

Agnès Desolneux Lionel Moisan  
Jean-Michel Morel

# **From Gestalt Theory to Image Analysis**

A Probabilistic Approach

**4: Springer**

# Contents

<b>Preface</b> .....	v
<b>1 Introduction</b> .....	1
1.1 Gestalt Theory and Computer Vision .....	1
1.2 Basic Principles of Computer Vision .....	3
<b>2 Gestalt Theory</b> .....	11
2.1 Before Gestaltism: Optic-Geometric Illusions .....	11
2.2 Grouping Laws and Gestalt Principles .....	13
2.2.1 Gestalt Basic Grouping Principles .....	13
2.2.2 Collaboration of Grouping Laws .....	17
2.2.3 Global Gestalt Principles .....	19
2.3 Conflicts of Partial Gestalts and the Masking Phenomenon .....	21
2.3.1 Conflicts .....	21
2.3.2 Masking .....	22
2.4 Quantitative Aspects of Gestalt Theory .....	25
2.4.1 Quantitative Aspects of the Masking Phenomenon .....	25
2.4.2 Shannon Theory and the Discrete Nature of Images .....	27
2.5 Bibliographic Notes .....	29
2.6 Exercise .....	29
2.6.1 Gestalt Essay .....	29
<b>3 The Helmholtz Principle</b> .....	31
3.1 Introducing the Helmholtz Principle: Three Elementary Examples .....	31
3.1.1 A Black Square on a White Background .....	31
3.1.2 Birthdays in a Class and the Role of Expectation .....	34
3.1.3 Visible and Invisible Alignments .....	36
3.2 The Helmholtz Principle and E-Meaningful Events .....	37
3.2.1 A First Illustration: Playing Roulette with Dostoevski .....	39
3.2.2 A First Application: Dot Alignments .....	41
3.2.3 The Number of Tests .....	42

3.3	Bibliographic Notes	...43
3.4	Exercise	44
3.4.1	Birthdays in a Class	...44
<b>4</b>	<b>Estimating the Binomial Tail</b>	47
4.1	Estimates of the Binomial Tail	47
4.1.1	Inequalities for $B(l, k, p)$	49
4.1.2	Asymptotic Theorems for $B(l, k, p) = P[S_i > k]$	50
4.1.3	A Brief Comparison of Estimates for $B(l, k, p)$	50
4.2	Bibliographic Notes	52
4.3	Exercises	52
4.3.1	The Binomial Law	52
4.3.2	Hoeffding's Inequality for a Sum of Random Variables	53
4.3.3	A Second Hoeffding Inequality	55
4.3.4	Generating Function	56
4.3.5	Large Deviations Estimate	57
4.3.6	The Central Limit Theorem	60
4.3.7	The Tail of the Gaussian Law	63
<b>5</b>	<b>Alignments in Digital Images</b>	65
5.1	Definition of Meaningful Segments	65
5.1.1	The Discrete Nature of Applied Geometry	66
5.1.2	The A Contrario Noise Image	67
5.1.3	Meaningful Segments	70
5.1.4	Detectability Weights and Underlying Principles	72
5.2	Number of False Alarms	74
5.2.1	Definition	74
5.2.2	Properties of the Number of False Alarms	75
5.3	Orders of Magnitudes and Asymptotic Estimates	76
5.3.1	Sufficient Condition of Meaningfulness	77
5.3.2	Asymptotics for the Meaningfulness Threshold $k(l)$	78
5.3.3	Lower Bound for the Meaningfulness Threshold $k(l)$	80
5.4	Properties of Meaningful Segments	81
5.4.1	Continuous Extension of the Binomial Tail	81
5.4.2	Density of Aligned Points	83
5.5	About the Precision $p$	86
5.6	Bibliographic Notes	87
5.7	Exercises	91
5.7.1	Elementary Properties of the Number of False Alarms	91
5.7.2	A Continuous Extension of the Binomial Law	91
5.7.3	A Necessary Condition of Meaningfulness	92

**6 Maximal Meaningfulness and the Exclusion Principle** ..... 95

- 6.1 Introduction ..... 95
- 6.2 The Exclusion Principle ..... 97
  - 6.2.1 Definition ..... 97
  - 6.2.2 Application of the Exclusion Principle to Alignments ..... 98
- 6.3 Maximal Meaningful Segments ..... 100
  - 6.3.1 A Conjecture About Maximality ..... 102
  - 6.3.2 A Simpler Conjecture ..... 103
  - 6.3.3 Proof of Conjecture 1 Under Conjecture 2 ..... 105
  - 6.3.4 Partial Results About Conjecture 2 ..... 106
- 6.4 Experimental Results ..... 109
- 6.5 Bibliographical Notes ..... 112
- 6.6 Exercise ..... 113
  - 6.6.1 Straight Contour Completion ..... 113

**7 Modes of a Histogram** ..... 115

- 7.1 Introduction ..... 115
- 7.2 Meaningful Intervals ..... 115
- 7.3 Maximal Meaningful Intervals ..... 119
- 7.4 Meaningful Gaps and Modes ..... 122
- 7.5 Structure Properties of Meaningful Intervals ..... 123
  - 7.5.1 Mean Value of an Interval ..... 123
  - 7.5.2 Structure of Maximal Meaningful Intervals ..... 124
  - 7.5.3 The Reference Interval ..... 126
- 7.6 Applications and Experimental Results ..... 127
- 7.7 Bibliographic Notes ..... 129
- 7.8 Exercises ..... 129
  - 7.8.1 Kullback-Leibler Distance ..... 129
  - 7.8.2 A Qualitative *a Contrario* Hypothesis ..... 130

**8 Vanishing Points** ..... 133

- 8.1 Introduction ..... 133
- 8.2 Detection of Vanishing Points ..... 133
  - 8.2.1 Meaningful Vanishing Regions ..... 134
  - 8.2.2 Probability of a Line Meeting a Vanishing Region ..... 135
  - 8.2.3 Partition of the Image Plane into Vanishing Regions ..... 137
  - 8.2.4 Final Remarks ..... 141
- 8.3 Experimental Results ..... 144
- 8.4 Bibliographic Notes ..... 145
- 8.5 Exercises ..... 150
  - 8.5.1 Poincaré-Invariant Measure on the Set of Lines ..... 150
  - 8.5.2 Perimeter of a Convex Set ..... 150
  - 8.5.3 Crofton's Formula ..... 150

<b>9 Contrasted Boundaries .....</b>	<b>153</b>
9.1 Introduction .....	153
9.2 Level Lines and the Color Constancy Principle .....	153
9.3 A <i>Contrario</i> Definition of Contrasted Boundaries .....	159
9.3.1 Meaningful Boundaries and Edges .....	159
9.3.2 Thresholds .....	162
9.3.3 Maximality .....	163
9.4 Experiments .....	164
9.5 Twelve Objections and Questions .....	168
9.6 Bibliographic Notes .....	174
9.7 Exercise .....	175
9.7.1 The Bilinear Interpolation of an Image .....	175
<b>10 Variational or Meaningful Boundaries<sup>7</sup> .....</b>	<b>177</b>
10.1 Introduction .....	177
10.2 The "Snakes" Models .....	177
10.3 Choice of the Contrast Function $g$ .....	180
10.4 Snakes Versus Meaningful Boundaries .....	185
10.5 Bibliographic Notes .....	188
10.6 Exercise .....	188
10.6.1 Numerical Scheme .....	188
<b>11 Clusters .....</b>	<b>191</b>
11.1 Model .....	191
11.1.1 Low-Resolution Curves .....	191
11.1.2 Meaningful Clusters .....	193
11.1.3 Meaningful Isolated Clusters .....	193
11.2 Finding the Clusters .....	194
11.2.1 Spanning Tree .....	194
11.2.2 Construction of a Curve Enclosing a Given Cluster .....	194
11.2.3 Maximal Clusters .....	196
11.3 Algorithm .....	196
11.3.1 Computation of the Minimal Spanning Tree .....	196
11.3.2 Detection of Meaningful Isolated Clusters .....	197
11.4 Experiments .....	198
11.4.1 Hand-Made Examples .....	198
11.4.2 Experiment an a Real Image .....	198
11.5 Bibliographic Notes .....	198
11.6 Exercise .....	201
11.6.1 Poisson Point Process .....	201
<b>12 Binocular Grouping .....</b>	<b>203</b>
12.1 Introduction .....	203
12.2 Epipolar Geometry .....	204
12.2.1 The Epipolar Constraint .....	204
12.2.2 The Seven-Point Algorithm .....	204

- 12.3 Measuring Rigidity ..... 205
  - 12.3.1 F-rigidity ..... 205
  - 12.3.2 A Computational Definition of Rigidity ..... 206
- 12.4 Meaningful Rigid Sets ..... 207
  - 12.4.1 The Ideal Case (Checking Rigidity) ..... 207
  - 12.4.2 The Case of Outliers ..... 208
  - 12.4.3 The Case of Nonmatched Points ..... 210
  - 12.4.4 A Few Remarks ..... 214
- 12.5 Algorithms ..... 215
  - 12.5.1 Combinatorial Search ..... 215
  - 12.5.2 Random Sampling Algorithm ..... 216
  - 12.5.3 Optimized Random Sampling Algorithm (ORSA) ..... 217
- 12.6 Experiments ..... 217
  - 12.6.1 Checking All Matchings ..... 217
  - 12.6.2 Detecting Outliers ..... 219
  - 12.6.3 Evaluation of the Optimized Random Sampling Algorithm ..... 219
- 12.7 Bibliographic Notes ..... 222
  - 12.7.1 Stereovision ..... 222
  - 12.7.2 Estimating the Fundamental Matrix from Point Matches ..... 223
  - 12.7.3 Robust Methods ..... 224
  - 12.7.4 Binocular Grouping ..... 224
  - 12.7.5 Applications of Binocular Grouping ..... 225
- 12.8 Exercise ..... 225
  - 12.8.1 Epipolar Geometry ..... 225
- 13 A Psychophysical Study of the Helmholtz Principle ..... 227**
  - 13.1 Introduction ..... 227
  - 13.2 Detection of Squares ..... 227
    - 13.2.1 Protocol ..... 227
    - 13.2.2 Prediction ..... 228
    - 13.2.3 Results ..... 230
    - 13.2.4 Discussion ..... 231
  - 13.3 Detection of Alignments ..... 231
    - 13.3.1 Protocol ..... 232
    - 13.3.2 Prediction ..... 233
    - 13.3.3 Results ..... 233
  - 13.4 Conclusion ..... 234
  - 13.5 Bibliographic Notes ..... 235
- 14 Back to the Gestalt Programme ..... 237**
  - 14.1 Partial Gestalts Computed So Far ..... 237
  - 14.2 Study of an Example ..... 240
  - 14.3 The Limits of Every Partial Gestalt Detector ..... 242
    - 14.3.1 Conflicts Between Gestalt Detectors ..... 242

14.3.2 Several Straight Lines or Several Circular Arcs? .....	244
14.3.3 Influence of the A-contrario Model .....	246
14.4 Bibliographic Notes .....	247
<b>15 Other Theories, Discussion .....</b>	<b>249</b>
15.1 Lindenbaum's Theory .....	249
15.2 Compositional Model and Image Parsing .....	250
15.3 Statistical Framework .....	252
15.3.1 Hypothesis Testing .....	252
15.3.2 Various False Alarms or Error Rates Compared to NFA ...	253
15.3.3 Comparison with Signal Detection Theory .....	254
15.4 Asymptotic Thresholds .....	255
15.5 Should Probability Be Maximized or Minimized? .....	256
<b>References .....</b>	<b>261</b>
<b>Index .....</b>	<b>271</b>