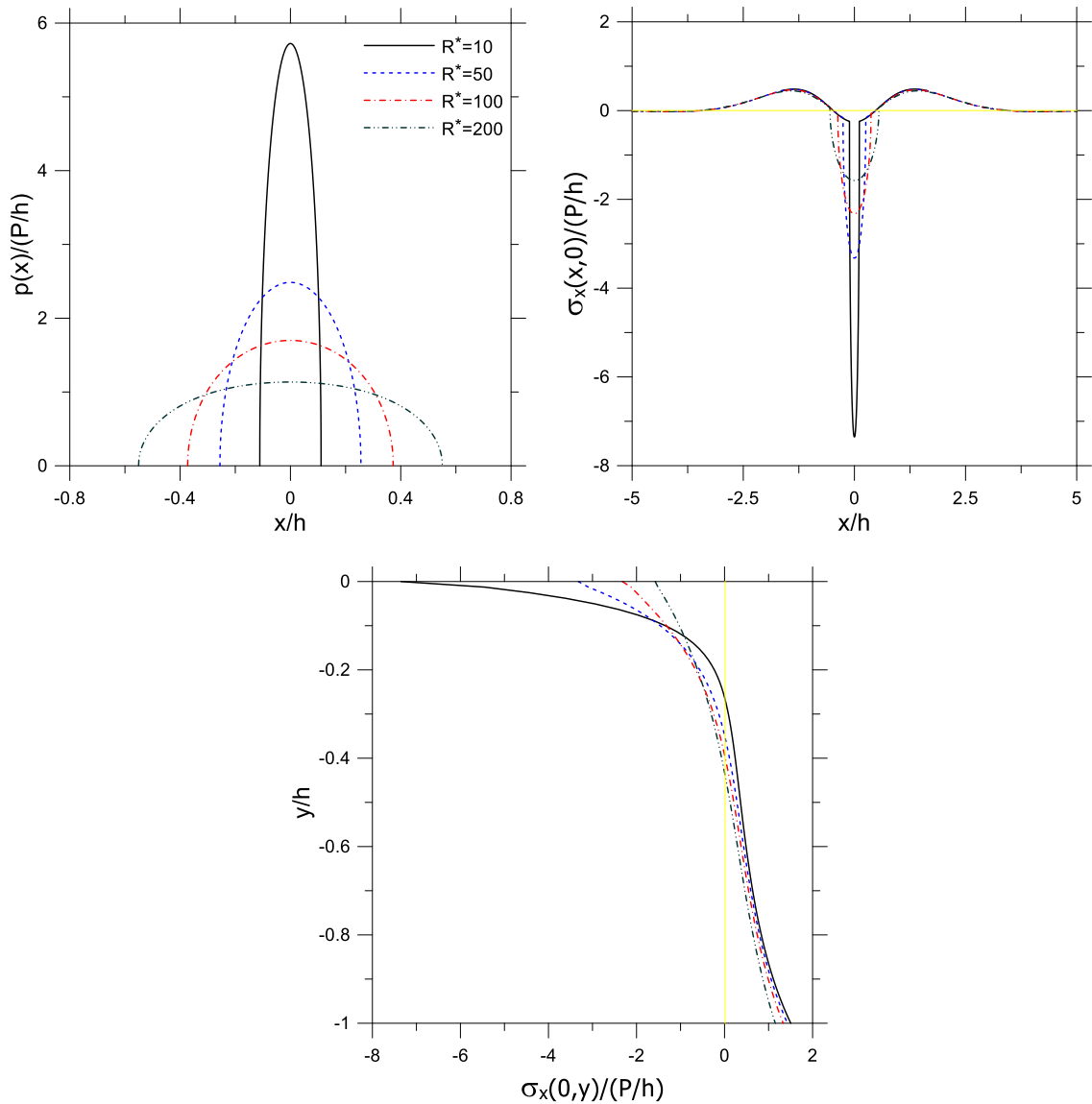
Fig 6. Contact stress and in-plane stress distributions for different values of the punch radius $R^* = R/h$. $(V^2 \rho / \mu = 0.2, k_w / \mu = 1, \mu / (P/h) = 500)$

6. Conclusion

This study is concerned with the frictionless moving contact problem between the rigid cylindrical punch and a homogeneous layer lying on a Winkler foundation. The plane contact problem is solved in the framework of linear elasticity and using Fourier integral transform technique. The effect of the moving velocity, the stiffness of the Winkler foundation, punch radius and external load on the contact area and distributions of the stress components are given.

The contact width tends to widen when the punch moves faster and the Winkler foundation getting flexible. The contact width also increases with increasing values of external load and punch radius. The contact stress increases with the increasing stiffness of the Winkler foundation and decreasing values of moving velocity. The in-plane stress distributions imply that the tensile peaks occur at the bottom and on the surface of the layer which should be considered for crack propagation problem. The tensile stress tends to increase when

the punch moves faster, and the Winkler foundation becomes flexible.

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Appendix A

Expressions of $k_1(x,\xi)$ and ϕ_1 appearing in (14) are given below:

$$k_1(x,\xi) = \frac{1}{\phi_1} \int_0^\infty M_1(\alpha) \sin \alpha(\xi - x) d\alpha \tag{A1}$$

$$M_1(\alpha) = -((-1+n_1^2)n_2(1+n_1^2)^2(-1+e^{2n_1h\alpha})(1+e^{2n_2h\alpha})\alpha$$
$$+n_2(-1+e^{2n_2h\alpha})((-1+n_1^2)(-1+e^{2n_1h\alpha})k_w/\mu$$
$$-4n_1(1+e^{2n_1h\alpha})\alpha)))/\Delta \tag{A2}$$

$$\Delta = ((1+n_1^2)^4(-1+e^{2n_1h\alpha})(-1+e^{2n_2h\alpha})\alpha$$
$$-4n_1n_2^2(-1+e^{2n_2h\alpha})((-1+n_1^2)(1+e^{2n_1h\alpha})k_w/\mu$$
$$-4n_1(-1+e^{2n_1h\alpha})\alpha)$$
$$+(1+n_1^2)^2n_2((-1+n_1^2)(-1+e^{2n_1h\alpha})(1+e^{2n_2h\alpha})k_w/\mu \tag{A3}$$
$$-8n_1(1+e^{2n_1h\alpha}+e^{2n_2h\alpha}-4e^{(n_1+n_2)h\alpha}+e^{2(n_1+n_2)h\alpha})\alpha)))$$

$$\varphi_1 = \lim_{\alpha\to\infty} M_1(\alpha) \tag{A4}$$