From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models*

Fernando Alvarez
University of Chicago
f-alvarez1 AT uchicago.edu

Martin Beraja
University of Chicago
maberaja AT uchicago.edu

Martin Gonzalez-Rozada
UTDT
mrozada AT utdt.edu

Andy Neumeyer
UTDT
pan AT utdt.edu

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Abstract

We review and extend comparative statics results about how inflation changes price setting behavior in a model with fixed costs of changing prices in which firms face real idiosyncratic shocks. These results are confronted with their empirical counterparts using the micro data underlying Argentina's consumer price index for 1988-1997, when inflation rates ranged from almost 200% per month to less than zero. We find empirical support for several theoretical predictions: (i) the steady state frequency of price changes is unresponsive to inflation for low inflation rates, while its elasticity with respect to inflation is between 1/2 and 2/3 as inflation becomes large; (ii) the frequency of price increases is unresponsive to inflation and equal to the frequency of price decreases for small inflation rates, while the frequency of price decreases converges to zero as inflation increases; (iii) the average magnitude of price changes is symmetric for price increases and decreases at low inflation rates; while for high rates of inflation the magnitude of price increases is increasing with the inflation rate (for price decreases is less clearly so in the data); (iv) the steady state dispersion of relative prices is unresponsive to inflation for low rates while it is an increasing function of inflation for high rates of inflation; and (v) the variability of the frequency of price changes across goods diminishes as inflation grows. Our findings in (i) confirm and extend the cross country evidence available in the literature.

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1 Introduction

Infrequent nominal price adjustments are at the center of a large number of models of aggregate fluctuations and monetary policy analysis. This paper studies properties of nominal price dynamics under different rates of inflation in a class of models in which monopolistic firms set prices subject to a fixed cost of adjustment. The predictions of this class of models are examined with a unique data set, which includes several years of price stability as well as sustained high rates of inflation. The evidence strongly supports the theoretical predictions.

We concentrate on simple comparative static results which apply as long as the fixed cost are small enough. The results compare the effect of different constant inflation rates on the average frequency of price changes and on the dispersion of relative prices, keeping everything else fixed. First, we show that in the neighborhood of zero inflation, the average frequency of price changes as well as the dispersion of relative prices are approximately unresponsive to inflation. Likewise, under the same conditions, at zero inflation both the frequency and absolute value of price increases and decreases are the same. Next, we present a set of results for the case when inflation rates are large relative to the volatility of idiosyncratic shocks. We find that the elasticity of the average frequency of price changes with respect to inflation is approximately $2/3$ and the elasticity of relative price dispersion with respect to inflation is positive, although the existence and magnitude of the later depends on the persistence of idiosyncratic shocks. Furthermore, at high inflation rates, most adjustments are price increases, or equivalently, price decreases become less frequent; while the magnitude of both types of price change becomes larger.

We review and extend the comparative static properties of models of monopolistic firms setting prices subject to a fixed cost of adjustment. These models have been introduced by Barro (1972) and Sheshinski and Weiss (1977), and augmented to include idiosyncratic firm level shocks by Bertola and Caballero (1990), Danziger (1999), Golosov and Lucas (2007), and Gertler and Leahy (2008), among others. The results for high inflation are from Sheshinski and Weiss (1977) and especially Benabou and Konieczny (1994), which we extend to the case with persistent idiosyncratic shocks. The results for low inflation are new, but closely related to some results in Alvarez, Lippi, and Paciello (2011) and Alvarez and Lippi (2014) for models with permanent idiosyncratic shocks.

We believe these predictions to be interesting from the perspective of menu cost models of price stickiness because they underlie the welfare costs of inflation and because they serve to test this class of models. In relation to the former, the cost of frequently changing prices is a direct welfare cost of inflation, since these resources are wasted. Secondly, the "extra" price dispersion created by nominal variation in prices is the other avenue for inefficiency in models with sticky prices, as explained in chapter 6 of Woodford (2003) or Burstein and Hellwig (2008), among many others.[1]

We illustrate the theoretical predictions reported above using the micro-data underlying the construction of the Argentinean CPI index from 1989 to 1997. The unique feature of this data is that in the initial years inflation was extremely high, almost 5000% during 1989 and almost 1500% during 1990. After the stabilization plan of 1991 there is a quick disinflation episode, and after 1992 there is virtually price stability, with some deflationary periods. The large variation of inflation in our data set makes it a good laboratory to test features of different models of price adjustment, and as such we view our finding interesting well beyond the effect of large inflation. We construct time series for the frequency of price changes, for the magnitude of price adjustments, for the dispersion of the frequency of price changes across industries, and for the dispersion of relative prices. Then, we examine how these statistics vary with the rate of inflation, in particular how they covary with inflation at low rates of inflation, and how they covary with inflation at high rates of inflation.

We report three type of findings. First, we find that the frequency of price chances is approximately uncorrelated with contemporaneous inflation for inflation rates below 10%, and that this frequency has an elasticity between 1/2 and 2/3 for higher inflation rates. These findings are robust to different treatment of sales, product substitutions, and missing values in the estimation of the frequency of price changes and with respect to the level of aggregation of price changes. They are also robust to using contemporaneous inflation or an estimate of expected future inflation, for the relevant time frame; and to excluding observations corresponding to periods with inflation above some threshold for which we have reasons to believe that discrete sampling might bias the estimates. Second, we find that the cross-industry dispersion of the frequency of price changes diminishes as inflation increases. We interpret this to be consistent with our hypothesis that as inflation increases, the determinants for price changes become more similar across industries, since aggregate inflation is common to all goods. Third, we find that the dispersion of relative prices is approximately uncorrelated with inflation for low values of inflation, but it is tightly related to inflation for large values, with an elasticity below 1/3.

The paper is organized as follows. Section 2 contains the theoretical analysis of the effect of inflation on the frequency of price changes and on the dispersion of relative prices. Section 3 describes our data set. Section 4 has an explanation of the method used to compute the frequency of price changes. Section 5.1 presents the estimates of the time series of the

[1]See Benabou (1992) and Burstein and Hellwig (2008) for earlier and recent examples of analysis that takes both effects of inflation into account, the latter using heterogeneous consumers that search for products and homogenous firms, and the latter using differentiated products in the demand side and heterogeneity in the firms cost.

frequency of price changes and its relation to inflation. section 5.1.1 discusses the relation with other studies that analyze the frequency of price changes and inflation. Section 5.2 analyzes the cross industry dispersion of the frequency of price changes. Section 5.3 presents a decomposition of inflation into its extensive and intensive margins for positive and negative price changes and analyzes how this decomposition varies with the level of inflation. Section 5.4 presents the estimates of the dispersion of relative prices vs. inflation. In section 5.1.2 we perform extensive robustness checks to evaluate the sensitivity of our estimates. Several appendices discuss other methodological issues, given more details of the estimates, as well as a description of the data-base. Appendix G contains a short description of the history of economic policy and inflation in Argentina for the years before and during our sample.

2 Comparative Static Properties of Menu Cost Models

In this section we study how inflation affects price dynamics and the distribution of relative prices in menu cost models. To that end, we show several theoretical predictions of a class of models where a single competitive monopolist firm faces a fixed cost of changing its nominal price in the presence of idiosyncratic real cost shocks and constant inflation.

We first show that in a neighborhood of zero inflation: (i) the average frequency of price changes is unresponsive to inflation, (ii) the dispersion of relative prices is unresponsive to inflation, (iii) the frequency of price increases is equal to the frequency of price decreases, (iv) conditional on a price change, the size of price increases is equal to the size of price decreases, and (v) ninety percent of changes in inflation are accounted for by changes in the frequency of price changes. The second set of results is concerned with economies with high inflation. We show that for inflation rates that are large relative to the size of idiosyncratic shocks, when the idiosyncratic shocks are persistent: (i) the frequency of price adjustment is the same for all firms, (ii) the elasticity of the average frequency of price changes with respect to inflation converges to two thirds, (iii) the elasticity of the average size of price changes with respect to inflation converges to one third.

This section hast two parts. In section 2.1 we write down a simple set up we obtain the main analytical results. In section 2.1.1 and section 2.1.2 we obtain the results for low and high inflation, respectively, explain the nature of the assumptions needed for the results, remark on which form of these results are already present in the literature and on the pros and cons of applying these comparative statics results to time series data. In section 2.2 we illustrate the results with an example that follows Golosov and Lucas (2007), where we show that theoretical results of section 2.1 are applicable to inflations rates on the range observed in our data set.

2.1 The Basic Menu Cost Model

This section we presents a benchmark model to study the sensitivity of the firm's pricing decisions to changes in the rate of inflation. For two extreme cases, when inflation is in a neighborhood of zero and when inflation is very high, propositions 1-3 show the results mentioned above.

We study the problem of a monopolist adjusting the nominal price of its product in an environment with inflation, idiosyncratic real cost shocks, and a fixed cost (the menu cost) of changing nominal prices. We think of this problem as similar to the firm's problem in Golosov and Lucas (2007).

We assume that the instantaneous profit of the monopolist depends on its price relative to the economy (or industry) wide average price and on an idiosyncratic shock. We let $F(p, z)$ be the real value of the profit per period as a function of the log of the nominal price charged by the firm relative to the the nominal economy (or industry) wide price, p, and the idiosyncratic shock, z. We assume that the economy wide price grows at a constant inflation rate π so that when the firm does not change its nominal price its relative price evolves according to $dp = -\pi dt$. We also assume that F is strictly concave in its first argument. The variable $z \in Z$ is a shifter of the profit function. We also allow the menu cost to depend on z, in which case we write $C_t = c\,\zeta(z_t)$, where $c \geq 0$ is a constant, so $c = 0$ represents the frictionless problem. We assume that $\{z_t\}$ is a diffusion with coefficients $a(\cdot)$ and $b(\cdot)$:

$$dz = a\,(z)\;\, dt + \;b\,(z)\,\sigma\; dW \tag{1}$$

where $\{W(t)\}$ is a standard Brownian Motion so $W(t) - W(0) \sim N(0, t)$. We keep the parameter σ separately from $b(\cdot)$ so that when $\sigma = 0$ the problem is deterministic. We use $r \geq 0$ for the real discount rate of profits and adjustment costs. We let $\{\tau_i\}$ be the stopping times at which prices are adjusted and $\{\Delta p(\tau_i)\}$ the corresponding price changes, so that the problem of the firm can be written as

$$V\,(p, z) = \max_{\{\tau_i, \Delta p_i\}_{i=0}^{\infty}} \mathbb{E}\left[\int_0^\infty e^{-rt} F\,(p(t), z(t))\, dt \;-\; \textstyle\sum_{i=0}^{\infty} e^{-r\tau_i} c\,\zeta(z(t))\; \Big| z(0) = z\right] \tag{2}$$

with $p(t) = p(0) + \sum_{i=0}^{\tau_i < t} \Delta p(\tau_i) - \pi t$ for all $t \geq 0$ and the initial state is given by $p(0)$.

The optimal policy that solves this problem can be described by the vector

$$\Psi(z; \pi, \sigma^2) = \left[\underline{\psi}(z; \pi, \sigma^2), \bar{\psi}(z; \pi, \sigma^2); \hat{\psi}(z; \pi, \sigma^2)\right].$$

Each element of the vector is a function of z and we are including the parameters π and σ^2 as explicit arguments of the decisions rules to conduct some comparative statics. The functions

$\underline{\psi}(z;\pi,\sigma^2)$ and $\bar{\psi}(z;\pi,\sigma^2)$ define the inaction set

$$\mathcal{I}(\pi,\sigma^2) = \left\{(p,z) \in \mathbb{R} \times Z : \underline{\psi}(z;\pi,\sigma^2) \le p \le \bar{\psi}(z;\pi,\sigma^2)\right\}.$$

If the firm's relative price is within the inaction set, $(p,z) \in \mathcal{I}$, then it is optimal not to change prices. Outside the interior of the inaction set the firm will adjust prices so that its relative price just after adjustment is given by $p = \hat{\psi}(z;\pi,\sigma^2)$. Since $\{z(t)\}$ has continuous paths, with additional regularity conditions, all the adjustment will occur at the boundary of the inaction set. For instance, a firm with a relative price p and an idiosyncratic shock z such that the relative price hits the lower boundary of the inaction set—i.e. $p = \underline{\psi}(z;\pi,\sigma^2)$—will raise its price by $\Delta p = \hat{\psi}(z;\pi,\sigma^2) - \underline{\psi}(z;\pi,\sigma^2)$.

Using the optimal decision rules we can compute the density of the invariant distribution of the state, $g(p,z;\pi,\sigma^2)$, as well as the expected time between adjustments $\mathcal{T}(p,z;\pi,\sigma^2)$ starting from the state (p,z). Note that using $g(\cdot)$ we can readily find the distribution of relative prices in the economy (or industry) and we can compute the expected time elapsed between consecutive adjustments under the invariant distribution, and its reciprocal, the expected number of adjustments per unit of time, which we denote by $\lambda_a(\pi,\sigma^2)$.

We denote by $\lambda_a^+(\pi,\sigma^2)$ and $\lambda_a^-(\pi,\sigma^2)$ the frequencies of price increases and decreases respectively. Furthermore, we let $\Delta_p^+(\pi,\sigma^2)$ be the expected size of price changes, conditional on of having an increase, and $\Delta_p^-(\pi,\sigma^2)$ the corresponding expected size of price changes, conditional on having a decrease. The expectation is $\Delta_p^+(\pi,\sigma^2) = \int_Z \left[\hat{\psi}(z) - \underline{\psi}(z)\right] \frac{g\left(\underline{\psi}(z),z\right)}{\int_Z g\left(\underline{\psi}(z'),z'\right)dz'}\, dz$ where we omit (π,σ^2) as arguments of $g, \hat{\psi}$ and $\underline{\psi}$ to simplify notation.

2.1.1 Comparative Statics with Low Inflation.

In this section we show that when inflation is zero and firms face idiosyncratic profit shocks, changes in the rate of inflation do not have a first order effect neither on the frequency of price changes nor on the distribution of relative prices. The intuition for this result is that at zero inflation, price changes are triggered by idiosyncratic shocks and small variations in inflation have only a second order effect. We also show that in this case there is a strong symmetry in the sense that the frequency of price increases and of price decreases as well as the size of price increases and decreases are the same.

In order to prove this result we assume that the idiosyncratic shock z has strictly positive volatility and we make some mild symmetry assumptions on the firm's problem. We assume that the process for the shocks is symmetric around zero, and that the profit function is symmetric in the log of the static profit maximizing relative price as well as in its shifter.

More precisely we define symmetry as follows. Assume that $z \in Z = [-\bar{z}, \bar{z}]$ for some strictly positive $\bar{z}$. Let the profit maximizing relative price given z be $p^*(z) \equiv \arg\max_x F(x,z)$. We say that $a(\cdot), b(\cdot), \zeta(z)$ and $F(\cdot)$ are *symmetric* if

$$
\begin{aligned}
a(z) &= -a(-z) \le 0 \text{ and } b(z) = b(-z) > 0 \text{ for all } z \in [0,\bar{z}] \;, & (3)\\
p^*(z) &= -p^*(-z) \ge 0 \text{ for all } z \in [0,\bar{z}] & (4)\\
F(\hat{p}+p^*(z),z) &= F(-\hat{p}+p^*(-z),-z) + f(z) \;\text{ for all } z \in [0,\bar{z}] \text{ and } \hat{p} \ge 0 \;, & (5)\\
\zeta(z) &= \zeta(-z) > 0 \text{ for all } z \in [0,\bar{z}] \;, & (6)
\end{aligned}
$$

for some function $f(z)$ and with the normalization $p^*(0) = 0$.

Let $\mu(z)$ be the density of the invariant distribution of z, when it exists. Equation (3) implies that the invariant distribution μ as well as the transition densities of the exogenous process $\{z_t\}$ are symmetric around $z = 0$. Equations (4)-(5) state that the profit function is symmetric around the (log) maximizing price and its cost shifter. Thus if the price is $\hat{p}$ *higher* than the optimal for a firm with z, profits deviate from its optimal value by the same amount as with prices $\hat{p}$ *lower* than the optimal when the shifter is $-z$. The function f allows to have an effect of the shifter z on the level of the profits that is independent of the price.

An example of a symmetric case is

$$
a(z) \;=\; -a_0 z \;,\; b(z) = b_0,\; \zeta(z) = \zeta_0,\; F(p,z) = d_0 - e_0\,(p-z)^2 - f_0 z \;\text{ so } p^*(z) = z \quad (7)
$$

for non-negative constants $a_0, b_0, \zeta_0, e_0, d_0$ and f_0. One way to think about the symmetry assumption is to consider a second order approximation of the profit function around the profit maximizing price, so that

$$
F(p,z) = F\left(p^*\left(z\right),z\right) + \frac{1}{2}F_{pp}\left(p^*\left(z\right),z\right)\left(p-p^*\left(z\right)\right)^2 + o\left(\left(p-p^*\left(z\right)\right)^2\right). \quad (8)
$$

We note that when the fixed adjustment cost c is small, then the firm will adjust the prices frequently enough so that $(p - p^*(z))^2$ will be small, and hence the quadratic approximation should be increasingly accurate as c become small.

We let $h(\hat{p};\pi,\sigma^2) = \int g(\hat{p},z;\pi,\sigma^2)\,dz$ be the invariant distribution of the relative prices $\hat{p}$, for an economy, or industry, with (π,σ). Using h we can compute several statistic of interest, such as $\bar{\sigma}(\pi,\sigma^2)$ the standard deviation of the relative prices $\hat{p} = p - \bar{p}$. As in the case of the frequency of price changes, we include (π,σ^2) explicitly as arguments of this statistic.

PROPOSITION 1. Assume that $z \in Z = [-\bar{z},\bar{z}]$ for some strictly positive $\bar{z}$ and that $F(\cdot), a(\cdot), b(\cdot)$ and $\zeta(\cdot)$ satisfy the symmetry conditions (3)-(6). Then,
(i) the frequency of price changes is insensitive to inflation at $\pi = 0$, i.e. if the frequency of

price changes $\lambda_a(\pi, \sigma^2)$ is differentiable at $\pi = 0$, then

$$\frac{\partial}{\partial \pi} \lambda_a(0, \sigma^2) = 0 \,,$$

(ii) the dispersion of relative prices under the invariant distribution is insensitive to inflation at $\pi = 0$, i.e. if the density of the invariant $h(\hat{p}; \cdot, \sigma^2)$ is differentiable at $\pi = 0$, then

$$\frac{\partial}{\partial \pi} \bar{\sigma}(0, \sigma^2) = 0 \,,$$

(iii) the frequencies of price changes and the size of price adjustment are symmetric at $\pi = 0$ in the sense that

$$\lambda_a^+(0, \sigma^2) = \lambda_a^-(0, \sigma^2), \quad \frac{\partial \lambda_a^+(0, \sigma^2)}{\partial \pi} = -\frac{\partial \lambda_a^-(0, \sigma^2)}{\partial \pi} \text{ and}$$

$$\Delta_p^+(0, \sigma^2) = \Delta_p^-(0, \sigma^2), \quad \frac{\partial \Delta_p^+(0, \sigma^2)}{\partial \pi} = -\frac{\partial \Delta_p^-(0, \sigma^2)}{\partial \pi}$$

where λ_a^+ is the frequency of price increases, Δ_p^+ is the average size of price increases and λ_a^-, Δ_p^- are the analogous concepts for price decreases.

The proof is in appendix A.1. The main idea is to use the symmetry of F to show that the expected number of adjustments is symmetric around zero inflation, i.e. that $\lambda_a(\pi, \sigma^2) = \lambda_a(-\pi, \sigma^2)$ for all π. Symmetry implies that if λ_a is differentiable, then $\frac{\partial}{\partial \pi}\lambda_a(\pi, \sigma^2) = -\frac{\partial}{\partial \pi}\lambda_a(-\pi, \sigma^2)$, which establishes the first result.

Given the symmetry of the profit function we view this property as quite intuitive: a 1% inflation should give rise to as much price changes as a 1% deflation. Analogously, for the distribution of relative prices, the main idea is to show that the marginal distribution of relative prices is symmetric in the sense that $h(\hat{p}; \pi, \sigma^2) = h(-\hat{p}, -\pi, \sigma^2)$ for all $\hat{p}, \pi$, i.e. the probability of high relative prices with inflation is the same as the one of low relative prices with deflation. As these symmetric functions are locally unchanged with respect to π when $\pi = 0$, inflation has no first order effect on the second moment of inflation at $\pi = 0$.[2]

[2] The assumption of differentiability of λ_a and $\bar{\sigma}$ with respect to π is not merely a technical condition. The function $\lambda_a(\cdot, \sigma^2)$ could have have a local minimum at $\pi = 0$ without being smooth, as it is in the case of $\sigma^2 = 0$ to which we will turn in the next proposition. Nevertheless, we conjecture, but have not proved at this level of generality, that as long as $\sigma^2 > 0$, the problem is regular enough to become smooth, i.e. the idiosyncratic shocks will dominate the effect of inflation. For several examples one can either compute all the required functions or show that they are are smooth, given the elliptical nature of the different ODE's involved. Based on this logic, as well as on computations for different models, we believe that the length of the interval for inflations around zero for which $\lambda_a(\cdot, \sigma^2)$ is approximately flat is increasing in the value of σ^2.

Similarly, the symmetry of the frequency and of the average size of price increases and of price decreases also follow from the symmetry assumptions.

Remarks and relation to the literature

We find Proposition 1's theoretical predictions interesting because they extend an important result on the welfare cost of inflation from sticky price models with exogenous price changes (e.g. the Calvo model) to menu cost models with endogenous frequency of price changes. The result is that in cashless economies with low inflation there is no first order welfare effect of inflation, i.e welfare cost of inflation can be approximated by a "purely quadratic" function of inflation.

In cashless economies, inflation imposes welfare costs through two channels. First, the "extra" price dispersion created by inflation is an avenue for inefficiency in models with sticky prices, since it creates "wedges" between the marginal rates of substitution in consumption and the marginal rates of transformation in production. See, for example, chapter 6 of Woodford (2003) and references therein for the analysis of this effect. Part (ii) of proposition 1 extends this result to the menu cost model. Second, the endogenous frequency of adjustment is an obvious source of a welfare loss due to inflation when changing prices is costly. Part (i) of proposition 1 establishes that this second channel is also negligible for low inflation rates.[3]

The results of proposition 1 are likely to apply to a wider class of models. The alert reader will realize that the essential assumption is the symmetry of the profit function around the profit maximizing price, and hence if this is maintained, the result should hold. For example, a version of proposition 1 applies to models with both observations and menu cost, Alvarez, Lippi, and Paciello (2011), to models that have multi-product firms, Alvarez and Lippi (2014), and to models that combine menu costs and Calvo type adjustments (Nakamura and Steinsson (2010) and Alvarez, Bihan, and Lippi (2014)). In those papers the second order approximation of the profit function stated in equation (8) is used.[4] Note that one can either assume symmetry directly, or obtain it approximately if the menu cost is small, since in this case a second order approximation around the static profit maximizing price is symmetric. Indeed in section 2.2 we solve numerically a version of the model that is not symmetric, but with cost that are empirically reasonable, and confirm the results of this section. In section 2.2 we also show that for reasonable parameter values the functions λ_a and $\bar{\sigma}$ are

[3]There are other papers that take more than one effect into account. Burstein and Hellwig (2008) compute numerical examples in a model closer to ours, which also includes the traditional money demand cost. Benabou (1992) uses a different framework, with heterogeneous consumers that search for products and homogeneous firms subject to menu costs but constant production costs.

[4]Furthermore $p^*(z) = z$ is assumed to follow a random walk with no drift, so that $\bar{z} = \infty, a(z) = 0$ and $b(z) = 1$.

approximately flat for a wide value of inflation rates around zero.

We don't know of other theoretical results analyzing the sensitivity of $\lambda_a(\pi)$ and $\bar{\sigma}(\pi)$ to inflation around $\pi = 0$ in this set-up. However, there is a closely related model that contains a complete analytical characterization by Danziger (1999). In fact, we can show that for a small cost of changing prices, proposition 1 holds in Danziger's characterization.

2.1.2 Comparative Statics with High Inflation.

Now we turn to the analysis of the elasticity of the frequency of adjustment, λ_a, and of the cross-sectional dispersion of relative prices, $\bar{\sigma}$, to inflation, π, for large values of inflation. In highly inflationary environments, the main reason for firms to change nominal prices is to keep their relative price in a target zone as the aggregate price level grows. Idiosyncratic shocks in the high inflation case become less important and, therefore, the analysis of the deterministic case is instructive. This intuition leads us to proceed in two steps. First we study the sensitivity of price dynamics and that of the distribution of relative prices with respect to inflation in the deterministic case—i.e. when $\sigma^2 = 0$. This is a version of the problem studied by Sheshinski and Weiss (1977). Then, we study the conditions under which the comparative statics for deterministic case is the same as for the case of $\sigma^2 > 0$ and very large π.

Sheshinski and Weiss (1977) study a menu cost model similar to the deterministic case in our basic setup. The firm's problem is to decide when to change prices and by how much when aggregate prices grow at the rate π. In Sheshinski and Weiss's (1977) model the time elapsed between adjustments is simply a constant, which we denote by $\mathcal{T}(\pi)$. Sheshinski and Weiss (1977) find sufficient conditions so that the time between adjustments decreases with the inflation rate (see their Proposition 2), and several authors have further refined the characterization by concentrating on the case where the fixed cost c is small. Let $p^* = \arg\max_p F(p, 0)$ be the log the static monopolist maximization profit, where $z = 0$ is a normalization of the shifter parameter which stays constant. In the deterministic set-up the optimal policy for $\pi > 0$ is to let the log of the price reach a value s at which time it adjusts to S, where $s < p^* < S$. The time between adjustments is then $\mathcal{T}(\pi) = (S - s)/\pi$. We also note another implication obtained in the Sheshinski and Weiss's (1977) model, i.e. the set-up with $\sigma^2 = 0$. The distribution of (log) of relative price is uniform in the interval $[s, S]$. Thus the standard deviation of the log of the relative prices in this economy, denoted by $\bar{\sigma}$, is given by $\bar{\sigma} = \sqrt{1/12}\,(S - s)$. As established in Proposition 1 in Sheshinski and Weiss (1977), the range of prices $S - s$ is increasing in the inflation rate π. Obviously the elasticities of λ_a and of $\bar{\sigma}$ with respect to π are related since $S - s = \pi\mathcal{T}$ and $\lambda_a = 1/\mathcal{T}$.

LEMMA 1. Assume that $\sigma^2 = 0$ and $\pi > 0$. Then it follows immediately that $\lambda_a^-(\pi, \sigma^2) = 0$ and that $\Delta_p^+(\pi, \sigma^2) = S - s$. Furthermore assume that $F(\cdot, 0)$ is three times differentiable, then

$$\lim_{c \to 0} \frac{\partial \lambda_a}{\partial \pi} \frac{\pi}{\lambda_a} = \frac{2}{3} \quad \text{and} \tag{9a}$$

$$\lim_{c \to 0} \frac{\partial \bar{\sigma}}{\partial \pi} \frac{\pi}{\bar{\sigma}} = \frac{1}{3} \tag{9b}$$

Proof. See section A.2 in the appendix.

The lemma establishes that in the deterministic case when menu costs c are small, there are no price decreases, and the magnitude of price increases, $S - s$, increases with inflation at a rate of 1/3. Also, as inflation increases the time between consecutive price changes shrinks and the frequency of price adjustment increases with an elasticity of 2/3.

Next, lemma 2 analyzes the conditions under which the limiting values of the elasticities in lemma 1 for the Sheshinski and Weiss (1977) model are the same as for the case with idiosyncratic costs, $\sigma > 0$, and very large π. Lemma 2 establishes that when the idiosyncratic shocks z are very persistent and interest rates and menu costs are very small, the frequency of price adjustment is homogeneous of degree one in (π, σ^2) so that it can be written as a function of the ratio σ^2/π.

For the next results we write the frequency of price adjustment as a function of the rate of inflation, π, the variance of the idiosyncratic shock, σ^2, the discount factor, r, and the inverse of the menu cost, $1/c$, that is, $\lambda_a(\pi, \sigma^2, r, \frac{1}{c})$. We also write the policy rules as functions of the parameters for each z; $\Psi(\pi, \sigma^2, r, \frac{1}{c}; z) = \left[\underline{\psi}(z; \pi, \sigma^2, r, \frac{1}{c}), \bar{\psi}(z; \pi, \sigma^2, r, \frac{1}{c}), \hat{\psi}(z; \pi, \sigma^2, r, \frac{1}{c})\right]$ and the expected price change functions as $\Delta_p^+(\pi, \sigma^2, r, \frac{1}{c})$ and $\Delta_p^-(\pi, \sigma^2, r, \frac{1}{c})$.

LEMMA 2. Let $a(z) = 0$ for all z. Then $\lambda_a(\pi, \sigma^2, r, \frac{1}{c})$ is homogenous of degree one and the policy functions $\Psi(\pi, \sigma^2, r, \frac{1}{c}; z)$ are homogeneous of degree zero in all the parameters for a fixed z. Therefore,

$$\lim_{\pi \to \infty} \left[\lim_{r \downarrow 0, c \downarrow 0} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi} \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \right]_{\sigma > 0} = \lim_{\sigma \to 0} \left[\lim_{r \downarrow 0, c \downarrow 0} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi} \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \right]_{\pi > 0}, \tag{10a}$$

for each z,

$$\lim_{\pi\to\infty}\left[\lim_{r\downarrow 0,c\downarrow 0}\Psi(\pi,\sigma^2,r,\frac{1}{c};z)\right]_{\sigma>0}=\lim_{\sigma\to 0}\left[\lim_{r\downarrow 0,c\downarrow 0}\Psi(\pi,\sigma^2,r,\frac{1}{c};z)\right]_{\pi>0} \tag{10b}$$

and

$$\lim_{\pi\to\infty}\left[\lim_{r\downarrow 0,c\downarrow 0}\Delta_p^+(\pi,\sigma^2,r,\frac{1}{c})\right]_{\sigma>0}=\lim_{\sigma\to 0}\left[\lim_{r\downarrow 0,c\downarrow 0}\Delta_p^+(\pi,\sigma^2,r,\frac{1}{c})\right]_{\pi>0}. \tag{10c}$$

Proof. See section A.3 in the appendix.

The intuition underlying lemma 2's proof is that multiplying r, π, σ^2 and the functions $a(\cdot)$ and $F(\cdot)$ in the firm's problem— equation (2)—by a constant $k > 0$ is akin to changing the units in which we measure time. Moreover, the objective function in the right hand side of equation (2) is homogeneous of degree one in $F(\cdot)$ and c and, hence, the policy function is the same whether we multiply $F(\cdot)$ by k or divide c by it. Thus, if the function $a(\cdot)$ equals zero, then λ_a is homogeneous of degree one in $(\pi, \sigma^2, r, \frac{1}{c})$. Likewise, $\lambda_a(\pi, \sigma^2)$ is homogeneous of degree one in (π, σ^2) when menu costs are small, $c \downarrow 0$, the interest rate is very low, $r \downarrow 0$, and shocks are very persistent, $a(\cdot) = 0$. The interpretation of r going to zero is that instead of maximizing the expected discounted profit, the firm is maximizing the expected average profit, a case frequently analyzed in stopping time problems. If the idiosyncratic shock process has $a(\cdot) = 0$ and $b(\cdot)$ is bounded, then z is a Martingale, i.e. the shifter has permanent shocks.

Lemma 2 extends the result on the elasticity of the frequency of price adjustment with respect to inflation of equation (9a) in lemma 1 to the case with $\sigma > 0$ and with an arbitrarily large π. This lemma requires that the shifter z has only permanent shocks, i.e. that $a(z) = 0$. A similar argument does not apply to the cross sectional dispersion of relative prices $\bar{\sigma}$ since setting $a(z) = 0$ implies that there is no invariant distribution of z, and hence no invariant distribution of relative prices.

Using lemma 1 and lemma 2 we obtain the following result for the high inflation case comparing firms with different parameters:

Proposition 2. Assume that $a(z) = 0$ for all z and F is three times differentiable.

Consider two firms with $\sigma_1, \sigma_2 > 0$. Then,

$$\lim_{\pi\to\infty}\left[\frac{\lim_{r\downarrow 0, c\downarrow 0}\lambda_a(\pi,\sigma_1^2)}{\lim_{r\downarrow 0, c\downarrow 0}\lambda_a(\pi,\sigma_2^2)}\right] = 1 \tag{11a}$$

$$\lim_{\pi\to\infty}\left[\lim_{r\downarrow 0, c\downarrow 0}\frac{\partial\lambda_a(\pi,\sigma_i^2)}{\partial\pi}\frac{\pi}{\lambda_a(\pi,\sigma_i^2)}\right] = \frac{2}{3} \quad \text{for } i = 1,2 \quad \text{and} \tag{11b}$$

$$\lim_{\pi\to\infty}\left[\lim_{r\downarrow 0, c\downarrow 0}\frac{\partial\Delta_p^+(\pi,\sigma_i^2)}{\partial\pi}\frac{\pi}{\Delta_p^+(\pi,\sigma_i^2)}\right] = \frac{1}{3} \quad \text{for } i = 1,2. \tag{11c}$$

Proposition 2 contains strong predictions about the limiting behavior of the frequency of price adjustment as inflation becomes large. The first part is a direct consequence of lemma 2. It implies that if we think that different industries have systematically different idiosyncratic shocks, we would expect the variance of these shocks to differ across industries and, hence, the frequency of price adjustment to be different across industries when inflation is low. Equation (11a) implies that differences in the frequency of price adjustment observed with low inflation should wash away as inflation becomes large. This is illustrated in the numerical example in the next section (see figure 2) and verified in the data (see section 5.2). The intuition is that with low inflation the main driver of idiosyncratic nominal price changes are idiosyncratic shocks and with high inflation the main driver of price changes is the growth of aggregate prices. The second part of proposition 2 has a sharp prediction about the rate at which firms change the frequency of price adjustment when inflation grows. It states that this elasticity should be $2/3$ in the limit when $\pi \to \infty$. Finally, equation (11c) states that the elasticity of price increases converges with respect to inflation converges to $1/3$ as $\pi \to \infty$. It follows from the fact that Δ_p^+ is $S - s = \pi/\lambda_a$ when $\sigma = 0$.

The results of proposition 2 apply to a wider set of models such as those mentioned in the comments to proposition 1 .

2.1.3 Decomposition of Changes in the Rate of Inflation

This section shows how steady state changes in the rate of inflation can be decomposed into changes in the extensive and intensive margins of price adjustment—i.e. changes due to the frequency of price changes and changes due to the size of price changes conditional on a price change taking place. Our main result is that for low inflation the extensive margin accounts for ninety percent of changes in inflation and for larg inflations it accounts for two thirds of price changes.

We decompose the inflation rate as the difference between the product of the frequency of price increases times the average size of price increases, and the product of the frequency of price decreases times the average size of price decreases, i.e:

$$\pi = \Delta_p^+ \lambda_a^+ - \Delta_p^- \lambda_a^-$$

Totally differentiating the previous expression with respect to the inflation rate, we can decompose the changes into those accounted for by changes on average size, denoted by δ, and those accounted by for changes in the frequency, denoted by $1-\delta$ as follows:

$$1 = \underbrace{\frac{\partial \Delta_p^+}{\partial \pi} \lambda_a^+ - \frac{\partial \Delta_p^-}{\partial \pi} \lambda_a^-}_{\substack{\text{Intensive Margin} \\ \delta(\pi)}} + \underbrace{\frac{\partial \lambda_a^+}{\partial \pi} \Delta_p^+ - \frac{\partial \lambda_a^-}{\partial \pi} \Delta_p^-}_{\substack{\text{Extensive Margin} \\ 1-\delta(\pi)}}$$

We derive the decomposition of inflation for $\pi = 0$ and $\pi \to \infty$ for the undiscounted case with a quadratic profit function[5] in the special case where z represents the (log of the) product cost and follows a drift-less continuous time random walk.

PROPOSITION 3. Assume that $a(z) = 0$ and $b(z) = 1$, $\sigma > 0$ and $F(p - \bar{p}, z) = B\ (p - z)^2$. Consider the limit of the policy functions $\lim_{r \to 0} \Psi(\pi, \sigma^2, r, \frac{1}{c}, B; z) = \Psi(\pi, \sigma^2, \frac{1}{c}, B; z)$. Then,

$$\delta(0) = \frac{1}{10} \text{ and } \lim_{\pi \to \infty} \delta(\pi) = \frac{1}{3}$$

Proof. See section A.4 ■

The key insight in the proof of proposition 3 is that $\lim_{r \to 0} rV(x, r)$ is finite and independent of x. This allows us to obtain an analytical solution of the value function and to characterize optimal policies as simple functions of the ratio σ^2/π.

It is interesting to note that while proposition 3 states that ninety percent of changes in inflation at zero are accounted for by the extensive margin of price adjustment, proposition 1 states that when inflation is zero the frequency of price changes is insensitive to inflation. To gain insight into the interplay between the two propositions observe that for zero inflation there is no trend in relative prices and symmetry implies that the frequency of price increases and decreases are the same, $\lambda_a^+ = \lambda_a^-$. Also, that $\Delta_p^+ = \Delta_p^-$. Proposition 1 implies that $\frac{\partial \lambda_a^+}{\partial \pi} = -\frac{\partial \lambda_a^-}{\partial \pi}$ so the extensive margin at zero is $1 - \delta(0) = 2\Delta_p^+ \frac{\partial \lambda_a^+}{\partial \pi}$. Since inflation introduces a negative trend in relative prices, it induces them to hit more often the lower limit of the

[5] Equivalently, we can write the result for small a fixed cost c, so that prices are close to the profit maximizing value, and thus a second order expansion of the profit function is accurate.

inaction set, prompting more price increases and less price decreases. The characterization of optimal policies in the proof of proposition 3 shows that these changes in the frequency of price increases and decreases accounts for *ninety percent* of changes in the rate of inflation at $\pi = 0$. A similar argument holds for the decomposition of the change of inflation to a mild deflation.

Proposition 3 is useful to distinguish the menu cost model of price adjustment from the Calvo model. While in both models the frequency of price changes, λ_a, is insensitive to inflation for low inflation, the difference between the frequency of price increases and the frequency of price decreases is informative. In particular, while in the Calvo model the difference between the frequency of price increases and the frequency of price decreases is insensitive to inflation, $\frac{\partial(\lambda_a^+ - \lambda_a^-)}{\partial\pi} = 0$, in the menu cost model proposition 3 predicts that $\frac{\partial(\lambda_a^+ - \lambda_a^-)}{\partial\pi} > 0$ accounts for ninety percent of changes in inflation.

The second part of proposition 3 restates the results in proposition 2 under slightly different assumptions. The results in proposition 2 hold in the limit when $c \to 0$ while proposition 3 holds for a specific functional form of the profit function F. Not surprisingly, for small adjustment costs the quadratic profit function in proposition 3 is a good approximation of F.

General Remarks

We finish section 2.1 with a few remarks on the applicability of these comparative static results to the time series variation in our data set. The two propositions in this section were obtained under the assumption that inflation is to remain *constant* at the rate π, and that the frequency of price changes is computed under the invariant distribution. Thus, strictly speaking, our proposition are *not* a prediction for time series variation, but just a comparative static result. We give three comments on this respect.

First, this should be less of a concern for very high inflation, since the model becomes close to static, i.e. firms change prices very often and thus the adjustment to the invariant distribution happens very fast.

Second, when we analyze the Argentinean data we correlate the current frequency of price changes with an average of the current and future inflation rates. We experiment with different definition of these averages and find that the estimates of the elasticities in the two propositions of this section are not sensitive to this. Finally, with Argentina's experience in mind, Beraja (2013) studies the transition dynamics in a menu cost model where agents anticipate a disinflation in the future and performs the same comparative statics with artificial data generated from such model. He finds that the theoretical results in this section are robust to conducting the analysis in a non-stationary economy during a disinflation process calibrated to the Argentine economy.

Third, in section 2.2 we solve numerically a standard version of the menu cost model. In this section we derive several results for statistics of interest using limit values, both of menu cost c, discount rates r and inflation π. In section 2.2 we show that this limit values predict correctly the behavior of the statistics of interest for reasonable parameter values for c and r, which are positive but small, and for finite but large inflation rates π, of the order that are observed in Argentina.

2.2 Illustrating the theory with an example

In this section we specify a version of the firm's problem studied in section 2.1 to illustrate the theory. We characterize the solution of the model analytically and numerically and show how changes in the rate of inflation affect optimal pricing rules, the frequency of price changes, and the size of price adjustments.

We compute this example to verify the robustness of the analytical predictions obtained so far. In section 2.1 we obtained sharp analytical results under a variety of simplifying assumption such as limit values of parameters (e.g. vanishing menu cost c and or discount rate r), or the shape of profit functions (e.g. symmetry of F). Also our analytical results were obtained at two extreme values of inflation. In this section we check the robustness of the simplifying assumptions by computing a version of the model away from the limit cases, and also consider values of inflation within Argentina's experience.

The example assumes a constant elasticity of demand, a constant returns to scale production technology, idiosyncratic shocks to marginal cost that are permanent, an exponentially distributed product life and a cost of changing prices that is proportional to current profits (but independent of the size of the price change). This version of the Golosov and Lucas (2007) model is identical to the one in Kehoe and Midrigan (2010).[6]

We assume that the period profits are given by a demand with constant elasticity η and with a constant return to scale technology with marginal cost given by e^z, so that

$$F(p, z) = e^{-\eta p} \left(e^p - e^z \right) \ ,$$

where $p - z$ is the log of the gross markup (the net markup).

[6] We zero out the transitory shock that gives rise to sales in Kehoe and Midrigan (2010) and set it in continuous time.

The shocks on the log of the cost are permanent in the sense that

$$dz = \mu_z dt + \sigma dW - z dN.$$

where N is the counter of a Poisson process with constant arrival rate per unit of time ρ. We interpret this as products dying with Poisson arrival rate $\rho > 0$ per unit of time, at which time they are replaced by a new one which starts with $z = 0$ and must set its initial price. A positive value of μ_z can be interpreted as a vintage effect, i.e. the technology for new products grows at rate μ_z. The disappearance of products assures the existence of an ergodic distribution of relative prices and we view this device as realistic given the rate at which product are substituted in most data sets.

The menu cost, $\zeta(z)$, is assumed to be $\zeta(z) = c\ F(p^*(z), z)$ for some constant $c > 0$. Here, $p^*(z) = z + m$ where $m = \log(\frac{\eta}{\eta-1})$ is the log of the gross optimal static markup. Thus $F(p^*(z), z) = e^{(1-\eta)z} \left(\frac{\eta-1}{\eta}\right)^{\eta} \frac{1}{\eta-1}$. Note that $F(p^*(z), z)$ is decreasing and strictly convex on z for $\eta > 1$.

We will assume that $\eta > 1$ so that the static monopolist problem as a solution, and that

$$r + \rho \geq (1 - \eta) \left[\mu_z + (1 - \eta) \frac{\sigma^2}{2}\right] \tag{12}$$

so that the profits of the problem with zero fixed cost, $\hat{c} = 0$ are finite. Since for $\eta > 1$, period profits are decreasing and convex on z, and hence discounted expected profits are finite if the discount rate $r + \rho$ is high enough, or if the cost increases at a high enough rate μ_z, or if $\sigma^2/2$ is low enough.[7]

The firm's optimal pricing policy for the $\sigma > 0$ case can be characterized in terms of three constants $X \equiv (\underline{x}, \bar{x}, \hat{x})$. This is due to the combination of assumptions of constant elasticity of demand, constant returns to scale and permanent shocks to cost while the product last. In this example the policy function takes the simple form $\Psi(z) = X + z$. Let $x = p - z$ be the log of the real gross mark-up, which we refer to as the net markup. We can write the inaction set in terms of the net markups as $\mathcal{I} = \{x : \underline{x} < x < \bar{x}\}$. It is optimal to keep the price unchanged when the net markup x is in the interval $(\underline{x}, \bar{x})$. When prices are not changed, the real markup evolves according to $dx = -(\mu_z + \pi)dt + \sigma dW$. When the real markup hits either of the two thresholds, prices are adjusted so that the real markup is $\hat{x}$ and thus the optimal return relative price is $\hat{\psi}(z) = \hat{x} + z$. In the case where $\sigma = 0$ and $\pi + \mu_z > 0$, we obtain a version of Sheshinski and Weiss's (1977) model, and the optimal

[7] In this case the expected discounted value of profits, starting with z_0 is given by $\mathbb{E}_0 \int_0^\infty e^{-(r+\rho)t} F\left(p^*\left(z_t\right), z_t\right) dt = \frac{(\eta-1)^{\eta-1} e^{(1-\eta)z_0}}{\eta^\eta} \int_0^\infty e^{\left[-(r+\rho)+(1-\eta)\mu_z+(1-\eta)^2 \frac{\sigma^2}{2}\right]t} dt$.

policy can be characterized simply by two thresholds $s \equiv \underline{x} < \hat{x} \equiv S$.[8]

In appendix B we present several propositions with an analytical characterization of the solution of this model. A novel contribution of this paper is to derive a system of three equations in three unknowns for X, as well as the explicit solution to the value function, which depends on the parameters $\Theta \equiv (\pi, \mu_z, \sigma^2, \rho, r, \eta, c)$. We also derive an explicit solution for the expected number of adjustment per unit of time λ_a given a policy X and parameters $(\pi, \mu_z, \sigma^2, \rho)$ and we characterize the density g for the invariant distribution of (p, z) implied by the policy X and the parameters $(\pi, \mu_z, \sigma^2, \rho, \eta)$.

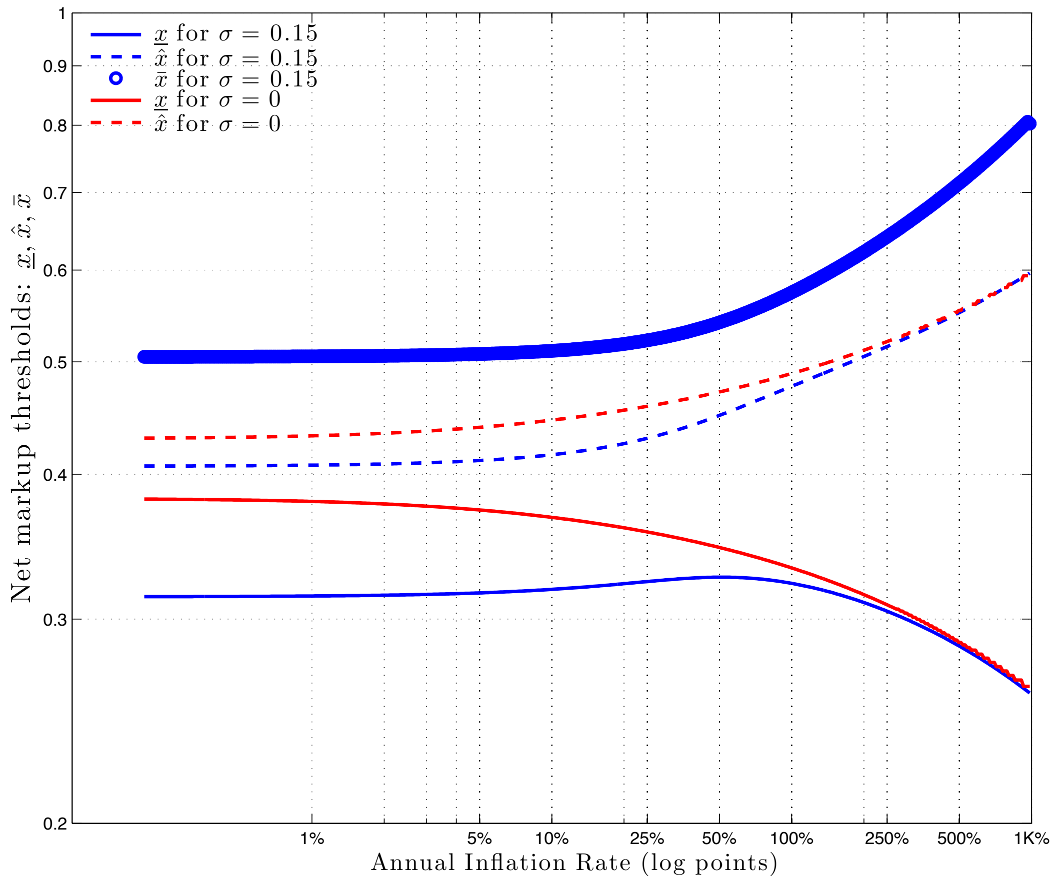
The remainder of this section contains several figures that describe numerically how changes in the rate of inflation affect the optimal pricing rules (the triplet X), the frequency of price changes, and the size of price adjustments. For the numerical examples we follow Kehoe and Midrigan (2010) and use $\eta = 3$, which implies a very large markup, but it is roughly inline with marketing/IO estimates of demand elasticities. We set $\rho = 0.1$ so products have a lifetime of 10 years, and $r = 0.06$ so yearly interest rates are 6%. We let $c = 0.002$ so that adjustment cost is 20 basis point of yearly frictionless profits. We let $\mu_z = 0.02$, i.e. a 2 percent per year increase in cost (or a 2% increase in vintage productivity). We consider three values for $\sigma \in \{0, 0.15, 0.20\}$, the first corresponds to Sheshinski and Weiss's (1977) model, and the others are 15% and 20% standard deviation in the change in marginal cost, at annual rates. The values of $c/\left(\eta(\eta-1)\right)$ and $\sigma = 0.15$ were jointly chosen so that at zero inflation the model matches both the average number of price changes $\lambda_a = 2.7$ and the average size of price changes of $\Delta_p = 0.10$, roughly the values corresponding to zero inflation in our data set.[9]

Figure 1 illustrates how the optimal threshold policies vary with inflation for two cases, $\sigma = 0.15$ and $\sigma = 0$. The blue lines depict Golosov and Lucas's (2007) case with $\sigma > 0$. The dashed center line is the optimal return mark-up and the outer lines are the boundaries of the inaction set. With no inflation, starting at the optimal mark-up $\hat{x}$ the mark-up will drift driven by the idiosyncratic shock. As long as it doesn't hit the boundaries the firm will keep its nominal price fixed. Once the mark-up hits either boundary, $\underline{x}$ or $\bar{x}$, the firm resets the price and the mark-up returns to $\hat{x}$. The red lines depict the optimal thresholds for Sheshinski and Weiss's (1977) case with $\sigma = 0$. When there are no idiosyncratic shocks after the firm resets its nominal price, mark-ups always fall. Hence, the upper limit of the

[8] In this case $\lambda_a = \rho / \left[1 - \exp\left(-\frac{\rho}{\pi+\mu_z}(\hat{x} - \underline{x})\right)\right] = (\pi + \mu_z)/(\hat{x} - \underline{x}) + o(\rho/(\pi + \mu_z))$, so that it coincides with the expression used for $\sigma = 0$ if ρ is small relative to π.

[9] Using results in Alvarez, Lippi, and Paciello (2011), when z is a random walk and the fixed cost is proportional to profits, there are mappings from observations to parameters $\lambda_a = \left[\frac{\sigma^2}{2} \frac{(1/2)\,\eta\,(\eta-1)}{c}\right]^{\frac{1}{2}}$ and $\Delta_p = \left[\sigma^2 \frac{c}{(1/2)\,\eta\,(\eta-1)}\right]^{\frac{1}{4}} \sqrt{\frac{2}{3.14159\cdots}}$

Figure 1: Optimal thresholds for different inflation rates



inaction set becomes irrelevant. The firm resets its nominal price to $\hat{x} + z$ when the mark-up hits the lower bound $\underline{x}$ and waits for it to fall again.

Figure 1 shows several properties of the model.[10] At very low inflation rates, and when $\sigma > 0$, the thresholds are symmetric, i.e. the distance between $\underline{x}$ and $\hat{x}$ is the same as the distance between $\bar{x}$ and $\hat{x}$. This symmetry implies that the size of price increases is equal to the size of price decreases, $\Delta_p^+(0, \sigma) = \Delta_p^-(0, \sigma)$, and that the frequency of price increases is equal to the one of price decreases, $\lambda_a^+(0, \sigma) = \lambda_a^-(0, \sigma)$. These are the results obtained in part (iii) of proposition 1. At very high inflation rates, the models with $\sigma > 0$ and with $\sigma = 0$ are equivalent in the sense that the critical values $\underline{x}$ and $\hat{x}$ in Golosov and Lucas's (2007) model converge to the Ss bands in Sheshinski and Weiss's (1977) model as established in -equation (10b) in lemma 2. As a result, the magnitude of price changes in the two models

[10] This example does not exactly satisfies all the assumptions of the model in section 2.1 since the profit function F derived from a constant elasticity demand is not symmetric. Yet, for small cost c the terms in the quadratic expansion, which are symmetric by construction, should provide an accurate approximation.

is the same as $\Delta_p^+(\pi, 0) = S - s = \hat{x} - \underline{x}$—equation (10c) in lemma 2. For rates of inflation above 250% per year figure 1 also reveals that the elasticity of $\Delta_p^+(\pi, 0)$ with respect to inflation is close to 1/3—equation (11c) in proposition 2

Figure 2: Frequency of Price Adjustments λ_a and of Price Increases λ_a^+

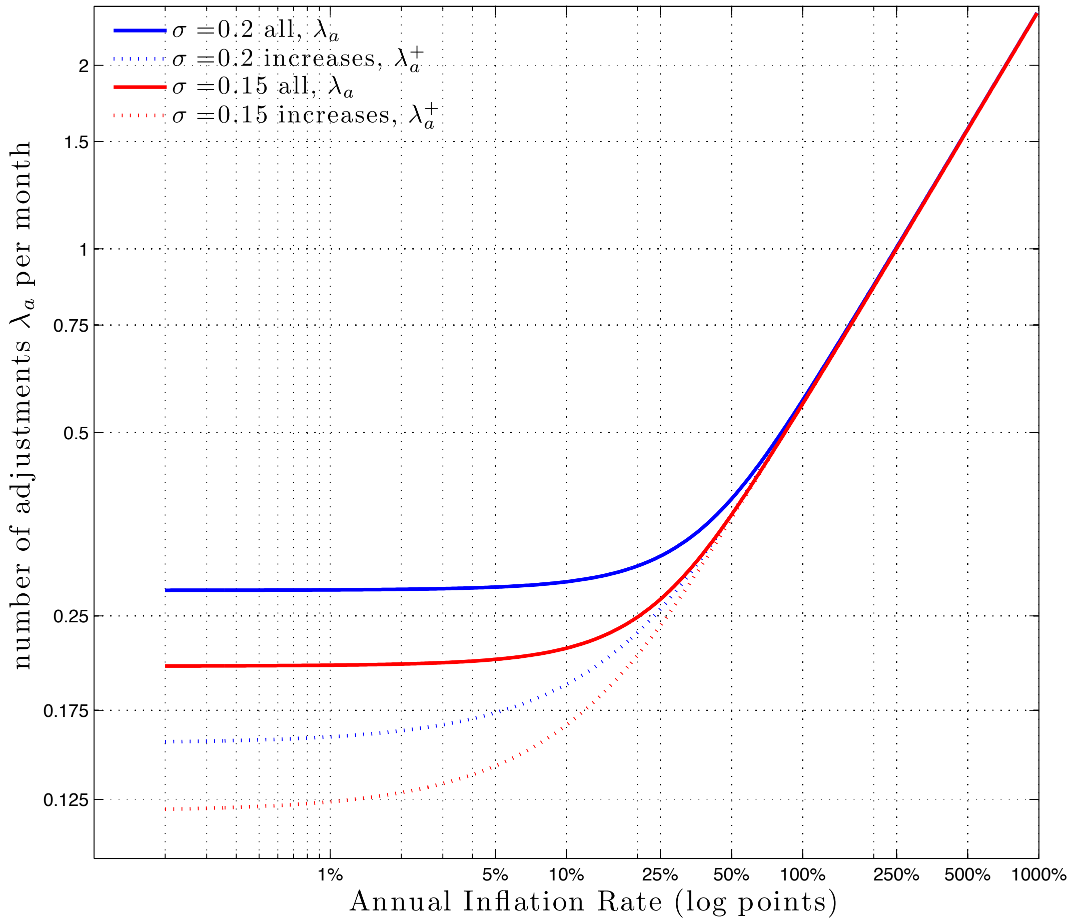


Figure 2 displays the frequency of price increases λ_a^+, together with the frequency of all adjustments λ_a, for two values of the cost volatility σ. There are several interesting observations about this figure. First, the frequency λ_a is insensitive to inflation in the neighborhood of zero inflation as established in part (i) of proposition 1. Second, the length of the inflation interval around $\pi = 0$ for which λ_a is approximately constant is higher for a higher σ—see the discussion in footnote 2. Third, the last part of proposition 1 predicts that the frequency of price increases and price decreases is the same when $\pi = 0$. The figure shows that for low inflation the frequency of price increases is about half of the frequency of price changes, indicating that half the price changes are increases and half are decreases. Fourth, for values of inflation above 250% per year, the frequency of price changes λ_a for different values of σ are approximately the same, consistent with the limiting results in equation (11a) of proposition 2. Fifth, since the graph is in log scale, it is clear that the common slope is approximately constant for large inflation, and close to 2/3 as established in equation (11b)in proposition 2. Finally, as inflation become large all price adjustments are price increases—as it can be seen by the fact that λ_a^+ converges to λ_a for each value of σ.

We now turn to the distribution of relative prices (which is derived in appendix B). Let $g\left(p, z; \pi, \sigma\right)$ be the invariant joint distribution of p and z and let the marginal distribution of p be $h(p) = \int_{p-\bar{x}}^{p-\underline{x}} g\left(p, z; \pi, \sigma\right) dz$. We are mostly interested in the *marginal*, or unconditional distribution of relative prices $h(p)$, as opposed to the conditional distribution $g(p, z)$ since the marginal (or unconditional) distribution is the object we can hope to measure in actual data because z is not readily observable.

Our focus is the analysis of the standard deviation of relative prices, especially its elasticity with respect to inflation. There are two sources of dispersion for relative prices: the idiosyncratic cost shocks and the asynchronous price adjustments to inflation. It is helpful to to decompose the unconditional variance of relative prices $\bar{\sigma}\left(p; \pi, \cdot\right)$ for a given inflation rate π as follows:

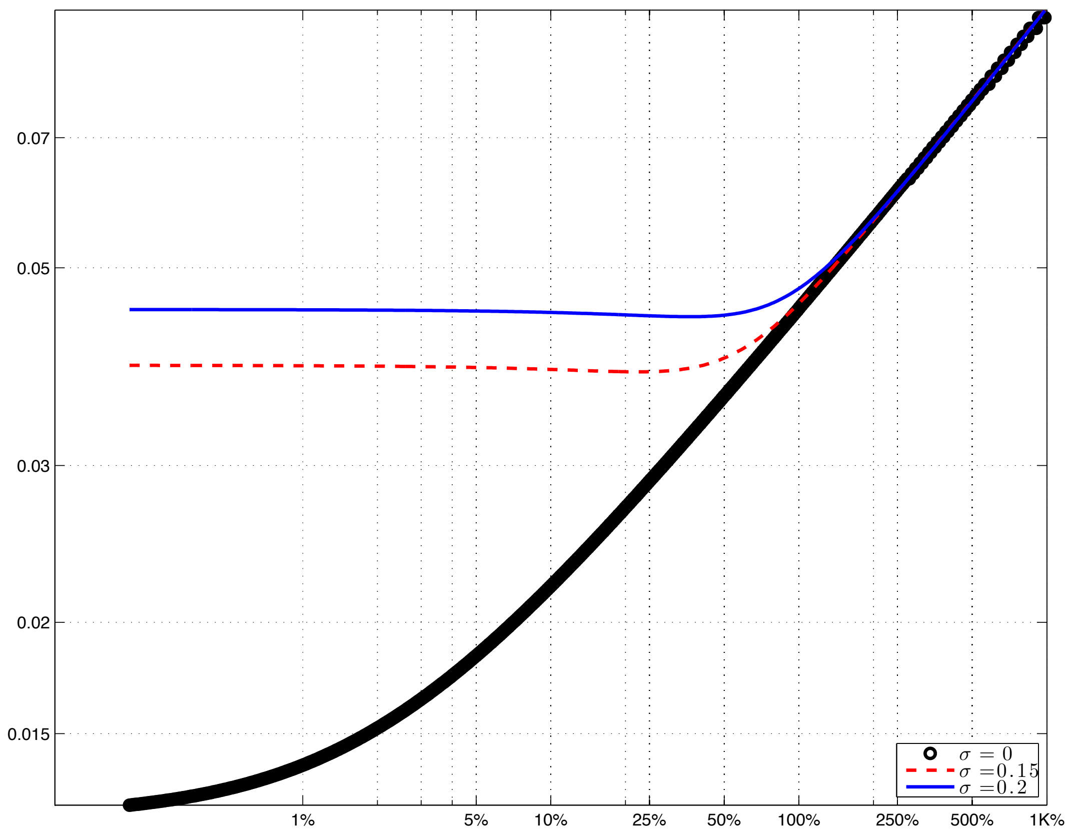
$$\sigma\left(p; \pi, \cdot\right) = \mathbb{E}\left[\mathbb{V}ar\left(p|z; \pi, \cdot\right)\right] + \mathbb{V}ar\left[\mathbb{E}\left(p|z; \pi, \cdot\right)\right] \tag{13}$$

We first explain the behavior of the first term in equation (13). For this note that the distribution $g(\cdot, z)$ for π close to zero is single peaked in p at $\hat{x}$ with support on $[\underline{x}, \bar{x}]$, roughly symmetric with densities decaying up to zero at the two extremes. The distribution $g(\cdot, z)$ for large π is almost uniform between $[\underline{x}, \hat{x}]$. The latter is the distribution for the case with $\sigma = 0$ (i.e. Sheshinski and Weiss's (1977) case), which again is equivalent to the case of large π. The results in appendix B shows that the distribution $g(\cdot, z)$ has the same shape for any value of z. In figure 3 we fix $z = 0$ and plot the standard deviation of p corresponding to the distribution $g(\cdot, 0)$ and $\sigma > 0$ for a range of value of π. The figure shows that the elasticity of the standard deviation of relative prices conditional on z with respect to inflation is approximately 0 for $\pi = 0$ (as in proposition 1) and it is approximately 1/3 for large π (as in lemma 1). The figure also shows that for large π, the variance of relative prices is independent of σ (as in lemma 2).

Now we discuss the second term in equation (13). This dispersion is not systematically related to inflation, it is mostly given by the cross sectional distribution of cost z. For example in the case of zero menu cost the variance of the average price is equal to the cross sectional dispersion of z for all values of π.

Now putting the two effect together we have that for low values of inflation neither of the

Figure 3: Standard deviation of log relative prices, conditional on $z = 0$.



two terms of the unconditional variance changes with inflation. For large enough inflation rates the width of the inaction range will swamp the effect the variation of z. In our numerical examples, however, it takes inflation rates even much higher than the ones observed in the peak months in Argentina for this to happen(see figure 13 in appendix B).

We briefly comment on the difference between the behaviour of λ_a and of $\bar{\sigma}$ as functions of π. In this model the value for the frequency of price changes $\lambda_a(\pi, \sigma^2)$ converges to $\lambda_a(\pi, 0)$ as inflation increases much faster than $\bar{\sigma}(\pi, \sigma^2)$ does. The reason is that given the permanent nature of the shocks to a product cost, z, the expected time until the next adjustment $\mathcal{T}$ is only a function of $x = p - z$. Recall that $\lambda_a = 1/\mathcal{T}$ so that the cross sectional distribution of z is essentially irrelevant for the frequency of adjustment. Instead, the standard deviation of relative prices $\bar{\sigma}$ depends on the cross sectional distribution of z on a crucial way. Indeed, if the idiosyncratic shocks were completely permanent, there will be no invariant distribution of relative prices. In our example, the reason why there is an invariant distribution is that $\rho > 0$, so products are returned to $z = 0$ at exponentially distributed times.

3 Description of the Dataset

Our dataset contains $8,618,345$ price quotes that underlie the consumer price index for the Buenos Metropolitan Area in the period December 1988-September 1977. Each price quote represents an *item*, that is, a good/service of a determined brand sold in a specific outlet in a specific period of time. Goods[11] and outlets are chosen to be representative of consumer expenditure in the 1986 consumer expenditure survey[12]. Price quotes are for 506 goods that account for about 84% of household expenditures.

Goods are divided into two groups: homogeneous and differentiated goods. Differentiated goods represent 50.5% of the expenditure in our sample while homogeneous goods account for the remaining 49.5%[13]. Prices are collected every two weeks for all homogeneous goods and for those differentiated goods sold in super-market chains; and are gathered every month for the rest of the differentiated goods. The data set contains 233 prices collected every two weeks and 302 prices collected every month. There are 29 goods gathered both monthly and biweekly.[14]

An important feature of the dataset is the rich cross section of outlets where prices are recorded at each point in time. Over the whole sample there are $11,659$ outlets. Roughly around 3200 outlets per month for homogeneous goods and about the same number for differentiated goods. On average across the 9 years there are 166 outlets per good (81 outlets per product collected monthly and 265 per good collected bimonthly). Online appendix C contains further information on data collection and on the classification of goods.

We exclude from the sample price quotes for baskets of goods, rents and fuel prices. Baskets correspond to around 9.91% of total expenditure and are excluded because their prices are gathered for any good in a basket, i.e., if one good is not available, it is substituted by any another in the basket. Examples are medicines and cigarettes. Rents are sampled monthly for a fixed set of representative properties. Reported prices are for the average of the sampled properties and include what is paid on that month, as opposed to what is paid for a new contract. Rents represent 2.33% of household expenditure. Fuel prices account for 4% of total expenditure and we exclude them because they were gathered in separate data base that we do not have access to.

The dataset has some missing observations and flags for stock-outs, price substitutions

[11] To simplify the exposition, when it is clear, we use goods to refer to either goods or services.

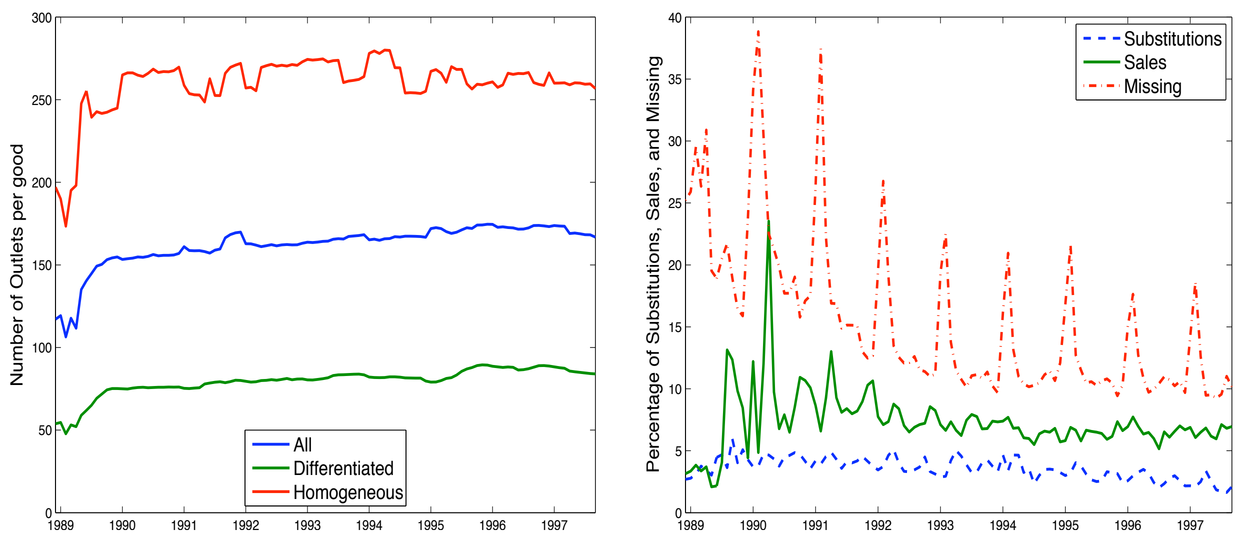
[12] Encuesta Nacional de Gasto de los Hogares

[13] Examples of homogeneous goods are: barley bread, chicken, lettuce, etc. Examples of differentiated goods are moccasin shoes, utilities, tourism, and professional services.

[14] The outlets are divided into 20 waves, corresponding to the 20 working days of the month. Each outlet is visited, roughly in the same working day every 10 working days in the case of homogeneous goods and differentiated gathered at super-markets. The dataset includes the particular day when each price is gathered.

and sales. We treat stock-outs (10.5% of observations) and price quotes with no recorded information (2.25% of observations) as missing observations. The statistical agency substitutes the price quote of an item for a similar item, typically when the good is either discontinued by the producer or not sold any longer by an outlet. Using this definition, across the 9 years of our data set we have an average of 2.39% of price quotes that have been substituted. The data set contains an indicator of whether an item was on sale or not. Around 5% of items have a sale flag. This is small compared with the 11% frequency of sales reported by Klenow and Kryvtsov (2008) for the US. 70% of the sales corresponds to homogeneous items (this is similar to Klenow and Kryvtsov (2008), who report that sales are more frequent for food items). The time series data for the number of outlets per good and for the frequencies of missing observations, substitutions and sales are depicted in figure 4.

Figure 4: Number Outlets per Good, and Frequencies of Substitution and Sales



Note: For the homogeneous goods during a month we count a sale or substitution if there was one such event in any of the two fifteen days subperiods. Missing includes stock-outs.

4 Estimating the Frequency of Price Changes

We extend the methodology of Klenow and Kryvtsov (2008) to the case of time varying frequencies of price changes. We assume a constant probability of a price change per unit of time (a month for differentiated goods and two weeks for homogeneous goods) so that the arrival rate of a price change follows a Poisson process. In this case, the maximum likelihood estimator of the frequency of price changes is

$$\lambda_t = -\ln\left(1 - \text{fraction of outlets that changed price between t and t-1}\right). \tag{14}$$

The fraction of outlets that changed price between periods can be calculated for individual goods or for the aggregate by pooling the data for all outlets and all goods together. In this computation we drop observations with missing price quotes. This *simple estimator* just counts the fraction of price changes in a period of time, and transform it into a per unit of time rate, λ. We refer to λ as the "instantaneous" frequency of price changes, which has the dimension of the number of price changes per month.

Later we perform robustness checks by using different methods of aggregation across goods, by considering different treatments for sales, substitutions and missing observations, and by dropping the assumption that price changes follow a Poisson process.

Figure 5 plots the monthly time series of the simple pooled estimator of λ as well as of the expected inflation rate. It assumes that all homogeneous and all differentiated goods have the same frequency of price changes and estimates this aggregate frequency by using the simple pooled estimator for the homogeneous and for the differentiated goods. The bi-weekly estimates of the homogeneous goods are aggregated to a monthly frequency[15], and the plot shows the weighted average of these two estimators, using the share of household expenditures as weights.

5 Argentina's Evidence on Menu Cost Models of Price Dynamics

In section 2 we uncovered several properties of menu cost models that can be contrasted with data. The presentation of the empirical results in this is section is organized around those predictions.

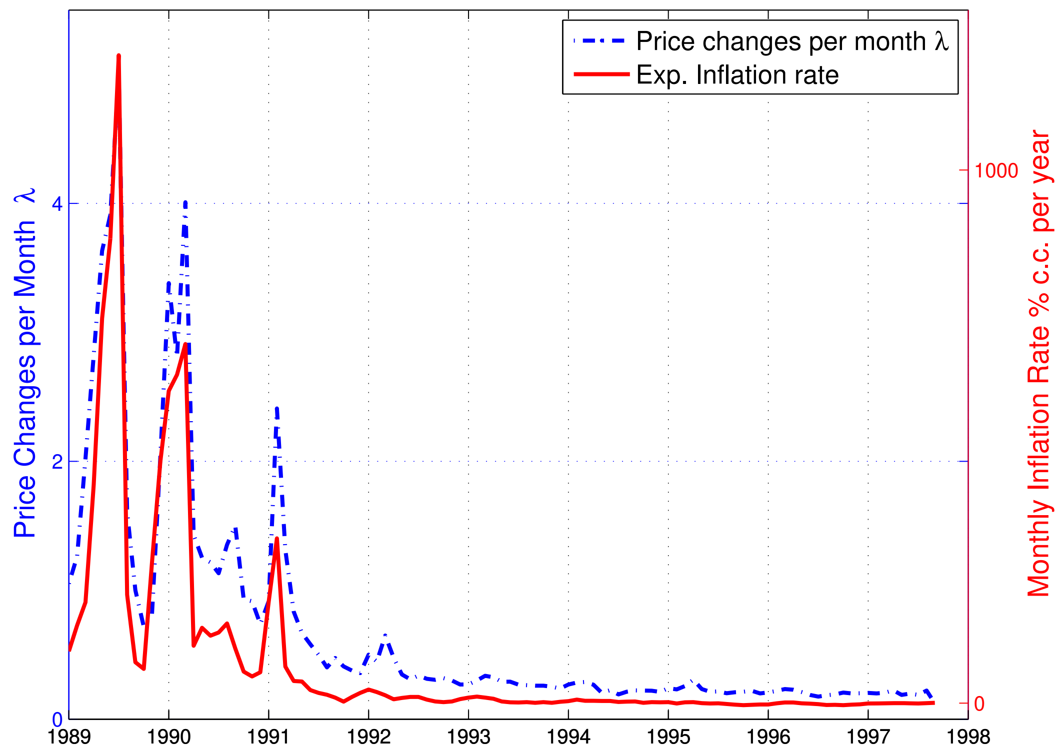
1. The elasticity of the frequency of price changes λ with respect to changes in the rate of inflation is zero at low inflation rates and it approximates two thirds as inflation becomes very large[16].

2. The dispersion of the frequency of price changes across goods decreases with inflation. It should be zero when inflation goes to infinity and the model converges to the Sheshinski

[15] The monthly frequency is the sum of the bi-weekly frequencies of each month.

[16] In models in which the menu cost is zero at some random times, so that they combine menu costs and Calvo type price adjustments, this elasticity can be lower than 2/3 at high inflation. See, for example Nakamura and Steinsson (2010) and Alvarez, Bihan, and Lippi (2014)

Figure 5: Estimated Frequency of Price Changes λ and Expected Inflation



Note: Simple estimator of λ, $\hat{\lambda} = -\log(1 - f_t)$, where f_t is the fraction of outlets that changed price in period t. λ is estimated separately for homogeneous goods (bi-weekly sample) and for differentiated goods (monthly sample). Homogeneous goods frequencies are converted to monthly by adding the bi-weekly ones for each month pair. The aggregate number is obtained by averaging with the respective expenditure shares in the Argentine CPI. Inflation is the average of the log-difference of monthly prices multiplied by 1200 and weighted by expenditure shares. Expected inflation is the average inflation rate $1/\hat{\lambda}_t$ periods ahead.

and Weiss (1977)) model with no idiosyncratic shocks

3. Intensive and extensive margins of price increases and decreases.

    (a) The frequency of price increases and of price decreases is similar at low inflation rates.

    (b) The magnitude of price increases and of price decreases is similar at low inflation rates.

    (c) At low inflation rates as inflation grows the frequency of price changes remains constant while the frequency of price increases rises and the frequency of price decreases falls.

(d) For high inflation rates the fraction of firms raising prices converges to λ and the fraction of firms lowering prices converges to zero.

(e) The absolute value of price changes is an increasing function of the inflation rate.

4. The elasticity of the dispersion of prices across stores with respect to inflation is zero for low inflation rates and approaches one third when inflation goes to infinity. This elasticity, however, may be smaller than one third for high inflation rates if idiosyncratic shocks are persistent.

We now look at each of these dimensions of the Argentina data.

5.1 The Frequency of Price Changes and Inflation

We now report how the estimated frequency of price changes to reacts to changes in the rate of inflation. As predicted by the menu cost model, at low rates of inflation the frequency of price changes is insensitive to the inflation rate. For high inflation rates we find that the elasticity of the frequency of price changes with respect to the rate of inflation is between one half and two thirds.

Figure 6 plots the frequency of price changes against the rate of inflation using log scale for both variables.[17] On the right axis we indicate the implied instantaneous duration, i.e. $1/\lambda$. In interpreting this figure, as well as the other estimates presented below, it is worth noting that $1/\lambda_t$ is the expected duration of prices at time t if λ_t will be unchanged into the future, and provided that the probability of a price change is the same within the smaller period of observation (1 month for differentiated goods, 2 weeks for homogeneous goods).

Motivated by the theoretical considerations of section 2, as well as by the pattern we think evident in figure 6, we fit (by non-linear least squares) the following statistical model to the data:

$$\log \lambda = a + \epsilon \min\{\pi - \pi^c, 0\} + \nu(\min\{\pi - \pi^c, 0\})^2 + \gamma \max\{\log \pi - \log \pi^c, 0\}. \tag{15}$$

This model assumes that $\log \lambda$ is a quadratic function of inflation for inflation rates below the critical value, π^c and that $\log \lambda$ is a linear function of $\log \pi$ for inflation rates above π^c. In figure 6 we indeed observe that λ is insensitive to inflation at low inflation rates. Increasing inflation from 0 to 1% per year increases the frequency of price changes by only 0.04%. Morover, the behavior of λ is symmetric around zero. For high inflation rates the elasticity of λ with respect to inflation is captured by the parameter γ, which we estimate at a large

[17] See section 2 for the caveats on these results and on the interpretation of contemporaneous correlations.

value, of at least 1/2, but it is still smaller than the theoretical limit 2/3. Also, as predicted by the menu cost model, as inflation rises this elasticity becomes constant—i.e. the linear fit for $\log \lambda$ as a function of $\log \pi$ works well for high inflation rates. In this estimation, the critical value π^c in the statistical model, which has no theoretical interpretation, is of 14% per year.[18] The expected duration of price spells for zero inflation is 4.5 months which, as we shall see next, is consistent with the international evidence.

5.1.1 International Evidence on the Frequency of Price Changes and Inflation

The previous section shows that Argentine price dynamics are consistent with the predictions of the menu cost model: the elasticity of the frequency of price adjustment is close to zero at low inflation rates and close to 2/3 for high inflation. Here we show that the Argentine data is of special interest because on one hand it spans and extends the existing literature and, on the other, it is consistent with previous findings.

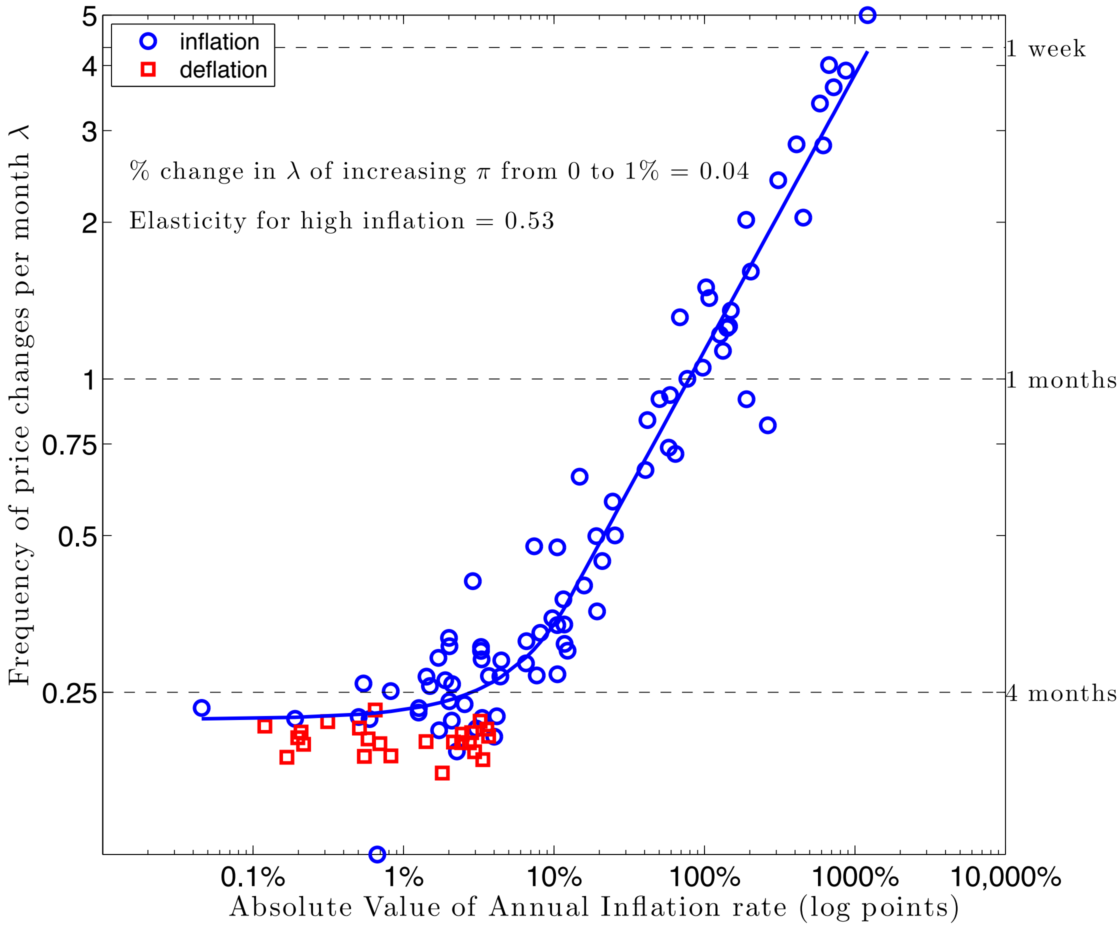
There are several studies that estimate the frequency of price changes for countries experiencing different inflation rates. Figure 7 provides a visual summary of these studies and compares them to ours by adding the international evidence to figure 6. First, observe how the wide range of inflation rates covered by our sample makes this paper unique: none of the other papers covers inflation rates ranging from a mild deflation to 7.2 million percent per year (annualized rate of inflation in July 1989). This is what enables us to estimate the elasticity of the frequency of price changes with respect to inflation at low and at high inflation rates. In the other samples it is hard to test these hypothesis. Second, observe how the patterns of the data for each country are consistent with the two predictions of the menu cost model. Third, observe that in most cases the level of the estimated frequency of price changes is similar to Argentina's. The similarity between our results and the existing literature is remarkable since the other studies involve different economies, different goods and different time periods. It is a strong indicator that our results are of general interest, as the theory suggests, and are not a special feature of Argentina.[19]

The studies included in the figure are all the ones we could find covering a wide inflation range. For the low inflation range we included studies for the United States by Bils and Klenow (2004), Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) and for

[18] The comparative static of the models discussed in section 2 does not imply a kink as the one in equation (15), we merely use this specification because it is a low dimensional representation of interesting patterns in the data that provides a good fit and has properties at the extreme values that are consistent with our interpretation of the theory.

[19] Table 5 in the on-line appendix provides a succinct comparison of the data sets used in these studies and of the inflationary environment in place in each case. The table shows that in addition to covering a wider range of inflation rates our data set is special due to its broad coverage that includes more than 500 goods representing 85% of Argentina's consumption expenditures.

Figure 6: The Frequency of Price Changes (λ) and Expected Inflation.



Note: Simple estimator of λ, $\hat{\lambda} = -\log(1 - f_t)$, where f_t is the fraction of outlets that changed price in period t. λ is estimated separately for homogeneous goods (bi-weekly sample) and for differentiated goods (monthly sample). Homogeneous goods frequencies are converted to monthly by adding the bi-weekly ones for each month pair. The aggregate number is obtained by averaging with the respective expenditure shares in the Argentine CPI. Inflation is the average of the log-difference of monthly prices weighted by expenditure shares. Expected inflation is computed as the simple average of inflation rates $1/\lambda$ months ahead. The fitted line is $\log \lambda = a + \epsilon \min\{\pi - \pi^c, 0\} + \nu(\min\{\pi - \pi^c, 0\})^2 + \gamma \max\{\log \pi - \log \pi^c, 0\}$. The red squares represent negative expected inflation rates and the blue circles positive ones.

the Euro Area by Álvarez et al. (2006). Our estimates of the frequency of price changes are consistent with all of them.[20] We have three data points for Israel corresponding to an inflation rate of 16% per year in 1991-2 (Baharad and Eden (2004)) and to inflation rates of

[20] There are other studies for low inflation countries, especially for the Euro area, but as they mostly yield estimates similar to those of Álvarez et al. (2006) we do not report them (see Álvarez et al. (2006) and Klenow and Malin (2011) for references to these studies).

64% per year in 1978-1979 and 120% per year in 1981-1982 (Lach and Tsiddon (1992)). The frequency of price adjustment for these three points is well aligned with the Argentine data. The same is true for the Norwegian data (Wulfsberg (2010)) that ranges from 0.5% to 14% per year. For Poland, Mexico and Brazil we were able to obtain monthly data for a wide range of inflation rates. The Polish sample ranges from 18% to 249% per year (Konieczny and Skrzypacz (2005)) and the Mexican one ranges from 3.5% to 45% per year (Gagnon (2009)). In both cases the triangles are aligned with the Argentina sample. The Brazilian (Barros et al. (2009)) data yields higher frequencies of price changes than the other studies, but the slope of the Brazilian cloud of points is consistent with ours.

5.1.2 Robustness

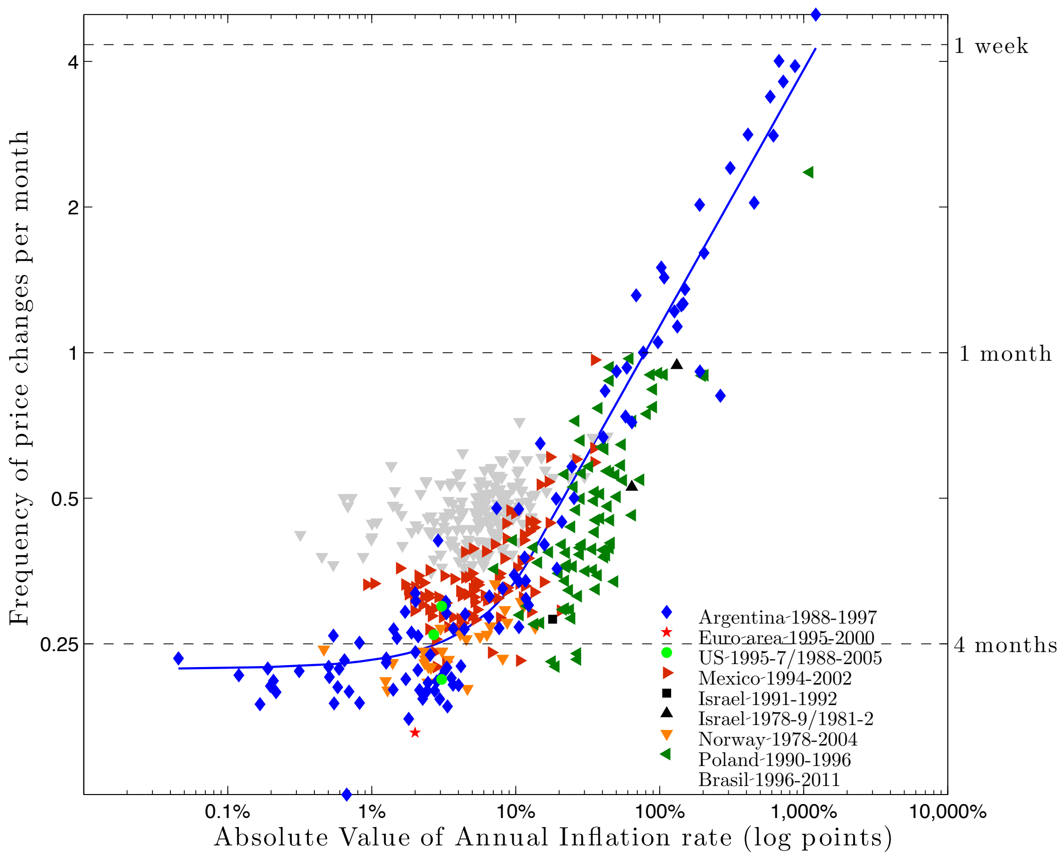
In appendix E we conduct a battery of robustness checks to evaluate the sensitivity of the main results in this section. The first set of checks consists in dealing with recurrent issues when analyzing micro-price datasets such as missing observations and price changes due to substitutions or sales. Secondly, we discuss issues of aggregation across products. Third, we address biases resulting from discrete sampling. Fourth, we present results using different-measure of expected inflation instead of contemporaneous inflation. Finally, we address the possibility that the theoretical propositions which hold in the steady state are a poor description of the argentinean experience in the high inflation period leading to the stabilization plan in 1991 where agents are likely to have anticipated the strong disinflation that followed. To summarize, the empirical findings at low inflation go through intact. At high inflation, we observe some quantitative but not qualitative differences. Most notable, depending on the estimator used, the elasticity of the frequency of price changes can range from approximately 1/2 to the theoretical 2/3.

5.2 Inflation and the Dispersion of the Frequency of Price Changes

This section reports how the dispersion of the frequency of price changes varies as inflation grows. Proposition 2 tells us that, under some conditions, the firm's pricing decisions with very high inflation are independent of the variance of its idiosyncratic shocks. This implies that as inflation becomes higher it swamps the effect of idiosyncratic differences across firms that result in differences in the frequency with which they change prices. Figure 2 illustrates this point in the numerical example of our version of Golosov and Lucas (2007) model in section 2.2.

We can think that the comparative static result in proposition 2 also applies to the individual industries within which monopolistic firms compete. In table 1 we estimate λ

Figure 7: The Frequency of Price Changes (λ) and Expected Inflation: International Evidence



Note: price changes per month for Argentina are the simple pooled estimator of λ. For the other cases we plot $-log(1-f)$, where f is the reported frequency of price changes in each study. The (λ, π) pairs for Argentina, Mexico and Brazil are estimated once a month and for the other countries once a year. Expected inflation is the average inflation $1/\lambda$ months ahead. Data for the Euro area is from Álvarez et al. (2006), for the US from Bils and Klenow (2004), Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008), for México from Gagnon (2009), for Israel from Baharad and Eden (2004), and Lach and Tsiddon (1992), for Poland from Konieczny and Skrzypacz (2005), for Brazil Barros et al. (2009), and for Norway from Wulfsberg (2010). Logarithmic scale for the both axis.

for each narrowly defined industry (at a 5-digit level of aggregation)[21], calculate the implied average duration $\frac{1}{\lambda}$ and present two measures of the dispersion of λs across such industries. We observe a significant decline in dispersion as inflation rises both across homogeneous and differentiated good industries. For homogeneous goods the 90-10 percentile difference in the λs with inflation above 500% per year is only 12% of the one for single digit inflation. For differentiated goods the 90-10 percentile difference of λs with high inflation is 1.6% of the

[21]Examples of 5 digit aggregation are citric fruits, soaps and detergents. See table 2 in appendix C.

one with low inflation.

Table 1: Cross Industry Dispersion of Duration $\frac{1}{\lambda}$

Annual Inflation Range (%)	Median Duration	75-25 pct Difference	90-10 pct Difference
Homogeneous Goods			
< 10	6.37	5.47	10.30
$[10, 100)$	3.36	2.70	5.25
$[100, 500)$	2.29	1.67	3.26
≥ 500	1.03	0.77	1.28
Differentiated Goods			
< 10	11.55	11.81	24.67
$[10, 100)$	3.82	2.84	5.39
$[100, 500)$	1.05	0.65	1.23
≥ 500	0.39	0.20	0.41

Note: duration is in months and calculated as $\frac{1}{\lambda}$ for each 5-digit industry. The cross-industry statistic, e.g. 75-25 pct, is the average over all observations corresponding to inflation rates in the interval.

5.3 Inflation and the Intensive and Extensive Margins of Price Adjustments

In this section we take the theoretical predictions of the menu cost model on the behavior of the intensive and extensive margins[22] of price adjustment to the data.

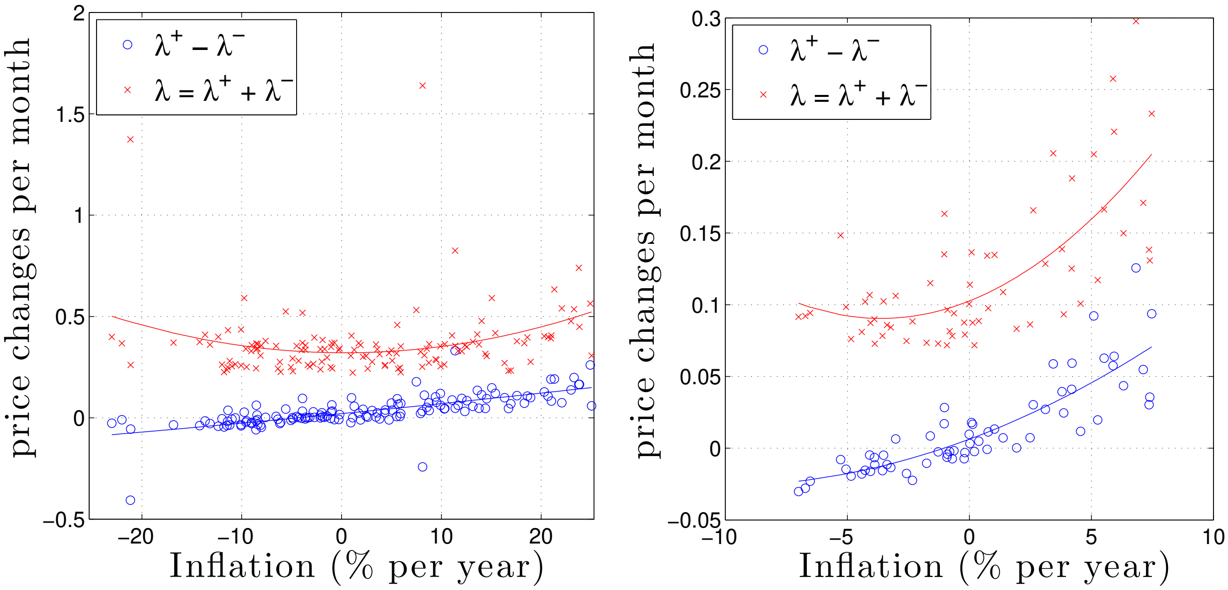
We first look at the predictions of the theory (propositions 1 and 3) with respect to the frequency of price adjustment (extensive margin) for near zero inflation rates. According to the theory, for near zero inflation, the frequency of price increases and of price decreases is the same, the frequency of price changes is insensitive to inflation and the frequency of price increases (decreases) rises (falls) when inflation increases. Figure 2 illustrates these properties of the menu cost model in the numerical example in section 2.2.

Figure 8 takes these predictions to the data for our two groups of goods. The red crosses plot our estimates of the frequency of price changes, λ, against inflation while the blue circles represent the difference between the frequency of price increases and that of price decreases, $\lambda^+ - \lambda^-$. The range of inflation in the figure was chosen by picking the lowest rate of inflation

[22]See section 2.1.3 for a definition of these margins

Figure 8: Decomposition of inflation for low inflation rates

(a) Homogeneous goods (b) Heterogeneous Goods



Note: λ is the frequency of price changes per month. λ^+ (λ^-) is the frequency of price increases (decreases) per month. Inflation is the annualized log difference of the average price between two consecutive periods. The inflation range is chosen by picking the 1-percentile inflation (minimum inflation rate removing outliers) and its positive opposite. Lines are least squares second degree polynomials.

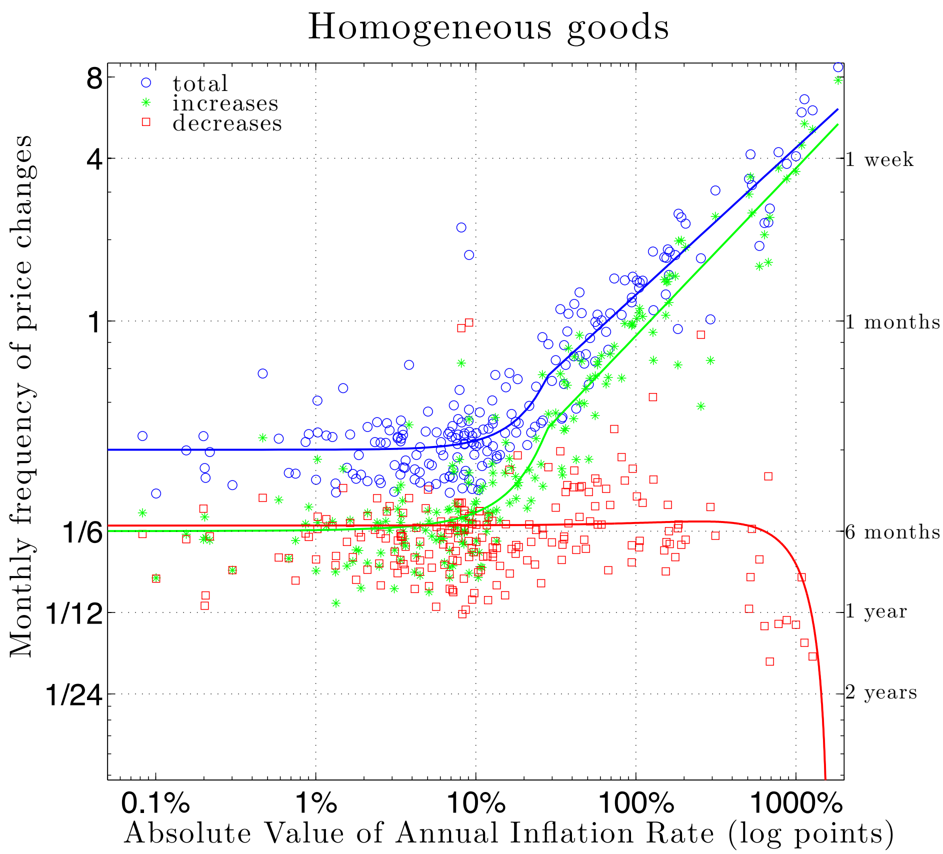
excluding outliers, which is negative, and its positive opposite. For heterogeneous goods a 20% inflation/deflation represents month to month price change of 1.7%. For homogeneous goods a 5% inflation/deflation represents bi-weekly price changes of 0.2%. The lines are a quadratic least squares fit to the data intended as a visual aid. The symmetry between the frequency of price increases and of price decreases is reflected in the data as the fitted line for the $\left(\pi_t, \lambda_t^+ - \lambda_t^-\right)$ pairs is essentially zero for zero inflation. Figure 9 shows the same fact by plotting the frequency of price increases (green stars) and the frequency of price decreases (red squares) against inflation. The two panels in figure 8 also show that the prediction in proposition 3 stating that that the derivative of $\lambda^+ - \lambda^-$ with respect to inflation is positive for low rates of inflation seems to be consistent with the data. Finally, figure 8 shows that when inflation is close to zero, the frequency of price adjustment is insensitive to inflation.[23] [24]

[23]Unlike figure 6 and figure 9 the axes in figure 8 are not in log scale assuring that the insensitivity of the frequency of price adjustment to inflation is not an artifact of the scale.

[24]This strong results in figure 6 are surprising as the model applies to steady states in which the inflation rate has been at the same level for a long time. As a robustness check we replicated figure 6 using expect inflation instead of inflation. The results are similar to those in figure 6 , but somewhat weaker.

The menu cost model predicts that for high inflation rates the frequency of price increases, λ^+, converges to λ and that the frequency of prices decreases, λ^- converges to zero (lemma 1 and lemma 2). Figure 9 shows that this is the case in the Argentine data.

Figure 9: Inflation and the Extensive Margin of Price Adjustments

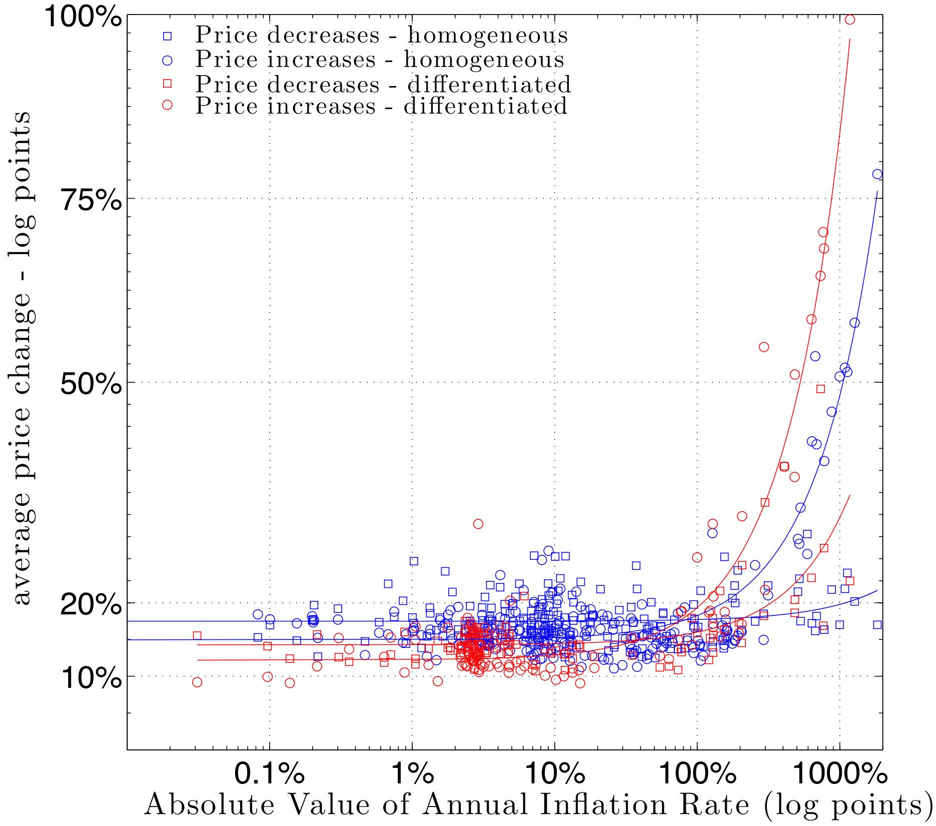


Note: The frequency of price increases and decreases is calculated as $-log(1-f)$, where f is the fraction of outlets increasing or decreasing price in a given date.

The empirical behavior of the intensive margin is described in figure 10, which shows the absolute value of the magnitude of price increases and of price decreases as a function of the absolute value of inflation (in semilog scale).[25] Figure 10 shows that, for low inflation rates, the size of price changes plotted against inflation raes is flat and that the magnitude of price increases and decreases is the same and approximately 10%. This is consistent with the last part of proposition 1. As inflation rises, the magnitude of price increases and decreases rises, with the magnitude of price increases becoming larger than that of price decreases. This is consistent with the properties of our numerical example, shown in figure 1 and in figure 12.

[25] Figure 12 in appendix B.1 is the analog of figure 10, which we computed with the numerical example described in 2.2. The theoretical and the empirical figures are are qualitatively very similar

Figure 10: Inflation and the Intensive Margin of Price Adjustments



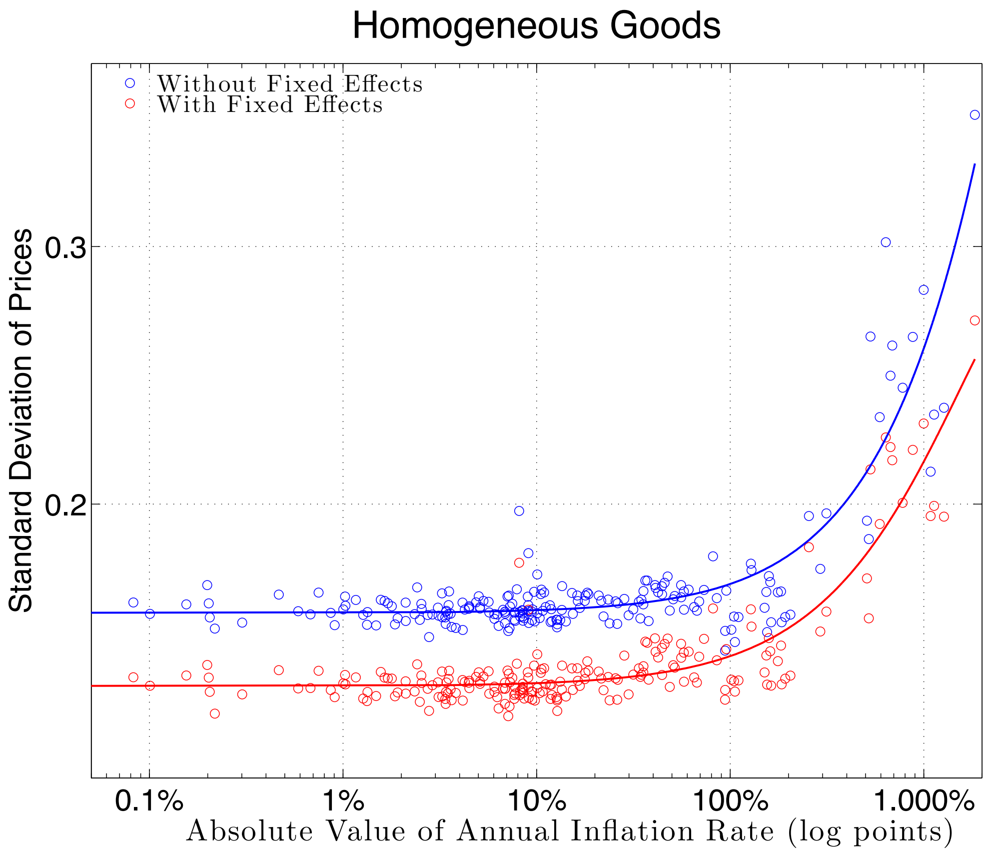
Note: The average price change is the log difference in prices, conditional on a price change taking place, averaged with expenditure weights over all homogeneous and differentiated goods in a given date.

5.4 Inflation and the Dispersion of Relative Prices

In section 2 we analyze the effect of inflation on the variance of relative prices. Summarizing our discussion there, inflation should have a very small effect on the dispersion of relative prices at low level of inflation, and for higher levels the dispersion should become an increasing function of inflation.

In this section we explore the association between average price dispersion across goods and inflation. We measure the price dispersion across outlets selling the same good or service at a given month. We then report a weighted average of the dispersion of prices, where the weights are given by each product's expenditure share in the consumer survey. To be

Figure 11: Average Dispersion of Relative Prices and Inflation



Standard deviation of prices at t is $\bar{\sigma}_t = \sum_{j=1}^{N} \omega_j \left[\sum_{i \in O_j} \left[(\log p_{i,j,t})^2 - \left(\frac{1}{\# O_j} \sum_{i \in O_j} \log p_{i,j,t} \right) \right]^2 \right]^{\frac{1}{2}}$, where O_j are the set of outlets that sell the good j, ω_j the expenditure share of the good j and $p_{i,j,t}$ is the price of the good j sold at outlet i at time t.

concrete the average dispersion of relative prices at time t is given by

$$\bar{\sigma}_t = \sum_{j=1}^{N} \omega_j \left[\sum_{i \in O_j} \left[(\log p_{i,j,t})^2 - \left(\frac{1}{\# O_j} \sum_{i \in O_j} \log p_{i,j,t} \right) \right]^2 \right]^{\frac{1}{2}},$$

where O_j are the set of outlets that sell the good j, ω_j the expenditure share of the good j and $p_{i,j,t}$ is the price of the good j sold at outlet i at time t.[26]. We compute the time series for $\bar{\sigma}_t$ among differentiated goods, and among homogeneous goods. In figure 11 we plot, in a log scale, these measures of average dispersion of relative prices against the corresponding

[26]In the computation of $\bar{\sigma}$ the set of outlets that sell a given good vary across time. Also, we exclude the goods whose prices are missing

inflation for homogeneous goods and differentiated goods. For robustness, we also introduce a similar dispersion measure where we control for store fixed effects.[27]

Figure 11 presents the empirical counterpart to the comparative statics illustrated in figure 3. It plots the standard deviation of prices for homogeneous goods against the absolute value of inflation. Deflation points are represented by squares and inflation points by circles. As predicted by proposition 1 the variance of relative prices whether we control or not for store fixed effects is unresponsive to inflation at low inflation rates. Moreover, it is symmetric around 0. For high inflation rates, the theory predicts that in the limit at infinity the elasticity of the standard deviation of relative prices to inflation converges to 1/3. However, in our simulations, for the ranges of inflation experienced by Argentina, the variance of the idiosyncratic shocks swamped the variance of relative prices within the inaction set that arises from asynchronicity in price adjustment. In the data, price dispersion increases for inflation rates above 50% per year (in log point).

We conclude that the dispersion of relative prices caused by inflation, and emphasized in chapter 6 of Woodford (2003) and in Burstein and Hellwig (2008) as a welfare cost of inflation, is likely to be relevant only for high rates of inflation.[28]

6 Conclusions

In this paper we studied the properties of menu cost models of nominal price adjustment in which monoplistic firms set prices subject to a fixed adjustment cost in the limits when inflation is near zero and when inflation tends to infinity relative to the firm's idiosynchratic shock to profits.

We then examined the properties of these class of models with a micro-data set for Argentina's economy in the 1989-1997 period. This data set is unique as it spans periods of price stability with periods of sustained high inflation. The evidence strongly supports the predictions of menu cost models.

[27]We also tried other measures of dispersion such as the 90-10 percentile difference with very similar qualitative results.

[28] We remind the reader that the "extra" price dispersion created by nominal variation in prices is one of the main avenues for inefficiency in models with sticky prices.

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From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models

SUPPLEMENTARY MATERIAL FOR ONLINE PUBLICATION

A Proofs of comparative static results

A.1 Proof of proposition 1

First we establish that the value function, the optimal adjustment function and the inaction sets are all symmetric. We show that:

$$
\begin{aligned}
V(\hat{p}+p^*(z),z;\pi,\sigma^2) &= V(-\hat{p}+p^*(-z),-z;-\pi,\sigma^2)+v(z)\ , \\
\hat{\psi}(z;-\pi,\sigma^2) &= -\hat{\psi}(-z;\pi,\sigma^2),\ \text{and} \\
(\hat{p}+p^*(z),z)\in\mathcal{I}(\pi,\sigma^2) &\implies (-\hat{p}+p^*(-z),-z)\in\mathcal{I}(-\pi,\sigma^2)
\end{aligned}
$$

for all $z\in[0,\bar{z}]$, $\hat{p}\geq 0$ and $\pi\in(-\bar{\pi},\bar{\pi})$. The symmetry of these three objects can be established using a guess and verify argument in the Bellman equation. This argument has two parts, one deals with the instantaneous return and the second with the probabilities of different paths of $z's$. First we show that the instantaneous return satisfy the analogous property that the required symmety property for the value function stated above. We note that for $t\leq\tau$, where we let 0 the time where prices were last set, we have:

$$
\begin{aligned}
F\left(p(t)-\bar{p}(t)\ ,\ z(t)\right) &= F\left(\ p(0)-\bar{p}(0)-\pi t-p^*\left(z(t)\right)\ +p^*\left(z(t)\right),z(t)\right) \\
&= F\left(-p(0)+\bar{p}(0)+\pi t-p^*\left(-z(t)\right)+p^*\left(-z(t)\right),-z(t)\right)+f(z(t)) \\
&= F\left(-p(0)+\bar{p}(0)+\pi t+p^*\left(z(t)\right)+p^*\left(-z(t)\right),-z(t)\right)+f(z(t))\ ,
\end{aligned}
$$

where the second equality holds by symmetry of $F(\cdot)$ setting $\hat{p}(t)=p(0)-\bar{p}(0)-p^*(z(t))-\pi t$. Thus fixing the path of $\{z(t)\}$ for $0\leq t\leq\tau$, starting with $p(0)-\bar{p}(0)$ and $z(0)$ and having inflation π, gives the same profits, assuming symmetry of $F(\cdot)$, than starting with $-p(0)+\bar{p}(0)$ and having inflation $-\pi$ and $-z(0)$. Finally the probability of the path $\{z(t)\}$ for $t\in[0,\tau]$ conditional on $z(0)$, given the symmetry of $a(\cdot)$ and $b(\cdot)$ is the same as the one for the path $\{-z(t)\}$ conditional on $-z(0)$. From here one obtains that the inaction set is symmetric. Likewise, from this property it is easy to see that the optimal adjustment is also symmetric. If with inflation π a firm adjust with current shock z setting $p=\bar{p}+\hat{\psi}(z;\pi,\sigma^2)$, then with inflation $-\pi$ and current shock $-z$ it will adjust to $p=\bar{p}+\hat{\psi}(-z,-\pi)=\bar{p}-\hat{\psi}(z,-\pi)$. To se this, let $t = 0$ be a date where an adjustment take place, let $p(0)$ the price right after the adjustment, and let τ the stopping time until the next adjustment. The value of $p(0)$ maximizes

$$\begin{aligned}
p(0) &= \arg\max_{\tilde{p}} \mathbb{E}\left[\int_0^{\tau} e^{-rt} F(\tilde{p} - \bar{p}(0) - \pi t, z(t)) \; |z(0)\right] \\
&= \arg\max_{\tilde{p}} \mathbb{E}\left[\int_0^{\tau} e^{-rt} F(\tilde{p} - \bar{p}(0) - \pi t - p^*(z(t)) + p^*(z(t)), z(t)) \; |z(0)\right] \\
&= \arg\max_{\tilde{p}} \mathbb{E}\left[\int_0^{\tau'} e^{-rt} F(-\tilde{p} + \bar{p}(0) + \pi t - p^*(-z(t)) + p^*(-z(t)), -z(t)) \; | - z(0)\right] \\
&= \arg\max_{\tilde{p}} \mathbb{E}\left[\int_0^{\tau'} e^{-rt} F(-\tilde{p} + \bar{p}(0) + \pi t, -z(t)) \; | - z(0)\right] .
\end{aligned}$$

where τ' is the stopping time obtained from τ but defined flipping the sign of the $z's$.

Given the symmetry of the inaction set and optimal adjustment it is relatively straightforward to establish the symmetry of the expected time to adjustment $\mathcal{T}$ and of the invariant density g. With the $\mathcal{T}$ and g symmetric, it is immediate to establish that λ_a is symmetric. Finally, if λ_a is differentiable, then $\frac{\partial}{\partial \pi}\lambda_a(\pi, \sigma^2) = -\frac{\partial}{\partial \pi}\lambda_a(-\pi, \sigma^2)$, which establish part (i) of the proposition.

Now we show that inflation has no first order effect of $\bar{\sigma}$, i.e. we establish part (ii). For that we first use that the symmetry of the decision rules and of the invariant distribution of the shocks (which follows from the symmetry of and a and b) implies that $h(\hat{p}, \pi) = h(-\hat{p}, -\pi)$ where for simplicity we suppress σ^2 as an argument. Differentiating this expression with respect to π and evaluating at $\pi = 0$ we obtain: $h_\pi(\hat{p}, 0) = -h_\pi(-\hat{p}, 0)$. Let $f(\hat{p}, \pi)$ be any symmetric differentiable function in the sense that $f(\hat{p}) = f(-\hat{p})$. Then writing the expected value of f as $\mathbb{E}[f|\pi] = \int_{-\infty}^{0} f(\hat{p})h(\hat{p}, \pi)d\hat{p} + \int_0^{\infty} f(\hat{p})h(\hat{p}, \pi)d\hat{p}$ and differentiating both terms with respect to π and evaluating it at $\pi = 0$, using the implications for symmetry for the derivatives of h and the symmetry of f we have $\frac{\partial}{\partial \pi}\mathbb{E}[f|0] = 0$. Applying this to $f(\hat{p}) = \hat{p}^2$ we obtain that inflation does not have a first order effect on the second non-centered moment of the relative prices. Finally, to examine the effect of inflation on the variance of the relative prices, we need to examine the effect of inflation on the square of the average relative price,

$$\frac{\partial}{\partial \pi}\left[\int_{-\infty}^{\infty} \hat{p}\ h(\hat{p},\pi)d\hat{p}\right]^2 |_{\pi=0} = 2\left[\int_{-\infty}^{\infty} \hat{p}\ h(\hat{p},0)d\hat{p}\right]\left[\int_{-\infty}^{\infty} \hat{p}\ \frac{\partial}{\partial \pi}h(\hat{p},0)d\hat{p}\right] = 0,$$

since by symmetry of $h(\cdot,0)$ around $\hat{p}=0$ we have $\int_{-\infty}^{\infty} \hat{p}\ h(\hat{p},0)d\hat{p}=0$. Then, we have shown that inflation has no first order effect on the variance of relative prices around $\pi=0$.

Finally, observe that by symmetry we have that for any inflation rate $\lambda_a^+(\pi) = \lambda_a^-(-\pi)$ and $\Delta_p^+(\pi) = \Delta_p^-(-\pi)$. Differentiating with respect to inflation yields $\frac{\partial \lambda_a^+(\pi)}{\partial \pi} = -\frac{\partial \lambda_a^-(-\pi)}{\partial \pi}$ and $\frac{\partial \Delta_p^+(\pi)}{\partial \pi} = -\frac{\partial \Delta_p^-(-\pi)}{\partial \pi}$. Evaluating it at $\pi=0$ gives the result. Q.E.D.

A.2 Proof of lemma 1

Benabou and Konieczny (1994) compute the value of following an sS policy assuming that the period return function $F(p,0)$ is cubic in terms of deviations from the profit maximizing price, i.e. $p-p^*$. This allows for explicit computation of the value of the policy and to obtain the first order conditions at s and S for any value of π. Adding equations (8) and (14) in Benabou and Konieczny (1994) we get the expression $S-s = 2\delta + \frac{2}{3}\left(\frac{r}{\pi}-a\right)\delta^2$ for $\delta = \left(\frac{3}{2}\frac{c\pi}{-F''}\right)^{\frac{1}{3}}$ and $a = -\frac{F'''}{2F''}$. Thus,

$$S-s = 2\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{1}{3}}\pi^{\frac{1}{3}} + \frac{2}{3}\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{2}{3}} r\pi^{\frac{-1}{3}} - \frac{2}{3}\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{2}{3}} a\pi^{\frac{2}{3}}$$

and

$$\frac{d(S-s)}{d\pi}\frac{\pi}{S-s} = \omega_1\frac{1}{3} + \omega_2\frac{-1}{3} + \omega_3\frac{2}{3},$$

where $\omega_1 = 2\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{1}{3}}\pi^{\frac{1}{3}}/(S-s)$, $\omega_2 = \frac{2}{3}\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{2}{3}} r\pi^{\frac{-1}{3}}/(S-s)$, and $\omega_3 = -\frac{2}{3}\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{2}{3}} a\pi^{\frac{2}{3}}/(S-s)$. Using L'hospital's rule $lim_{c\to 0}\omega_1 = 1$, and $lim_{c\to 0}\omega_2 = lim_{c\to 0}\omega_3 = 0$, so that $\frac{d(S-s)}{d\pi}\frac{\pi}{S-s} = 1/3$.

The same argument for $\mathcal{T} = (S-s)/\pi$ yields

$$\mathcal{T} = 2\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{1}{3}}\pi^{-\frac{2}{3}} + \frac{2}{3}\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{2}{3}} r\pi^{-\frac{4}{3}} - \frac{2}{3}\left(-\frac{3}{2}\frac{c}{F''}\right)^{\frac{2}{3}} a\pi^{-\frac{1}{3}}$$

As $\lambda_a = \mathcal{T}^{-1}$, taking limits as $c\to 0$ yields $lim_{c\to 0}\frac{d\lambda_a}{d\pi}\frac{\lambda_a}{\pi} = \frac{2}{3}$. QED.

A.3 Proof of lemma 2

First notice that if in the problem described in equation (2) we multiply r, π, σ^2 and the functions $a(\cdot), F(\cdot)$ by a constant $k > 0$, we are just changing the units at which we measure time. Moreover, the objective function in the right hand side of equation (2) is homogeneous of degree one in $F(\cdot)$ and c and, hence, the policy function is the same whether we multiply $F(\cdot)$ by k or divide c by it. Thus, if the function $a(\cdot)$ equals zero, λ_a is homogeneous of degree one in $(\pi, \sigma^2, r, \frac{1}{c})$. Using this homogeneity:

$$\frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})}{\partial \pi} = \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi}$$

Taking limits for $\sigma > 0$:

$$\begin{aligned} &\lim_{r\downarrow 0, c\downarrow 0} \frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})}{\partial \pi} = \frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, 0, \infty)} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, 0, \infty)}{\partial \pi} \\ &= \lim_{r\downarrow 0, c\downarrow 0} \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi} = \frac{\pi}{\lambda_a(\pi, \sigma^2, 0, \infty)} \frac{\partial \lambda_a(\pi, \sigma^2, 0, \infty)}{\partial \pi} \end{aligned}$$

Thus

$$\lim_{\pi\to\infty} \left[\lim_{r\downarrow 0, c\downarrow 0} \frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})}{\partial \pi} \right]_{\sigma^2>0} = \lim_{\sigma\to 0} \left[\lim_{r\downarrow 0, c\downarrow 0} \frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})}{\partial \pi} \right]_{\pi>0} .$$

For the same reason all the elements of Ψ for a given z are homogeneous of degree 0 so, equation (10b) follows.

Finally we establish equation (10c). Using the same notation we index the invariant density as follows $g\,(p - \bar{p}, z; \pi, \sigma^2, r, 1/c)$. Note that since $a(z) = 0$, scaling the four parameters scales the units of time of z. Moreover, scaling the four parameters does not change Ψ. Thus for each $(p - \bar{p}, z)$ the invariant density is homogeneous of degree zero in $(\pi, \sigma^2, r, 1/c)$. The result follows from the definition of Δ_p^+ and Δ_p^- in terms of g and Ψ. QED.

A.4 Proof of proposition 3

Let $x = p - z$ be the log of the real gross mark-up, which we refer to as the net markup. The firm's optimal pricing policy, in this case, can be characterized in terms of three constants $X \equiv (\underline{x}, \bar{x}, \hat{x})$. The policy function takes the simple form $\Psi(z) = X + z$. We can write the inaction set in terms of the net markups as $\mathcal{I} = \{x : \underline{x} < x < \bar{x}\}$. It is optimal to keep the price unchanged when the net markup x is in the interval $(\underline{x}, \bar{x})$. When prices are not changed, the real markup evolves according to $dx = -\pi dt + \sigma dW$. When the real markup hits either of the two thresholds, prices are adjusted so that the real markup is $\hat{x}$ and thus the optimal return relative price is $\hat{\psi}(z) = \hat{x} + z$. Price increases are equal to $\Delta_p^+ = \hat{x} - \underline{x}$ and price decreases are equal to $\Delta_p^- = \bar{x} - \hat{x}$. In the case where $\sigma = 0$ and $\pi > 0$, we obtain a version of Sheshinski and Weiss's (1977) model, and the optimal policy can be characterized simply by two thresholds $\underline{x} < \hat{x}$.

We present a series of lemmas that yield the proof of proposition 3. First, Lemma 3 simplifies the Hamilton-Jacobi-Bellman for the undiscounted case. It shows that $\lim_{r\to 0} rV(x,r)$ in problem 2 is a constant. Second, lemma 4 represents the value function as a power series and finds its coefficients as functions of π and σ. Lemma 5 is an analytical solution for the zero inflation case that characterizes $(\underline{x}, \bar{x}, \hat{x})$, the size of prize changes and the frequency of price increases and of price decreases. Lemma 6 characterizes the derivatives of these elements with respect to inflation at $\pi = 0$. Finally, lemma 7 is a complete analytical characterization of Sheshinski and Weiss's (1977) case.

Lemma 3. The limit as $r \downarrow 0$, of the value function and of the thresholds $\underline{x}, \hat{x}, \bar{x}$ can be obtained by solving for a constant A and a function $v : [\underline{x}\,,\, \bar{x}] \to \mathbb{R}$ which satisfy:

$$A = Bx^2 - \pi v'(x) + \frac{\sigma^2}{2} v''(x) \text{ for all } x \in [\underline{x}, \bar{x}] \tag{16}$$

and the boundary conditions:

$$v(\bar{x}) = v(x^*) + c\,,$$
$$v(\underline{x}) = v(x^*) + c$$
$$v'(\bar{x}) = 0\,,$$
$$v'(\underline{x}) = 0\,,$$
$$v'(\hat{x}) = 0\,.$$

Remark: $v(\hat{x})$ can be normalized to zero, and A has the interpretation of the expected profits per unit of time net of the expected cost of changing prices.

Proof. The solution of the firm's problem (2) can be characterized by the equation

$$rV(x_0, r) = F(x) - \pi V'(x, r) + \frac{\sigma^2}{2} V''(x, r) \text{ for all } x \in [\underline{x}, \bar{x}] \tag{17}$$

and the following boundary conditions: two value matching conditions $V(\hat{x}, r) - V(\bar{x}, r) = V(\hat{x}, r) - V(\underline{x}, r) = c$ and the smooth pasting and optimal return conditions $V'(\hat{x}, r) = V'(\bar{x}, r) = V'(\underline{x}, r) = 0$.

Let $v(x) = \lim_{r \to 0} V(x, r)$ for $x \in [\underline{x}, \bar{x}]$ and $v'(x) = \lim_{r \to 0} V'(x, r)$ for $x \in [\underline{x}, \bar{x}]$.

We show that when $r \to 0$ the left hand side of equation (17) is a constant—i.e.$\lim_{r \to 0}\, rV(x, r) = A$—so it becomes equation (16). Write $V(x, r)$ as $V(x, r) = V_1(x, r) + V_2(x, r)$, where the first term is the present value of expected profits and the second is the present value of the expected adjustment costs. Multiplying the former by r we get

$$rV_1(x_0, r) = \lim_{T \to \infty} E \left[\int_0^T re^{-rt} F(x(t))\; dt \mid x = x_0 \right]$$

for some profit function $F(x(t))$. Observe that rV_1 is a weighted average of $F(x(t))$ with positive weights, re^{-rt} that satisfy $\int_0^T re^{-rt} dt = 1$. As $r \to 0$, the terms re^{-rt} become a constant, which has to be $1/T$ to still integrate to 1. Then,

$$\lim_{r\to 0} E\left[\lim_{T\to\infty}\int_0^T re^{-rt}F(x(t))dt \mid x = x_0\right] = \lim_{T\to\infty}\int_0^\infty \frac{1}{T}\ E\left[F(x(t)) \mid x = x_0\right]\ dt.$$

If the path of $x(t)$ that solves the firm's problem (2) is ergodic then the average $E\left[F(x(t)) \mid x = x_0\right]$ is independent of the state so

$$\lim_{T\to\infty}\int_0^\infty \frac{1}{T}\ E\left[F(x(t)) \mid x = x_0\right]\ dt \to E\left[F(x)\right] \text{ for all } x(0) \in [\underline{x}, \bar{x}].$$

Now write the second part of the value function in problem (2), the expected costs of price adjustments, as $V_2(x_0, r) = \lim_{T\to\infty} E\left[\sum_{i=1}^{N(T)} e^{-r\tau_i}c \mid x = x(0)\right]$, where τ_i is the time of each price adjustment and $N(T) = \max[i : \tau_i \leq T]$ is the number of price adjustments before T. Letting τ_N be the time of the N^{th} adjustment, we can write $V_2(x, r)$ as $V_2(x_0, r) = E\left[\sum_{i=1}^{N} e^{-r\tau_i}c \mid x = x_0\right] + E\left[e^{-r\tau_N}\sum_{i=N+1}^{\infty} e^{-r\tau_i}c \mid x = x_0\right]$. Multiplying both terms by r, noticing that immediately after an adjustment x reverts to the reset value $\hat{x}$, adding and subtracting $V(\hat{x}, r)$ on the left side of the equality, and collecting terms yields

$$\begin{aligned} rV_2(x_0, r) = {} & \frac{r}{1 - E[e^{-r\tau_N} \mid x = x_0]} E\left[\sum_{i=1}^{N} e^{-r\tau_i}c \mid x = x(0)\right] - \\ & \frac{r}{1 - E[e^{-r\tau_N} \mid x = x_0]}\left[V_2(x_0, r) - V_2(\hat{x}, r)\right]. \end{aligned}$$

Taking limits as $r \to 0$, we get $\lim_{r\to 0}\sum_{i=1}^{N} e^{-r\tau_i} = N$ and $\lim_{r\to 0}\frac{r}{1-E[e^{-rT\tau_N}|x=x_0]} = \frac{1}{E[\tau_N|x=x_0]}$ so that

$$\lim_{r\to 0} rV_2(x_0, r) = c\frac{N}{E[\tau_N \mid x = x_0]} - \frac{1}{E[\tau_N \mid x = x_0]}\left[V_2(x_0, r) - V_2(\hat{x}, r)\right]$$

For $N \to \infty$, the first term in the right hand side converges to λ_a by the strong law of large numbers of renewal theory, and the second term vanishes since $|V_2(x_0, r) - V_2(\hat{x}, r)| \leq 1$ for all $r > 0$, so that

$$\lim_{r\to 0} rV_2(x_0, r) = \lambda_a \text{ for all } x_0$$

Equation (16) together with its boundary conditions is a functional equation to find a constant A, the values $(\underline{x}, \hat{x}, \bar{x})$ and a twice differentiable function $v : [\underline{x}, \bar{x}] \to \mathbb{R}$. Moreover, A is the expected profit net of the expected cost of adjustment—i.e.$A = E[F(x)] - \lambda_a c$.

In the case with $\sigma = 0$, the integral $\lim_{T\to\infty} \int_0^\infty \frac{1}{T} \, F(x(t))dt$ converges to a value that is independent of $x(0)$ for the path of $x(t)$ that solves the firm's problem and a similar argument applies. Analogously for the expected costs of price adjustment V_2.

■

Lemma 4. Given $\underline{x}$ and $\bar{x}$, the function v described in lemma 3 is the power series

$$v(x) = \sum_{i=1}^{\infty} \alpha_i \, x^i. \tag{18}$$

In the case with $\pi = 0$ and $\sigma > 0$ the coefficients are

$$\alpha_2 = \frac{A}{\sigma^2}, \;\; \alpha_4 = -\frac{1}{6}\frac{B}{\sigma^2} \text{ and } \alpha_i = 0 \text{ for all } i \neq 2, 4$$

In the case with $\pi > 0$ and $\sigma = 0$, the coefficients are

$$\alpha_1 \;\; = -\frac{A}{\pi}, \;\; \alpha_3 = \frac{1}{3}\frac{B}{\pi} \text{ and } \alpha_i = 0 \text{ for } i = 2 \text{ and for } i \geq 4.$$

Proof. We look for the coefficients of equation (18) that solve equation (16) for all $x \in [\underline{x}, \bar{x}]$. Substituting for v' and for v'' in equation (16) and matching coefficients yields the results. When $\sigma > 0$, the conditions for matching coefficients are

$$\begin{aligned}
A &= -\alpha_1 \pi + \alpha_2 \sigma^2 \\
\alpha_3 &= \frac{1}{3}\left(\frac{2\pi}{\sigma^2}\right)\alpha_2 \\
\alpha_4 &= -\frac{1}{6}\frac{B}{\sigma^2} + \frac{1}{12}\left(\frac{2\pi}{\sigma^2}\right)^2 \alpha_2 \\
\alpha_i &= \left[-2\frac{B}{\sigma^2} + \left(\frac{2\pi}{\sigma^2}\right)^2 \alpha_2\right] \frac{2}{i!}\left(\frac{2\pi}{\sigma^2}\right)^{i-4} \quad \text{for } i \geq 5.
\end{aligned}$$

For $\pi = 0$ we have that $\alpha_i = 0$ for all i except for α_2 and α_4. For $i = 3$ and for $i \geq 5$ this follows directly from the conditions above. $\alpha_1 = 0$ is a condition for the symmetry of v when $\pi = 0$. $\blacksquare$

Finding $v(x)$ then requires to solve a two dimensional problem in α_1 and α_2 when $\sigma > 0$ (or in α_1 and α_3 when $\sigma = 0$) using the boundary conditions in lemma 3.

Lemma 5. The solution for the thresholds when $\pi = 0$ is $\hat{x} = 0$, $\bar{x} = -\underline{x} = \left(6c\frac{\sigma^2}{B}\right)^{\frac{1}{2}}$ and the constant $A = \left(\frac{2}{3}Bc\sigma^2\right)^{\frac{1}{2}}$.

Proof. We will use the normalization $v(\hat{x}) = 0$, the smooth pasting condition $v'(\bar{x}) = 0$ and the value matching conditions $v(\bar{x}) = v(\underline{x}) = c$ together with lemma 4. From lemma 4 we know that for the case of $\pi = 0$ the only terms in equation (18) are the powers 2 and 4 of x with $A = \alpha_2\sigma^2$ and $\alpha_4 = -\frac{1}{6}\frac{B}{\sigma^2}$. This implies that for $v'(\bar{x}) = 0$ we need $A = B/3\bar{x}^2$, which, in turn, implies that $v(\bar{x}) = 1/6\frac{B}{\sigma^2}\bar{x}^4 = c$ from where we obtain the expressions for $\bar{x}$ and A. $\hat{x} = 0$ then follows from $v(\hat{x}) = 0$. Later we use the fact the $\alpha_2 = \frac{1}{3}B\frac{\bar{x}^2}{\sigma^2}$ $\blacksquare$

Lemma 6. The derivatives of the thresholds are for $\pi = 0$ are given by:

$$\frac{\partial \bar{x}}{\partial \pi} = \frac{\partial \underline{x}}{\partial \pi} = \frac{2}{15}\frac{1}{\lambda_a(0)} \text{ and } \frac{\partial \hat{x}}{\partial \pi} = \frac{21}{90}\frac{1}{\lambda_a(0)}$$

Corollary 1. The derivative of the size of price changes with respect to inflation for $\pi = 0$ is

$$\frac{\partial \Delta_p^+(0)}{\partial \pi} = -\frac{\partial \Delta_p^-(0)}{\partial \pi} = 0.1\ \frac{1}{\lambda_a(0)}$$

Proof. We solve first for the derivatives of α_1 and α_2 with respect for π at $\pi = 0$.

Derivative of α_2: From the representation of $v(x;\pi)$ in equation (18) we have that $\frac{\partial^2}{(\partial x)^2}v(0;\pi) = 2\alpha_2$. The symmetry of the value function implies that $v(x,\pi) = v(-x,-\pi)$, which implies $\frac{\partial^3}{(\partial x)^2\partial\pi}v(x,\pi) = -\frac{\partial^3}{(\partial x)^2\partial\pi}v(-x,-\pi)$ for all $x \in [\underline{x},\bar{x}]$ and for all π. Hence, $\frac{\partial^3}{(\partial x)^2\partial\pi}v(0,0) = -\frac{\partial^3}{(\partial x)^2\partial\pi}v(0,0) = 0$. Finally, $\frac{\partial^3}{(\partial x)^2\partial\pi}v(0,0) = 2\frac{\partial\alpha_2}{\partial\pi} = 0$.

Derivative of α_1: From the representation of $v(x,\pi)$ in equation (18), using $\frac{\partial \alpha_2}{\partial \pi} = 0$ at $\pi = 0$, we have that

$$\frac{\partial v(x,0)}{\partial \pi} = \frac{\partial \alpha_1(0)}{\partial \pi} x + \frac{2\alpha_2}{3\sigma^2} x^3 - \frac{1}{15} \frac{B}{\sigma^4} x^5$$

Differentiating value matching, $v(\hat{x}(\pi),\pi) + c = v(\bar{x}(\pi),\pi)$ with respect to inflation and using smooth pasting we get $\frac{\partial v(\hat{x},\pi)}{\partial \pi} = \frac{\partial v(\bar{x},\pi)}{\partial \pi}$. Evaluating at $\pi = 0$, $\frac{\partial v(\bar{x},0)}{\partial \pi} = \frac{\partial v(0,0)}{\partial \pi}$, which is equal to zero since by symmetry $\frac{\partial v(0,0)}{\partial \pi} = 0$. Therefore, evaluating the first equation at $\bar{x}$ and dividing by $\bar{x}$ we get

$$0 = \frac{\partial \alpha_1(0)}{\partial \pi} + \frac{2}{3\sigma^2} \alpha_2 \bar{x}^2 - \frac{1}{15} \frac{B}{\sigma^4} \bar{x}^4$$

On the other hand, smooth pasting evaluated at $\pi = 0$, yields $\frac{\partial v(\bar{x},0)}{\partial x} = 2\alpha_2 \bar{x} - 4\frac{1}{6}\frac{B}{\sigma^2}\bar{x}^3 = 0$, implying $\alpha_2 \bar{x}^2 = \frac{1}{3}\frac{B}{\sigma^2}\bar{x}^4$. Substituting above implies

$$0 = \frac{\partial \alpha_1(0)}{\partial \pi} + \frac{2}{3\sigma^2} \frac{1}{3} \frac{B}{\sigma^2} \bar{x}^4 - \frac{1}{15} \frac{B}{\sigma^4} \bar{x}^4$$

and we get

$$\frac{\partial \alpha_1(0)}{\partial \pi} = -\frac{7}{45} B \frac{\bar{x}^4}{\sigma^4}.$$

Now we are ready to take the derivative of the thresholds X with respect to π at $\pi = 0$.

Derivative of $\hat{x}$. Using the implicit function theorem on the smooth pasting condition $\frac{\partial v(\hat{x},0)}{\partial x} = 0$ and recalling that at $\pi = 0$ $\hat{x} = 0$ we get

$$\frac{\partial \hat{x}(0)}{\partial \pi} = -\frac{\frac{\partial^2 v(0,0)}{\partial x \partial \pi}}{\frac{\partial^2 v(0,0)}{(\partial x)^2}} = -\frac{\frac{\partial \alpha_1(0)}{\partial \pi}}{2\alpha_2(0)} = \frac{\frac{7}{45} B \frac{\bar{x}^4}{\sigma^4}}{\frac{1}{3} B \frac{\bar{x}^2}{\sigma^2}} = \frac{21}{90} \frac{\bar{x}^2}{\sigma^2}$$

Derivative of $\bar{x}$ *and of* $\underline{x}$. Using the implicit function theorem on the smooth pasting condition $\frac{\partial v(\bar{x},0)}{\partial x} = 0$ we get

$$\frac{\partial \bar{x}(0)}{\partial \pi} = -\frac{\frac{\partial^2 v(\bar{x},0)}{\partial x \partial \pi}}{\frac{\partial^2 v(\bar{x},0)}{(\partial x)^2}}$$

Using the expression of α_2 for $\pi = 0$ in the proof of lemma 5 and the result for the derivative of α_1, the numerator is

$$\begin{aligned}\frac{\partial^2 v(\bar{x},0)}{\partial x \partial \pi} &= \frac{\partial \alpha_1(0)}{\partial \pi} + \frac{2\alpha_2}{3\sigma^2}\bar{x}^2 - \frac{1}{15}B\frac{\bar{x}^4}{\sigma^4} \\ &= -\frac{7}{45}B\frac{\bar{x}^4}{\sigma^4} + \frac{1}{3}B\frac{\bar{x}^4}{\sigma^4} = \frac{8}{45}B\frac{\bar{x}^4}{\sigma^4}\end{aligned}$$

Using equation (18) with the coefficients in lemma 4 evaluated at $\pi = 0$ as well as the expression for α_2 for $\pi = 0$, the denominator is

$$\begin{aligned}\frac{\partial^2 v(\bar{x},0)}{(\partial x)^2} &= 2\alpha_2 - 2B\frac{\bar{x}^2}{\sigma^2} \\ &= \frac{2}{3}B\frac{\bar{x}^2}{\sigma^2} - 2B\frac{\bar{x}^2}{\sigma^2} = -\frac{4}{3}B\frac{\bar{x}^2}{\sigma^2}.\end{aligned}$$

Symmetry implies that $\frac{\partial \bar{x}(0)}{\partial \pi} = \frac{\partial \underline{x}(0)}{\partial \pi}$ Therefore,

$$\frac{\partial \bar{x}(0)}{\partial \pi} = \frac{\partial \underline{x}(0)}{\partial \pi} = \frac{2}{15}\frac{\bar{x}^2}{\sigma^2}$$

Size of price changes. Recall that the size of price increases and of price decreases is given by

$$\Delta_p^+(\pi) = \hat{x}(\pi) - \underline{x}(\pi) \text{ and } \Delta_p^-(\pi) = \bar{x}(\pi) - \hat{x}(\pi)$$

Using the previous results $\frac{\partial \Delta_p^+(0)}{\partial \pi} = \frac{\partial \hat{x}(0)}{\partial \pi} - \frac{\partial \bar{x}(0)}{\partial \pi} = \frac{21}{90}\frac{\bar{x}^2}{\sigma^2} - \frac{2}{15}\frac{\bar{x}^2}{\sigma^2} = 0.1\frac{\bar{x}^2}{\sigma^2}$. We also know that $\lambda_a(\pi)$ for $\pi = 0$ is $\lambda_a(0) = \frac{\sigma^2}{\bar{x}^2}$. Hence,

$$\frac{\partial \Delta_p^+(0)}{\partial \pi} = -\frac{\partial \Delta_p^-(0)}{\partial \pi} = 0.1\frac{1}{\lambda_a(0)}$$

■

Lemma 7. Assume that $\sigma = 0$ and $\pi > 0$ or, equivalently, that $\pi/\sigma \to \infty$ for $\sigma > 0$. Then,

$$\hat{x}(\pi) = -\underline{x}(\pi) = \frac{1}{2}\Delta_p^+(\pi) = \left(\frac{3}{4}\frac{c}{B}\pi\right)^{1/3}$$

$$\lambda_a = \lambda_a^+ = \frac{\pi}{\Delta_p^+(\pi)} = \frac{1}{2}\left(\frac{3}{4}\frac{c}{B}\right)^{-1/3}\pi^{2/3}$$

Proof. Consider the case in which $\sigma = 0$. This is equivalent to taking the limit for $\pi \to \infty$ with $\sigma > 0$—see lemma 2 or lemma 4. The state moves deterministically from $\hat{x}$ to $\underline{x}$ at speed π. We find closed form solutions for these thresholds. From lemma 4 we know that for $\sigma = 0$ the value function is $v(x) = \alpha_1\pi + \alpha_3 x^3$ with $\alpha_1 = -A/\pi$ and $\alpha_3 = \frac{B}{3\pi}$. We use smooth pasting at $\hat{x}$ and $\underline{x}$, together with value matching to solve for α_1, $\hat{x}$ and $\underline{x}$. Smooth pasting, $v'(\hat{x}) = 0$ and $v'(\underline{x}) = 0$ implies that $\hat{x} = -\underline{x} = \sqrt{\frac{-\alpha_1\pi}{B}}$. Value matching, $v(\hat{x}) - v(\underline{x}) = c$, then implies $\alpha_1 = -\left(c\frac{3}{4}\right)^{2/3}\left(\frac{B}{\pi}\right)^{1/3}$. The results of the lemma follow. ■

B Analytical characterization of the model in section 2.2

In what follows we use $x = p - z$ for the log of the real gross mark-up. In proposition 4 we show that when $\sigma > 0$, the inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$ and that the optimal return point is given by $\hat{\psi}(z) = \hat{x} + z$ for three constants $X \equiv (\underline{x}, \hat{x}, \bar{x})$. This is due to the combination of a assumptions of constant elasticity of demand, constant returns to scale and permanent shocks to cost while the product last. This means that it is optimal to keep the price unchanged when the real markup x is in the interval $(\underline{x}, \bar{x})$. When prices are not changed, the real markup evolves according to $dx = -(\mu_z + \pi)dt + \sigma dW$. When the real markup hits either of the two thresholds, prices are adjusted so that the real markup is $\hat{x}$. Proposition 4 derives a system of three equations in three unknowns for X, as well as the explicit solution to the value function, as function of the parameters $\Theta \equiv (\pi, \mu_z, \sigma^2, \rho, r, \eta, c)$. Proposition 5 derives an explicit solution for the expected number of adjustment per unit of time λ_a given a policy X and parameters $(\pi, \mu_z, \sigma^2, \rho)$. Proposition 6 characterizes the density g for invariant distribution of (p, z) implied by the policy X and the parameters $(\pi, \mu_z, \sigma^2, \rho, \eta)$.

For future reference we define $\hat{c}$ implicitely $\zeta(z) = \hat{c}\; e^{z(1-\eta)}$

Proposition 4. Assume that $\sigma > 0, c > 0$ and that equation (12) holds. The inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$. The optimal return point is given by $\hat{\psi}(z) = \hat{x} + z$. The value function in the range of inaction and the constants $X \equiv (\underline{x}, \hat{x}, \bar{x})$ with $\underline{x} < \hat{x} < \bar{x}$ solve

$$V(p, z) = e^{z(1-\eta)} V(p - z, 0) \equiv e^{z(1-\eta)} v(p - z) \tag{19}$$

$$v(x) = a_1\; e^{x(1-\eta)} + a_2\; e^{-x\eta} + \sum_{i=1}^{2} A_i\; e^{\nu_i x} \tag{20}$$

where the coefficients a_i, ν_i are given by

$$\begin{aligned}
0 &= -b_0 + b_1 \nu_i + b_2 (\nu_i)^2 \\
a_1 &= \frac{1}{b_0 - \;(1-\eta)\; b_1 - \;(1-\eta)^2\; b_2} \text{ and } a_2 = -\frac{1}{b_0 + \eta\; b_1 - \;(\eta)^2\; b_2} \text{ where} \\
b_0 &= r + \rho - \mu_z (1-\eta) - (1-\eta)^2 \frac{\sigma^2}{2},\; b_1 = -\left[\mu_z + \pi + 2(1-\eta)\frac{\sigma^2}{2}\right], b_2 = \frac{\sigma^2}{2}\;.
\end{aligned}$$

and where the five values A_1, A_2, X solve the following five equations:

$$\begin{aligned}
\hat{c}\;\; -a_1 \left(e^{\hat{x}(1-\eta)} - e^{\bar{x}(1-\eta)}\right) - \; a_2 \left(e^{-\hat{x}\eta} - e^{-\bar{x}\eta}\right) &= \sum_{i=1}^{2} A_i \left(e^{\nu_i \hat{x}} - e^{\nu_i \bar{x}}\right), \\
\hat{c}\;\; -a_1 \left(e^{\hat{x}(1-\eta)} - e^{\underline{x}(1-\eta)}\right) - \; a_2 \left(e^{-\hat{x}\eta} - e^{-\underline{x}\eta}\right) &= \sum_{i=1}^{2} A_i \left(e^{\nu_i \hat{x}} - e^{\nu_i \underline{x}}\right), \\
0 = a_1\; (1-\eta) e^{\hat{x}(1-\eta)} - a_2\; \eta e^{-\hat{x}\eta} &+ \sum_{i=1}^{2} A_i\; \nu_i e^{\nu_i \hat{x}}\;, \\
0 = a_1\; (1-\eta) e^{\bar{x}(1-\eta)} - a_2\; \eta e^{-\bar{x}\eta} &+ \sum_{i=1}^{2} A_i\; \nu_i e^{\nu_i \bar{x}}\;, \\
0 = a_1\; (1-\eta) e^{\underline{x}(1-\eta)} - a_2\; \eta e^{-\underline{x}\eta} &+ \sum_{i=1}^{2} A_i\; \nu_i e^{\nu_i \underline{x}}\;.
\end{aligned}$$

The first two equations are linear in (A_1, A_2), given X.

Proof. To simplify the notation we evaluate all the expressions when $\bar{p} = 0$, so the relative price and the nominal price coincide.

The Bellman equation in the inaction region $(p, z) \in \mathcal{I}$ is

$$(r + \rho)V(p, z) = e^{-\eta p}\left(e^p - e^z\right) - \pi V_p(p, z) + V_z(p, z)\mu_z + V_{zz}(p, z)\frac{\sigma^2}{2}$$

for all $p \in [\underline{p}(z), \bar{p}(z)]$. The boundary conditions are given by first order conditions for the optimal return point:

$$V_p(\hat{\psi}(z), z) = 0 \tag{21}$$

the value matching conditions, stating that the value at each of the two boundaries is the same as the value at the optimal price after paying the cost:

$$V(\underline{p}(z), z) = V(\hat{\psi}(z), z) - \zeta(z),\ V(\bar{p}(z), z) = V(\hat{\psi}(z), z) - \zeta(z). \tag{22}$$

and the smooth pasting conditions, stating that the value function should have the same slope at the boundary than the value function in the control region (which is flat), so:

$$V_p(\underline{p}(z), z) = 0,\ V_p(\bar{p}(z), z) = 0, \tag{23}$$

Under this conditions, the value function and optimal policies are homogeneous in the sense that:

$$V(p, z) = e^{z(1-\eta)}V(p - z, 0) \equiv e^{z(1-\eta)}v(p - z) \tag{24}$$

$$\underline{p}(z) = \underline{z} + z,\ \bar{p}(z) = \bar{x} + z\ ,\ \text{and}\ \hat{\psi}(z) = \hat{x} + z\ , \tag{25}$$

where $\underline{x}, \bar{x}$ and $\hat{x}$ are three constant to be determined.

Using the homogeneity of the value function in equation (24) we can compute the derivatives

$$
\begin{aligned}
V_p(p,z) &= e^{z(1-\eta)}v'(p-z) \\
V_z(p,z) &= (1-\eta)e^{z(1-\eta)}v(p-z) - e^{z(1-\eta)}v'(p-z) \\
V_{zz}(p,z) &= (1-\eta)^2 e^{z(1-\eta)}v(p-z) - 2(1-\eta)e^{z(1-\eta)}v'(p-z) + e^{z(1-\eta)}v''(p-z)
\end{aligned}
$$

Replacing this derivatives in the Bellman equation for the inaction region we get

$$
\begin{aligned}
(r+\rho)v(p-z) &= e^{-(p-z)\eta}\left(e^{p-z}-1\right) - \pi v'(p-z) + [(1-\eta)v(p-z) - v'(p-z)]\mu_z \\
&+ [(1-\eta)^2 v(p-z) - 2(1-\eta)v'(p-z) + v''(p-z)]\frac{\sigma^2}{2}
\end{aligned}
$$

or

$$
\begin{aligned}
&\left[r+\rho-\mu_z(1-\eta)-(1-\eta)^2\frac{\sigma^2}{2}\right]v(p-z) = e^{(p-z)(1-\eta)} - e^{-\eta(p-z)} \\
-\; & v'(p-z)[\mu_z+\pi+2(1-\eta)\frac{\sigma^2}{2}] + v''(p-z)\frac{\sigma^2}{2}
\end{aligned}
$$

We write $x = p - z$ be the log of the gross markup, or the net markup. Consider the free boundary ODE:

$$
\begin{aligned}
b_0\ v(x) &= e^{x(1-\eta)} - e^{-\eta x} + b_1\ v'(x) + b_2\ v''(x) \text{ for all } x \in [\underline{x}, \bar{x}] \\
v\left(\underline{x}\right) &= v\left(\hat{x}\right) - \hat{c}, \quad v\left(\bar{x}\right) = v(\hat{x}) - \hat{c}, \\
v'\left(\underline{x}\right) &= 0, \quad v'\left(\bar{x}\right) = 0, \quad v'\left(\hat{x}\right) = 0\ ,
\end{aligned}
$$

where

$$
\begin{aligned}
b_0 &= \left[r+\rho-\mu_z(1-\eta)-(1-\eta)^2\frac{\sigma^2}{2}\right]\ , \\
b_1 &= -\left[\mu_z+\pi+2(1-\eta)\frac{\sigma^2}{2}\right]\ , \\
b_2 &= \frac{\sigma^2}{2}\ .
\end{aligned}
$$

The value function is given by the sum of the particular solution and the solution of the homogeneous equation:

$$v(x) = a_1 \; e^{x(1-\eta)} \; + \; a_2 \; e^{-x\eta} \; + \; \sum_{i=1}^{2} \; A_i \; e^{\nu_i x}$$

where ν_i are the roots of the quadratic equation

$$0 = -b_0 + b_1 \nu_i + b_2 (\nu_i)^2$$

and where the coefficients for the particular solution are

$$\begin{aligned} a_1 &= \frac{1}{b_0 - \;(1-\eta)\; b_1 - \;(1-\eta)^2\; b_2} \\ a_2 &= -\frac{1}{b_0 + \eta\; b_1 - \;(\eta)^2\; b_2}\;, \end{aligned}$$

since

$$\begin{aligned} b_0 \; a_1 \; e^{x(1-\eta)} &= e^{x(1-\eta)} + a_1 \; (1-\eta) e^{x(1-\eta)} \; b_1 + a_1 \; (1-\eta)^2 e^{x(1-\eta)} \; b_2 \\ b_0 \; a_2 \; e^{-x\eta} &= -e^{-x\eta} - a_2 \; \eta e^{-x\eta} \; b_1 + a_2 \; (\eta)^2 e^{-x\eta} \; b_2 \; . \end{aligned}$$

The five constants A_1, A_2 and $X \equiv (\underline{x}, \bar{x}, \hat{x})$ are chosen to satisfies the 2 value matching conditions equation (22), the two smooth pasting conditions equation (23) and the optimal return point equation (21). It is actually more convenient to solve the value function in two steps. First to solve for the constants $A_i(X)$ for $i = 1, 2$ using the two value matching conditions. Mathematically, the advantage of this intermediate step is that, given X, the equations for the A_1, A_2 are linear. Conceptually, the advantage is that the solution represent the value of the policy described by the triplet $X = (\bar{x}, \underline{x}, \hat{x})$. Then we solve for $(\bar{x}, \underline{x}, \hat{x})$ using the conditions for the optimality of the thresholds, namely the two smooth pasting equation (23) and the f.o.c. for the return point equation (21).

Solving A_1, A_2 for a given policy X amount to solve the following linear system:

$$
\begin{aligned}
\hat{c} - a_1 \left(e^{\hat{x}(1-\eta)} - e^{\bar{x}(1-\eta)}\right) - \; a_2 \left(e^{-\hat{x}\eta} - e^{-\bar{x}\eta}\right) &= \sum_{i=1}^{2} A_i \left(e^{\nu_i \hat{x}} - e^{\nu_i \bar{x}}\right) \\
\hat{c} - a_1 \left(e^{\hat{x}(1-\eta)} - e^{\underline{x}(1-\eta)}\right) - \; a_2 \left(e^{-\hat{x}\eta} - e^{-\underline{x}\eta}\right) &= \sum_{i=1}^{2} A_i \left(e^{\nu_i \hat{x}} - e^{\nu_i \underline{x}}\right)
\end{aligned}
$$

Given $A_1(X), A_2(X)$ we need to solve the following three equations:

$$
\begin{aligned}
0 &= a_1 \; (1-\eta) e^{\hat{x}(1-\eta)} \; - \; a_2 \; \eta e^{-\hat{x}\eta} \; + \; \sum_{i=1}^{2} \; A_i(X) \; \nu_i e^{\nu_i \hat{x}} \; , \\
0 &= a_1 \; (1-\eta) e^{\bar{x}(1-\eta)} \; - \; a_2 \; \eta e^{-\bar{x}\eta} \; + \; \sum_{i=1}^{2} \; A_i(X) \; \nu_i e^{\nu_i \bar{x}} \; , \\
0 &= a_1 \; (1-\eta) e^{\underline{x}(1-\eta)} \; - \; a_2 \; \eta e^{-\underline{x}\eta} \; + \; \sum_{i=1}^{2} \; A_i(X) \; \nu_i e^{\nu_i \underline{x}} \; .
\end{aligned}
$$

■

The expected number of adjustments per unit of time is given in the next proposition:

Proposition 5. Given a policy described by $X = (\underline{x}, \hat{x}, \bar{x})$ the expected number of adjustment per unit of time λ_a and the expected number of price increases λ_a^+ are given by

$$
\begin{aligned}
\lambda_a &= 1 \Big/ \left[\frac{1}{\rho} + \sum_{i=1}^{2} \; B_i \; e^{q_i \hat{x}} \right] \\
\lambda_a^+ &= 1 \Big/ \left[\frac{1}{\rho} + \sum_{i=1}^{2} B_{l,i} \; e^{q_i \hat{x}} \right]
\end{aligned}
$$

where q_i are the roots of $\rho = -(\pi + \mu_z) q_i + \frac{\sigma^2}{2}(q_i)^2$ and where B_i and $B_{l,i}, B_{H,i}$ solve the following system of linear equations:

$$
\begin{aligned}
0 &= \frac{1}{\rho} + \sum_{i=1}^{2} \; B_i \; e^{q_i \underline{x}} = \frac{1}{\rho} + \sum_{i=1}^{2} \; B_i \; e^{q_i \bar{x}}, \\
\frac{1}{\rho} &= -B_{h,1} \; e^{q_1 \hat{x}} - B_{h,2} \; e^{q_2 \hat{x}} \; , \; 0 = B_{h,1} \left(e^{q_1 \hat{x}} - \; e^{q_1 \bar{x}}\right) + B_{h,2} \left(e^{q_2 \bar{x}} - \; e^{q_2 \hat{x}}\right) \; , \\
-\frac{1}{\rho} &= B_{l,1} \; e^{q_1 \underline{x}} + B_{l,2} \; e^{q_2 \underline{x}} \; , \; 0 = B_{l,1} \; q_1 \; e^{q_1 \hat{x}} + B_{l,2} \; q_2 \; e^{q_2 \hat{x}} - B_{h,1} \; q_1 \; e^{q_1 \hat{x}} - B_{h,2} \; q_2 \; e^{q_2 \hat{x}} \; .
\end{aligned}
$$

Proof. The expected time until the next adjustment solves the following Kolmogorov equation:

$$\rho\mathcal{T}(p,z) = 1 - \pi\mathcal{T}_p(p,z) + \mathcal{T}_z(p,z)\mu_z + \mathcal{T}_{zz}(p,z)\frac{\sigma^2}{2}$$

for all p such that $\underline{p}(z) < p < \bar{p}(z)$, and all z. The boundary conditions are that time reaches zero when it hits the barriers:

$$\mathcal{T}(\bar{p}(z), z) = \mathcal{T}(\underline{p}(z), z) = 0 \ .$$

Given the homogeneity of the policies we look for a function satisfying

$$\mathcal{T}(p,z) = T(p-z)$$

Given the form of the expected time we have:

$$\mathcal{T}_p(p,z) = T'(p-z),\ \mathcal{T}_z(p,z) = -T'(p-z) \text{ and } \mathcal{T}_{zz}(p,z) = T''(p-z) \ ,$$

so the Kolmogorov equation becomes:

$$\rho T(x) = 1 - (\pi+\mu_z)T'(x) + T''(x)\frac{\sigma^2}{2} \ \text{ for all } x \in (\underline{x}, \bar{x}) \ .$$

The solution of this equation, given $\underline{x}, \bar{x}$ is:

$$T(x) = \frac{1}{\rho} \ + \ \sum_{i=1}^{2} \ B_i \ e^{q_i x} \ \text{ for all } x \in (\underline{x}, \bar{x})$$

where q_i are roots of

$$\rho = -(\pi+\mu_z)q_i + \frac{\sigma^2}{2}(q_i)^2 \ , \tag{26}$$

and where the B_1, B_2 are chosen so that the expected time is zero at the boundaries:

$$0 = \frac{1}{\rho} + \sum_{i=1}^{2} B_i \; e^{q_i \underline{x}} \tag{27}$$

$$0 = \frac{1}{\rho} + \sum_{i=1}^{2} B_i \; e^{q_i \bar{x}} \tag{28}$$

Given the solution of this two linear equations $B_1(\underline{x}, \bar{x}), B_2(\underline{x}, \bar{x})$ the expected number of adjustments per unit of time λ_a is given by

$$\lambda_a = \frac{1}{T(\hat{x})} = \frac{1}{\frac{1}{\rho} + \sum_{i=1}^{2} \; B_i(\underline{x}, \bar{x}) \; e^{q_i \hat{x}}},$$

Finally, we derive the expression for the frequency of price increases. The time until the next price increase is the first time until x hits $\underline{x}$ or the product dies while $\underline{x} < x < \hat{x}$. If x hits $\bar{x}$, or the product dies exogenously while $\bar{x} > x > \hat{x}$, then x then is returned to $\hat{x}$. Thus the expected time until the next increase in price solves the following Kolmogorov equation:

$$\rho \mathcal{T}(p, z) = \begin{cases} 1 - \pi \mathcal{T}_p(p, z) + \mathcal{T}_z(p, z)\mu_z + \mathcal{T}_{zz}(p, z)\frac{\sigma^2}{2} & \text{if } p < z + \hat{x} \\ 1 + \rho \mathcal{T}(z + \hat{x}, z) - \pi \mathcal{T}_p(p, z) + \mathcal{T}_z(p, z)\mu_z + \mathcal{T}_{zz}(p, z)\frac{\sigma^2}{2} & \text{if } p > z + \hat{x} \end{cases}$$

for all p such that $\underline{p}(z) < p < \bar{p}(z)$, and all z. The boundary conditions are that time reaches zero when it hits the barriers:

$$\mathcal{T}(\bar{x} + z, z) = \mathcal{T}(\hat{x} + z, z) \text{ and } \mathcal{T}(\underline{x} + z, z) = 0 \; .$$

We look for a solution that is continuous and once differentiable at $(p, z) = (\hat{x} + z, z)$, and otherwise twice continuously differentiable. To do so we let $\mathcal{T}(p, z) = T_h(x)$ for $x \in [\hat{x}, \bar{x}]$

and $\mathcal{T}(p,z) = T_l(x)$ for $x \in [\underline{x}, \hat{x}]$ and

$$\begin{aligned}
\rho T_l(x) &= 1 - (\pi + \mu_z) T_l'(x) + T_l''(x)\frac{\sigma^2}{2} \\
\rho T_h(x) &= 1 + \rho T_h(\hat{x}) - (\mu_z + \pi) T_h'(x) + T_h''(x)\frac{\sigma^2}{2} \\
T_l(\hat{x}) &= T_h(\hat{x}) \ , T_l'(\hat{x}) = T_h'(\hat{x}) \\
T_h(\hat{x}) &= T_h(\bar{x}) \ , T_l(\underline{x}) = 0 \ .
\end{aligned}$$

The solution for T_j for $j = h, l$ are:

$$T_l(x) = \frac{1}{\rho} + \sum_{i=1}^{2} B_{l,i}\, e^{q_i x} \ \text{ and } \ T_h(x) = \frac{1}{\rho} + \sum_{i=1}^{2} B_{h,i}\, e^{q_i x} + \left(\frac{1}{\rho} + \sum_{i=1}^{2} B_{l,i}\, e^{q_i \hat{x}} \right)$$

The four boundary conditions become the following four linear equations of the constants $B's$:

$$\begin{aligned}
\sum_{i=1}^{2} B_{l,i}\, e^{q_i \hat{x}} &= \sum_{i=1}^{2} B_{h,i}\, e^{q_i \hat{x}} + \sum_{i=1}^{2} B_{l,i}\, e^{q_i \hat{x}} + \frac{1}{\rho} \\
\sum_{i=1}^{2} B_{l,i}\, q_i\, e^{q_i \hat{x}} &= \sum_{i=1}^{2} B_{h,i}\, q_i\, e^{q_i \hat{x}} \\
\sum_{i=1}^{2} B_{l,i}\, e^{q_i \underline{x}} &= 0 \\
\sum_{i=1}^{2} B_{h,i}\, e^{q_i \hat{x}} &= \sum_{i=1}^{2} B_{h,i}\, e^{q_i \bar{x}}
\end{aligned}$$

Hence, the frequency of price increases λ_a^+ is given by

$$\lambda_a^+ = 1/T_l(\hat{x}) = 1 \Big/ \left(1/\rho + \sum_{i=1}^{2} B_{l,i} e^{q_i \hat{x}} \right) \ . \blacksquare$$

Now we turn to the density of the invariant distribution

Proposition 6. Given a policy described by $X = (\underline{x}, \hat{x}, \bar{x})$ the density of the invariant distribution $g(p,z)$ is given by

$$g(p,z) = \begin{cases} e^{\phi_1 z}\left[U_1^+ e^{\xi_1(p-z)} + U_2^+\right] & \text{if } p-z \in (\hat{x},\bar{x}]\ ,\ z>0 \\ e^{\phi_1 z}\left[L_1^+ e^{\xi_1(p-z)} + L_2^+\right] & \text{if } p-z \in [\underline{x},\hat{x}]\ ,\ z>0 \\ e^{\phi_2 z}\left[U_1^- e^{\xi_2(p-z)} + L_2^-\right] & \text{if } p-z \in (\hat{x},\bar{x}]\ ,\ z<0 \\ e^{\phi_2 z}\left[L_1^- e^{\xi_2(p-z)} + L_2^-\right] & \text{if } p-z \in [\underline{x},\hat{x}]\ ,\ z<0 \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

where $\{\phi_1, \phi_2, \xi_1, \xi_2\}$ are given by

$$\begin{aligned} \rho &= -\mu_z \phi_j + \frac{\sigma^2}{2}\phi_j^2 \ \text{ for each of the roots } j=1,2 \ \text{ and} \\ \xi_j &= -\frac{\pi + \mu_z - 2\phi_j \frac{\sigma^2}{2}}{\sigma^2/2} \end{aligned}$$

and where the coefficients $\{U_i^+, L_i^+, U_i^-, L_i^-\}_{i=1,2}$ solve 8 linear equations:

$$\begin{aligned} 0 &= U_1^+\ e^{\xi_1 \bar{x}} + U_2^+ = L_1^+\ e^{\xi_1 \underline{x}} + L_2^+ \\ 0 &= U_1^-\ e^{\xi_2 \bar{x}} + U_2^- = L_1^-\ e^{\xi_2 \underline{x}} + L_2^- \\ \frac{\phi_1\phi_2}{\phi_1 - \phi_2} &= \frac{L_1^+}{\xi_1}\left[e^{\xi_1 \hat{x}} - e^{\xi_1 \underline{x}}\right] + L_2^+\left[\hat{x} - \underline{x}\right] + \frac{U_1^+}{\xi_1}\left[e^{\xi_1 \bar{x}} - e^{\xi_1 \hat{x}}\right] + U_2^+\left[\bar{x} - \hat{x}\right] \\ \frac{\phi_1\phi_2}{\phi_1 - \phi_2} &= \frac{L_1^-}{\xi_2}\left[e^{\xi_2 \hat{x}} - e^{\xi_2 \underline{x}}\right] + L_2^-\left[\hat{x} - \underline{x}\right] + \frac{U_1^-}{\xi_2}\left[e^{\xi_2 \bar{x}} - e^{\xi_2 \hat{x}}\right] + U_2^-\left[\bar{x} - \hat{x}\right] \\ & U_1^+\ e^{\xi_1 \hat{x}} + U_2^+ = L_1^+\ e^{\xi_1 \hat{x}} + L_2^+ \\ & U_1^-\ e^{\xi_2 \hat{x}} + U_2^- = L_1^-\ e^{\xi_2 \hat{x}} + L_2^-\ . \end{aligned}$$

Proof. The density of the invariant distribution for (p,z) solves the forward Kolmogorov p.d.e.:

$$\rho g(p,z) = \pi g_p(p,z) - \mu_z g_z(p,z) + g_{zz}(p,z)\frac{\sigma^2}{2} \tag{30}$$

for all $(p,z) \neq (\hat{x}+z, z) = (\hat{\psi}(z), z)$ and all $p : \underline{p}(z) = \underline{x} + z \leq p \leq \bar{x} + z = \bar{p}(z)$ and all z. The pde does not apply at the optimal return point, since local consideration cannot determine g there. The other boundary conditions are zero density at the lower and upper boundaries of adjustments, and that g integrates to one:

$$
\begin{aligned}
g(\underline{x} + z, z) &= g(\bar{x} + z, z) = 0 \text{ for all } z \\
1 &= \int_{\infty}^{\infty} \int_{\underline{x}+z}^{\bar{x}+z} g(p, z)\ dp\ dz
\end{aligned}
$$

We will show that g can be computed dividing the state space in four regions given by whether (p, z) is such that $z > 0$ and $z < 0$ and given z whether $p \in [\underline{x}+z, \hat{x}+z]$ and $p \in [\hat{x}+z, \bar{x}+z]$.

As a preliminary step we solve for the marginal on z of the invariant distribution, which we denote by $\tilde{g}$. This is the invariant distribution of the process $\{z\}$ which with intensity ρ is re-started at zero and otherwise follows $dz = \mu_z dt + \sigma dW$. It can be shown that $\tilde{g}$ is given by

$$
\tilde{g}(z) \equiv \int_{\underline{x}+z}^{\bar{x}+z} g(p, z)\ dp = \begin{cases} \frac{\phi_1\phi_2}{\phi_1-\phi_2} e^{\phi_2 z} & \text{if } z < 0 \\ \frac{\phi_1\phi_2}{\phi_1-\phi_2} e^{\phi_1 z} & \text{if } z > 0, \end{cases} \tag{31}
$$

where $\phi_1 < 0 < \phi_2$ are the two real roots of the characteristic equation

$$
\rho = -\mu_z \phi + \frac{\sigma^2}{2} \phi^2. \tag{32}
$$

We conjecture that g can be written as follows:

$$
g(p, z) = \begin{cases} e^{\phi_1 z} k(p - z) & \text{if } z < 0 \\ e^{\phi_2 z} k(p - z) & \text{if } z > 0, \end{cases} \tag{33}
$$

In this case we compute the derivatives as:

$$
\begin{aligned}
g_p(p, z) &= e^{\phi z} k'(p - z) \ , \\
g_z(p, z) &= e^{\phi z} \phi\ k(p - z) - e^{\phi z} k'(p - z) \ , \\
g_{zz}(p, z) &= e^{\phi z} \phi^2\ k(p - z) - 2e^{\phi z} \phi\ k'(p - z) + e^{\phi z} k''(p - z) \ .
\end{aligned}
$$

where $\phi = \phi_1$ for $z < 0$ and $\phi = \phi_2$ for $z > 0$. The p.d.e. then becomes:

$$\rho k(p-z) = \pi k'(p-z) - \mu_z \left[\phi\ k(p-z) - k'(p-z)\right] + \left[\phi^2\ k(p-z) - 2\phi\ k'(p-z) + k''(p-z)\right] \frac{\sigma^2}{2}$$

for $p - z \neq \hat{x}$ or

$$\left[\rho + \phi\mu_z - \phi^2 \frac{\sigma^2}{2}\right] k(p-z) = \left(\pi + \mu_z - 2\phi\frac{\sigma^2}{2}\right) k'(p-z) + k''(p-z)\frac{\sigma^2}{2}$$

Thus

$$k(p-z) = \begin{cases} U_1 e^{\xi_{1j}(p-z)} + U_2 e^{\xi_{2j}(p-z)} & \text{if } p - z \in (\hat{x}, \bar{x}] \\ L_1 e^{\xi_{1j}(p-z)} + L_2 e^{\xi_{2j}(p-z)} & \text{if } p - z \in [\underline{x}, \hat{x}] \end{cases}$$

where ξ_{1j}, ξ_{2j} solves the quadratic equation:

$$\left[\rho + \phi_j\mu_z - \phi_j^2 \frac{\sigma^2}{2}\right] = \left(\pi + \mu_z - 2\phi_j\frac{\sigma^2}{2}\right)\xi + \frac{\sigma^2}{2}\xi^2 \tag{34}$$

for each $j = 1, 2$ corresponding to $\phi = \phi_1$ and $\phi = \phi_2$, i.e. the positive and negative values of z. We note that, by definition of ϕ in equation (32), the left hand side of equation (34) equal zero, and hence one of the two roots is always equal to zero. Thus we label $\xi_{2j} = 0$ for $j = 1, 2$. The remaining root equals:

$$\xi_{1j} = -\frac{\pi + \mu_z - 2\phi_j \frac{\sigma^2}{2}}{\sigma^2/2} \ \text{ and } \ \xi_{2j} = 0 \ \text{ for } j = 1, 2\ . \tag{35}$$

We integrate $g(p, z)$ over p and equate it to $\tilde{g}(z)$ to obtain a condition for coefficients C.

First we consider the case of $z > 0$:

$$
\begin{aligned}
\tilde{g}(z) &= \int_{\underline{x}+z}^{\hat{x}+z} e^{\phi_1 z} \sum_{i=1}^{2} L_i^+ \; e^{\xi_{i1}(p-z)} \; dp + \int_{\hat{x}+z}^{\bar{x}+z} e^{\phi_1 z} \sum_{i=1}^{2} \; U_i^+ \; e^{\xi_{i1}(p-z)} \; dp \\
&= e^{\phi_1 z} \sum_{i=1}^{2} \frac{L_i^+ \; e^{-\xi_{i1} z}}{\xi_{i1}} \left[e^{\xi_{i1}(\bar{x}+z)} - e^{\xi_{i1}(\hat{x}+z)} \right] + e^{\phi_1 z} \sum_{i=1}^{2} \frac{U_i^+ \; e^{-\xi_{i1} z}}{\xi_{i1}} \left[e^{\xi_{i1}(\hat{x}+z)} - e^{\xi_{i1}(\underline{x}+z)} \right] \\
&= e^{\phi_1 z} \left(\sum_{i=1}^{2} \frac{L_i^+}{\xi_{i1}} \left[e^{\xi_{i1}\bar{x}} - e^{\xi_{i1}\hat{x}} \right] + \sum_{i=1}^{2} \frac{U_i^+}{\xi_{i1}} \left[e^{\xi_{i1}\hat{x}} - e^{\xi_{i1}\underline{x}} \right] \right) \\
&= e^{\phi_1 z} \left(\frac{L_1^+}{\xi_{11}} \left[e^{\xi_{11}\hat{x}} - e^{\xi_{11}\underline{x}} \right] + L_2^+ \left[\hat{x} - \underline{x} \right] + \frac{U_1^+}{\xi_{11}} \left[e^{\xi_{11}\bar{x}} - e^{\xi_{11}\hat{x}} \right] + U_2^+ \left[\bar{x} - \hat{x} \right] \right)
\end{aligned}
$$

where the last line uses that $\xi_{2,1} = 0$. The analogous expression for $z < 0$ is

$$
\tilde{g}(z) = e^{\phi_2 z} \left(\frac{L_1^-}{\xi_{12}} \left[e^{\xi_{12}\hat{x}} - e^{\xi_{12}\underline{x}} \right] + L_2^- \left[\hat{x} - \underline{x} \right] + \frac{U_1^-}{\xi_{12}} \left[e^{\xi_{12}\bar{x}} - e^{\xi_{12}\hat{x}} \right] + U_2^- \left[\bar{x} - \hat{x} \right] \right)
$$

The value of the density at the boundary of the range of inaction is given by

$$
g(\bar{x}+z, z) = \begin{cases} e^{\phi_1 z} \sum_{i=1}^{2} U_i^+ \; e^{\xi_{i1}\bar{x}} & \text{for } z > 0 \\ e^{\phi_2 z} \sum_{i=1}^{2} U_i^+ \; e^{\xi_{i2}\bar{x}} & \text{for } z < 0 \end{cases}
$$

$$
g(\underline{x}+z, z) = \begin{cases} e^{\phi_1 z} \sum_{i=1}^{2} L_i^+ \; e^{\xi_{i1}\underline{x}} & \text{for } z > 0 \\ e^{\phi_2 z} \sum_{i=1}^{2} L_i^+ \; e^{\xi_{i2}\underline{x}} & \text{for } z < 0 \end{cases}
$$

If the density g at $(p, z) = (\hat{\psi}(z), z) = (z + \hat{x}, z)$ is continuous on p for a given z we have:

$$
g(\hat{x}, z) = \begin{cases} e^{\phi_1 z} \left[\sum_{i=1}^{2} U_i^+ \; e^{\xi_{i1}\hat{x}} \right] = e^{\phi_1 z} \left[\sum_{i=1}^{2} L_i^+ \; e^{\xi_{i1}\hat{x}} \right] & \text{if } z > 0 \; , \\ e^{\phi_2 z} \left[\sum_{i=1}^{2} U_i^- \; e^{\xi_{i2}\hat{x}} \right] = e^{\phi_2 z} \left[\sum_{i=1}^{2} L_i^- \; e^{\xi_{i2}\hat{x}} \right] & \text{if } z < 0 \; . \end{cases} \tag{36}
$$

We summarize the results for the invariant density g here

$$g(p,z) = \begin{cases} e^{\phi_1 z}\left[U_1^+ e^{\xi_1(p-z)} + U_2^+\right] & \text{if } p-z \in (\hat{x},\bar{x}]\ ,\ z>0 \\ e^{\phi_1 z}\left[L_1^+ e^{\xi_1(p-z)} + L_2^+\right] & \text{if } p-z \in [\underline{x},\hat{x}]\ ,\ z>0 \\ e^{\phi_2 z}\left[U_1^- e^{\xi_2(p-z)} + L_2^-\right] & \text{if } p-z \in (\hat{x},\bar{x}]\ ,\ z<0 \\ e^{\phi_2 z}\left[L_1^- e^{\xi_2(p-z)} + L_2^-\right] & \text{if } p-z \in [\underline{x},\hat{x}]\ ,\ z<0 \end{cases} \tag{37}$$

where $\{\phi_1, \phi_2\}$ are the two roots of the quadratic equation (32) and where the use $\xi_1 \equiv \xi_{11}, \xi_2 \equiv \xi_{12}\}$ are given by the non-zero roots equation (35). The 8 values for $\{U_i^+, L_i^+, U_i^-, L_i^-\}_{i=1,2}$ solve two system of 4 linear equations, one for $\{U_i^+, L_i^+\}_{i=1,2}$ and one for $\{U_i^-, L_i^-\}_{i=1,2}$. The upper and lower boundary of the range of inaction has zero density for both positive and negative values of z:

$$0 = U_1^+\ e^{\xi_1 \bar{x}} + U_2^+ = L_1^+\ e^{\xi_1 \underline{x}} + L_2^+ \tag{38}$$
$$0 = U_1^-\ e^{\xi_2 \bar{x}} + U_2^- = L_1^-\ e^{\xi_2 \underline{x}} + L_2^- \tag{39}$$

The marginal distribution of the z computed using g coincides with $\tilde{g}$ for positive and negative values of z:

$$\frac{\phi_1\phi_2}{\phi_1-\phi_2} = \frac{L_1^+}{\xi_1}\left[e^{\xi_1\hat{x}} - e^{\xi_1\underline{x}}\right] + L_2^+\left[\hat{x}-\underline{x}\right] + \frac{U_1^+}{\xi_1}\left[e^{\xi_1\bar{x}} - e^{\xi_1\hat{x}}\right] + U_2^+\left[\bar{x}-\hat{x}\right] \tag{40}$$
$$\frac{\phi_1\phi_2}{\phi_1-\phi_2} = \frac{L_1^-}{\xi_2}\left[e^{\xi_2\hat{x}} - e^{\xi_2\underline{x}}\right] + L_2^-\left[\hat{x}-\underline{x}\right] + \frac{U_1^-}{\xi_2}\left[e^{\xi_2\bar{x}} - e^{\xi_2\hat{x}}\right] + U_2^-\left[\bar{x}-\hat{x}\right] \tag{41}$$

The density is continuous at $(p,z) = (\hat{\psi}(z), z) = (z+\hat{x}, z)$. Thus

$$U_1^+\ e^{\xi_1\hat{x}} + U_2^+ = L_1^+\ e^{\xi_1\hat{x}} + L_2^+ \tag{42}$$
$$U_1^-\ e^{\xi_2\hat{x}} + U_2^- = L_1^-\ e^{\xi_2\hat{x}} + L_2^-. \tag{43}$$

■

We are now ready to characterize the marginal density of prices, denoted by h.

$$h(p) \equiv \int_{-\infty}^{\infty} g(p,z)dz = \int_{p-\bar{x}}^{p-\underline{x}} g(p,z)dz = \int_{p-\bar{x}}^{p-\hat{x}} g(p,z)dz + \int_{p-\hat{x}}^{p-\underline{x}} g(p,z)dz \tag{44}$$

$$h(p) = \begin{cases} \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_1 z} \left[U_1^+ e^{\xi_1(p-z)} + U_2^+\right] dz + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_1 z} \left[L_1^+ e^{\xi_1(p-z)} + L_2^+\right] dz & \text{if } p > \bar{x} \\ \\ \int_{p-\bar{x}}^{0} e^{\phi_2 z} \left[U_1^- e^{\xi_2(p-z)} + U_2^-\right] dz + \int_{0}^{p-\hat{x}} e^{\phi_1 z} \left[U_1^+ e^{\xi_1(p-z)} + U_2^+\right] dz & \\ + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_1 z} \left[L_1^+ e^{\xi_1(p-z)} + L_2^+\right] dz & \text{if } p \in [\hat{x}, \bar{x}) \\ \\ \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_2 z} \left[U_1^- e^{\xi_2(p-z)} + U_2^-\right] dz + \int_{p-\hat{x}}^{0} e^{\phi_2 z} \left[L_1^- e^{\xi_2(p-z)} + L_2^-\right] dz & \\ + \int_{0}^{p-\underline{x}} e^{\phi_1 z} \left[L_1^+ e^{\xi_1(p-z)} + L_2^+\right] dz & \text{if } p \in [\underline{x}, \hat{x}) \\ \\ \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_2 z} \left[U_1^- e^{\xi_2(p-z)} + U_2^-\right] dz + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_2 z} \left[L_1^- e^{\xi_2(p-z)} + L_2^-\right] dz & \text{if } p < \underline{x} \end{cases} \tag{45}$$

Using that

$$\begin{aligned} \int_a^b e^{\phi z} \left[D_1 e^{\xi(p-z)} + D_2\right] dz &= D_1 e^{\xi p} \int_a^b e^{(\phi-\xi)z} dz + D_2 \int_a^b e^{\phi z} dz \\ &= D_1 \, \frac{e^{\xi p + (\phi-\xi)b} - e^{\xi p + (\phi-\xi)a}}{\phi - \xi} + D_2 \frac{e^{\phi b} - e^{\phi a}}{\phi} \end{aligned}$$

we can solve the integrals for each of the branches of h where $D_i \in \{U_i^+, U_i^-, L_i^+, L_i^-\}$ for $i = 1, 2$and $(\phi, \xi) \in \{\phi_1, \xi_1, \phi_2, \xi_2\}$ and a and b take different values accordingly.

For completeness we give the expression for the case with $\sigma = 0$, a version of Sheshinski and Weiss (1977) model.

Proposition 7. Assume that $\sigma = 0, c > 0, \pi + \mu_z > 0$ and equation (12) holds. The inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$. The optimal return point is given by $\hat{\psi}(z) = \hat{x} + z$. The value function in the range of inaction and the constants $X \equiv (\underline{x}, \hat{x})$ solve

$$
\begin{aligned}
V(p,z) &= e^{z(1-\eta)} V(p-z,0) \equiv e^{z(1-\eta)} v(p-z) \\
v(x) &= a_1 \, e^{x(1-\eta)} \; + \; a_2 \, e^{-x\eta} \; + \; A \, e^{\nu x}
\end{aligned}
$$

where the coefficients a_i, ν are given by

$$
\begin{aligned}
\nu &= \frac{b_0}{b_1} \; , \; a_1 = \frac{1}{b_0 - \; (1-\eta) \; b_1} \text{ and } a_2 = -\frac{1}{b_0 + \eta \; b_1} \; \text{ where} \\
b_0 &= r + \rho - \mu_z (1-\eta), \; b_1 = -\left[\mu_z + \pi\right] \; .
\end{aligned}
$$

and where the three values $A, X \equiv (\underline{x}, \hat{x})$ solve the following three equations:

$$
\begin{aligned}
\hat{c} \quad & -a_1 \left(e^{\hat{x}(1-\eta)} - e^{\underline{x}(1-\eta)}\right) - \; a_2 \left(e^{-\hat{x}\eta} - e^{-\underline{x}\eta}\right) = \; A \left(e^{\nu\hat{x}} - e^{\nu\underline{x}}\right) , \\
0 &= a_1 \; (1-\eta) e^{\hat{x}(1-\eta)} \; - \; a_2 \; \eta e^{-\hat{x}\eta} \; + \; A \; \nu e^{\nu\hat{x}} \; , \\
0 &= a_1 \; (1-\eta) e^{\underline{x}(1-\eta)} \; - \; a_2 \; \eta e^{-\underline{x}\eta} \; + \; A \; \nu e^{\nu\underline{x}} \; .
\end{aligned}
$$

Furthermore:

$$
\lambda_a = \frac{\rho}{1 - \exp\left(-\frac{\rho}{\pi+\mu_z}(\hat{x} - \underline{x})\right)}
$$

Proof. The expressions for the value function are obtained by setting $\sigma = 0$ and imposing the sS policy between the bands $\underline{x}, \hat{x}$. This problem is identical to the one in Sheshinski and Weiss (1977), where the discount rate is $r + \rho$. For the frequency of price adjustment we need to include the death and replacement of the products. For this we let $\mathcal{T}$ the expected time until an adjustment:

$$
\begin{aligned}
\rho \mathcal{T}(p,z) &= 1 + \mathcal{T}_z(p,z)\mu_z - \mathcal{T}_p(p,z)\pi \\
\rho T(x) &= 1 - T'(x)(\mu_z + \pi) \; .
\end{aligned}
$$

with boundary conditions $T(\underline{x}) = 0$. So the solution is $T(x) = 1/\rho + B \exp\left(-\frac{\rho}{\pi+\mu_z} x\right)$ with $B = -\exp\left(\frac{\rho}{\pi+\mu_z}\underline{x}\right)/\rho$ so

$$T(x) = \frac{1}{\rho}\left[1 - \exp\left(-\frac{\rho}{\pi+\mu_z}(x - \underline{x})\right)\right]$$

and hence

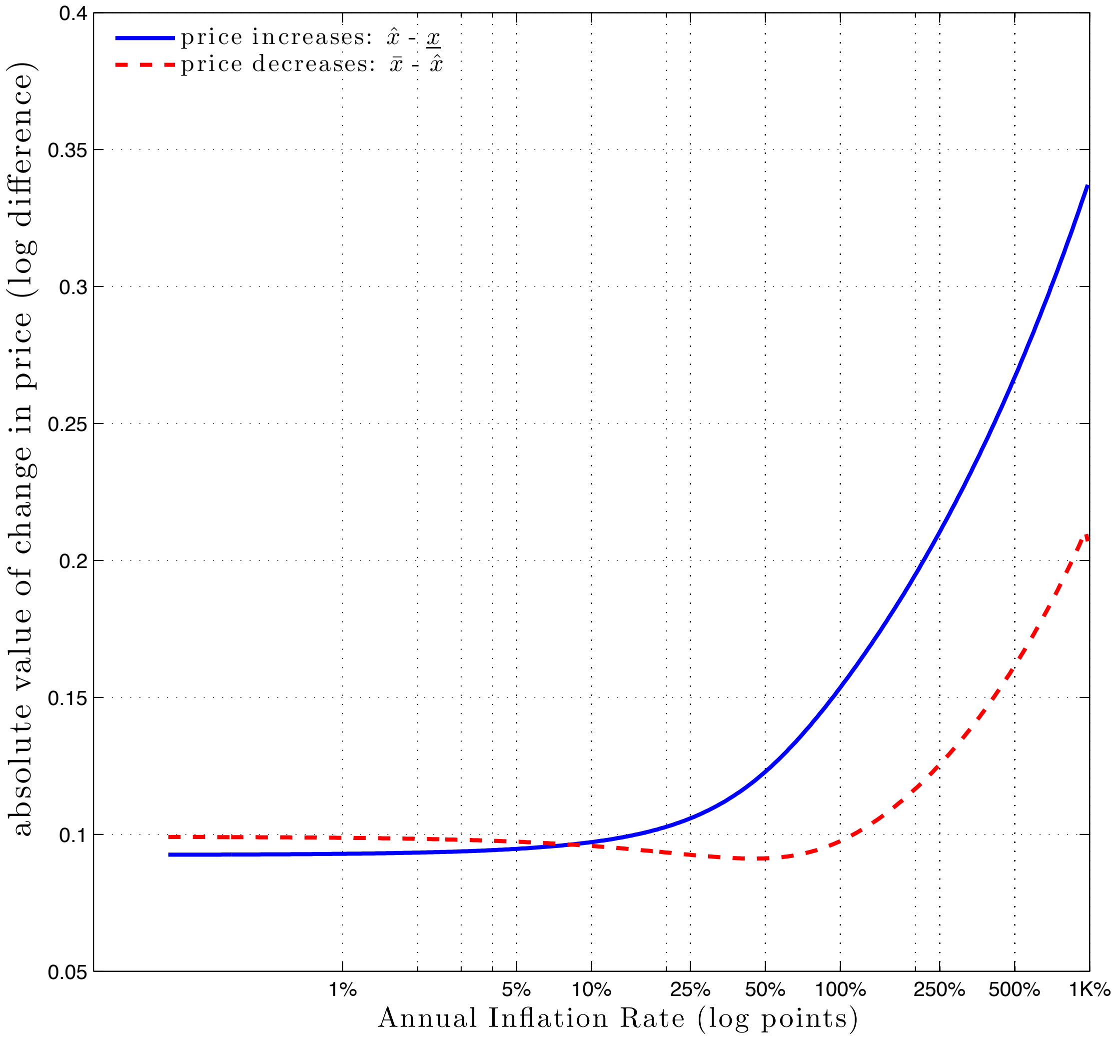
$$1/\lambda_a = T(\hat{x}) = \frac{1}{\rho}\left[1 - \exp\left(-\frac{\rho}{\pi+\mu_z}(\hat{x} - \underline{x})\right)\right] \qquad \blacksquare$$

B.1 Some graphical illustrations of the model in section 2.2

Figure 12 plots the size of the“regular” price increases and “regular” price decreases, for different inflation rates. Imitating the empirical literature, we define as regular price changes those not triggered by the jump shock that reset the value of z to zero. The figure shows that for low inflation rates the size of price increases and of price decreases is very similar as predicted by part (iii) of proposition 1.The figure also that the slope of the size of price changes with respect to inflation is very small, consistent with the prediction that $\frac{\partial \Delta_p^+\left(0,\sigma^2\right)}{\partial \pi} + \frac{\partial \Delta_p^-\left(0,\sigma^2\right)}{\partial \pi} = 0$. As inflation rises, the size of both price increases and decreases becomes larger. Figure 10 in section 5 is the empirical counterpart of figure 12.

Figure 13 plot the unconditional variance of relative prices as a function of inflation. For low values of inflation the unconditional variance is insensitive to inflation as predicted by the theory. For large enough inflation rates the width of the inaction range should swamp the effect the variation of z on σ. In our numerical examples, however, it takes inflation rates even much higher than the ones observed in the peak months in Argentina for this to happen.

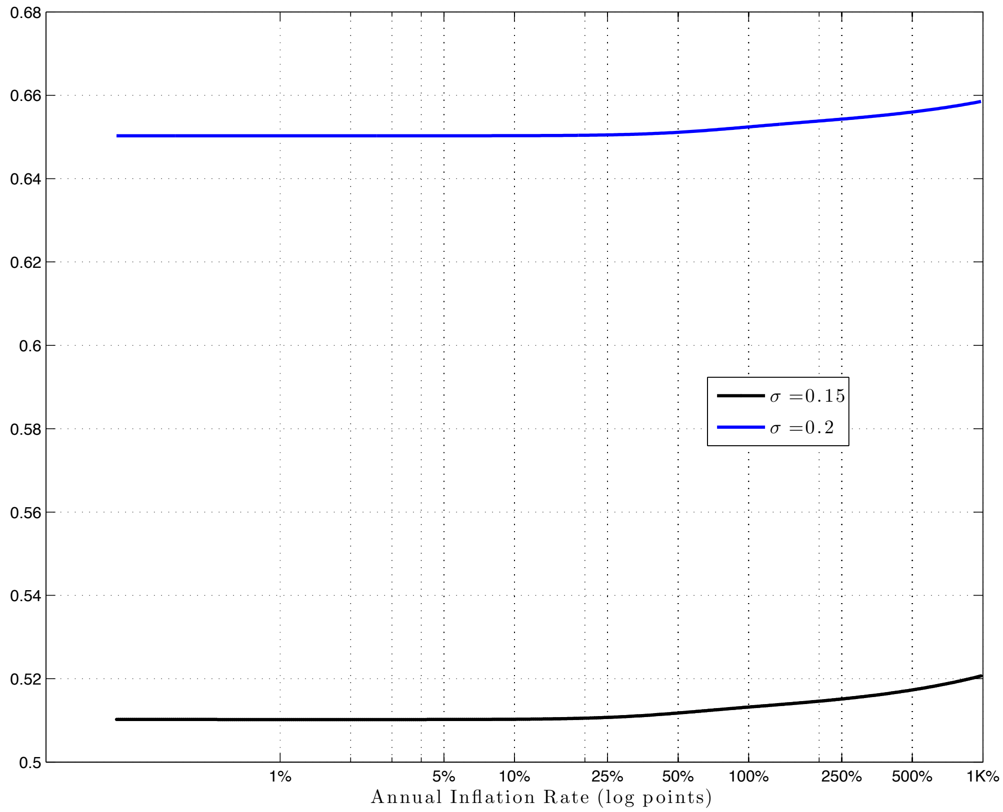
Figure 12: Average size of price increases Δ_p^+ and decreases Δ_p^-



C Classification of Gand Services and Data Collection

Goods/services in the dataset are classified according to the MERCOSUR Harmonized Index of Consumer Price (HICP) classification. The HICP uses the first four digit levels of the Classification of Individual Consumption According to Purpose (COICOP) of the United Nations plus three digit levels based on the CPI of the MERCOSUR countries. The *goods/services* in the database are the seven digit level of the HICP classification; six digit level groups are called *products*; five digit level groups are called *sub-classes*; four digit level categories are called *classes*; three digit level categories are called *groups* and two digit level groups are

Figure 13: Unconditional standard deviation of log relative prices $\bar{\sigma}$



called *divisions*. Table 2 shows two examples of this classification.

Table 2: Example of the Harmonized Index of Consumer Price Classification

Classification	Example 1	Example 2
Division	Food and Beverages	Household equipment and maintenance
Group	Food	Household maintenance
Class	Fruits	Cleaning tools and products
Sub-Class	Fresh Fruits	Cleaning products
Product	Citric Fruits	Soaps and detergents
Good	Lemons	Liquid soap

For most cases, the brand chosen for the product is the one most widely sold by the outlet, or the one that occupy more space in the stands, if applicable (hence brands can change from month to month or from two-weeks to two-weeks). For same cases, the brand is part of the attributes, the product is defined as one from a "top brand".

The weight of each good in the CPI is obtained from the 1986 National Expenditure Survey (Encuesta Nacional de Gasto de los Hogares). Weights are computed as the proportion of the households expenditure on each good over the total expenditure of the households. We re-normalize these weights to reflect the fact that our working dataset represents only 84% of expenditure. The weight of a particular good is proportional to the importance of its expenditure with respect to the total expenditure without taking into account the percentage of households buying it.

Table 3 shows the top 20 goods, in terms of the importance of their weights in our sample. As it can be seen from the table, most goods whose prices are gathered twice a month are represented by food and beverages while goods whose prices are gathered monthly include services, apparel and other miscellaneous goods and services.

Table 3: Goods ordered by weight whose prices are gathered once and twice per month

Differentiated)	Weight (%)	Homogeneous	Weight (%)
Lunch	2.02	Whole chicken	1.51
Lunch in the workplace	1.67	Wine	1.49
Car	1.52	French bread (less than 12 pieces)	1.38
Housemaid	1.48	Fresh whole milk	1.31
Monthly union membership	1.22	Blade steaks	0.96
Snack	0.91	Standing rump	0.93
Medical consultation	0.83	Eggs	0.87
Gas bottle	0.58	Short ribs (Roast prime ribs)	0.85
Laides hairdresser	0.56	Striploin steaks	0.78
Labor for construction	0.54	Apple	0.73
Adult cloth slippers	0.50	Oil	0.72
Color TV	0.50	Rump steaks	0.72
Funeral expenses	0.49	Potatos	0.71
Men's dress shirt	0.49	Soda (coke)	0.70
Dry cleaning and ironing	0.48	Cheese (quartirolo type)	0.70
Sports club fee	0.47	Tomatoes	0.63
Movie ticket	0.47	Minced meat	0.60
Men's denim pants	0.45	Sugar (white)	0.59
Disposable diapers	0.43	Coffee (in package)	0.57
Shampoo	0.40	Yerba mate	0.55

Table 4 shows the weight structure in our database classified by divisions. The table shows goods in terms of their weight with respect to the total weight in the sample (Total column), and with respect to the total weight of their belonging category (one or two visit goods) Food and non-alcoholic beverages represent almost 43% of the total weight in the sample and 82% of the total weight of goods whose prices are gathered twice a month. On the other hand, weights of one visit goods are less concentrated. Almost 12% of the total weight in the sample corresponds to furniture and household items and around 9% correspond to apparel. These percentages are around 24% and 19%, respectively, when computing percentages over the total weight in the one visit goods category.

Table 4: Weights for Divisions of the Harmonized Index of Consumer Price

	Weight		Weight	
Divisions	Total	One visit	Total	Two visits
Food and non-alcoholic beverages	4.94	9.91	42.67	82.39
Apparel	9.24	18.53		
Conservation and repair of housing plus gas in bottle	0.99	1.99		
Furniture and household items	11.86	23.79		
Medical products, appliances and equipment plus external medical services	3.39	6.79		
Transportation	2.60	5.22		
Recreation and culture	4.50	9.02		
Education	2.42	4.85		
Miscellaneous goods and services (Toiletries, haircut, etc.)	5.07	10.16		
Jewelry, clocks, watches plus other personal belongings	4.86	9.75		
Alcoholic beverages			3.94	7.62
Non-durable household goods			2.40	4.64
Other items and personal care products			2.77	5.36

C.1 Instructions to CPI's Pollsters

Pollsters record item's prices. Remember that an item is a good/service of a determined brand sold in a specific outlet in a specific period of time. Prices are transactional, meaning that the pollster should be able to buy the product in the outlet. Goods are defined by their attributes. For the majority of the goods, the brand is not an attribute. The brand of a specific item is determined the first time the pollster visits an outlet. The brand is the most sold/displayed by the outlet. Once the item is completely defined, the pollster collects the price of that item next time she visits the same outlet. After the first visit, in the following visits, the pollsters arrive to each outlet with a form that includes all items for which prices are to be collected.

For example, assume the good is *soda-cola top brand* and the attributes are *package: plastic bottle and weight:* 1.5 *litters.* The first time the pollster goes to, say, outlet A she ask for the cola top brand most sold in that outlet. Assume that *Coca-Cola* is the most sold soda in outlet A. Then the item is completely defined: *Coca-Cola* in plastic bottle of 1.5 litters in outlet A. Next time the pollster goes to outlet A she records the price of that item.

All prices are in Argentine pesos. Our dataset does not contain flags for indexed prices. In traditional outlets pollsters ask for the price of an item even when, for example it is displayed in a blackboard, because the good has to be available in order to record its price. In supermarket chains the price recorded is collected from the shelf/counter display area.

There are a number of special situations to be taken into account:

1. *Substitutions:* every time there is a change in the attributes of the good the pollster has to replace that particular good for another one. In this case, the pollster mark the price collected with a flag indicating a substitution has occurred. The goods that are substituted should be similar in terms of the type of brand and or quality.

2. *Stockouts:* every time the pollster could not buy the item, either because the good is out of stock in the outlet or because the same good or a similar one is not sold by outlet at the time the price has to be collected, she has to mark the item with a flag of stockout and she has to assign a missing price for that item. Stock-outs include what we label "pure stock-outs", the case where the outlet has depleted the stock of the good, including end of seasonal goods/services. Stock outs also include the case where the outlet no longer carries the same good/service and it does not offers a similar good/service of comparable quality/brand. Examples of stock-outs include many fruits and vegetables not available off-season, as well as clothing such as winter coats and sweaters during summer.

3. *Sales:* every time the pollster observes a good with a sale flag in an outlet, she has to mark the price of that particular item with a sale flag.

All pollsters were supervised at least once a month. Supervisors visit some of the outlets, visited earlier the same day by the pollster, and collect a sample of the prices that the pollster should have to collect. Then at the National Statistic Institute, another supervisor compared both forms.

D Comparison to other studies

Table 5 provides a succinct view of the samples used in other studies of price dynamics under high inflation. It shows the countries analyzed, the product coverage of each sample, and the range of inflation rates in each study. The most salient features of this paper are the broader coverage of our sample and the larger variation in inflation rates.

Table 5: Comparison with other Studies in Countries with High Inflation

Country	Authors	Sample product coverage	Observ. per month	Sample	Inflation (%. a.r.)	Monthly freq. (%)
Argentina	This paper	506 goods/servicies, representing 84% of Argentinian consumption expenditures	81,305 on average	1988-1997	$0 - 7.2 \times 10^6$	16-99
Brazil	Barros et al. (2009)	70% of Brazilian consumption expenditures	98,194 on average	1997-2010	2-13	39-50
Israel	Llach and Tsiddon (1992)	26 food products (mostly meat and alcoholic beverages)	250	1978-1979	64	41
Israel	Llach and Tsiddon (1992) Eden (2001),	26 food products (mostly meat and alcoholic beverages)	530	1981-1982	118	61
Israel	Baharada and Eden (2004)	up to 390 narrowly-defined products from the Israeli CPI	2800	1991-1992	16	24
Mexico	Gagnon (2007)	227 product categories, representing 54.1 percent of Mexican consumption expenditures	31,500	1994-2002	3.5-45	27-45
Poland	Konieczny and Skrzypacz (2005)	52 goods, including 37 grocery items, and 3 services	up to 2400	1990-1996	18-249	59-30

E Robustness

In this section we conduct a battery of robustness checks to evaluate the sensitivity of the main results in the paper. The first set of checks consists in dealing with recurrent issues when analyzing micro-price datasets such as missing observations and price changes due to substitutions or sales. Secondly, we discuss issues of aggregation across products. Third, we address biases resulting from discrete sampling. Fourth, we present results using a measure of expected inflation instead of contemporaneous inflation. Finally, we address the possibility that the theoretical propositions which hold in the steady state are a poor description of the argentinean experience in the high inflation period leading to the stabilization plan in 1991 where agents are likely to have anticipated the strong disinflation that followed. To summarize, the empirical findings at low inflation go through intact. At high inflation, we observe some quantitative but not qualitative differences. Most notable, depending on the estimator used, the elasticity of the frequency of price changes can range from approximately 1/2 to the theoretical 2/3.

E.1 Missing data, substitution, sales and aggregation

This subsection reports the sensitivity of the estimates of the elasticity γ, the semielasticity $\Delta\%\lambda$ and the duration at low inflation obtained with the simple estimator to different treatments of missing data, sales, product substitution and broad aggregation levels. The results obtained from using the different methodologies for estimating the frequencies of price adjustment are presented in table 6. In appendix E.1.1 we describe in detail the methodologies and the definitions of all estimators in table 6.

We report estimates of the three parameters of equation (15) for the sample of differentiated goods (monthly), for the sample of homogeneous goods (bi-weekly) and for the aggregate. The latter is obtained by averaging the estimated λs with their expenditure shares after converting the bi-weekly estimates to monthly ones.

The first and second columns show the elasticities at high and low inflation. The third block of columns shows the implied duration of price spells when inflation is low (below the threshold) under the assumption that the frequency of price adjustment is constant.

Table 6: The Frequency of Price Adjustment and Inflation: Robustness Checks.

Aggregation	Elasticity γ			Semi-Elasticity at zero $\Delta\%\lambda$			Expected Duration at $\pi = 0$ (months)		
	Diff.	Hom.	Agg	Diff.	Hom.	Agg	Diff.	Hom.	Agg
A. Simple Estimator (No information from missing price quotes)									
Pooled	0.51	0.5	0.53	0.08	-0.01	0.04	9.1	5.8	4.5
Weighted Average	0.52	0.48	0.52	0.07	-0.02	0.04	8.2	5.7	4.4
Median	0.64	0.64	0.68	0.1	0.03	0.05	15	14	9.3
Weighted Median	0.65	0.64	0.68	0.09	0.02	0.04	12.2	11	7.8
Pooled (excluding sales)	0.5	0.47	0.52	0.08	0.03	0.05	10	7.5	5.7
B. All price quotes									
Pooled	0.51	0.5	0.52	0.08	-0.01	0.04	8.8	5.9	5.1
Weighted Average	0.52	0.45	0.49	-0.05	-0.02	0.04	8.9	5.8	4.6
Median	0.62	0.58	0.65	0.09	-0.21	0.02	15.3	8.9	10.2
Weighted Median	0.62	0.65	0.65	0.09	0.02	0.04	12.9	10.8	7.5
C. Excluding substitution quotes									
Pooled	0.55	0.5	0.52	0.09	-0.01	0.03	7	6.3	6.2
Weighted Average	0.52	0.45	0.51	0.06	-0.02	0.03	10.7	6.1	6.7
Median	0.66	0.65	0.68	0.13	0.02	0.02	18.8	17.9	12.4
Weighted Median	0.66	0.62	0.66	0.07	-0.06	0.02	16.5	12.2	10.4
D. Excluding substitution spells									
Pooled	0.52	0.5	0.52	0.07	0	0.05	10.7	6	5.5
Weighted Average	0.53	0.44	0.49	-0.1	-0.02	0.04	8.6	5.8	4.8
Median	0.62	0.64	0.64	0.09	0.04	-0.02	18.4	16.6	10.9
Weighted Median	0.63	0.6	0.66	0.09	-0.13	-0.05	15.4	9.4	8.2
E. Excluding substitution and sales quotes									
Pooled	0.5	0.47	0.52	0.08	0.06	0.05	9.9	5.2	6.7

Note: *Diff.* denotes differentiated goods, which are samples once a month. *Hom.* denotes homogeneous goods, which are sampled twice a moth. *Agg* denotes the weighted average of the Differentiated and homogeneous goods, with weights given by the expenditure shares and where the homogeneous goods have been aggregated to monthly frequencies. For each case we use NLLS to fit: $\log \lambda_t = a + \epsilon \min\{\pi_t - \pi^c, 0\} + \nu(\min\{\pi_t - \pi^c, 0\})^2 + \gamma \max\{\log \pi_t - \log \pi^c, 0\} + \omega_t$. The semi-elasticity at zero $\Delta\%\lambda$ is the percentage change in λ when inflation goes from 0 to 1%. A. estimates λ with the simple estimator in equation (14) discarding information from missing prices, B. is the full information maximum likelihood estimator described in Appendix E.1.1, C. replaces price quotes with a product substitution by missing data, D. replaces price spells ending in a product substitution by missing data and E. replaces sales quotes by the previous price and product substitutions by a missing quote.

The first line in table 6 corresponds to the pooled simple estimator reported in figure 6. The estimates of the elasticity of the frequency of price adjustment with respect to inflation, γ, are very similar for the λs in the two samples and for the aggregate λ.The estimates for the semi-elasticity and expected duration at low inflation are markedly higher for differentiated goods in comparison to homogeneous goods. The other lines in the table provide estimates of the three parameters of interest for different aggregation methods and for the different treatments of missing observations, product substitutions and sales. The values for the elasticity γ across all these estimation techniques ranges from approximately 1/2 to 2/3. The variation in $\Delta\%\lambda$ estimates is much smaller across methodologies. Both differences in the estimators result from alternative aggregation methodologies and not from the treatment of sales, substitutions and missing values. For instance, the elasticity at high inflation when using the simple estimator with pooled data climbs from 0.53 to 0.68 when using the median estimate across industries.

The treatment of sales and substitutions does seem to have an effect on the estimates of the expected duration of price spells when inflation is low, as in other papers in the literature (see Klenow and Malin (2011)). For example, durations increase from 4.5 months to 5.7 months when sales price quotes are replaced by the price quote of the previous regular price. In Klenow and Kryvtsov (2008) durations go from 2.2 months to 2.8 after the sales treatment and in Nakamura and Steinsson (2008) they go from 4.2 months to 3.2. Time series for frequency of substitution, sales and missing values in the sample can be seen in figure 4.

What accounts for the differences in the estimates implied duration at low inflation between the sample of differentiated and homogeneous goods? Expected durations are much higher for differentiated goods than for homogeneous goods. In principle, we believe that this discrepancy can be attributed to two features: an intrinsic difference between the type of goods or due to the fact that the prices of homogeneous goods are sampled bi-monthly and prices for differentiated goods once a month. For the interested reader, in appendix **??** we try to elucidate this issue by conducting some further exercises which point both to a violation of the assumption of the hazard rate of price changes being constant in the duration of the price spell, and thus making sampling periodicity not innocuous and homogeneous goods having higher idiosyncratic volatility as well.

E.1.1 Missing data, substitutions, sales and aggregation

Here we describe in detail the methodologies and the definitions of all estimators in table 6 in section E.1.

We start by describing the assumptions used to estimate the probability of a price change when we observe missing prices. If between two observed prices there are some missing prices we use the following assumption. If the two observed prices are exactly equal we assume there has been *no* changes in prices in any times between these dates. This is the same assumption as in Klenow and Kryvtsov (2008). Instead, if the two observed prices are different we assume there has been *at least one change* in prices in between. The first assumption allow us to complete the missing prices in between two observed prices that are equal. From here on, assume that the missing prices in such string of prices have been replaced.

We will refer from now on to the sequence of prices between two different observed prices as a *spell of constant prices*, or for short a *spell of prices*. Without any missing prices, a spell of constant prices is just a sequence of repeated prices ending with a different price. Notice that the last (observed) price in a spell of constant prices is the first price of the next spell.

Next we describe the possible patterns of prices, and its implications for the estimation of the probability of a price change. After following the procedure described above, all spell of prices and missing observations have only two possible patterns. The first pattern is a spell of prices ending with a price change, but with no missing observations. We consider the second pattern in the following section, where we deal with the effect of missing prices. Consider the following example for a spell for an outlet i that contains no missing prices nor substitutions:

The braces on top of the values of λ are meant to remind the reader that λ_t refers to the constant probability of change in prices between $t-1$ and t. The indicator I_{it} adopts the value one if, in outlet i, the price in period t is different from the price at period $t-1$, and zero otherwise, except for the first temporal period of the first string of prices where it is missing[29]. In this example we have exactly no changes for the first four periods and at least

[29]We also include the indicator γ_t, which we explain below, for completeness.

Table 7: Example of a spell of constant prices without missing prices

p_t	10		10		10		10		10		15
	t		$t+1$		$t+2$		$t+3$		$t+4$		$t+5$
		$\overbrace{\lambda_{t+1}}$		$\overbrace{\lambda_{t+2}}$		$\overbrace{\lambda_{t+3}}$		$\overbrace{\lambda_{t+4}}$		$\overbrace{\lambda_{t+5}}$	
I_t	y		0		0		0		0		1
γ_t	y		1		1		1		1		1

one change in the next period. The probability of observing this completed spell of constant prices is thus:[30]

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times e^{-\lambda_{t+3}} \times e^{-\lambda_{t+4}} \times \left(1 - e^{-\lambda_{t+5}}\right) \tag{46}$$

It follows that in this simple case, assuming all the outlets selling the same good have the same λ_j, the likelihood function of prices observed for product j is

$$L_j = \prod_{i \in O_j} \left[e^{-\lambda_{j,t}}\right]^{(1-I_{it})} \times \left[1 - e^{\lambda_{j,t-\tau}}\right]^{I_{it}} \tag{47}$$

The maximum likelihood estimator of the arrival rate of price changes for product j between times t and $t+1$ in the simple case without missing prices and without substitutions is

$$\lambda_{j,t+1} = \log \left(\frac{\sum_{i \in O_j} 1}{\sum_{i \in O_j} (1 - I_{it})} \right) = -log(1 - f_{jt}) \tag{48}$$

where we let O_j denote the set of the outlets i of the product j and f_{jt} is the fraction of outlets of good j that changed prices in period t. In words, $\lambda_{j,t+1}$ is the log of the ratio of the number of outlets to the number of outlets that have not changed the price between t and $t+1$. Thus $\lambda_{j,t+1}$ ranges between zero, if no outlets have change prices, and infinite if all outlets have changed prices. The probability of at least one change in prices in period t for product j is $1 - e^{-\lambda_{jt}} = f_{jt}$.

[30] We assume that the number of price changes between $t-1$ and t follows a homogeneous Poisson process with arrival rate λ_t per unit of time. The probability of k occurrences is $e^{-\lambda}\lambda^k/k!$ and the waiting time between occurrences follows an exponential distribution.

E.1.2 Incorporating information on Missing Prices

Now we consider the case where there are some missing prices before the price change, but we postpone the discussion of the effect of price substitutions.

In general, a spell of constant prices is a sequence of $n+1$ prices that starts with an observed price p_t, possibly followed by a series of prices all equal to p_t, then followed, possibly, by a series of missing prices, that finally ends with an observed price at p_{t+n} that differs from the value of the initial price p_t. Notice that while we also refer to this sequence of prices as a spell of constant prices, it can include more than one price change if there were missing observations, a topic to which we return in detail below.

To deal with missing prices, the interesting patterns for a spell of constant prices are those which end with a price change, but that contain some missing price(s) just before the end of the spell. For example, consider a spell of prices for an outlet, i, in which there are exactly no changes in the first four periods and at least one change sometime during the next three periods.

Table 8: Example of a spell of constant prices with observed and missing prices

p_t	10		10		10		10		10		m		m		15
	t		$t+1$		$t+2$		$t+3$		$t+4$		$t+5$		$t+6$		$t+7$
		$\overbrace{}\lambda_{t+1}$		$\overbrace{}\lambda_{t+2}$		$\overbrace{}\lambda_{t+3}$		$\overbrace{}\lambda_{t+4}$		$\overbrace{}\lambda_{t+5}$		$\overbrace{}\lambda_{t+6}$		$\overbrace{}\lambda_{t+7}$	
I_t	y		0		0		0		0		x		x		1
γ_t	y		1		1		1		1		x		x		3
χ_t	0		0		0		0		0		1		1		0

In table 8 an x means that the variable is not defined for that case, the m denotes a missing/imputed price and the y denotes that for indicator I and counter γ the first observation in the spell of prices is missing because it depends on the prices in period $t-1$ which are not in the information of the table. The probability of observing this spell is:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times e^{-\lambda_{t+3}} \times e^{-\lambda_{t+4}} \times \left(1 - e^{-\lambda_{t+5}} \times e^{-\lambda_{t+6}} \times e^{-\lambda_{t+7}}\right) \tag{49}$$

The first four products are the probability of no change during the first four periods, and the last term is the probability of at least one change during the last three periods. The second term is the complement of the probability of no change in prices during the last three periods.

The likelihood of the sample of T periods (with $T+1$ prices) of all the outlets for the good j –denoted by the set O_j– is the product over all outlets i of the product of all spells for outlet i of the probability equation (49). To write the likelihood we define an indicator, χ_{it}, and a counter γ_{it}. The indicator χ_{it} adopts the value one if a price is missing for outlet i in period t, and zero otherwise. The value of γ_t counts the number of periods between two non-missing prices. The counter γ_{it} is Klenow and Kryvtsov (2008) duration clock . Then the likelihood function of the prices observed for product j is:

$$L_j = \prod_{i \in O_j} \prod_{t=1}^{T} \left[e^{-\lambda_{j,t}\gamma_{it}}\right]^{(1-I_{it})(1-\chi_{it})} \times \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}}\right]^{I_{it}(1-\chi_{it})} \tag{50}$$

Since the λ's are the probability of a price change and they are indexed at the end of a period, the first temporal observation of prices at $t = 0$ does not enter the likelihood. The log likelihood is:

$$\ell_j = \sum_{i \in O_j} \left(\sum_{t=1}^{T} (1-\chi_{it})(1-I_{it}) \times (-\lambda_{j,t}\gamma_{it}) + \sum_{t=1}^{T} (1-\chi_{it}) I_{it} \ln \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}}\right] \right) \tag{51}$$

To compute the contribution to the likelihood of a given value of $\lambda_{j,t}$ for $t = 1, ..., T$ we find convenient to introduce two extra counters: κ_{it} and η_{it} for any period t in which prices are missing/imputed. The variable κ_{it} counts the number of periods since the beginning of a string of missing/imputed prices. The variable η_{it} counts the number of periods of missing/imputed prices until the next price is observed. For example, consider the string of prices in table 9.

Table 9 shows an example of a spell of constant prices for a given variety and a given outlet. In equation (52) we highlight the contribution to the log-likelihood of the value of λ_t for a given outlet i:

Table 9: Example of a spell of constant prices, w/ missing prices and counters

p	10		10		m		m		m		m		m		15
	t		$t+1$		$t+2$		$t+3$		$t+4$		$t+5$		$t+6$		$t+7$
		λ_{t+1}		λ_{t+2}		λ_{t+3}		λ_{t+4}		λ_{t+5}		λ_{t+6}		λ_{t+7}	
I	y		0		x		x		x		x		x		1
χ	0		0		1		1		1		1		1		0
γ	y		1		x		x		x		x		x		6
κ	x		x		1		2		3		4		5		x
η	x		x		5		4		3		2		1		x

$$
\begin{aligned}
\ell_j = \cdots &+ (1-\chi_{it})(1-I_{it}) \times (-\lambda_{j,t}\gamma_{it}) + (1-\chi_{it}) I_{it} \ln\left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1}\lambda_{j,t-\tau}}\right] \\
&+ \chi_{it} \ln\left[1 - e^{-(\sum_{\tau=0}^{\kappa_{it}-1}\lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}}\lambda_{j,t+\tau}}\right] + \cdots \ . \qquad (52)
\end{aligned}
$$

The first two terms have the contribution to the likelihood of $\lambda_{j,t}$, if the price at time t is not missing. The first case corresponds to a price at the beginning of the spell of constant prices. The second to a case where the price is the last one of the spell, and uses γ_{it} to be able to write the corresponding probability. The third term, correspond to the contribution of $\lambda_{j,t}$, if the price at time t is missing, and uses κ_{it}, and η_{it} to write the corresponding probability.

Using equation (52) it is easy to write the FOC with respect to $\lambda_{j,t}$ of the sample as:

$$
\begin{aligned}
\frac{\partial\,\ell_j}{\partial\,\lambda_{j,t}} = &\sum_{i\in O_j} (1-\chi_{it})\,(1-I_{it}) \times (-\gamma_{it}) + \sum_{i\in O_j} (1-\chi_{it})\, I_{it}\ \frac{1}{e^{\left(\sum_{\tau=0}^{\gamma_{it}-1}\lambda_{j,t-\tau}\right)} - 1} \\
&+ \sum_{i\in O_j} \chi_{it}\ \frac{1}{e^{\left(\sum_{\tau=0}^{\kappa_{it}-1}\lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}}\lambda_{j,t+\tau}\right)} - 1} = 0\ , \qquad (53)
\end{aligned}
$$

for $t = 1, \ldots, T$.

Roughly speaking the estimator for $\lambda_{j,t}$ computes the ratio of the number of outlets i that a time t have change the prices with those that have not change prices or that have missing prices. This approximation is exact if no outlet has a missing/imputed price at period t or before. In this case $\chi_{is} = 0$ for $s = 1, 2, ..., t$ for all $i \in O_j$, then equation (53) becomes

$$\sum_{i \in O_j} (1 - I_{it}) \times \gamma_{it} = \sum_{i \in O_j} \left[\frac{I_{it}}{e^{\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} - 1} \right] ,$$

which, if we make $\lambda_{j,t} = \lambda_{j,t-1} = ... = \lambda_{j,t-\gamma_{it}+1}$ is the same expression than in Klenow and Kryvtsov (2008). In the case of no missing observations, and where all the $\lambda's$ are assumed to be the same, this maximum likelihood estimator coincide with the simple estimator introduced in equation (48).

E.1.3 Incorporating Missing Prices and Sales

Our data contains a flag indicating whether an item was on sale. We consider a procedure that disregards the changes in prices that occur during a sale. The idea behind this procedure is that sales are anomalies for the point of view of some models of price adjustment, and hence they are not counted as price changes. To explain this assumption we write an hypothetical example:

Table 10: Example of a spell of constant prices removing sales

p_t	10		10		10		m		8		15
	t		$t+1$		$t+2$		$t+3$		$t+4$		$t+5$
		$\overbrace{}\lambda_{t+1}$		$\overbrace{}\lambda_{t+2}$		$\overbrace{}\lambda_{t+3}$		$\overbrace{}\lambda_{t+4}$		$\overbrace{}\lambda_{t+5}$	
I_t	y		0		0		0		0		1
a_t	y		0		0		0		1		0

In table 10 the indicator a_t takes the value of one if the good is on sale on period t and if the price at the time t is smaller than the previous recorded price. In this case, the price in periods $t+3$ and $t+4$ is changed to 10, the value of the previous recorded price. The general principle is to consider a string of recorded prices, possible missing values, and a price that has a sale flag, and replace the price of the string of missing values and the period with a sale flag for the previous recorded price. In other words, we replace the price at the period with a sale flag for the previous recorded price and then using our first assumption on missing prices we complete the missing price in between two prices that are equal. When the sales are disregarded the number and duration of price spells can change. Once this procedure is implemented, the likelihood is the one presented above using the modified price series -indeed in table 10 the indicator I_t is the one that corresponds to the modified price string. We refer to the corresponding estimates as those that *excludes sale quotes*. This is a procedure used by many, e.g. Klenow and Kryvtsov (2008). By construction with this method the estimates for λ will be smaller.

E.1.4 Incorporating Missing Prices and Price Substitutions

In this section we discuss different assumptions on the treatment of missing data and good substitutions that allow us to construct four estimators of the frequency of price changes that we report later on.

We use the indicators $\tilde{e}_{it}, e^*_{it}, \bar{e}_{it}$ to consider two different assumptions on how to treat a price spell that ends in some missing values or price substitutions. In particular, consider the case of a substitution of a product or a missing price. As explained above, our data set contains the information of whether the characteristic of the product sold at the outlet has changed and was subsequently substituted by a similar product. We also have information on whether the price is missing (mostly due to a stock-out). To be concrete, consider the following example of a spell of constant prices in table 11. In this table, and $s_{it} = 1$ denotes the period where a price substitution has occurred. Thus, the example has a spell of 9 prices, with two periods (3 prices) with no change in prices, then 5 periods with missing prices, and finally in the last period there is an observed price that correspond to a substitution of the good.

As in the previous examples, for the first observation an y indicates that the value of the indicator cannot be decided based on the information in the table.

The issue is the interpretation of when and whether there has been a price change in the previous price spell. One interpretation is that there has been a price change somewhere between periods $t+2$ and $t+7$. A different interpretation is that, because the price spell ends with the substitution of the good, the price has not changed. The idea for this interpretation is that if the good would have not changed, the price could have been constant beyond $t+7$. The next three cases explain how to implement the first interpretation, and two ways to

Table 11: Example: spell of constant prices w/ counters for substitutions

p	10		10		10		m		m		m		m		15
	t		$t+1$		$t+2$		$t+3$		$t+4$		$t+5$		$t+6$		$t+7$
		λ_{t+1}		λ_{t+2}		λ_{t+3}		λ_{t+4}		λ_{t+5}		λ_{t+6}		λ_{t+7}	
I	y		0		x		x		x		x		x		1
χ	0		0		0		1		1		1		1		0
s	y		0		0		0		0		0		0		1
$\tilde{e}$	y		0		0		0		0		0		0		0
e^*	y		1		1		1		1		1		1		1
$\bar{e}$	y		0		0		1		1		1		1		1

implement the second one. The last two cases present two simple estimators, one that treats quotes with substitutions as regular price changes, and one that exclude them.

1. We disregard the information of the substitution of a good, and proceed as we have done so far: including all price quotes as if the good have been not changed. In the previous example, it consists on assuming that the price has changed between periods $t+2$ and $t+7$. In this case we say that the probability of observing the spell in the table is given by:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times \left(1 - e^{-\lambda_{t+3}-\lambda_{t+4}-\lambda_{t+5}-\lambda_{t+6}-\lambda_{t+7}}\right) \tag{54}$$

   In this case we set $\tilde{e}_{it} = 0$ for all periods, since we don't want to exclude any part of this price spell. We refer to these assumptions as *including all price quotes*.

2. We follow Klenow and Kryvtsov (2008) and others and exclude completely any spell of prices that ends with a substitution of a product. We implement this by defining the indicator $e^*_{it} = 1$ for any price corresponding to this spell, i.e. we exclude all the observations –with the exception of the first price, which is relevant for the previous string. In this case we have no associated probability for this spell. The idea behind this treatment is that if there would have been no substitution of the good the price could have stayed constant even beyond $t+7$. We refer to these assumptions as *excluding substitution spells* for short.

3. We introduce a new way to handle this information based on the following underlying assumption: if a spell of constant prices ends up in a substitution we interpret that the product has changed, and hence we cannot infer from the observed price whether the price has changed or not, as in the previous case. Yet, unlike the previous case, we do not discard the information at the beginning of the string, where the product was the same and its price was not changing. In this case the associated probability for this spell is:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \tag{55}$$

   In this case we use the indicator $\bar{e}_{it} = 0$ for the first two observations, among which we know that there was no price change, and $\bar{e}_{it} = 1$ for the rest of the observations where we can't conclude if there was a price change for the *same* product. We refer to these assumptions as *excluding substitution quotes* for short.

4. We present an alternative estimator to the maximum likelihood, which has the advantage of being simpler to describe and understand. This estimator imitates the one for the case where there are no missing price quotes in equation (48), and simply excludes the values of the missing. In this case, this estimator is:

$$\lambda_{j,t+1} = \log \left(\frac{\sum_{i \in O_j} (1-\chi_t)(1-\chi_{t+1})}{\sum_{i \in O_j} (1-\chi_t)(1-\chi_{t+1})(1-I_{it})} \right) \tag{56}$$

   In words, $\lambda_{j,t+1}$ is a non-linear transformation of the probability of the change of prices. This probability is estimated as the ratio of the outlets that have changed prices over all the outlets, including only those price quotes that are simultaneously not missing at t and $t+1$. While this estimator is simpler than the maximum likelihood, it does not use all the information of the missing values efficiently. We refer to this estimator as the *simple* estimator.

Finally, to completely state the notation for the likelihood function, we use the indicator ζ to deal with missing prices at the beginning of the sample. In particular, if for an outlet i the sample starts with $n+1$ missing prices, we exclude these observations from the likelihood since we cannot determine the previous price. We do this by setting the indicator $\zeta_{it} = 1$ for $t = 0, 1, ..., n$. Thus, depending of the assumption, the exclusion indicator e takes the values given by $\tilde{e}$, $\bar{e}$ or e^{*}, besides the value of 1 for all the missing observations at the beginning of the sample. The log likelihood function is then:

$$\ell_j = \sum_{i \in O_j} \sum_{t=1}^{T} (1 - e_{it}) \left((1 - \chi_{it})(1 - I_{it}) \, (-\lambda_{j,t}\gamma_{it}) + (1 - \chi_{it}) I_{it} \ln \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right] \right) \tag{57}$$

The first order condition for $\lambda_{j,t}$, using the counters κ and η is:

$$\begin{aligned} \frac{\partial \, \ell_j}{\partial \, \lambda_{j,t}} &= \sum_{i \in O_j} (1 - \chi_{it}) \, (1 - e_{it}) \, (1 - I_{it}) \times (-\gamma_{it}) + \\ & \sum_{i \in O_j} (1 - \chi_{it}) \, (1 - e_{it}) \, I_{it} \; \frac{1}{e^{\left(\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}\right)} - 1} \\ & + \sum_{i \in O_j} \chi_{it} \; (1 - e_{it}) \; \frac{1}{e^{\left(\sum_{\tau=0}^{\kappa_{it}-1} \lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}} \lambda_{j,t+\tau}\right)} - 1} = 0 \; , \end{aligned} \tag{58}$$

for $t = 1, \ldots, T$.

E.1.5 Aggregation: Weighted Average, Median, Weighted Median and Pooled Maximum Likelihood

In this section we deal with the issue of aggregation. So far we have described how to estimate the frequency of price adjustment for each good category separately.

Remember that those goods that fall in the homogeneous goods category are sampled bi-monthly and so will be our estimates. In this way, the first step in order to aggregate all categories is to convert them into monthly estimates, which is done simply by adding the two estimates in any given month (this results from the exponential assumption for our likelihood function).

Next, we compute three aggregated estimations. First, we calculate the weighted average of all monthly estimates (both differentiated and homogeneous goods), where the weights are the corresponding expenditure shares of each good category.

$$\lambda_t = \sum_{i=1}^{N} \omega_i \lambda_{it}$$

For future reference, we will call this *weighted average* or simply *WA*, followed by the specific treatment of missing, sales and substitutions. For example, *Weighted average excluding sales.*

The other two estimates are the median and weighted median of all monthly estimates (both differentiated and homogeneous goods). The aggregated median estimation (*median* for short) consists in taking the median λ of all products at each time period. The aggregate weighted median estimation is computed by sorting, at each time period, the expenditure weights of each product by the value of their associated λ from lowest to highest. Then we compute the accumulated sum of the weights until reaching 0.5. The aggregate weighted median of the frequency of price adjustment is the associated λ of the product whose weight makes the accumulated sum equal to or grater than 0.5. We refer to this estimate as *weighted median*.

Finally we consider a last aggregated estimation. As mentioned, the estimates for the frequency of price changes presented allow the value of λ to depend on the time period and the good. We now consider an estimate based on the assumption that the frequency price changes is common for all goods, but that that it can change between time periods. This simply puts together the outlets for all goods in our sample. Thus, the log likelihood is:

$$\ell(\lambda_1, ..., \lambda_T) = \sum_{j=1}^{N} \ell_j \left(\lambda_1, ..., \lambda_T\right)$$

where the $\ell_j \left(\lambda_1, ..., \lambda_T\right)$ corresponding to the log likelihood for each assumption about missing prices and or price substitutions as it has been introduced in the previous sections, and where N is the number of goods in our sample. We refer to this estimator as the *pooled maximum likelihood* or for short PML. Likewise, when we assume that all goods have the same frequency of price changes but use the simple estimator for λ, we refer to it as *simple pooled* estimator.

E.2 Estimation of the elasticities at a more disaggregated level

The theory predicts, and the data confirms, that the frequency of price adjustment is very different across goods. In fact, in table 1 we observe a considerable heterogeneity in the *simple* estimator of λ across 5-digit level industries. In this subsection we explore the robustness of the parameter estimates for the elasticity of the frequency of price changes with respect to inflation at high and low inflation rates by fitting equation (15) for each of the 5-digit industries, using the simple estimator of λ.[31]

Table 12 presents statistics describing the distribution of the coefficient estimates derived from equation (15) across 5-digit industries. The elasticity estimates confirm our previous findings: (i) the elasticity of the frequency of price changes at high inflation, γ, varies between 1/2 and 2/3; and (ii) the semi-elasticity $\Delta\%\lambda$ is approximately zero regardless of the industry. Consistent with the results in table 1, there is large variation in the expected duration at low inflation; particularly so for differentiated goods.

Table 12: Distribution of fitted coefficients at 5 digit level

	Elasticity γ		Semi-elasticity at zero $\Delta\%\lambda$		Duration at $\pi = 0$	
	Diff .	Hom.	Diff.	Hom.	Diff.	Hom.
Mean	0.58	0.56	0.03	0.01	18.5	7.5
Median	0.58	0.55	0.03	0	15	6.4
Perc 75	0.7	0.6	0.1	0.02	25.6	9.3
Perc 25	0.48	0.5	-0.03	-0.02	6.9	4.7
Std. Dev.	0.14	0.08	0.1	0.07	14.2	5.2

Note: *Diff.* denotes differentiated goods. *Hom.* denotes homogeneous goods. For each 5-digit industry we use NLLS to fit: $\log \lambda_t = a + \epsilon \min\{\pi_t - \pi^c, 0\} + \nu(\min\{\pi_t - \pi^c, 0\})^2 + \gamma \max\{\log \pi_t - \log \pi^c, 0\} + \omega_t$. The semi-elasticity at zero $\Delta\%\lambda$ is the percentage change in λ when inflation goes from 0 to 1%. λ is estimated with the simple estimator in equation (14).

[31] We performed the same exercise at a 6-digit level obtaining qualitatively similar results. See for table 2 disaggregation levels

E.3 Sampling Periodicity

So far we have been using the estimator of the theoretical frequency λ_a that has been proposed in the literature, $\hat{\lambda}_t = -\ln\left(1 - f_t\right)$, where f_t is the fraction that changed outlets in $(t-1, t]$. If price changes follow a Poisson process this is the maximum likelihood estimator of λ_a. However, since we only observe f_t at discrete times, a well known bias may arise if prices change more than once within the time interval and these changes are not independent. In particular, we would expect the bias to become larger as inflation increases and prices change more frequently.

In this section, we consider an alternative estimator $\lambda_t^{SW} = f_t$ and compare it to $\hat{\lambda}_t$. In the Sheshinski-Weiss model with no idiosyncratic shocks (or in the limit as inflation becomes very large compared to the volatility of the shocks) this is a maximum likelihood estimator of λ_a.

In figure 14 we present the results of Monte-Carlo simulations using data generated by the model in section 2. We sample observations every two-weeks and calculate both estimators of λ_a for different inflation rates. The true frequency of price adjustment, λ_a, is represented by the red line, the frequency, f, by the green line and the simple estimator $\hat{\lambda}_t$ by the blue line. The figure points to the existence of an upward bias in $\hat{\lambda}_t$ for high inflation rates, and as such, in the elasticity of the frequency of price adjustments to inflation.

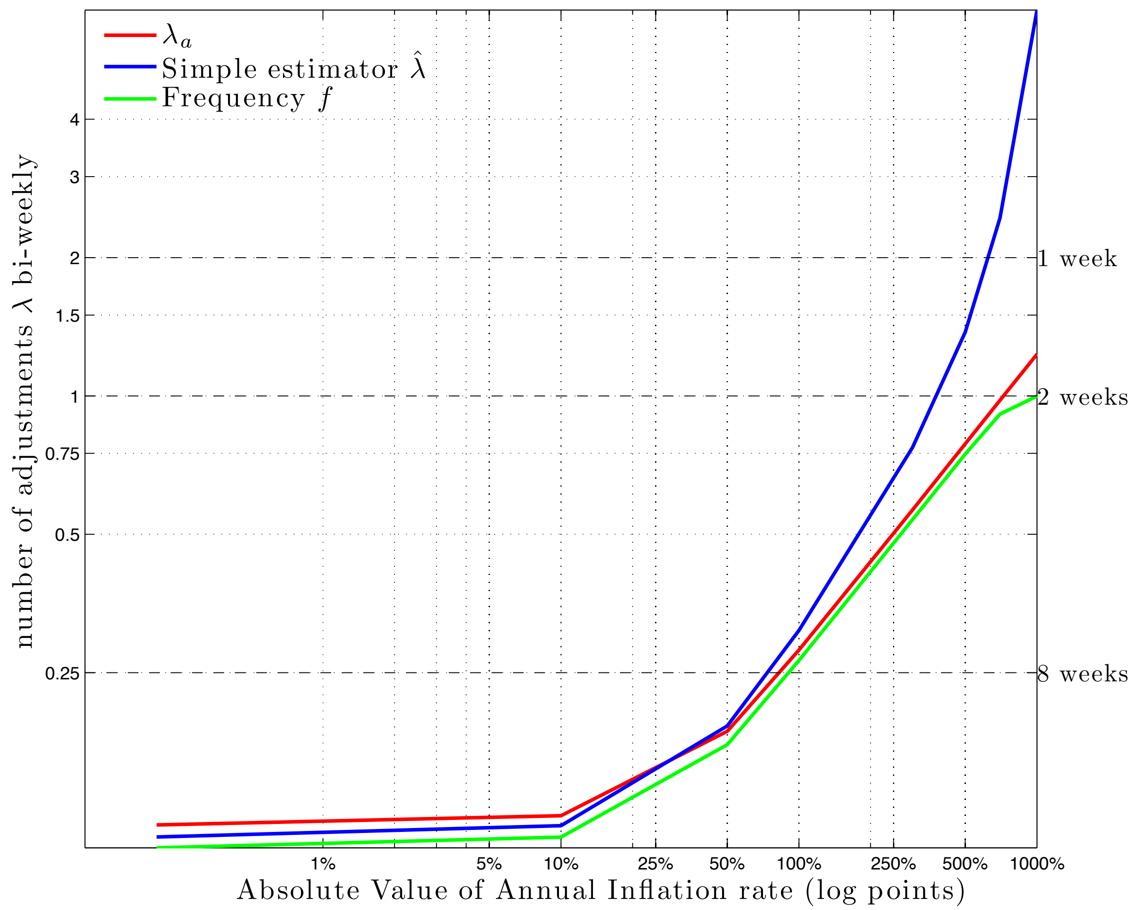
To reduce the incidence of such bias in our empirical estimates we proceed by re-estimating the elasticity of the frequency of price changes to inflation by excluding observations corresponding to inflation above some threshold. Figure 15 illustrates the results for a threshold inflation of 200 percent using simple estimators of λ and f. The estimated elasticities are 0.63 and 0.48 respectively, much in line with our benchmark estimates.[32]

E.4 Expected Inflation

The robustness check in this section pertains to the measure of expected inflation as opposed to our estimates of the frequencies of price adjustment. Theoretical models of price setting

[32] Excluding observations corresponding to inflation below 50 percent and above 200 percent or below 50 percent and above 100 percent result in estimated elasticities of 0.64 and 0.76 when using $\hat{\lambda}$; when using f instead, these are 0.44 and 0.59

Figure 14: Estimators $\hat{\lambda}$ and f , sampled every two weeks



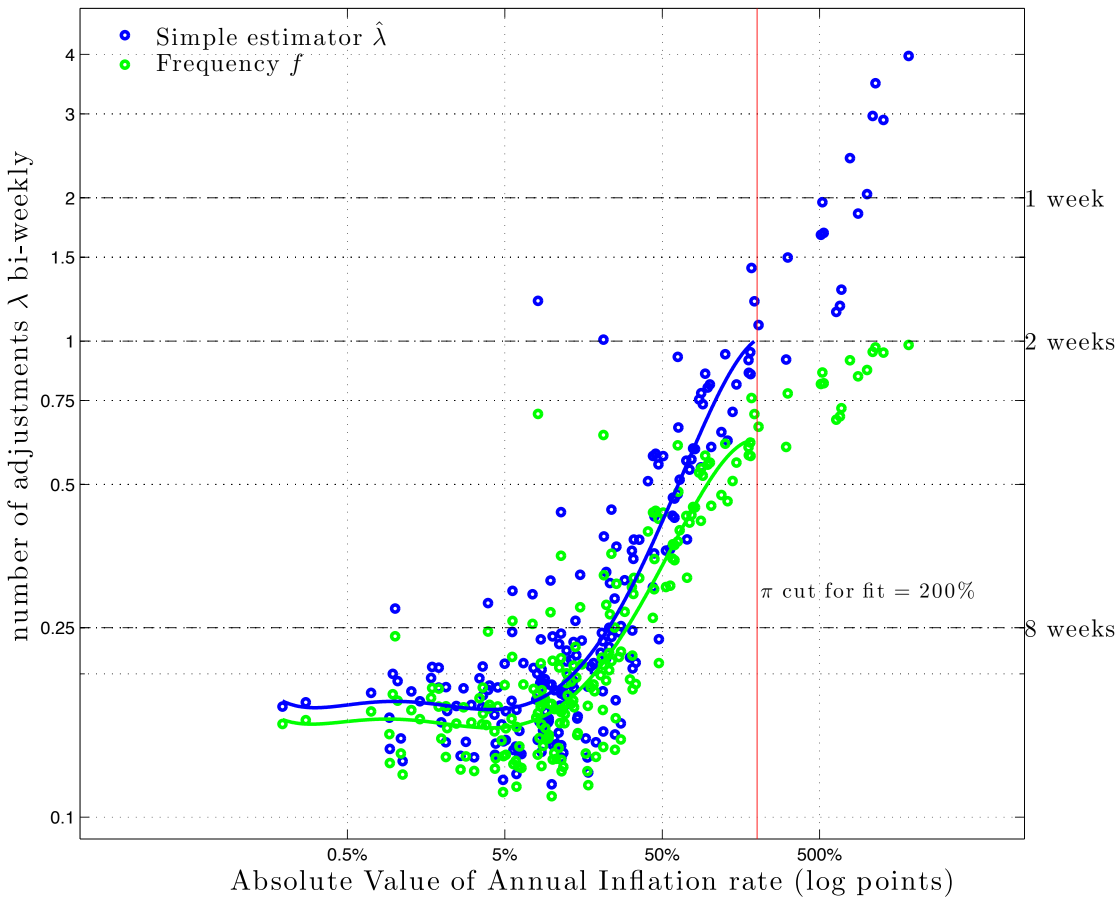
behavior, such as the menu cost model presented in section 2, predict that firms decision to change prices, and the magnitude of such change, depend on *expected* inflation between adjustments. Our measure of expected inflation so far was the average realized inflation for the expected duration of the price set in period t, $1/\lambda_t$.

We now consider a more flexible form for expected inflation as an average of the actual inflation rate of the following k_t months, where $k_t = [n/\lambda_t]$ and $[x]$ is the integer part of x; that is

$$\pi_t^e = \frac{1}{k_t} \sum_{s=t}^{t+k_t} \pi_s \tag{59}$$

We refer to n as the *forward looking factor*. Thus, as inflation falls (and implied durations

Figure 15: Estimators λ and f , homogeneous goods sampled twice a month



rise) in our sample agents put an increasing weight on future inflation. When $n = 0$ expected and actual inflation are the same.

Table 13: Elasticities and implied duration for different estimates of expected inflation.

Forward looking factor	Elasticity γ	Semi-elasticity at zero $\Delta\%\lambda$	Duration at $\pi = 0$	R^2
$n = 0$	0.57	0.01	4.4	0.959
$n = 0.5$	0.54	0.03	4.4	0.949
$n = 1$	0.53	0.04	4.5	0.953
$n = 1.5$	0.48	0.04	4.6	0.946
$n = 2$	0.46	0.04	4.7	0.944

Table 13 shows that the results presented in section 5.1 are not very sensitive to estimating equation (15) using different forward looking factors in equation (59). The first row of the table shows the results when we use the actual rate of inflation[33]. The following four rows shows that as expectations are more forward looking the threshold inflation and the elasticity fall slightly and the implied duration at the threshold remains constant. The R^2 of the regression is maximized for $n = 1$, so that the best fit is when we assume agents set expected inflation equal to the average of the perfect foresight future inflation rates for the expected duration of the prices. All the estimates of the elasticity γ are consistent with the theoretical prediction in section 2.

F Computation of estimates and standard errors of λ

Here we describe the details on the maximum likelihood estimator and the computation of the standard errors.

We compute the maximum likelihood estimator by using an iterative procedure. In this discussion we fix a good or item. We denote the iterations by superindex j. The initial guess for λ_t^0 is the log of the ratio of the outlets that change the price between $t-1$ and t.Then we solve for λ_t^{j+1} in equation (58) for the foc of λ_t taking as given the values of λ_{t-i}^j and $\lambda^j t + i$ for all $i \neq 0$. Notice that this equation has a unique solution. Also notice that the solution can be ∞, for instance if all prices change, or zero.

We have not derived expression for standard error for the different estimator of λ. Nevertheless to give an idea of the precision of our estimator, we note that for the simple pooled estimator, assuming that missing values are independent, we can use the expression for the standard error of a binomial distributed variable for the probability of a price change π, while using the estimate $\hat{\pi}_t =$ number of outlets with a price change $/N_t$ where N_t is the number of outlet with a price quote between t and t_1. In this case $se(\hat{\pi}) = \sqrt{\hat{\pi}(1-\hat{\pi})/N}$. Using the

[33] We use the inflation rate estimated by INDEC for the whole CPI instead of the inflation rate in our sample because we need forward looking values of inflation at the end of the sample that otherwise we cannot obtain.

delta method and $\lambda(p) = -\log(1-p)$ then we have:

$$se(\hat{\lambda}) = \frac{\sqrt{\exp(\hat{\lambda}) - 1}}{\sqrt{N}} \quad \text{and} \quad se(\log\hat{\lambda}) = \frac{1}{\hat{\lambda}} \frac{\sqrt{\exp(\hat{\lambda}) - 1}}{\sqrt{N}} \ . \tag{60}$$

To give an idea of the magnitudes for our estimated parameters we use some round numbers for both homogeneous and differentiated goods. For the case where we pool *all* the differentiated goods we can take $N_d = 230 \times 80 \times 0.80 = 14720 \approx \#$ diff. goods $\times avg.$ # outlets diff. goods $\times$ fraction of non-missing quotes. Pooling *all* the homogeneous goods we $N_h = 60000 = 300 \times 250 \times 0.80 \approx \#$ homog. goods $\times avg.$ # outlets homog. goods $\times$ fraction of non-missing quotes. So for high value of λ such as for $log\hat{\lambda} = \log 5$, we have that the standard errors are $se(\log\hat{\lambda}_d) = 0.02$ and $se(\log\hat{\lambda}_h) = 0.010$. Instead for low values of λ such as for $log\hat{\lambda} = \log 1/5$, we have that the standard errors are $se(\log\hat{\lambda}_d) = 0.019$ and $se(\log\hat{\lambda}_h) = 0.009$. Instead if we estimate λ at the level of each good, the standard errors should be larger by a factor $\sqrt{230} \approx 15$ and $\sqrt{330} \approx 17$ for differentiated and homogeneous goods respectively.

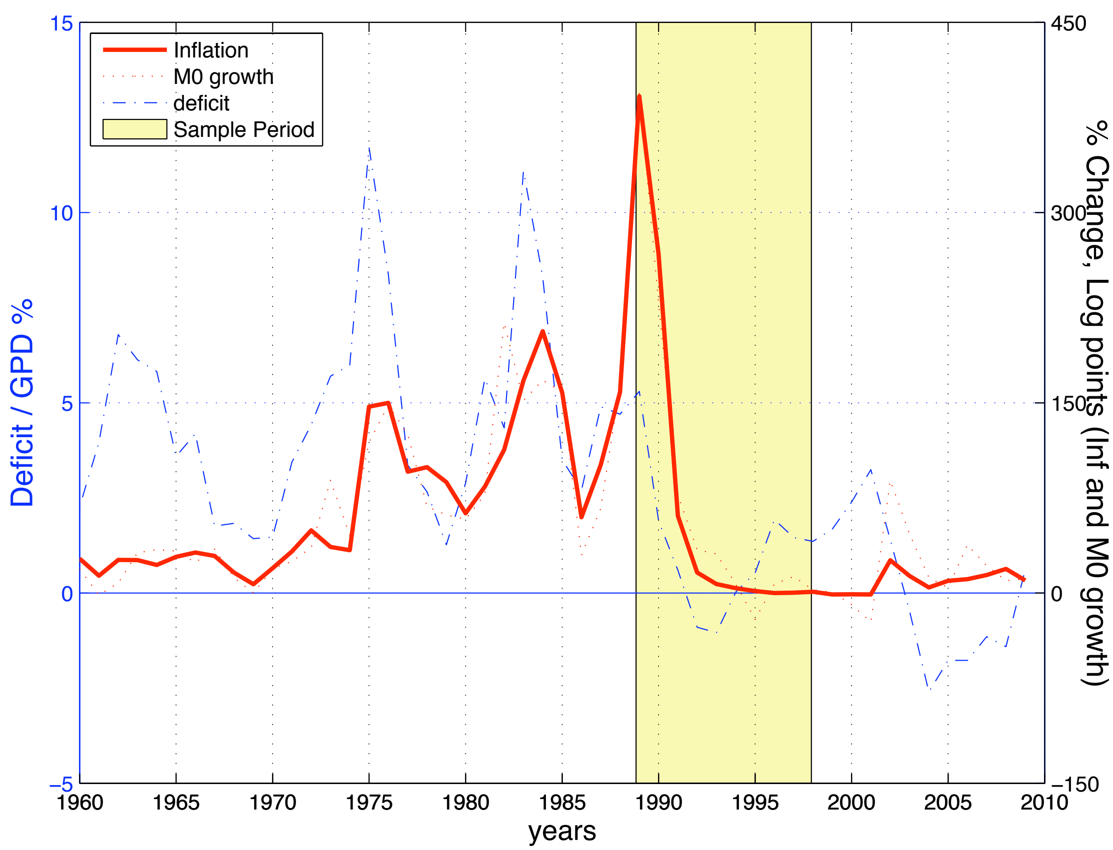
G Background on Inflation and Economic Policy

Here we give a brief chronology of economic policy to help readers understand the economic environment in our sample period, which goes from December 1988 to September 1997. The beginning of our sample coincides with decades of high inflation culminating in two years of extremely high inflation (typically referred as two short hyperinflations) followed by a successful stabilization plan, based on a currency board, started in April of 1991 which brought price stability in about a year, and stable prices until at least three more years after the end of our sample.

The years before the introduction of the currency board witnessed several unsuccessful stabilization plans, whose duration become shorter and shorter, and that culminated in the two short hyperinflations, all of these during a period of political turmoil. Several sources describe the inflation experience of Argentina since the 1970, such as Kiguel (1991) and Alvarez and Zeldes (2005) for the period before 1991 and Cavallo and Cottani (1997) for descriptions right after 1991. For a more comprehensive study see Buera and Nicolini (2010).

Argentina had a very high average inflation rate since the beginning of the 1970s. Institutionally, the Central Bank has been part of the executive branch with no independent powers, and typically has been one of the most important sources of finance for a chronic fiscal deficit. figure 16 plots inflation, money growth and deficits between 1960 and 2010 with our sample period highlighted in yellow. The deficit as a percentage of GDP was on average well above 5% from 1975 to 1990, see figure 16. At the beginning of the 1980s fiscal deficit and its financing by the Central Bank were large even for Argentinean standards. These years coincide with a bout of high inflation that started in the second half of the last military government, 1980 to 1982, and continued during the first two years of the newly elected administration of Dr. Alfonsin, 1983 to 1984.

Figure 16: Inflation, Money Growth and Deficits



In June of 1985 there was a serious attempt to control inflation by a new economic team which implemented what it is referred to as the Austral stabilization plan (the name comes from the introduction of the "Austral" currency in place of the "Argentine Peso"). The core of this stabilization plan was to fixed the exchange rate, to control the fiscal deficit and its financing from the Central Bank, and to introduce price and wage controls. While the Austral plan had some initial success, reducing the monthly inflation rate from 30% in June of 1985 to 3.1% in August of 1985, by mid 1986 the exchange rate was allowed to depreciate every month and inflation reached about 5% per month. By July of 1987 the monthly inflation rate was already above 10%. The same economic team started what is referred to as the "Primavera" stabilization plan in October 1988, when the inflation rate was again around 30% per month, at a time when the Alfonsin administration was becoming politically weak. The primavera plan was a new short lived exchange rate based stabilization plan that was abandoned in February of 1989 [34]. Our data set starts right after the beginning of the Primavera stabilization plan, in December of 1988.

After the collapse of the "Plan Primavera" the economy lost its nominal anchor and a perverse monetary regime was in place. Legal reserve requirements for banks where practically 100% and the Central Bank paid interest on reserves (most of the monetary base) printing money. Thus a self fulfilling mechanism for inflation was in place. High inflationary expectations, led to high nominal deposit rates, which turned into high rates of money creation that validated the inflationary expectations.

figure 17 displays the yearly percent continuously compounded inflation rate and interest rate for the first years of our sample, together with references to some of the main changes in economic policy during the period. Observe how interest rates and inflation skyrocketed after the plan Primavera's crawling peg was abandoned. In May 1989 a presidential election took place where the opposition candidate, Dr. Menem, was elected. The finance minister and the central bank president that carried the Primavera plan resigned in April 1989. Thereafter, Dr. Alfonsin's administration had two different finance ministers and two different central bank presidents, in the midst of a very weak political position and rampant uncertainty about the policies to be followed by the next administration. During the campaign for the presidential

[34] The peg started at about 12 units of argentine currency ("the Austral") per us dollar. To put it in perspective, at the beginning of the Austral plan, the peg was 0.8 units of argentine currency per US dollar.

election Dr. Menem proposed economic policies that can be safely characterized as populist, with a strong backing from labor unions. Indeed, the core of his proposed economic policy was to decree a very large generalized wage increase, “el salariazo”.[35] In July 1989 the elected president, Dr Menem, took office, several months before the stipulated transition date, in the midst of uncontrolled looting, riots and extreme social tension. The inflation rate at this time was the highest ever recorded in Argentina, almost 200% per month—2.3% per day.

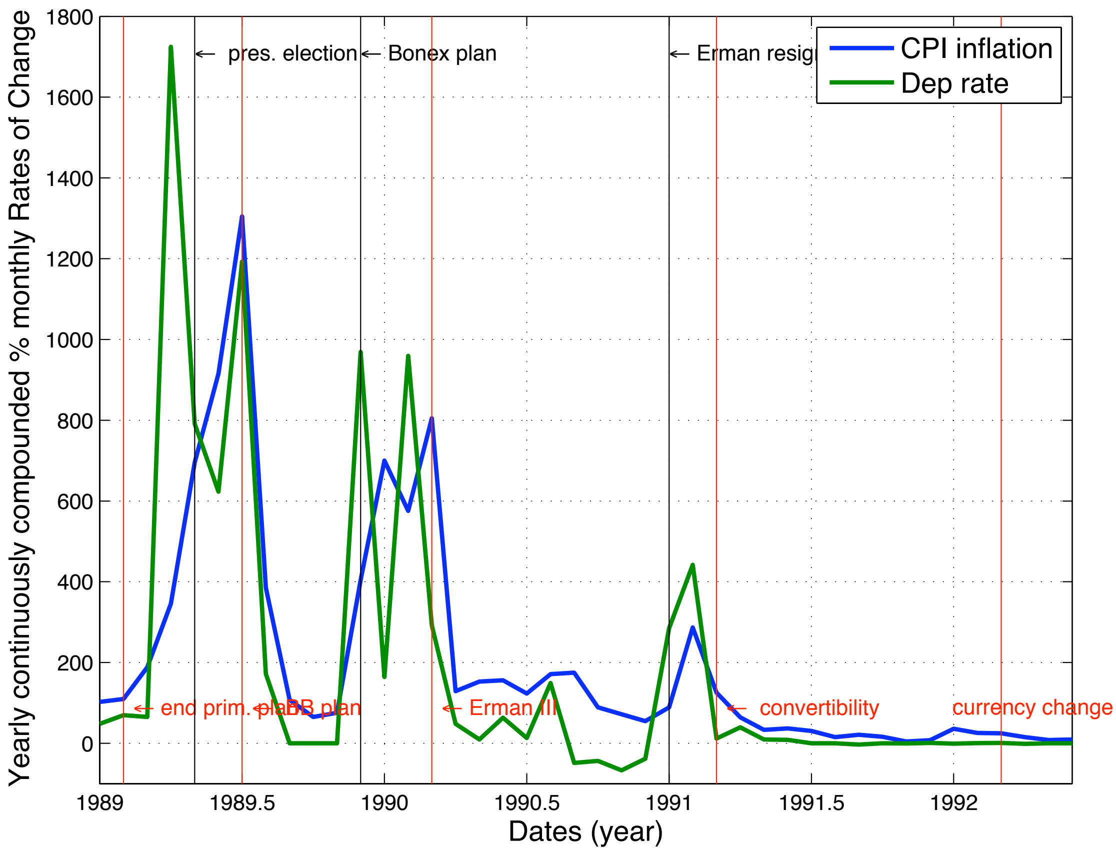
The beginning of the Menem administration started with a large devaluation of the argentine currency, in what is known as the BB stabilization plan, for the name of the company Bunge and Born, where the two first secretaries of the treasury came from. Indeed these appointments made by the Menem administration were a surprise to most observers, given the promises made in the campaign. The announcement of tight control of the fiscal deficit, and the management of the exchange rate of this plan were also surprising for most observers. During this time inflation transitorily fell.

In December 1989, amid large looses in the value of the argentine peso, a new finance minister was appointed, Dr. Erman Gonzalez, who started yet a new “stabilization plan” (referred to as Plan Bonex). The core of this plan was a big compulsory open market operation by which the central bank exchanged all time deposits in the Banking system (mostly peso denominated time deposits with maturities of less than a month) for 10 year US Dollar denominated government bonds (Bonex 1989). This big open market operation changed the monetary regime and allowed the Central Bank to regain control of the money supply, as the government no longer had to pay interest on money (reserves on time deposits) by printing money. During Dr. Gonzalez tenure there were several fiscal measures aimed at controlling the fiscal deficit. In march 1990 a renewed version of the stabilization plan was launched, with a stricter control of the money supply and of the fiscal deficit. The actual percentage inflation rates during 1989 and 1990 were 4924% and 1342% respectively!

In January of 1991, Dr. Gonzalez resigned and Dr. Cavallo was appointed as finance minister. During the first two months of his tenure there was a large devaluation of the currency and a large increase in the prices of government owned public utility firms. In April

[35] The slogan to summarize his proposed economic policy was “el salariazo”, i.e. “the huge wage increase” in Spanish.

Figure 17: Inflation and Depreciation Rate during HyperInflations



1^{st} of 1991 there was a regime shift that lasted until 2001. The new regime was a currency board that fixed the exchange rate and enacted the independence of the Central Bank, first by means of presidential decrees, and then by laws approved by congress. At this time the argentinean currency, the Austral, was pegged to the US dollar at 10,000 units per USD.[36]

On January 1^{st} 1992 there was a currency reform that introduced a new currency (the Peso Argentino) to replace the Austral, chopping four zeros of the latter (so that one peso was pegged to one dollar, and to 10,000 australes).

There are a host of changes that were introduced at this time, both in term of deregulation and in terms of fiscal arrangements (broadening of the value added tax's base, sale of state owned firms, etc.), which in the first years reduced the size of the fiscal deficit. There was

[36]To have an idea of the average inflation rate until 1991, notice that the exchange rate when the Austral was introduced in June of 1985 was 0.8 austral per US dollars.

also a renewed access to the international bond markets, and a constant increase in public debt. There were also an acceleration of the trade liberalization that started in the mid 80s and a liberalization of all price and wage controls. During the years covered in our sample, GDP grew substantially, despite the short and sharp recession during 1995, typically associated with the balance of payment crisis in Mexico. The exchanged rate remained fixed until January of 2002, where the exchange rate was depreciated in the midst of a banking run that started in the last quarter of 2001, a recession that started at least a year prior, and the simultaneous default of the public debt.

H Collection of some results with low inflation

We can write inflation as

$$\pi = \lambda^{+}\Delta^{+} - \lambda^{-}\Delta^{-} \tag{61}$$

From proposition 1 we know that

$$\frac{\partial \lambda}{\partial \pi} = 0$$
$$\lambda^{+} = \lambda^{-}$$
$$\Delta^{+} = \Delta^{-}$$

Arguments similar to proposition 1 imply

$$\frac{\partial \lambda^{+}}{\partial \pi} = -\frac{\partial \lambda^{-}}{\partial \pi} = \frac{1}{2}\frac{1}{\Delta^{+}} \tag{66}$$
$$\frac{\partial \Delta^{+}}{\partial \pi} = -\frac{\partial \Delta^{-}}{\partial \pi}0 \tag{67}$$

By symmetry we have that for any inflation rate:

$$\lambda_a^+(\pi) = \lambda_a^-(-\pi) \tag{68}$$

$$\Delta_p^+(\pi) = \Delta_p^-(-\pi) \tag{69}$$

Differentiating with respect to inflation

$$\frac{\partial \lambda_a^+(\pi)}{\partial \pi} = -\frac{\partial \lambda_a^-(-\pi)}{\partial \pi} \tag{70}$$

$$\frac{\partial \Delta_p^+(\pi)}{\partial \pi} = -\frac{\partial \Delta_p^-(-\pi)}{\partial \pi} \tag{71}$$

Evaluating it at $\pi = 0$ gives:

$$\frac{\partial \lambda_a^+(0)}{\partial \pi} = -\frac{\partial \lambda_a^-(0)}{\partial \pi} \tag{72}$$

$$\frac{\partial \Delta_p^+(0)}{\partial \pi} = -\frac{\partial \Delta_p^-(0)}{\partial \pi} \tag{73}$$

Differentiating equation (61), using equation (66) we get

$$1 = 2\left[\frac{\partial \lambda_a^+}{\partial \pi}\Delta_p^+ + \frac{\partial \Delta_a^+}{\partial \pi}\lambda_a^+\right] \tag{74}$$