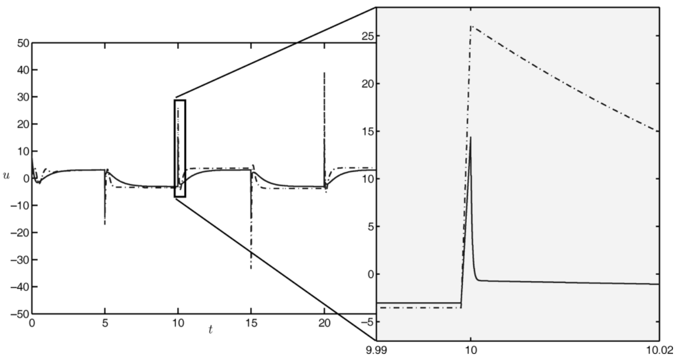

Fig. 8. Control action during tracking for the proposed controller (solid line) and the λ-tracker (dash-dotted line).

the reference. The proposed controller does not suffer this problem resulting in transient profiles that do not change over time and in far less actuator exploitation.

V. Conclusions

We have proposed an adaptive scheme, based on the Lyapunov derivative, which overcomes some limitations of existing high-gain controllers. An *a priori* upper bound for the transient can be determined and the scheme can be used jointly with standard optimal control techniques guaranteeing optimality if the model matches the system exactly. This in practice means that the control is nearly optimal under accurate modeling and when the system is close to 0. Extensions of the technique are expected in several directions. For instance it can be combined with Razumikhin techniques [5] to deal with functional differential equations. We may also consider the case in which $\kappa(t)$ is a matrix. Interesting results are expected in this direction but no formal results were available at the moment of writing this note. We did not consider explicit bounds for the control actions but we conjecture that these can be included in the scheme. An open problem is the output feedback one. We think that under standard minimum-phase and relative degree assumptions the extension is possible.

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From Continuous-Time Design to Sampled-Data Design of Observers

Iasson Karafyllis and Costas Kravaris

***Abstract*—In this work, a sampled-data nonlinear observer is designed using a continuous-time design coupled with an inter-sample output predictor. The proposed sampled-data observer is a hybrid system. It is shown that under certain conditions, the robustness properties of the continuous-time design are inherited by the sampled-data design, as long as the sampling period is not too large. The approach is applied to linear systems and to triangular globally Lipschitz systems.**

***Index Terms*—Input-to-output stability, nonlinear observers, sampled-data observers.**

I. Introduction

The problem of designing sampled-data nonlinear observers has attracted a lot of attention in the literature. Continuous-time nonlinear observer designs [11], [17], [19], [21], [22] are meant to be used only for very small sampling periods, whereas their potential "redesign" for the purpose of digital implementation, even though straightforward and popular for linear systems, poses significant challenges in the nonlinear case. For this reason, the main line of attack has been through the use of an exact or approximate discrete-time description of the dynamics as the starting point for observer design [2], [4], [6]–[8], [10], [13], [18], [22]. This is a reasonable point of view, but faces two important difficulties:

(i) from the moment that the continuous-time system description is abandoned and is substituted by a discrete-time description, the inter-sample dynamic behavior is lost;

(ii) any errors in the sampling schedule get transferred into errors in the discrete-time description.

As a consequence, available design methods (i) do not provide an explicit estimate of the error in between two consecutive sampling times and (ii) do not account for perturbations of the sampling schedule.

A hybrid observer design approach was recently proposed in [5]. In the present work, our proposed sampled-data observer will also be

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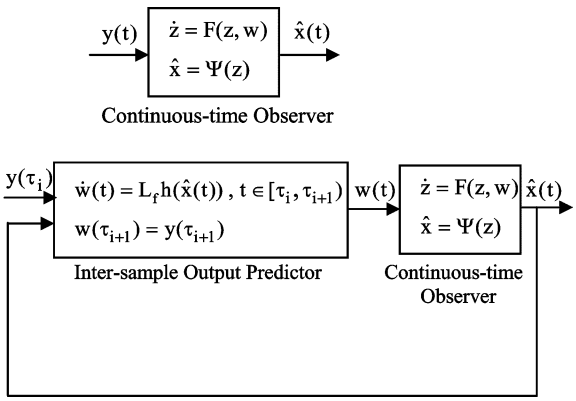

Fig. 1. Continuous-time observer (1.2) (top) versus sampled-data observer (1.3) (bottom).

a hybrid system; however, it will directly emerge from a continuous-time design of a nonlinear observer. Hybrid observer design is also accomplished by the continuous-discrete observer design methodologies, which take into account the discrete-time nature of the measurements (see [1], [9], [12], and [20]).

Consider a single-output continuous-time system

$$\begin{aligned} \dot{x} &= f(x), \quad x \in \Re^n \\ y &= h(x), \quad y \in \Re \end{aligned} \tag{1.1}$$

where $f \in C^1(\Re^n;\Re^n)$, $h \in C^2(\Re^n;\Re)$ with $f(0)=0$, $h(0)=0$. For this system, suppose that a continuous-time observer design is available

$$\begin{aligned} \dot{z} &= F(z,y), \quad z \in \Re^k \\ \hat{x} &= \Psi(z), \quad \hat{x} \in \Re^n \end{aligned} \tag{1.2}$$

where $F \in C^1(\Re^k \times \Re;\Re^k)$, $\Psi \in C^1(\Re^k;\Re^n)$ with $F(0,0)=0$, $\Psi(0)=0$. The question is whether this design would still be useful in the presence of sampled measurements $y(ih)$, $i=0,1,\ldots$, where h is the sampling period, or more generally, at some countable set of time instants $\pi = \{\tau_i\}_{i=0}^{\infty}$, not necessarily uniformly spaced, but satisfying $0 < \tau_{i+1} - \tau_i \le r$ for all $i=0,1,\ldots$ for some $r>0$. The present work has been motivated by the intuitive expectation that a continuous-time nonlinear observer design would still be useful in the presence of "medium-size" sampling periods, as long as special care is taken in the time-interval between measurements. Instead of holding the most recent measurement (zero-order hold), in the present paper, we propose a sampled-data observer consisting of the continuous-time observer, coupled with an output predictor for the time interval between two consecutive measurements (see Fig. 1)

$$\begin{aligned} \dot{z}(t) &= F(z(t),w(t)), \quad t \in [\tau_i,\tau_{i+1}) \\ \dot{w}(t) &= L_f h(\Psi(z(t))), \quad t \in [\tau_i,\tau_{i+1}) \\ w(\tau_{i+1}) &= y(\tau_{i+1}) \\ (z(t),w(t)) &\in \Re^k \times \Re \quad \hat{x}(t) = \Psi(z(t)), \quad \hat{x} \in \Re^n. \end{aligned} \tag{1.3}$$

Fig. 1 depicts the structure of the sampled-data observer (1.3) compared to the continuous-time observer (1.2).

It is important to point out that the entire system (1.1) with (1.3) is a hybrid system, which does not satisfy the classical semigroup property. However, the weak semigroup property holds (see [14] and [15]) and consequently it can be analyzed using the recent results in [14]–[16]. The main result of the present paper is that the properties of the observer (1.2) under continuous measurement are inherited by the observer (1.3) under arbitrary sampling schedules, as long as the sampling period is not too large. As far as we know, our results are new even in the linear case.

It would be interesting to compare the proposed approach to the continuous-discrete observer design methodologies proposed of [1], [9], [12], and [20]. The continuous-discrete observer design approach utilizes an open-loop continuous-time observer $\dot{z} = f(z)$ with a reset map G acting at the sampling times $z(\tau_{i+1}) = G\left(\lim_{t\to\tau_{i+1}^-} z(t), y(\tau_{i+1})\right)$. The reset map G depends heavily on the sampling schedule and therefore the continuous-discrete observer design does not guarantee convergence for arbitrary sampling schedules.

Notations: Throughout this paper, we adopt the notations of [16] and the following notations:

* By $B = \mathrm{diag}(b_1,\ldots,b_n)$ we denote the diagonal matrix $B \in \Re^{n\times n}$ with $b_1,\ldots,b_n$ in its diagonal.
* Let $D \subseteq \Re^l$ and $I \subseteq \Re^+$ be an interval. By $L^\infty_{\mathrm{loc}}(I;D)$ we denote the class of all Lebesgue measurable and locally bounded mappings $d:\Re^+ \to D$. Notice that $\sup_{\tau\in[0,t]}|d(\tau)|$ denotes the actual supremum of d on $[0,t]$.
* Let $f \in C^1(\Re^n;\Re^n)$, $h \in C^1(\Re^n;\Re)$. By $L_f h(x) := \nabla h(x) f(x)$ we denote the Lie derivative of the function $h \in C^1(\Re^n;\Re)$ along the vector field $f \in C^1(\Re^n;\Re^n)$.

II. Basic Notions

In the present work, we study systems of the form (1.1) under the following hypotheses:

(**H**) System (1.1) is Forward Complete.

It should be noted that hypothesis (H) in conjunction with the main result in [3] and characterizations of the notion of Robust Forward Completeness in [14], implies the existence of functions $\mu \in K^+$ and $a \in K_\infty$ such that for every $x_0 \in \Re^n$ the solution $x(t)$ of (1.1) with initial condition $x(0)=x_0$ satisfies

$$|x(t)| \le \mu(t) a(|x_0|), \quad \forall t \ge 0. \tag{2.1}$$

The following definition of the notion of robust observer for system (1.1) with respect to measurement errors is crucial to the development of the main results of the present work.

Definition 2.1: Consider the following system:

$$\begin{aligned} \dot{z} &= F(z,y), \quad z \in \Re^k \\ \hat{x} &= \Psi(z), \quad \hat{x} \in \Re^n \end{aligned} \tag{2.2}$$

where $F \in C^1(\Re^k \times \Re;\Re^k)$, $\Psi \in C^1(\Re^k;\Re^n)$ with $F(0,0)=0$, $\Psi(0)=0$. System (2.2) is called a **robust observer** for system (1.1) with respect to measurement errors, if the following conditions are met:

i) there exist functions $\sigma \in KL$, $\gamma, p \in N$, $\mu \in K^+$ and $a \in K_\infty$ such that for every $(x_0,z_0) \in \Re^n \times \Re^k$ and $v \in L^\infty_{\mathrm{loc}}(\Re^+;\Re)$, the solution $(x(t),z(t))$ of

$$\begin{aligned} \dot{x} &= f(x) \\ \dot{z} &= F(z,h(x)+v), \quad \hat{x} = \Psi(z) \end{aligned} \tag{2.3}$$

with initial condition $(x(0),z(0))=(x_0,z_0)$ corresponding to $v \in L^\infty_{\mathrm{loc}}(\Re^+;\Re)$ satisfies the following estimates for all $t \ge 0$:

$$|\hat{x}(t)-x(t)| \le \sigma(|x_0|+|z_0|,t) + \sup_{0\le\tau\le t}\gamma(|v(\tau)|) \tag{2.4a}$$

$$|z(t)| \le \mu(t)\left[a(|x_0|+|z_0|) + \sup_{0\le\tau\le t} p(|v(\tau)|)\right] \tag{2.4b}$$

ii) for every $x_0 \in \Re^n$ there exists $z_0 \in \Re^k$ such that the solution $(x(t),z(t))$ of (2.3) with initial condition $(x(0),z(0))=(x_0,z_0)$ corresponding to $v \equiv 0$, satisfies $x(t)=\Psi(z(t))$ for all $t \ge 0$.

We next define the corresponding notion of robust sampled-data observer. Notice that contrary to usual observers for which the output signal $y(t)$ of system (1.1) is available on-line, a sampled-data observer uses only the output values $y(\tau_i)$ at certain time instances $\pi = \{\tau_i\}_{i=0}^{\infty}$

with $0 < \tau_{i+1} - \tau_i \leq r$ for all $i = 0, 1, \ldots$. The number $r > 0$ is called the *upper diameter* of the sampling partition.

Definition 2.2: The system

$$\begin{aligned} \dot{z}(t) &= g(z(t), z(\tau_i), y(\tau_i)), \quad t \in [\tau_i, \tau_{i+1}) \\ z(\tau_{i+1}) &= G\left(\lim_{t \to \tau_{i+1}^-} z(t), y(\tau_{i+1})\right), \quad \hat{x}(t) = \Psi(z(t)) \end{aligned} \tag{2.5}$$

where $g \in C^1(\Re^k \times \Re^k \times \Re; \Re^k)$, $G \in C^0(\Re^k \times \Re; \Re^k)$, $\Psi \in C^1(\Re^k; \Re^n)$ with $g(0,0,0) = 0$, $G(0,0) = 0$, $\Psi(0) = 0$, is called a **robust sampled-data observer** for (1.1) with respect to measurement errors, if the following conditions are met.

i) There exist functions $\sigma \in KL, \gamma, p \in N, \mu \in K^+$ and $a \in K_\infty$ such that for every $(x_0, z_0, d, v) \in \Re^n \times \Re^k \times L^\infty_{\mathrm{loc}}(\Re^+; \Re^+) \times L^\infty_{\mathrm{loc}}(\Re^+; \Re)$, the solution $(x(t), z(t))$ of

$$\begin{aligned} \dot{x}(t) &= f(x(t)) \\ \dot{z}(t) &= g(z(t), z(\tau_i), h(x(\tau_i)) + v(\tau_i)), \quad t \in [\tau_i, \tau_{i+1}) \\ z(\tau_{i+1}) &= G\left(\lim_{t \to \tau_{i+1}^-} z(t), h(x(\tau_{i+1})) + v(\tau_{i+1})\right) \\ \tau_{i+1} &= \tau_i + r\exp(-d(\tau_i)), \quad \hat{x}(t) = \Psi(z(t)) \end{aligned} \tag{2.6}$$

with initial condition $(x(0), z(0)) = (x_0, z_0)$ corresponding to $d \in L^\infty_{\mathrm{loc}}(\Re^+; \Re^+)$, $v \in L^\infty_{\mathrm{loc}}(\Re^+; \Re)$ satisfies the following estimates for all $t \geq 0$:

$$|\hat{x}(t) - x(t)| \leq \sigma\left(|x_0| + |z_0|, t\right) + \sup_{0 \leq \tau \leq t} \gamma(|v(\tau)|) \tag{2.7a}$$

$$|z(t)| \leq \mu(t)\left[a\left(|x_0| + |z_0|\right) + \sup_{0 \leq \tau \leq t} p(|v(\tau)|)\right]. \tag{2.7b}$$

ii) For every $x_0 \in \Re^n$ there exists $z_0 \in \Re^k$ such that for all $d \in L^\infty_{\mathrm{loc}}(\Re^+; \Re^+)$ the solution $(x(t), z(t))$ of (2.6) with initial condition $(x(0), z(0)) = (x_0, z_0)$ corresponding to $d \in L^\infty_{\mathrm{loc}}(\Re^+; \Re^+)$ and $v \equiv 0$, satisfies $x(t) = \Psi(z(t))$ for all $t \geq 0$.

III. Main Results

We are now in a position to state our main result.

Theorem 3.1: Consider system (1.1) under hypothesis (H) and suppose that system (2.2) is a robust observer for system (1.1) with respect to measurement errors. Moreover, suppose that there exists a constant $C \geq 0$ and a function $\bar{\sigma} \in KL$ such that for every $(x_0, z_0) \in \Re^n \times \Re^k$ and $v \in L^\infty_{\mathrm{loc}}(\Re^+; \Re)$, the solution $(x(t), z(t))$ of (2.3) with initial condition $(x(0), z(0)) = (x_0, z_0)$ corresponding to $v \in L^\infty_{\mathrm{loc}}(\Re^+; \Re)$ satisfies the following estimate for all $t \geq 0$:

$$|L_f h(\Psi(z(t))) - L_f h(x(t))| \leq \bar{\sigma}\left(|x_0| + |z_0|, t\right) + C \sup_{0 \leq \tau \leq t} |v(\tau)|. \tag{3.1}$$

Finally, suppose that $rC < 1$, where $r > 0$ is the upper diameter of the sampling partition and $C \geq 0$ is the constant involved in estimate (3.1). Then (1.3) is a robust sampled-data observer for system (1.1) with respect to measurement errors.

Proof: By virtue of Definition 2.2, it suffices to show that the following hybrid system:

$$\begin{aligned} \dot{x}(t) &= f(x(t)) \\ \dot{z}(t) &= F(z(t), w(t)), \quad t \in [\tau_i, \tau_{i+1}) \\ \dot{w}(t) &= L_f h(\Psi(z(t))), \quad t \in [\tau_i, \tau_{i+1}) \\ w(\tau_{i+1}) &= h(x(\tau_{i+1})) + v(\tau_{i+1}) \\ \tau_{i+1} &= \tau_i + r\exp(-d(\tau_i)) \\ Y(t) &= \Psi(z(t)) - x(t) \end{aligned} \tag{3.2}$$

satisfies the uniform input-to-output stability property from the input $v \in L^\infty_{\mathrm{loc}}(\Re^+; \Re)$, i.e., it suffices to show that system (3.2) is robustly forward complete from the input $v \in L^\infty_{\mathrm{loc}}(\Re^+; \Re)$, $0 \in \Re^n \times \Re^k \times \Re$ is a robust equilibrium point from the input $v \in L^\infty_{\mathrm{loc}}(\Re^+; \Re)$ (see [14]–[16]) and that there exist functions $\sigma \in KL$, $\mu \in K^+$, $\tilde{\gamma} \in N$ and $a \in K_\infty$ such that for every $(x_0, z_0, w_0, d, v) \in \Re^n \times \Re^k \times \Re \times L^\infty_{\mathrm{loc}}(\Re^+; \Re^+) \times L^\infty_{\mathrm{loc}}(\Re^+; \Re)$ the solution $(x(t), z(t), w(t))$ of (3.2) with initial condition $(x(0), z(0), w(0)) = (x_0, z_0, w_0)$ corresponding to $d \in L^\infty_{\mathrm{loc}}(\Re^+; \Re^+)$, $v \in L^\infty_{\mathrm{loc}}(\Re^+; \Re)$ satisfies for all $t \geq 0$

$$|Y(t)| \leq \sigma\left(|x_0| + |z_0| + |w_0|, t\right) + \sup_{0 \leq \tau \leq t} \tilde{\gamma}(|v(\tau)|). \tag{3.3}$$

The reader should notice that for every $x_0 \in \Re^n$ there exists $(z_0, w_0) \in \Re^k \times \Re$ with $w_0 = h(x_0)$ such that for all $d \in L^\infty_{\mathrm{loc}}(\Re^+; \Re^+)$ the solution $(x(t), z(t), w(t))$ of (3.2) with initial condition $(x(0), z(0), w(0)) = (x_0, z_0, w_0)$ corresponding to $d \in L^\infty_{\mathrm{loc}}(\Re^+; \Re^+)$, $v \equiv 0$, satisfies $x(t) = \Psi(z(t))$ for all $t \geq 0$.

Since system (2.2) is a robust observer for system (1.1) with respect to measurement errors and since hypothesis (H) holds, it follows from (2.1), (2.4a) and (2.4b) and (3.1) that for every $(x_0, z_0, w_0, d) \in \Re^n \times \Re^k \times \Re \times L^\infty_{\mathrm{loc}}(\Re^+; \Re^+)$ the solution $(x(t), z(t), w(t)) \in \Re^n \times \Re^k \times \Re$ of (3.2) with initial condition $(x(0), z(0), w(0)) = (x_0, z_0, w_0)$ satisfies the following estimates for all $t \in [0, t_{\max})$ (see (3.4)–(3.6), as shown at the bottom of the page) for appropriate functions $\sigma, \bar{\sigma} \in KL$, $\gamma, p \in N$, $\mu \in K^+$ and $a \in K_\infty$, where $t_{\max} \in (0, +\infty]$ is the maximal existence time of the solution. Let $\pi = \{\tau_i\}_{i=0}^\infty$ be the partition of $\Re^+$ generated by the recursive formula $\tau_{i+1} = \tau_i + r\exp(-d(\tau_i))$ with $\tau_0 = 0$. Taking into account that $w(\tau_i) = y(\tau_i) + v(\tau_i)$ for all $\tau_i \in \pi$ with $i \geq 1$ and that $\dot{w}(t) = L_f h(\Psi(z(t)))$, $t \in [\tau_i, \tau_{i+1})$, we get for all $t \in [\tau_i, \tau_{i+1}) \cap [0, t_{\max})$ with $i \geq 1$

$$|w(t) - h(x(t))| = \left| v(\tau_i) + \int_{\tau_i}^t L_f h(\Psi(z(s)))ds - \int_{\tau_i}^t L_f h(x(s))ds \right|.$$

$$|Y(t)| \leq \sigma\left(|x_0| + |z_0|, t\right) + \sup_{0 \leq \tau \leq t} \gamma(|w(\tau) - h(x(\tau))|) \tag{3.4}$$

$$|L_f h(\Psi(z(t))) - L_f h(x(t))| \leq \bar{\sigma}\left(|x_0| + |z_0|, t\right) + C \sup_{0 \leq \tau \leq t} |w(\tau) - h(x(\tau))| \tag{3.5}$$

$$|z(t)| + |x(t)| \leq \mu(t)\left[a\left(|x_0| + |z_0|\right) + \sup_{0 \leq \tau \leq t} p(|w(\tau) - h(x(\tau))|)\right] \tag{3.6}$$

The above equality in conjunction with the fact that $0 < \tau_{i+1} - \tau_i \leq r$ and estimate (3.5) implies for all $t \in [\tau_i, \tau_{i+1}) \cap [0, t_{\max})$ with $i \geq 1$

$$\begin{aligned} |w(t) - h(x(t))| &\leq r \sup_{\tau_i \leq s \leq t} |L_f h(\Psi(z(s))) - L_f h(x(s))| \\ &\quad + |v(\tau_i)| \\ &\leq r\bar{\sigma}(|x_0| + |z_0|, \tau_i) \\ &\quad + rC \sup_{0 \leq \tau \leq t} |w(\tau) - h(x(\tau))| + |v(\tau_i)| \\ &\leq \sigma_1(|x_0| + |z_0|, t) + rC \\ &\quad \times \sup_{0 \leq \tau \leq t} |w(\tau) - h(x(\tau))| + \sup_{0 \leq \tau \leq t} |v(\tau)| \end{aligned} \tag{3.7}$$

where $\sigma_1(s,t) := \bar{\sigma}(s, t-r)$ for $t \geq r$ and $\sigma_1(s,t) := \exp(r-t)\bar{\sigma}(s,0)$ for $t < r$. On the other hand taking into account that $w(\tau_0) = w(0) = w_0$ and that $\dot{w}(t) = L_f h(\Psi(z(t))), t \in [0, \tau_1)$, we get for all $t \in [0, \tau_1) \cap [0, t_{\max})$

$$|w(t) - h(x(t))| \leq |w_0 - h(x_0)| + \left| \int_0^t L_f h(\Psi(z(s)))ds - \int_0^t L_f h(x(s))ds \right|.$$

Continuity of h in conjunction with the fact that $h(0) = 0$ implies the existence of a function $\rho \in K_\infty$ such that

$$|w - h(x)| \leq \rho(|w| + |x|), \quad \forall (x, w) \in \Re^n \times \Re. \tag{3.8}$$

The previous inequalities in conjunction with the fact that $0 < \tau_{i+1} - \tau_i \leq r$ and estimate (3.5) imply for all $t \in [0, \tau_1) \cap [0, t_{\max})$

$$\begin{aligned} |w(t) - h(x(t))| &\leq \rho(|w_0| + |x_0|) \\ &\quad + r \sup_{0 \leq s \leq t} |L_f h(\Psi(z(s))) - L_f h(x(s))| \\ &\leq \rho(|w_0| + |x_0|) + r\bar{\sigma}(|x_0| + |z_0|, 0) \\ &\quad + rC \sup_{0 \leq \tau \leq t} |w(\tau) - h(x(\tau))| \\ &\leq \sigma_2(|x_0| + |z_0| + |w_0|, t) \\ &\quad + rC \sup_{0 \leq \tau \leq t} |w(\tau) - h(x(\tau))| \end{aligned} \tag{3.9}$$

for appropriate $\sigma_2 \in KL$. Combining estimates (3.7) and (3.9), we conclude that the following estimate holds for all $t \in [0, t_{\max})$:

$$\begin{aligned} |w(t) - h(x(t))| &\leq \sigma_2(|x_0| + |z_0| + |w_0|, t) \\ &\quad + rC \sup_{0 \leq \tau \leq t} |w(\tau) - h(x(\tau))| + \sup_{0 \leq \tau \leq t} |v(\tau)|. \end{aligned} \tag{3.10}$$

Using (3.8), (3.10), and (2.1), we obtain for all $t \in [0, t_{\max})$:

$$\begin{aligned} |w(t)| &\leq \rho\left(\mu^2(t)\right) + \rho\left((a(|x_0|))^2\right) \\ &\quad + \sigma_2(|x_0| + |z_0| + |w_0|, 0) \\ &\quad + rC \sup_{0 \leq \tau \leq t} |w(\tau) - h(x(\tau))| + \sup_{0 \leq \tau \leq t} |v(\tau)|. \end{aligned}$$

The above inequality in conjunction with (3.6) gives for all $t \in [0, t_{\max})$:

$$\begin{aligned} |x(t)| + |z(t)| + |w(t)| &\leq \phi(t) + \tilde{a}(|x_0| + |z_0| + |w_0|) \\ &\quad + \sup_{0 \leq \tau \leq t} q(|w(\tau) - h(x(\tau))|) + \sup_{0 \leq \tau \leq t} |v(\tau)| \end{aligned} \tag{3.11}$$

where $q(s) := (p(s))^2 + rCs$, $\phi(t) := \mu^2(t) + \rho\left(\mu^2(t)\right)$, and $\tilde{a}(s) := \rho\left((a(s))^2\right) + (a(s))^2 + \sigma_2(s, 0)$. Using (3.10) and the fact that $rC < 1$, we obtain

$$\begin{aligned} \sup_{0 \leq \tau < t_{\max}} |w(\tau) - h(x(\tau))| &\leq \frac{1}{1 - rC}\sigma_2\Bigg(|x_0| + |z_0| \\ &\quad + |w_0|, 0\Bigg) + \frac{1}{1 - rC} \sup_{0 \leq \tau \leq t_{\max}} |v(\tau)|. \end{aligned} \tag{3.12}$$

Exploiting (2.1), (3.6), (3.8), and (3.12) and the Boundedness-Implies-Continuation property for system (3.2) we may conclude that $t_{\max} = +\infty$. It follows that all the above inequalities hold for all $t \geq 0$. Moreover, taking into account that system (3.2) is autonomous, we may utilize (2.1), (3.6), (3.8) and (3.12) in order to show that $0 \in \Re^n \times \Re^k \times \Re$ is a robust equilibrium point from the input $v \in L^\infty_{\text{loc}}(\Re^+; \Re)$, i.e., for every $\varepsilon > 0$, $T \in \Re^+$ there exists $\delta := \delta(\varepsilon, T) > 0$ such that for all $(x_0, z_0, w_0, d, v) \in \Re^n \times \Re^k \times \Re \times L^\infty_{\text{loc}}(\Re^+; \Re^+) \times L^\infty_{\text{loc}}(\Re^+; \Re)$, with $|x_0| + |z_0| + |w_0| \sup_{t \geq 0} |v(t)| < \delta$ it holds that the solution $(x(t), z(t), w(t))$ of (3.2) with initial condition $(x(0), z(0), w(0)) = (x_0, z_0, w_0)$ corresponding to $(d, v) \in L^\infty_{\text{loc}}(\Re^+; \Re^+) \times L^\infty_{\text{loc}}(\Re^+; \Re)$ exists for all $t \in [t_0, t_0 + T]$ and

$$\sup\{|(x(t), z(t), w(t))| : t \in [0, T]\} < \varepsilon.$$

Using (3.10) and (3.11) [16, Theor. 3.1] in conjunction with [16, Remarks 3.2 and 3.6], inequality $rC < 1$ and the fact that system (3.2) is autonomous, we conclude that system (3.2) is robustly forward complete and there exist functions $\sigma_3 \in KL$, $\bar{\gamma} \in N$ such that for every $(x_0, z_0, w_0, d, v) \in \Re^n \times \Re^k \times \Re \times L^\infty_{\text{loc}}(\Re^+; \Re^+) \times L^\infty_{\text{loc}}(\Re^+; \Re)$ the solution $(x(t), z(t), w(t))$ of (3.2) with initial condition $(x(0), z(0), w(0)) = (x_0, z_0, w_0)$ corresponding to $d \in L^\infty_{\text{loc}}(\Re^+; \Re^+)$, $v \in L^\infty_{\text{loc}}(\Re^+; \Re)$ satisfies for all $t \geq 0$

$$\begin{aligned} |w(t) - h(x(t))| &\leq \sigma_3(|x_0| + |z_0| + |w_0|, t) \\ &\quad + \sup_{0 \leq \tau \leq t} \bar{\gamma}(|v(\tau)|). \end{aligned} \tag{3.13}$$

Using (3.4), (3.11), and (3.13) [16, Theor. 3.1] in conjunction with [16, Remark 3.2] and the fact that system (3.2) is autonomous, we conclude that there exist functions $\sigma \in KL$, $\tilde{\gamma} \in N$ such that for every $(x_0, z_0, w_0) \in \Re^n \times \Re^k \times \Re$, $(d, v) \in L^\infty_{\text{loc}}(\Re^+; \Re^+) \times L^\infty_{\text{loc}}(\Re^+; \Re)$ the solution $(x(t), z(t), w(t))$ of (3.2) with initial condition $(x(0), z(0), w(0)) = (x_0, z_0, w_0)$ corresponding to $(d, v) \in L^\infty_{\text{loc}}(\Re^+; \Re^+) \times L^\infty_{\text{loc}}(\Re^+; \Re)$ satisfies (3.3). The proof is complete. ◁

IV. Applications

In this section, we present applications of Theorem 3.1 to certain classes of systems.

A. Linear Detectable Systems

Consider the linear detectable system

$$\dot{x} = Ax, \quad y = c'x, \quad x \in \Re^n, \; y \in \Re. \tag{4.1}$$

Detectability implies that there exists a vector $k \in \Re^n$ such that the matrix $(A + kc')$ is Hurwitz. Consequently, there exists a positive definite symmetric matrix $P \in \Re^{n \times n}$ and constants $\mu, \gamma > 0$ such that

$$\begin{aligned} &x'P(A + kc')x + x'(A' + ck')Px - 2x'Pkv \\ &\quad \leq -2\mu x'Px + \gamma |v|^2 \\ &\forall (x, v) \in \Re^n \times \Re. \end{aligned} \tag{4.2}$$

It follows that for every $(x_0, z_0, v) \in \Re^n \times \Re^n \times L^\infty_{\text{loc}}(\Re^+;\Re)$, the solution of (4.1) with

$$\dot{z} = Az + k(c'z - y - v) \tag{4.3}$$

and initial condition $(x(0), z(0)) = (x_0, z_0) \in \Re^n \times \Re^n$ corresponding to $v \in L^\infty_{\text{loc}}(\Re^+;\Re)$ satisfies the estimates for all $t \geq 0$

$$|z(t) - x(t)| \leq \exp(-\mu t)\sqrt{\frac{K_2}{K_1}}|z_0 - x_0| + \sqrt{\frac{\gamma}{2\mu K_1}}\sup_{0\leq\tau\leq t}|v(\tau)| \tag{4.4}$$

$$|c'Az(t) - c'Ax(t)| \leq |c'A|\exp(-\mu t)\sqrt{\frac{K_2}{K_1}}|z_0 - x_0| + |c'A|\sqrt{\frac{\gamma}{2\mu K_1}}\sup_{0\leq\tau\leq t}|v(\tau)| \tag{4.5}$$

where $K_1, K_2 > 0$ are constants such that $K_1|x|^2 \leq x'Px \leq K_2|x|^2$ for all $x \in \Re^n$. It follows from Theorem 3.1 that the following system:

$$\begin{aligned} \dot{z}(t) &= Az(t) + k(c'z(t) - w(t)) \\ \dot{w}(t) &= c'Az(t), \quad t \in [\tau_i, \tau_{i+1}) \\ w(\tau_{i+1}) &= y(\tau_{i+1}), \quad \hat{x} = z \end{aligned} \tag{4.6}$$

is a robust sampled-data observer for system (4.1) with respect to measurement errors provided that the upper diameter of the sampling partition $r > 0$ satisfies the inequality

$$r|c'A|\sqrt{\frac{\gamma}{2\mu K_1}} < 1. \tag{4.7}$$

B. Triangular Globally Lipschitz Systems

Consider the system

$$\begin{aligned} \dot{x}_i &= f_i(x_1, \ldots, x_i) + x_{i+1}, \quad i = 1, \ldots, n-1 \\ \dot{x}_n &= f_n(x_1, \ldots, x_n), \quad y = x_1 \end{aligned} \tag{4.8}$$

where $f_i : \Re^i \to \Re$ $(i = 1, \ldots, n)$ with $f_i(0) = 0$ $(i = 1, \ldots, n)$ are globally Lipschitz functions, i.e., there exists a constant $L \geq 0$ such that the following inequalities hold for $i = 1, \ldots, n$, $(x_1, \ldots, x_i) \in \Re^i$, $(z_1, \ldots, z_i) \in \Re^i$:

$$|f_i(x_1, \ldots, x_i) - f_i(z_1, \ldots, z_i)| \leq L|(x_1 - z_1, \ldots, x_i - z_i)|. \tag{4.9}$$

The reader should notice that all linear observable systems can be written in the form (4.8) with $f_i : \Re^i \to \Re (i = 1, \ldots, n)$ being linear functions. Notice that systems of the form (4.8) are Forward Complete and satisfy hypothesis (H), since for every $x_0 \in \Re^n$ the solution of (4.8) with initial condition $x(0) = x_0$ satisfies the estimate

$$|x(t)| \leq \exp(ct)|x_0|, \quad \forall t \geq 0 \tag{4.10}$$

where $c := nL + n - 1$. A high-gain observer design for (4.8) is described in [11]: first a vector $k = (k_1, \ldots, k_n)' \in \Re^n$ is found so that the matrix $(A + kc') \in \Re^{n\times n}$ is Hurwitz, where $c := (1, 0, \ldots, 0)' \in \Re^n$ and $A \in \Re^{n\times n}$ is the matrix $A = \{a_{i,j} : i = 1, \ldots, n, j = 1, \ldots, n\}$ with $a_{i,i+1} = 1$ for $i = 1, \ldots, n-1$ and $a_{i,j} = 0$ if otherwise. The proposed observer is of the form

$$\begin{aligned} \dot{z}_i &= f_i(z_1, \ldots, z_i) + z_{i+1} + \theta^i k_i(c'z - y) \\ & i = 1, \ldots, n-1 \\ \dot{z}_n &= f_n(z_1, \ldots, z_n) + \theta^n k_n(c'z - y) \\ \hat{x} &= z, \quad z = (z_1, \ldots, z_n)' \in \Re^n \end{aligned} \tag{4.11}$$

where $\theta \geq 1$ is a constant sufficiently large. The proof is based on the quadratic error Lyapunov function $V(e) := e'\Delta_\theta^{-1}P\Delta_\theta^{-1}e$, where $e := z - x$, $\Delta_\theta := \text{diag}(\theta, \theta^2, \ldots, \theta^n)$ and $P \in \Re^{n\times n}$ is a symmetric positive definite matrix that satisfies $P(A+kc')+(A'+ck')P+2\mu I \leq 0$ for certain constant $\mu > 0$. Specifically, using the identities $\Delta_\theta^{-1}A = \theta A\Delta_\theta^{-1}$, $c' = \theta c'\Delta_\theta^{-1}$ and the inequalities $\theta^{-i}|f_i(x_1 + e_1, \ldots, x_i + e_i) - f_i(x_1, \ldots, x_i)| \leq L|\Delta_\theta^{-1}e|$ for $i = 1, \ldots, n$ [which follows from (4.9)], $2e'\Delta_\theta^{-1}Pkv \leq \theta\mu/2|P|V(e) + 2|P|^2|k|^2/\theta\mu|v|^2$ for all $(x, e, v) \in \Re^n \times \Re^n \times \Re$, we get for $\theta \geq \max(1, 2|P|L\sqrt{n}/\mu)$

$$\nabla V(e)\left[(A + \Delta_\theta kc')e + \Delta_\theta kv + g(x, e)\right] \leq -\frac{\theta\mu}{2|P|}V(e) + \frac{2|P|^2|k|^2}{\theta\mu}|v|^2 \tag{4.12}$$

where $g(x, e) = (f_1(x_1 + e_1) - f_1(x_1), \ldots, f_n(x + e) - f_n(x))'$. Inequality (4.12) implies that for all $(x_0, z_0, v) \in \Re^n \times \Re^n \times L^\infty_{\text{loc}}(\Re^+;\Re)$ the solution of (4.8) with

$$\begin{aligned} \dot{z}_i &= f_i(z_1, \ldots, z_i) + z_{i+1} + \theta^i k_i(c'z - c'xv) \\ & i = 1, \ldots, n-1 \\ \dot{z}_n &= f_n(z_1, \ldots, z_n) + \theta^n k_n(c'z - c'xv) \\ & z = (z_1, \ldots, z_n)' \in \Re^n \end{aligned} \tag{4.13}$$

and initial condition $(x(0), z(0)) = (x_0, z_0) \in \Re^n \times \Re^n$ corresponding to $v \in L^\infty_{\text{loc}}(\Re^+;\Re)$ satisfies the estimates for all $t \geq 0$

$$\begin{aligned} |z_i(t) - x_i(t)| &\leq \theta^{i-1}\sqrt{\frac{K_2}{K_1}}\exp\left(-\frac{\theta\mu}{4|P|}\right)|z_0 - x_0| \\ &+ 2\theta^{i-1}\frac{|P||k|}{\mu}\sqrt{\frac{K_2}{K_1}}\sup_{0\leq\tau\leq t}|v(\tau)| \\ & i = 1, \ldots, n \end{aligned} \tag{4.14}$$

where $K_1, K_2 > 0$ are constants such that $K_1|x|^2 \leq x'Px \leq K_2|x|^2$ for all $x \in \Re^n$. It follows from (4.14) and (4.10) that system (4.11) is a robust observer for system (4.8) with respect to measurement errors. Moreover, using inequality (4.9) for $i = 1$ and (4.14), we obtain that for all $(x_0, z_0, v) \in \Re^n \times \Re^n \times L^\infty_{\text{loc}}(\Re^+;\Re)$ the solution of (4.8) with (4.13) and initial condition $(x(0), z(0)) = (x_0, z_0) \in \Re^n \times \Re^n$ corresponding to $v \in L^\infty_{\text{loc}}(\Re^+;\Re)$ satisfies the estimate for all $t \geq 0$:

$$\begin{aligned} &|f_1(z_1(t)) + z_2(t) - f_1(x_1(t)) - x_2(t)| \\ &\leq (L + \theta)\sqrt{\frac{K_2}{K_1}}\exp\left(-\frac{\theta\mu}{4|P|}\right)|z_0 - x_0| \\ &+ 2(L + \theta)\frac{|P||k|}{\mu}\sqrt{\frac{K_2}{K_1}}\sup_{0\leq\tau\leq t}|v(\tau)|. \end{aligned} \tag{4.15}$$

It follows from Theorem 3.1 that the following system:

$$\begin{aligned} \dot{z}_i(t) &= f_i(z_1(t), \ldots, z_i(t)) + z_{i+1}(t) \\ &+ \theta^i k_i(c'z(t) - w(t)), \\ & i = 1, \ldots, n-1 \\ \dot{z}_n(t) &= f_n(z_1(t), \ldots, z_n(t)) + \theta^n k_n(c'z(t) - w(t)) \\ \dot{w}(t) &= f_1(z_1(t)) + z_2(t), \quad t \in [\tau_i, \tau_{i+1}) \\ w(\tau_{i+1}) &= y(\tau_{i+1}) \\ \hat{x} &= z, \quad z = (z_1, \ldots, z_n)' \in \Re^n \end{aligned} \tag{4.16}$$

is a robust sampled-data observer for system (4.8) with respect to measurement errors provided that the upper diameter of the sampling partition $r > 0$ satisfies the inequality

$$2r(L+\theta)\frac{|P||k|}{\mu}\sqrt{\frac{K_2}{K_1}} < 1. \tag{4.17}$$

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Hammerstein Systems Identification in Presence of Hard Nonlinearities of Preload and Dead-Zone Type

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***Abstract*—Hammerstein system identification is considered in presence of preload and dead-zone nonlinearities. The discontinuous feature of these nonlinearities makes it difficult to get a single system parameterization involving linearly all unknown parameters (those of the linear subsystem and those of the nonlinearity). Therefore, system identification has generally been dealt with using multiple stage schemes including different parametrizations and several data-acquisition experiences. However, the consistency issue has only been solved under restrictive assumptions regarding the identified system. In this paper, a new identification scheme is designed and shown to be consistent under mild assumptions.**

***Index Terms*—Hammerstein systems, hard nonlinearities, parametric identification.**

I. INTRODUCTION AND PROBLEM STATEMENT

We are considering discrete-time systems that can be described by the Hammerstein model

$$A(q^{-1})y(t) = B(q^{-1})w(t) + \xi(t) \tag{1a}$$
$$w(t) = F(v(t)) \tag{1b}$$

where $v(t), y(t)$ denote the system input and output, respectively; $w(t)$ is an internal signal that is not accessible to measurement. The noise $\xi(t)$ is a zero-mean stationary and ergodic sequence of stochastically independent variables. $A(q^{-1})$ and $B(q^{-1})$ are nth order polynomials in the backward shift operator q^{-1} such that $A(0) = 1$ and $B(0) = 0$. The linear subsystem (1a), is supposed to be controllable and BIBO stable. Controllability is required for persistent excitation purpose [1]. All parameters of the linear subsystem are unknown except for the order n. The input nonlinearity $F(\cdot)$ is a preload/dead-zone function (Fig. 1), that is fully characterized by the parameters $(D_1, D_2, S_1, S_2, L_1, L_2)$ which all are unknown except that $\max\{|D_1|, |D_2|\} < D_M$, for some known real$D_M$. In addition to parameter uncertainty, the difficulty lies in the fact that the function $F(v)$ is discontinuous and assumes different mathematical expressions in each of the intervals $(-\infty, -D_2]$, $(-D_2, D_1)$ and $[D_1, +\infty)$. Moreover, these intervals are in turn uncertain because so are D_1 and D_2. This makes the function $F(v)$ depend nonlinearly on its characteristics $(D_1, D_2, S_1, S_2, L_1, L_2)$. Input nonlinearities of this kind are commonly called 'hard'. As a matter of fact, most previous works on parametric Hammerstein system identification have focused on the case of "soft" input nonlinearities, see, e.g., [2] and reference list therein. Then, the nonlinearity is assumed to be a polynomial or a series of orthogonal functions (e.g., Fourier series). As hard input nonlinearities are more difficult to cope with, few results have been reported on this case. A first solution was proposed in [3] and [4] where an identification scheme estimates alternately the relevant parameters and some auxiliary variables, based on a pseudo-linear regression. No formal consistency analysis was made and the solution efficiency was only illustrated by simulations. In [5], a wide range of nonlinearities

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