From Individual to Aggregate Labor Supply: A Quantitative Analysis based on a Heterogeneous Agent Macroeconomy*

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Abstract

We investigate the mapping from individual to aggregate labor supply using a general equilibrium heterogeneous-agent model with an incomplete market. The nature of heterogeneity among workers is calibrated using wage data from the PSID. The gross worker flows between employment and nonemployment and the cross-sectional earnings and wealth distributions in our model are comparable to those in the micro data. We find that the aggregate labor supply elasticity of such an economy is around 1, bigger than micro estimates but smaller than those often assumed in aggregate models.

Keywords: Aggregate Labor Supply Elasticity, Heterogeneity, Indivisible Labor

JEL Classifications: E24, E32, J21, J22

1 Introduction

Despite enormous heterogeneity in the workforce, economists, for simplicity and tractability, often postulate and analyze an economy populated by identical agents. A fully specified representative-agent model has become a workhorse in macroeconomics, and it is common practice to rely on micro evidence to pin down the key parameters of highly aggregated models (Kydland and Prescott, 1982; King, Plosser, and Rebelo, 1988).

However, this practice often creates a tension between micro and macro observations.[1] A prominent example is the differences in measurement of labor supply elasticity. One of the stylized facts in aggregate fluctuations is that total hours worked varies greatly over the business cycle without much variation in wages.[2] To be consistent with the observed movement in hours and wages, a representative-agent model emphasizing the intertemporal substitution of leisure, pioneered by Lucas and Rapping (1969), requires the use of labor supply elasticity which is beyond admissible estimates based on micro data (Ghez and Becker, 1975; MaCurdy, 1981; Altonji, 1986; Abowd and Card, 1989).[3]

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[1]Browning, Hansen, and Heckman (1999) raise warning flags about the current use of micro evidence in calibrating macro models.

[2]The importance of labor supply elasticity is not limited to the business cycle analysis; for example, it plays a key role for the timing and effect of fiscal policy (Aucherbach and Kotlikoff, 1987; Judd, 1987).

[3]In his survey paper, Pencavel (1986) reports that most estimates are between 0 and 0.45 for men. In their parallel survey of research on the labor supply of women, Killingsworth and Heckman (1986) present a wide range of estimates, from -0.3 to 14; they do not venture a guess as to which is correct but conclude that the elasticity is probably somewhat higher for women than men. See Blundell and MaCurdy (1999) for a more recent review of the literature. See also Mulligan (1998) or Rupert, Rogerson, and Wright (2000) on how the current micro estimates may underestimate workers' willingness to substitute leisure over time. An alternative equilibrium approach is to introduce shifts in labor supply through shifts in preference (Bencivenga, 1992), home technology (Benhabib, Rogerson, and Wright, 1991; Greenwood and Hercowitz, 1991) or government spending (Christiano and Eichenbaum 1992). Even

The participation margin, the so-called extensive margin, has been recognized as a potential resolution. Hours fluctuations are accounted for mainly by movement in and out of employment by workers (Coleman, 1984; Heckman, 1984) with different reservation wages. Under this environment, the slope of the aggregate labor supply curve has little to do with intertemporal substitution but rather with the distribution of reservation wages across workers. The well-known lottery economy by Rogerson (1988) and Hansen (1985) is a special case where the reservation wage distribution is degenerate, yielding a very high elasticity, which is in fact infinity.

In this paper, we investigate the mapping from individual to aggregate labor supply using a general equilibrium model economy in which workers face idiosyncratic productivity shocks and the capital market is incomplete.[4] The heterogeneity of the workforce, more precisely the stochastic process of idiosyncratic productivity, is calibrated to be consistent with the wage data from the Panel Study of Income Dynamics (PSID) for 1971-1992. As the reservation wage distribution is crucial – but cannot be observed in practice – for our analysis, we test the model heterogeneity indirectly. The gross worker flows in and out of employment in the model are comparable to those in the Current Population Survey (CPS) for 1967:II-2000:IV. Cross-sectional distributions of earnings and wealth are comparable to those from the PSID and Survey of Consumer Finance (SCF) although the wealth distribution in the model is somewhat less skewed than that in the data.

From the reservation wage distribution, we uncover the upper bound of aggregate labor supply elasticity of our heterogeneous-agent economy. We find that elasticities range from .89 to 1.97

in a disequilibrium approach where the role of labor supply is dismissed in the short run, its slope is still important for the welfare cost departing from an equilibrium.

[4] An economy with indivisibility at the micro level may be approximated by a representative-agent economy with divisible labor, as the indivisibility is smoothed by an aggregation over heterogeneous agents. While this point is well illustrated in Mulligan (2001), we have yet to investigate its quantitative implications because the mapping from the micro to the macro function depends crucially on the heterogeneity of the workforce.

depending on the nature of heterogeneity. These values are bigger than typical micro estimates, but smaller than those often assumed in aggregate models. We also show that our model implies a small compensated labor-supply elasticity (between .37 and .69) at the individual level and a moderate elasticity (between .86 and 1.13) at aggregate level. In reference to the real-business-cycle analysis, our heterogeneous model economy is comparable to the representative-agent economy with the compensated labor-supply elasticity of 2.

The closest to our work are Kydland (1984), Cho and Rogerson (1988), Cho (1995), and Gomes, Greenwood and Rebelo (2001). Kydland constructs an economy with two types of workers, skilled and unskilled, and reproduces some labor-market regularities in relative wages and hours. However, this approach does not reflect the participation margin, a dominant source of the variation in total hours. Cho and Rogerson consider an economy which is populated by a continuum of identical families consisting of two members and show that the aggregate labor supply depends on the relative productivity among family members. While the female labor supply is indeed an important source of variation in aggregate hours, our analysis extends to a more general cross-sectional heterogeneity. Cho incorporates *ex post* heterogeneity into a standard real-business cycle framework. This considerably simplifies the computation as consumption is shared among workers. It is, however, clear in the data that persons with greater hours or greater earnings per hour consume more.[5] Gomes et al. also analyze the non-convexity of labor supply in an incomplete market with aggregate fluctuations. They focus on the cyclical behavior of unemployment rates, whereas we look into the mapping from individual to aggregate labor supply functions.

Other important works on the labor-market heterogeneity in the context of stochastic general

[5]For example, according to the Consumer Expenditure Survey data for 1990-1994, for single households, a one percent increase in hourly wage is associated with .6 percent increase in total consumption cross-sectionally; for married households, a one percent increase in household wage (the average wages of husband and wife if both are working) is associated with .29 percent increase in total consumption.

equilibrium include those of Andolfatto and Gomme (1996), Castañeda, Díaz-Giménez, and Ríos-Rull (1998), Merz (1999), and den Haan, Ramey, and Watson (2000). Andolfatto and Gomme study the unemployment insurance policy; Castañeda et al., the income distribution and unemployment spells; Merz, the cyclical behavior of labor turnover; den Haan et al., the propagation mechanism under labor-market matching and job destruction.

The paper is organized as follows. Section 2 lays out the model economy. In Section 3 we calibrate the model parameters consistent with various micro data. In Section 4, we investigate the aggregate labor supply of the model in both steady state and fluctuations. We also provide comparison with the representative-agent model. The conclusion, Section 5, summarizes our findings. Appendix collects the computational details and data sources.

2 The Model

2.1 Environment

The model economy is a version of the stochastic-growth model with a large (measure one) population of infinitely lived workers. Individual workers differ from each other in productivity. [6]

Each worker maximizes the expected discounted lifetime utility:

$$U = \max_{\{c_t,h_t\}_{t=0}^{\infty}} E_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_t,h_t)\right\},$$

with

$$u(c_t,h_t) = \ln c_t + B\frac{(1-h_t)^{1-1/\gamma}}{1-1/\gamma},$$

[6] Ideally, one would allow for heterogeneity both in the market and preference (or non-market productivity). However, it would be necessarily controversial to make an assumption about preference heterogeneity. We focus on the heterogeneity in the market productivity only whose process is inferred from the individual earnings data.

where $E_0[\cdot]$ denotes the expectation operator conditional on information available at time 0, β is the discount factor, c_t consumption, and h_t hours worked at time t. The utility is separable across times and between consumption and leisure. This assumption about the form of utility is popular in both business cycle analysis and empirical labor supply literature. The parameter γ denotes the intertemporal substitution elasticity of leisure. Log utility in consumption supports a balanced growth path.

According to our production technology, which will be specified below, labor input enters simply as efficiency units. Thus, a worker who supplies h_t units of time earns $w_t x_t h_t$, where w_t is the market wage rate for an efficiency unit of labor, and x_t represents the worker's productivity. We assume that individual productivity x_t exogenously varies over time according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. Since participation is the dominant source of variation in total hours worked in the data, we abstract from an intensive margin and assume that labor supply is indivisible; i.e., h_t takes either zero or $\bar{h}(< 1)$. A worker can save and borrow by trading a claim for physical capital, which yields the rate of return r_t and depreciates at rate δ. The capital market is incomplete; the physical capital is the only asset available to insure against idiosyncratic risks in x, and workers face a borrowing constraint; the asset a_t can take a negative value but cannot go below $\bar{a}$ at any time. A worker's budget constraint is:

$$c_t = w_t x_t h_t + (1 + r_t)a_t - a_{t+1},$$

and

$$a_{t+1} \geq \bar{a}.$$

Firms produce output according to a constant-returns Cobb-Douglas technology in capital, K_t, and efficiency units of labor, L_t:

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha},$$

where λ_t is aggregate productivity, following a stochastic process with a transition probability distribution function, $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$.

It is useful to consider a recursive equilibrium. Suppose $\mu(a, x)$ denotes the distribution (measure) of workers.[7] Let V^E and V^N denote the values of being employed and nonemployed, respectively. If a worker decides to work, she solves the following Bellman equation by choosing the next period asset holding a':

$$\begin{aligned} V^E(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \Big\{ & u(c, 1 - \bar{h}) \\ & + \beta E\Big[\max\left\{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \right\} \big| x, \lambda \Big] \Big\} \end{aligned} \tag{1}$$

subject to

$$c = wx\bar{h} + (1 + r)a - a',$$

$$a' \geq \bar{a},$$

and

$$\mu' = \mathbf{T}(\lambda, \mu).$$

where $\mathbf{T}$ denotes a transition operator for μ.

If the worker decides not to work, her Bellman equation is:

$$\begin{aligned} V^N(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \Big\{ & u(c, 1) \\ & + \beta E\Big[\max\left\{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \right\} \big| x, \lambda \Big] \Big\} \end{aligned} \tag{2}$$

subject to

$$c = (1 + r)a - a',$$

$$a' \geq \bar{a},$$

[7] Let $\mathcal{A}$ and $\mathcal{X}$ denote sets of all possible realizations of a and x, respectively. The measure $\mu(a, x)$ is defined over a σ-algebra of $\mathcal{A} \times \mathcal{X}$.

and

$$\mu' = \mathbf{T}(\lambda, \mu).$$

Having solved (1) and (2), it is straightforward to deal with worker's labor supply decision:

$$V(a, x; \lambda, \mu) = \max_{h \in \{0, \bar{h}\}} \Big\{ V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu) \Big\}. \tag{3}$$

2.2 Equilibrium

Equilibrium consists of a set of value functions, $\{V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu), V(a, x; \lambda, \mu)\}$, a set of decision rules for consumption, asset holdings, and labor supply, $\{c(a, x; \lambda, \mu), a'(a, x; \lambda, \mu), h(a, x; \lambda, \mu)\}$, aggregate inputs, $\{K(\lambda, \mu), L(\lambda, \mu)\}$, factor prices, $\{w(\lambda, \mu), r(\lambda, \mu)\}$, and a law of motion for the distribution $\mu' = \mathbf{T}(\lambda, \mu)$ such that:

1. Individual optimization:

   Given $w(\lambda, \mu)$ and $r(\lambda, \mu)$, the individual decision rules $c(a, x; \lambda, \mu)$, $a'(a, x; \lambda, \mu)$, and $h(a, x; \lambda, \mu)$ solve (1), (2), and (3).

2. The firm's profit maximization:

$$w(\lambda, \mu) = F_1\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big) \tag{4}$$

$$r(\lambda, \mu) = F_2\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big) - \delta \tag{5}$$

   for all (λ, μ).

3. The goods market clears:

$$\int \{a'(a, x; \lambda, \mu) + c(a, x; \lambda, \mu)\} d\mu = F\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big) + (1 - \delta)K \tag{6}$$

   for all (λ, μ).

4. Factor markets clear:

$$L(\lambda, \mu) = \int xh(a, x; \lambda, \mu) d\mu \tag{7}$$

$$K(\lambda, \mu) = \int a d\mu \tag{8}$$

for all (λ, μ).

5. Individual and aggregate behaviors are consistent:

$$\mu'(A^0, X^0) = \int_{A^0, X^0} \left\{ \int_{\mathcal{A}, \mathcal{X}} \mathbf{1}_{a'=a'(a,x;\lambda,\mu)} \; d\pi_x(x'|x) d\mu \right\} da' dx' \tag{9}$$

for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

3 Calibration

Individual productivity x is assumed to follow an AR(1) process in logs:

$$\ln x' = \rho_x \ln x + \varepsilon_x, \qquad \varepsilon_x \sim N(0, \sigma_x^2). \tag{10}$$

As we view x to reflect a broad measure of earnings ability in the market, the stochastic process of x is estimated by the individual wages from the PSID for 1971-1992. Appendix A.1 describes in detail the data we use. According to the model, the log wage for individual i at time t, denoted by $\ln w_t^i$, can be written as $\ln w_t^i = \ln w_t + \ln x_t^i$. When quasi-differenced, individual wage evolves as:

$$\ln w_t^i = \rho_x \ln w_{t-1}^i + (\ln w_t - \rho_x \ln w_{t-1}) + \varepsilon_{x,t}^i. \tag{11}$$

Equation (11) is estimated by the OLS with year dummies in the regression, capturing aggregate effects including $\ln w_t - \rho_x \ln w_{t-1}$. The annual estimates are $\widetilde{\rho}_x = .818$ (with a standard error of .0025) and $\widetilde{\sigma}_x = .291$.[8] According to the frequency conversion procedure described in Appendix

[8]Our estimate is slightly lower than, but comparable to, the persistence of idiosyncratic earnings risks in Storesletten, Telmer, and Yaron (1999). The difference is due to their decomposition of idiosyncratic shocks into a persistent AR(1) and purely temporary *i.i.d.* components, whereas we assume a single AR(1) process.

A.2, the corresponding quarterly values are $\rho_x = .95$ and $\sigma_x = .225$, which we refer to as the benchmark economy.

We consider two possible deviations from these values. The dispersion of productivity distribution may be larger than that of σ_x obtained from the wage distribution because the workers at the very low end of the productivity distribution are less likely to participate. The second model we consider has a larger dispersion in productivity: σ_x is magnified by 25%, yielding $\sigma_x = .28125$.[9]

In our model, x reflects the heterogeneity in earnings – both permanent and temporary – in the population. Thus, we have not controlled for individual characteristics in the regression. The persistence of wage may differ across population, especially for women who play an important role in the variation of total hours. Table 1 presents the estimate of ρ_x for various groups. Wages exhibit a smaller persistence for women. For example, the persistence is .722 for married women whereas it is .809 for married men. When we control for individual characteristics – gender, age, age squared, and years of schooling – in the regression, the persistence also decreases. For example, the annual estimate decreases from .818 to .743 for all workers. According to Pesaran and Smith (1995), when coefficients differ across groups, pooling and aggregation tend to lead to a higher estimates. Also, as wages tend to reflect good realizations of productivity, they appear more persistent than underlying productivity. Thus, we consider a less persistent productivity, $\rho_x = .92$, for our third model. In fact, with $\rho_x = .92$ (and $\sigma_x = .225$), the model shows the persistence of .816 and the standard deviation of innovation of .291 for individual wage regression in (11), which are almost identical to

[9]To understand the magnitude of selection bias, consider an extreme case where employment is completely ordered by the current productivity, that is, a worker with the highest productivity is hired first and so forth. In this case, the (observed) wage distribution is a truncated distribution of x. Under log-normality, when the bottom 40 percent (the average nonemployment rate in the CPS for 1967-2000) is truncated, the standard deviation of the underlying distribution is 1.5 times larger than that of the truncated distribution (See Maddala 1983). Given that labor supply depends on preference and wealth as well as wages, we magnify the estimate by 25%.

those in the PSID.

Other parameters of the model economies are in accord with business cycle analysis and empirical labor supply literature. According to the Michigan Time-Use Survey, a typical household allocates about 33 percent of its discretionary time for paid compensation (Hill, 1984; Juster and Stafford, 1991): $\bar{h} = 1/3$. Most micro estimates of intertemporal substitution elasticity of leisure fall between 0 and .5: we use $\gamma = .2$. With a discrete choice of hours of work (i.e., labor is indivisible), the value of γ is not so important for the aggregate labor supply elasticity since it mostly depends on the shape of the reservation wage distribution. The labor share, α, is .64, and the quarterly depreciation rate, δ, is 2.5 percent. We search for the weight parameter on leisure, B, such that the steady state employment rate is 60 percent, the average from the CPS for 1967:II-2000:IV. The discount factor β is chosen so that the quarterly rate of return to capital is 1 percent.[10] The borrowing constraint $\bar{a}$ is set to -2 which is approximately two quarters' earnings for a worker with the average productivity in our model economy.[11] Table 2 summarizes the parameter values. Finally, when we investigate the model economy with aggregate fluctuations, we introduce exogenous shifts in labor demand through aggregate technology shocks λ_t. We assume that $\ln \lambda_t$ follows an AR(1) process of which persistence is .95 and the standard deviation of innovation is .7 percent, which is consistent with the linearly de-trended post-war total factor productivity. We solve the equilibrium of the model economy in a discrete state space. Appendix A.4 provides a detailed description of the computational procedure.

[10] The discount factor is lower than that in the representative-agent model, because market incompleteness increases savings as noted in Aiyagari (1994).

[11] Given the persistent idiosyncratic earnings process, the size of the borrowing constraint itself does not affect the main result of the paper; for example, we obtain a similar aggregate labor supply elasticity with $\bar{a} = 0$.

4 Results

4.1 Steady State

We first characterize the steady state of the model economy where $\mu(x, a)$ is invariant. As an indirect diagnostic test, we ask whether the model generates a reasonable labor market mobility and cross-sectional distributions in wealth and earnings. Even in the absence of aggregate fluctuations there are constant flows of workers in and out of employment due to individual productivity shocks. Table 3 presents employment rate, gross-worker flows, and hazard rates from the model and the CPS. The statistics for the CPS are quarterly averages for 1967:II-2000IV.

As described in the previous section, the utility parameter B is calibrated to match the average employment rate of 60 percent. The quarterly gross-worker flows in the CPS are computed using Abraham and Shimer's (2001) monthly hazard rates as described in Appendix A.3. On average, 7.07 percent of the population moved from employment to nonemployment each quarter; 6.88 percent of the population moved in the opposite direction, from nonemployment (unemployment plus non-labor force) to employment.[12] In our first model, these flows are 5.92 percent, somewhat lower than those in the CPS data. With different degrees of idiosyncratic shocks, they are 5.64 (σ_x=.28125) and 6.85 (ρ_x=.92).[13] Overall, the worker flows and hazard rates are somewhat lower than, but comparable to, those in the CPS.

Wealth and earnings, excluding preference and non-market opportunity which are hard to

[12]These numbers are slightly higher than those in Blanchard and Diamond (1990) due to a different sample period and adjustment method. Also, we do not make a distinction between nonemployment and unemployment. According to Shimer, as well as Blanchard and Diamond, the flows between employment and non-labor force are as big as those between employment and unemployment.

[13]For each model, we adjust the utility parameter B and the discount factor β so that the employment rate is 60 percent and the quarterly rate of return to capital is 1 percent in steady state.

measure, are probably the most important factors for labor-market participation decision. Figure 1 exhibits the Lorenz curves of the wealth distributions from the 1984 PSID and three model economies.[14] Family wealth in the PSID reflects the net worth of house, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. According to Table 4, the Gini coefficient of wealth is .76 in the PSID, whereas those from the models are .64 for the benchmark, .65 (σ_x=.28125), and .58 (ρ_x=.92).

Figure 2 shows the Lorenz curves of earnings. The data is based on family earnings (earnings of head of household and spouse) also from the 1984 PSID. The model and the PSID exhibit similar inequality, but there are more zero earners at the bottom of the distribution in the model; 40 percent of population in the model and 20 percent in the PSID. In the PSID, a family with at least one family member working at some point during the survey year recorded positive earnings, whereas the model is calibrated to match the average employment rate of 60 percent. This makes the Gini index of the model, between .57 and .68, somewhat higher than .53, the Gini index in the PSID (See Table 4). However, when we use positive earnings only, the Gini indices from the models are .39 (benchmark), .47 (σ_x=.28125), and .29 (ρ_x=.92), comparable to .42 in the PSID.

Table 5 summarizes the detailed information on wealth and earnings from the SCF, PSID, and benchmark model.[15] Since the wealth-earnings distributions between the PSID and SCF are similar, we discuss the comparison between the model and PSID only. For each quintile group of wealth distribution, we calculate the wealth share, the ratio of group average to economy-wide

[14] In the PSID, information on family wealth is available for 1984, 1989, and 1996 survey years. We use the 1984 survey because the date falls in the mid point of our sample period. The degree of inequality does not vary significantly across the three surveys.

[15] In terms of Gini indices, the wealth and earnings distributions from the PSID are slightly less concentrated than those in the SCF. According to Díaz-Giménez, Quadrini, and Ríos-Rull, Gini indices are .78 and .63 for wealth and earnings, respectively, in the 1992 SCF.

average, and the earnings share. Both in the data and model, the poorest 20 percent of the wealth distribution owns almost nothing. In fact, households in the first quintile of the wealth distribution are in debt both in the model and data. On the contrary, households in the 4th and 5th quintile of the PSID own 18.74 and 76.22 percent of total wealth, respectively. According to the model, they own 24.48 and 66.31 percent, respectively. The average wealth of the 4th and 5th quintile are, respectively, .93 and 3.81 times larger than that of a typical household, while these ratios are 1.22 and 3.33 according to our model. The 4th and 5th quintile group of the wealth distribution earn, respectively, 24.21 and 38.23 percent of total earnings in the PSID. The corresponding groups earn 23.44 and 30.39 percent, respectively, in the model. Overall, the wealth distribution is more skewed in the data. In particular, the model fails to match the highly concentrated wealth in the right tail of the distribution. About half of total wealth—43 and 53 percent in the PSID and SCF, respectively—is held by the top 5 percent of the population (not shown in the Table). In our model, only 20 percent of total wealth is held by them. However, our primary objective is not to explain the behavior of the top 1 or 5 percent of the population.[16] We argue that the model economy possesses a reasonable heterogeneity—especially for the 2nd and 3rd quintile of the distribution, probably the most relevant group for the business cycle fluctuations—to study the average response of hours, as the stochastic process of productivity is estimated from the panel data, and the cross-sectional earnings distribution are, by and large, consistent with the data counterparts.

The shape of reservation wage distribution is crucial for the mapping from individual to aggregate labor supply. In Figure 3, we plot the reservation wage schedule of the benchmark model for all asset levels (Panel A) and for assets less than \$200,000 (Panel B). At a given asset level, workers

[16] As is well known, accounting simultaneously for the earnings and wealth in the U.S. economy is no easy task given the extreme wealth concentration observed in the data. For studies on the wealth distribution in a dynamic general equilibrium environment, see among others Huggett (1996), Krusell and Smith (1998), Quadrini (2000), or Castañeda, Ana, Javier Díaz-Giménez, and José-Víctor Ríos-Rull (2000).

with the wage (productivity) above the line choose to work. The reservation wage increases as the asset level increases. To illustrate, we adjust the units such that the mean asset of the model matches the average asset in the 1984 PSID survey, \$60,524.[17] Thus, the values are in 1983 dollars. Consider a worker whose assets are \$29,234, the median of the wealth distribution from the model. According to the model, he is indifferent between working and not working at quarterly earnings of \$3,814. A worker whose assets are equivalent to the average asset holding of the economy, \$60,524 (which belongs to the 66th percentile of the wealth distribution in our model and to the 72th percentile in the PSID) is indifferent in working at \$4,871 per quarter.

Based on the reservation wage schedule and invariant distribution $\mu(x, a)$, we infer the elasticity of aggregate labor supply. In Figure 4 we plot the inverse cumulative distribution of reservation wages for three model economies. In practice, the reservation wage distribution is neither observed nor constant over time. In Table 6 we compute the elasticities of employment with respect to the reservation wage around the steady state employment rate of 60 percent. These values may be viewed as upper bounds for aggregate employment response as they assume that the *entire* wealth distribution is held constant. For the benchmark case, the elasticities are 1.19, 1.09, and 1.0, respectively, at the employment rates of 58, 60, and 62 percent. The elasticities are somewhat smaller with a bigger heterogeneity ($\sigma_x = .28125$) as the reservation wage distribution is more dispersed. With a lower persistence ($\rho_x = .92$) – which generates a wage process similar to those in the PSID – the elasticities are 1.88, 1.78, and 1.61, respectively, at the employment rate of 58, 60 and 62 percent. Although these values are bigger than typical micro estimates, they remain at moderate range. In particular, a very high elasticity – in fact, infinity – generated by a lottery economy with a homogeneous workforce examined by Hansen and Rogerson, does not survive a serious heterogeneity.

[17]The mean asset in our model is 12.63 units. The reservation wages in the vertical axis reflect quarterly earnings (the reservation wage rate multiplied by $\bar{h}$).

4.2 Fluctuations

While the labor supply elasticity plays an important role in many issues such as timing of taxes and government spending (Auerbach and Kotlikoff, 1987; Judd, 1987), it is one of the most extensively debated parameter in business cycle literature. In this section, we examine the fluctuations of the model economy in the presence of exogenous shifts in aggregate productivity. We do not take a stand on the sources of the business cycle here. However, we intentionally exclude other types of aggregate disturbances, especially those that shift the labor supply curve, as we are interested in the response of labor supply. Aggregate productivity shocks serve as an instrument, exogenously shifting the labor-demand curve, to identify the response of labor supply.

Computing the equilibrium fluctuations of an economy of this sort requires a considerable degree of approximation. We use the so-called "bounded rationality method" developed by Krusell and Smith, in which agents are assumed to make use of a finite set of moments of the distribution μ. The justification of this method is that by using partial information about μ, households do almost as well as by using all the information in μ when predicting future prices. In fact, Krusell and Smith show that use of the first moment provides a good approximation in a stochastic-growth model. Gomes et al. and Castañeda et al. have shown that this method can be applied to various economic environments. The procedure used in this section closely follows those suggested in these works. The details of computation are provided in Appendix A.4.

We provide a comparison with the representative-agent (with divisible labor) model in terms of second moments of key aggregate variables. A common way to characterize the behavior of labor supply is using a Frisch function (Frisch, 1959).[18] For a representative agent-economy with the same type of utility function as ours, the Frisch labor supply function linearized around the steady

[18] See Heckman (1974), MaCurdy (1982) or McLaughlin (1995) for relationships among various measures of labor supply elasticities.

state is

$$\widehat{h}_t = \psi(\widehat{w}_t - \widehat{c}_t), \qquad \psi = \frac{1-\bar{h}}{\bar{h}}\gamma, \tag{12}$$

where the circumflex denotes the variable's percentage deviation from its steady state value. The compensated labor supply elasticity (Frisch elasticity) ψ represents the elasticity of hours with respect to wages, holding consumption (or wealth) constant. We consider four representative economies with the compensated labor supply elasticity, ψ, equaling to .4, 1, 2, and 4. With $\bar{h} = 1/3$, these values correspond to γ of .2, .5, 1, and 2, respectively. In reference to the real-business cycle analysis, Prescott (1986) corresponds to $\psi = 2$, King, Plosser and Rebelo (1988) to $\psi = 4$, and Hansen (1985) to $\psi = \infty$. The representative-agent economies have the same parameter values as our benchmark model summarized in Table 2 except for the intertemporal substitution elasticity γ and B.[19]

Table 7 displays the statistics of five model economies (our benchmark economy and four representative-agent economies) and the U.S economy. The upshot is that the response is similar to those of the representative-agent economies with $\psi = 2$. The volatility of output of our benchmark economy is 1.50, slightly smaller than that of the economy with $\psi = 2$ (1.53). The volatility of consumption is bigger in our model, but the investment volatility is close to that with $\psi = 2$. Hours are fairly volatile: the standard deviation of hours both in its absolute term and relative to output and labor productivity are close to those of the representative agent model with ψ of 4. This is partially due to a compositional bias. Typically new workers are less productive than existing workers, making employment more volatile than total hours in efficiency units. In our model, the volatility of hours in efficiency unit is .78, 30% smaller than that of employment, comparable to previous models with a complete market (Cho and Rogerson, 1988; Chang, 2000). Likewise, the standard deviation of marginal product of labor is .88, 15% bigger than the average labor

[19]For each representative-agent economy, the parameter B is adjusted to yield $\bar{h} = 1/3$ in the steady state.

productivity (.77) comparable to Bils (1985).

The response of aggregate hours to shifts in demand is moderate as the reservation wage distribution is scattered. For example, the dispersion of individual productivity, measured by the cross-sectional standard deviation of log wages in the PSID (.549), is larger than that of aggregate productivity, measured by the time-series standard deviation of aggregate TFP (.0224) by a factor of nearly 23.[20]

Due to incomplete financial market and borrowing constraint, the aggregation theorem cannot be applied in our model. However, it still might be of interest to estimate the equation (12) as if the aggregate time series were generated by a representative-agent model. When we use 3,000 periods of aggregate time series from our benchmark economy, the estimate for ψ is 1.03 by the OLS.[21] The estimates are .86 and 1.13, respectively, for the models with $\sigma_x = .28125$ and $\rho_x = .92$.[22] These numbers are not far from those we obtained based on the reservation wage distribution. For comparison to micro labor supply elasticities, we also construct annual panel data from our model and estimate the ψ for individual workers. Since we have an extensive margin only, our estimates are approximations relying on time aggregation: the variation of annual hours stems from the changes in quarterly labor-market participation. The OLS estimate based on the panel data consisting of 10,000 workers for 30 years from the benchmark model is .37. Table 8 summarizes the estimate of ψ for individual and aggregate labor supply for each model.[23] The aggregate elasticities tend to be

[20] We abstract from the variation of hours per worker to isolate the effect of participation margin only. Allowing for an intensive margin may generate a bigger response of labor supply. However, under the small intertemporal substitution elasticity of leisure, the effect on aggregate labor supply would be small.

[21] In general, estimation of labor supply using aggregate time series data suffers an identification problem due to difficulty in finding a good instrument. Hall (1980) estimates the aggregate labor supply elasticity with instruments such as military spending, political party of the president, and oil prices.

[22] R^2's of the regressions are between 0.33 and 0.40.

[23] With a discrete choice of labor supply, R^2's of panel data regressions are very low. They are between 0.03 and

bigger than individual elasticities, which is in line with typical micro estimates in the literature.

5 Conclusion

Labor supply elasticity is at the heart of macroeconomic research. It is a cornerstone of the equilibrium approach that relies on intertemporal substitution of leisure. In a disequilibrium approach, in which the role of labor supply is dismissed in the short run, its slope is still crucial for the welfare loss of the economy departing from the equilibrium.

Aggregate models based on the intertemporal substitution of leisure often assume a high aggregate labor supply elasticity, despite the low estimates from empirical studies based on individual data. The fact that fluctuations of hours are mainly accounted for by participation suggests that the aggregate labor supply has little to with the intertemporal substitution, but rather with the distribution of reservation wages among heterogeneous workers.

We investigate the mapping from individual to aggregate labor supply using a general equilibrium where heterogeneous agents decide on labor-market participation and the capital market is incomplete. The nature of heterogeneity among workers is calibrated using panel data on individual wages. Worker flows between employment and nonemployment, and cross-sectional distributions of earnings and wealth in our model are comparable to those in the U.S. data. While the model economy is parsimonious, we find that the aggregate labor supply elasticity of such an economy is around 1, bigger than micro estimates but smaller than those often assumed in aggregate models. As the model abstracts from other important factors affecting labor supply decisions, it would be interesting to incorporate preference heterogeneity (e.g., life-cycle effect or home production), partial insurance (joint labor supply of the married), or returns to working other than current wages (learning by doing) into the model.

0.08 whereas t-statistics are well over 200.

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A Appendix

A.1 The PSID Data

We use the non-poverty sample of heads of households and spouses for 1971-1992. The wage data for spouses are available only since 1979. Wages are annual hourly earnings (annual labor incomes divided by annual hours). Nominal wages are deflated by the Consumer Price Index. The base year is 1983. Workers who worked less than 100 hours per year or whose hourly wage rate was below $1 (in 1983 dollars) are viewed as nonemployed even though their employment status is reported as employed in the survey. We use workers who were employed in non-agricultural sectors and were not self-employed. We also restrict the sample to hourly earnings less than or equal to $500. In the PSID, the wealth data are available for 1984, 1989 and 1996 only. We use the 1984 data as the date falls around the mid point of our sample. The distributions are similar across the three surveys. The wealth is defined as the sum of net worth of all family members resulting from the aggregation of the following components: house (main home), other real estate, vehicles, farms and businesses, stocks, bonds, cash accounts, and other assets. Family earnings is the sum of earnings of head and spouses. The descriptive statistics for our PSID data are summarized in Table A.1.

Table A.1: Summary Statistics for the PSID Data

Variable	Mean	S.D.	Min	Max	Obs.
Real Wage Rate (in 1983 $)	10.96	6.81	1.00	175.19	44,717
Log Real Wage	2.24	.549	.00	5.16	44,717
Annual Hours of work	2062.1	551.7	100	5,000	44,717
Age	38.2	11.17	18	65	44,717
Years of schooling	13.03	2.51	0	17	44,717
Gender (male =1)	.615	.48	0	1	44,717
Family Wealth	60,524	231,103	-134,556	9,560,000	5,515
Family Earnings	17,485	19,181	0	530,000	5,515

Note: Statistics are based on 1971-1992 surveys. Family wealth and earnings are based on those reported in the 1984 survey.

A.2 Conversion between Annual and Quarterly Variances

After controlling for aggregate effect, the individual wage evolves according to x. Since the wages in the PSID are annual averages:[24]

$$\ln \widetilde{x}_\tau = \frac{1}{4}\Big\{ \ln x_{(\tau,1)} + \ln x_{(\tau,2)} + \ln x_{(\tau,3)} + \ln x_{(\tau,4)} \Big\},$$

where $\widetilde{x}_\tau$ is annual average and $x_{(\tau,q)}$ denotes the wage of the qth quarter in year τ. With AR(1) process for quarterly x, the stochastic process for the annual average is:

$$\ln \widetilde{x}_\tau = \rho^4 \ln \widetilde{x}_{\tau-1} + \widetilde{\varepsilon}_{x,\tau}, \quad \widetilde{\varepsilon}_{x,\tau} = \frac{1}{4}\sum_{j=1}^{4}\left\{ \varepsilon_{x,(\tau,j)} \sum_{k=0}^{4-j} \rho^k \right\}.$$

The quarterly values of ρ_x and σ_x are computed from the annual estimates using $\widetilde{\rho}_x = \rho_x^4$ and $\widetilde{\sigma}_x = \sigma_x \frac{1}{4}\sqrt{\sum_{j=1}^{4}\left\{\sum_{k=0}^{4-j}\rho^k\right\}^2}$.

A.3 The Worker-Flow Data

We compute the quarterly worker flows from the seasonally adjusted monthly hazard rates in the CPS for 1967:II-2000:IV, obtained from Robert Shimer, as follows. There are three possible labor-market states: employment, unemployment, and non-labor-force, denoted by e, u, and n, respectively. The flow of workers from labor-market status i to j during the quarter f_{ij} is computed as:

$$f_{ij} = \bar{i} \times \left\{ \sum_{k,l \in \{e,u,n\}} h^1_{ik} h^2_{kl} h^3_{lj}, \right\} \quad i, j \in \{e, u, n\},$$

where $\bar{i}$ denotes the number of workers in status i in the beginning of the quarter, and h^m_{kl} is the monthly hazard rate from status k to l in the m-th month of the quarter. This takes into account all possible paths, direct and indirect, from i to j during a quarter. This also avoids a potential double counting in a simple sum of monthly flows. Because of survey redesigns and privacy restrictions, hazard rates are not available in January 1976, January 1978, July 1985, October 1985, January

[24]Note that $\frac{1}{4}\sum_{q=1}^{4}\ln x_{(\tau,q)}$ can be interpreted as a log-linear approximation of the arithmetic average $\ln \widetilde{x}_\tau = \ln[\frac{1}{4}\sum_{q=1}^{4} x_{(\tau,q)}]$.

1994, and June to October 1995. For these months we interpolate the values from the nearby periods.

A.4 Computational Procedures

A.4.1 Steady State Equilibrium

The distribution of workers $\mu(x, a)$ as well as factor prices are invariant in the steady state. In finding the invariant μ, we use the algorithm suggested by Ríos-Rull (1999). We search for the discount factor β that clears the capital market given the quarterly rate of return of 1 percent. Computing the steady state equilibrium amounts to finding the value functions, the associated decision rules, and time-invariant measure of workers. Details are as follows:

1. First, we choose the grid points for x and a. The number of grids are denoted by N_x and N_a. For the benchmark model, $N_x = 17$ and $N_a = 1,163$. The asset holding a_i is in the range of $[0, 250]$, where the average asset holding is 12.63. The grid points of assets are not equally spaced. We assign more points on the lower asset range to better approximate savings decisions of workers with lower assets. For example, at the asset range close to the borrowing constraint, the grid points are as fine as .02, which is approximately 2.5% of the average quarterly labor income (these individuals have negligible interest income); at a high end of the asset range, the grid increases by .4, which corresponds to $10-20$ percent of the average quarterly total income (these individuals have larger interest income than labor income). For productivity, x_j, we construct grid vectors of N_x equally spaced points in which $\ln x_j$'s lie on the range of $\pm 3\sigma_x/\sqrt{1-\rho_x^2}$.

2. Given β, we solve the individual optimization problem in (1), (2), and (3) at each grid point of the individual states. In this step, we also obtain the optimal decision rules for asset holding $a'(a_i, x_j)$ and labor supply $h(a_i, x_j)$. This step involves the following procedure:

(a) Initialize value functions $V_0^E(a_i, x_j)$, $V_0^N(a_i, x_j)$, and $V_0(a_i, x_j)$.

(b) Update value functions by evaluating the discretized versions of (1), (2), and (3):

$$\begin{aligned} V_1^E(a_i, x_j) = \max \Big\{ & u\big(w\bar{h}x_j + (1+r)a_i - a', 1 - \bar{h}\big) \\ & + \beta \sum_{j'=1}^{N_x} \max \Big[V_0^E(a', x_{j'}), V_0^N(a', x_{j'}) \Big] \pi_x(x_{j'}|x_j) \Big\}, \end{aligned} \tag{A.4.1}$$

$$\begin{aligned} V_1^N(a_i, x_j) = \max \Big\{ & u\big((1+r)a_i - a', 1\big) \\ & + \beta \sum_{j'=1}^{N_x} \max \Big[V_0^E(a', x_{j'}), V_0^N(a', x_{j'}) \Big] \pi_x(x_{j'}|x_j) \Big\}, \end{aligned} \tag{A.4.2}$$

and

$$V_1(a_i, x_j) = \max \Big\{ V_1^E(a_i, x_j), V_1^N(a_i, x_j) \Big\}, \tag{A.4.3}$$

where $\pi_x(x_{j'}|x_j)$ is the transition probabilities of x, which is approximated using Tauchen's (1986) algorithm.

(c) If V_1 and V_0 are close enough for all grid points, then we found the value functions. Otherwise, set $V_0^E = V_1^E$ and $V_0^N = V_1^N$, and go back to step 2-(b).

3. Using $a'(a_i, x_j)$, $\pi_x(x_{j'}|x_j)$ obtained from step 2, we obtain time-invariant measures $\mu^*(a_i, x_j)$ as follows:

(a) Initialize the measure $\mu_0(a_i, x_j)$.

(b) Update the measure by evaluating the discretized version of (9):

$$\mu_1(a_{i'}, x_{j'}) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \mathbf{1}_{a_{i'} = a'(a_i, x_j)} \mu_0(a_i, x_j) \pi_x(x_{j'}|x_j) \tag{A.4.4}$$

(c) If μ_1 and μ_0 are close enough for all grid points, then we found the time-invariant measure. Otherwise, replace μ_0 with μ_1, and go back to step 3(b).

4. We calculate the real interest rate as a function of β, i.e., $r(\beta) = \alpha\big(K(\beta)/L(\beta)\big)^{1-\alpha} - \delta$, where $K(\beta) = \sum_{i=1}^{N_a}\sum_{j=1}^{N_x} a_i \mu^*(a_i, x_j)$ and $L(\beta) = \sum_{i=1}^{N_a}\sum_{j=1}^{N_x} x_j h(a_i, x_j)\mu^*(a_i, x_j)$. Other aggregate variables of interest are calculated using μ^* and decision rules. If $r(\beta)$ is close enough to the assumed value of the real interest rate, we find the steady state. Otherwise, we choose a new β and go back to step 2.

A.4.2 Equilibrium with Aggregate Fluctuations

Approximating the equilibrium in the presence of aggregate fluctuations requires us (i) to include the measure of workers and the aggregate productivity shock in the list of state variables, and (ii) to keep track of the evolution of the measure μ over time. Since μ is an infinite dimensional object, it is almost impossible to implement these tasks as they are. We follow the procedure suggested by Krusell and Smith (1998); agents are assumed to make use of its first moment only in predicting the law of motion for μ. Therefore, computing the equilibrium with aggregate fluctuations amounts to finding the value functions, decision rules, and law of motion for the aggregate capital within the class of log-linear functions in K and λ. The same method is used in Gomes et al. in their analysis on equilibrium unemployment rates. Details are as follows:

1. In addition to the grids for individual state variables specified above, we choose 11 grid points for the aggregate capital K in the range of $[.9K^*, 1.1K^*]$, where K^* denotes the steady state aggregate capital. In our numerous simulations, the capital stock has never reached the upper or lower bound. The aggregate productivity λ has 9 grid points and its transition probability $\pi_\lambda(\lambda'|\lambda)$ is calculated using Tauchen (1986)'s algorithm.

2. Let the parametric law of motion for the aggregate capital take a log linear in K and λ:

$$\ln K_{t+1} = \kappa_0^0 + \kappa_1^0 \ln K_t + \kappa_2^0 \ln \lambda_t. \tag{A.4.5}$$

   In order for individuals to make their decisions on savings and labor supply they have to know (or predict) the interest rate and wage rate for an effective unit of labor. While the factor prices depend on aggregate capital and labor, aggregate labor input is not known to individuals at the moment when they make decisions. Thus, individuals need to predict the factor prices. These predictions on factor prices, in turn, must be consistent with the outcome of individual actions, the factor market clearing in (7) and (8). We also assume that individuals predict the market wage and the interest rate using a log-linear function of K and λ:

$$\ln w_t = b_0^0 + b_1^0 \ln K_t + b_2^0 \ln \lambda_t. \tag{A.4.6}$$

and

$$\ln (r_t + \delta) = d_0^0 + d_1^0 \ln K_t + d_2^0 \ln \lambda_t \tag{A.4.7}$$

3. We choose the initial values for the coefficients κ^0's, b^0's and d^0's. Good initial values may come from a representative-agent model.

4. Given the law of motion for the aggregate capital and the prediction functions for factor prices, we solve the individual optimization problem in (1), (2), and (3). This step is analogous to step 2 in the steady state computation:

   (a) We have to solve for the value functions and the decision rules over a bigger state space. Now the state variables are (a, x, K, λ).

   (b) Computation of the conditional expectation involves the evaluation of the value functions not on the grid points along the K dimension since K' predicted by (A.4.5) need not be a grid point. We polynomial-interpolate the value functions along K dimension when necessary.

5. Using $a'(a_i, x_j, K_l, \lambda_m)$, $h(a_i, x_j, K_l, \lambda_m)$, $\pi_x(x_{j'}|x_j)$, $\pi_\lambda(\lambda_{m'}|\lambda_m)$, and the assumed law of motion for the aggregate capital, we generate a set of artificial time series data $\{K_t, w_t, r_t\}$

of the length of 3,000 periods. Each period, $\{K_t, w_t, r_t\}$ is calculated by aggregating labor supply and assets of 50,000 individuals.

6. We obtain new values for coefficients κ^1's, b^1's and d^1's by the OLS from the simulated data. If κ^1's, b^1's and d^1's are close enough to κ^0's, b^0's, and d^0's, respectively, we find the law of motion. Otherwise, we update coefficients by setting $\kappa^0 = \kappa^1$, $b^0 = b^1$'s and $d^0 = d^1$'s, and go back to step 4.

   The estimated law of motion for capital and prediction functions and their accuracy, measured by R^2 for the prediction equations are as follows.

   - the law of motion for aggregate capital in equation (A.4.5):

   $$\ln K_{t+1} = .1247 + .9508 \ln K_t + .0997 \ln \lambda_t, \qquad R^2 = .9999$$

   - the market wage rate in equation (A.4.6):

   $$\ln w_t = -.2621 + .4442 \ln K_t + .8068 \ln \lambda_t, \qquad R^2 = .9940$$

   - the interest rate in equation (A.4.7):

   $$\ln (r_t + \delta) = -1.3491 - .7897 \ln K_t + 1.3434 \ln \lambda_t, \qquad R^2 = .9691$$

   The law of motion for aggregate capital provides the highest accuracy. The wage function is more accurate than the interest rate function. Overall, predictions functions are fairly precise as R^2's are close to 1. Finally, as the agents make decisions based on the predicted prices, the actual employment may not be necessarily consistent with the predicted prices. We also used the method suggested in Ríos-Rull in which labor market clearing is imposed as an extra step. (See Ríos-Rull (1999) for details.) The result with a two-step process was very similar to the one reported here as the predicted prices approximate the actual prices very closely.

Table 1: Estimates of Stochastic Process for Idiosyncratic Shocks

	Annual Estimates $\widetilde{\rho}_x$	Annual Estimates $\widetilde{\sigma}_x$	Quarterly Values ρ_x	Quarterly Values σ_x	obs
All	.818 (.002)	.291	.951	.225	47,114
All - with gender dummy	.792 (.002)	.288	.944	.225	47,114
Male					
Married	.809 (.003)	.265	.948	.206	24,294
Single	.760 (.010)	.328	.934	.259	3,265
Female					
Married	.722 (.006)	.311	.922	.249	11,684
Single	.773 (.008)	.298	.938	.234	5,525
With individual characteristics					
All	.743 (.003)	.284	.928	.225	47,114
Male					
Married	.761 (.004)	.261	.934	.206	24,294
Single	.760 (.011)	.321	.915	.259	3,265
Female					
Married	.717 (.006)	.311	.920	.249	11,684
Single	.708 (.009)	.291	.917	.234	5,525

Note: The annual estimates are based on OLS of equation (11) using the wage data from the PSID 1971-1992. Numbers in parenthesis are standard errors. The corresponding quarterly values are calculated as described in the Appendix. For the second set of estimates, individual characteristics are controlled by including gender, age, age square, and years of schooling in the regression.

Table 2: Parameters of the Benchmark Economy

Parameter	Description
$\alpha = .64$	Labor share in production function
$\beta = .979852$	Discount factor
$\gamma = .2$	Intertemporal substitution elasticity of leisure
$B = 1.025$	Utility parameter
$\overline{h} = 1/3$	Steady state hour
$\rho_x = .95$	Persistence of idiosyncratic productivity shock
$\sigma_x = .225$	Standard deviation of innovation to idiosyncratic productivity
$\bar{a} = -2.0$	Borrowing constraint

Table 3: Labor-Market Steady States

Variable	CPS	Model		
		$\sigma_x = .225$ $\rho_x = .95$	$\sigma_x = .28125$ $\rho_x = .95$	$\sigma_x = .225$ $\rho_x = .92$
Employment rate	60.15	60.21	60.20	60.21
Flow out of employment	7.07	5.92	5.64	6.85
Flow into employment	6.88	5.92	5.64	6.85
Hazard rate out of nonemployment	17.75	14.89	14.16	17.22
Hazard rate out of employment	11.80	9.84	9.36	11.38

Note: All variables are in percentage. The CPS statistics are quarterly averages for 1967:II-2000:IV as described in Appendix A.3. σ_x and ρ_x denote, respectively, the standard deviation of innovations and persistence of idiosyncratic productivity shocks.

Table 4: Gini Indices for Wealth and Earnings

Variable	PSID	Models		
		$\sigma_x = .225$ $\rho_x = 95$	$\sigma_x = .28125$ $\rho_x = .95$	$\sigma_x = .225$ $\rho_x = .92$
Wealth	.76	.64	.65	58
Earnings	.53	.63	.68	.57
Earnings (non-zeros)	.42	.39	.47	.29

Note: The PSID statistics reflect the family wealth and earnings in the 1984 survey.

Table 5: Characteristics of Wealth Distribution

	Quintile 1st	2nd	3rd	4th	5th	Total
SCF						
Share of wealth	-.39	1.74	5.72	13.43	79.49	100
Group average/population average	-.02	.09	.29	.67	3.97	1
Share of earnings	7.05	14.50	16.48	20.76	41.21	100
PSID						
Share of wealth	-.52	.50	5.06	18.74	76.22	100
Group average/population average	-.02	.03	.25	.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
Benchmark Model						
Share of wealth	-2.68	1.78	10.11	24.48	66.31	100
Group average/population average	-.13	.09	.51	1.22	3.33	1
Share of earnings	10.79	15.94	19.43	23.44	30.39	100

Note: The SCF statistics are from Díaz-Giménez, Quadrini, and Ríos-Rull (1997). The PSID statistics reflect the family wealth and earnings from the 1984 survey.

Table 6: Implied Elasticity from the Reservation Wage Distribution

Model	Employment Rate $E = 58\%$	$E = 60\%$	$E = 62\%$
$\sigma_x = .225,\ \rho_x = .95$	1.19	1.09	1.00
$\sigma_x = .28125,\ \rho_x = .95$	.96	.89	.82
$\sigma_x = .225,\ \rho_x = .92$	1.88	1.78	1.61

Note: The numbers reflect the elasticity of labor-market participation rate with respect to reservation wage (evaluated at employment rates of 58, 60, and 62 percent) based on the reservation wage distribution in the steady state.

Table 7: Comparison with Representative-Agent Economies

	Benchmark	Divisible Labor				U.S. Data
		$\psi = .4$	$\psi = 1$	$\psi = 2$	$\psi = 4$	1948:I-2000:IV
$\sigma(Y)$	1.50	1.22	1.38	1.54	1.71	2.22
$\sigma(C)$	.57	.41	.45	.49	.52	.96
$\sigma(I)$	4.61	3.72	4.26	4.81	5.39	4.67
$\sigma(N)$	1.00	.25	.50	.75	1.01	1.78
$\sigma(N)/\sigma(Y)$	.67	.20	.36	.49	.59	.80
$\sigma(N)/\sigma(Y/N)$	1.31	.23	.55	.91	1.37	1.61

Note: All variables are de-trended by the H-P filter. "Divisible Labor" denotes the representative-agent model with different Frisch labor supply elasticities (ψ). The statistics for data are based on per capita values (divided by civilian noninstitutional population over 16) from the Citibase: Y = nonfarm business GDP (GPBUQF); C = consumption of non durables and services (GCNQ+GCSQ); I = non-residential fixed private investment (GIFQ); N= total employed hours in private non-agricultural sector based on the establishment survey (LPMHU).

Table 8: Compensated Labor Supply Elasticities From the Model-Generated Data

Model	Individual	Aggregate
$\sigma_x = .225, \ \rho_x = .95$	.37	1.03
$\sigma_x = .28125, \ \rho_x = .95$	.44	.86
$\sigma_x = .225, \ \rho_x = .92$	.69	1.13

Note: All estimates are based on the OLS of equation (12) using model-generated data. The individual labor supply elasticities are based on the annual panel data of 10,000 workers for 30 years. The aggregate estimates are based on the quarterly time series of 3,000 periods.

Figure 1: Lorenz Curves for Wealth

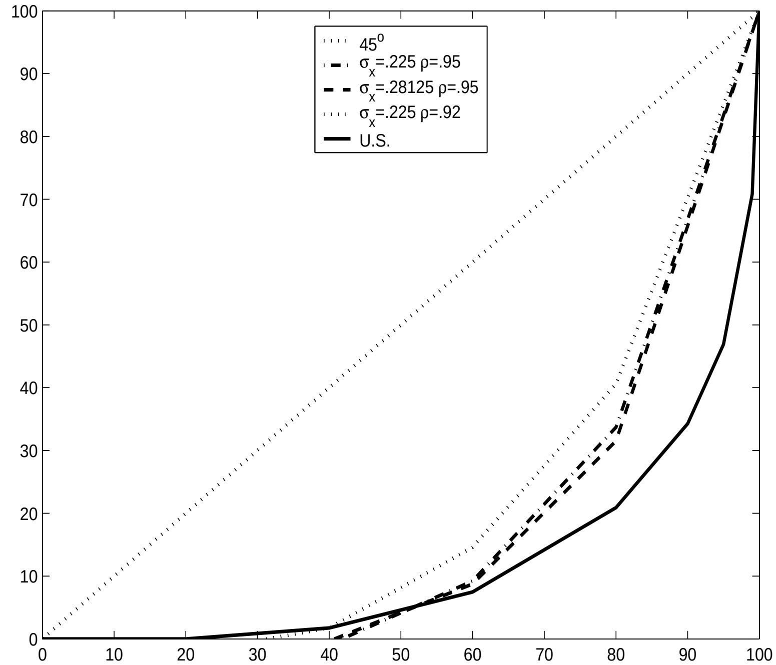


Figure 2: Lorenz Curves for Earnings

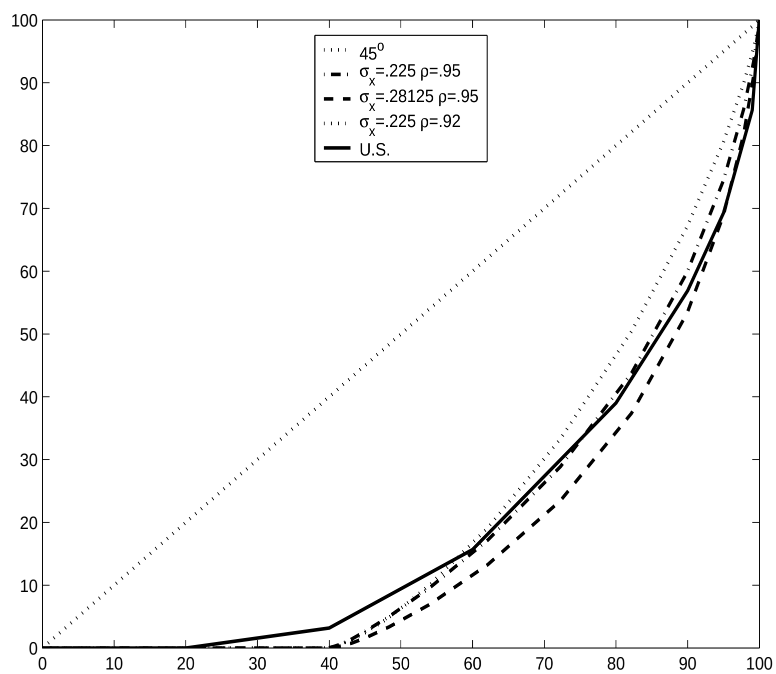
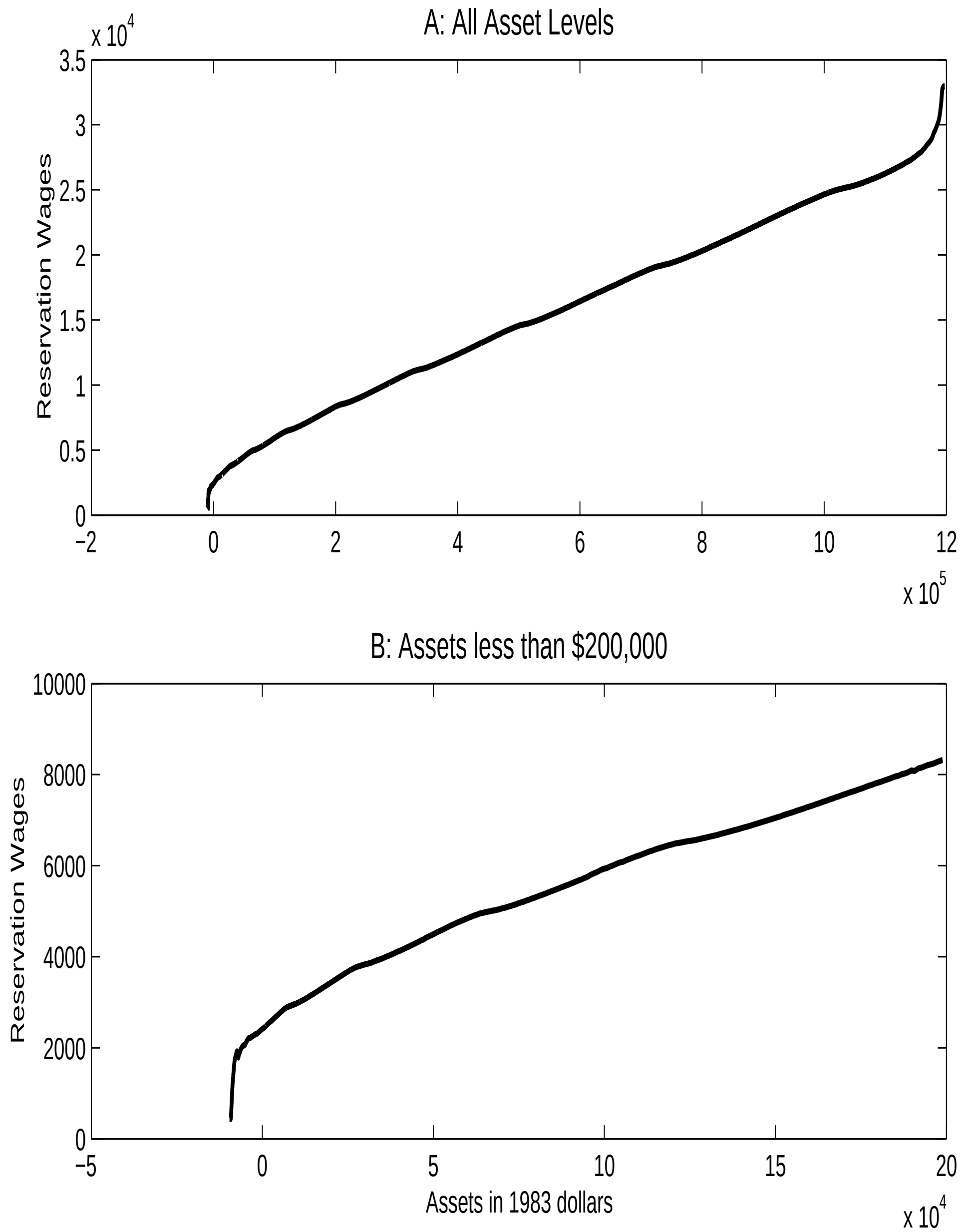
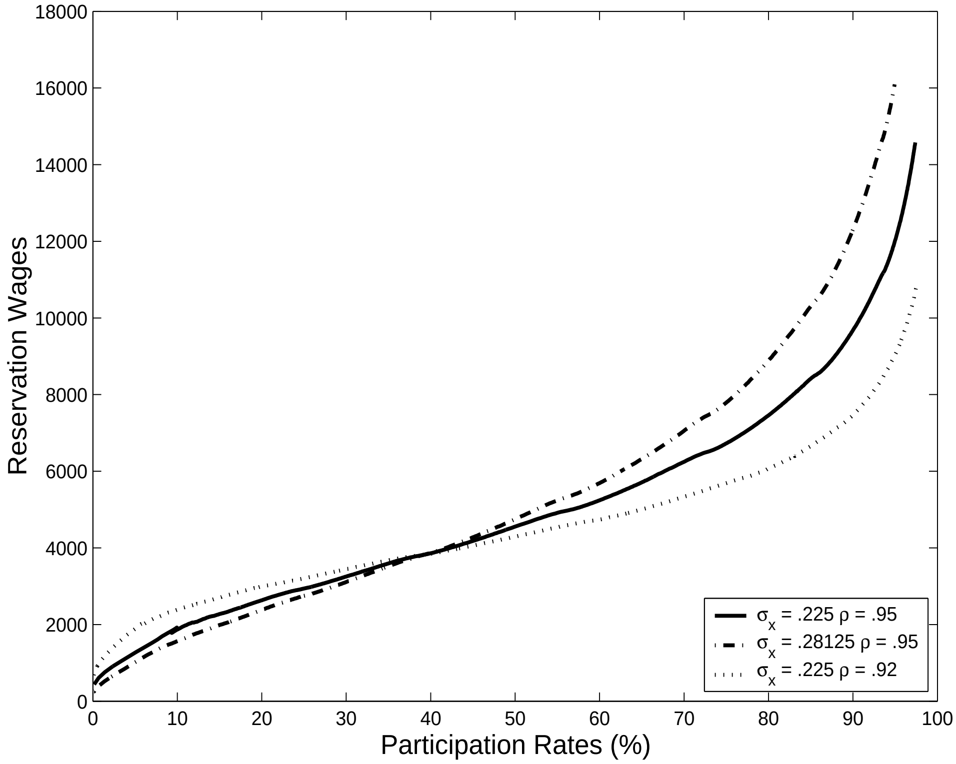


Figure 3: Reservation Wage Schedule



Note: The graph denotes the reservation wage schedule of the benchmark model. Wages (quarterly earnings) and assets are in 1983 dollars.

Figure 4: Reservation Wages and Participation Rates



Note: The graph denotes the inverse cumulative distribution functions of reservation wages. Wages are quarterly earnings in 1983 dollars.