

## **From Simplistic to Complex Systems in Economics**

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### **Abstract**

*The applicability of complex systems theory in economics is evaluated and compared with standard approaches to economic theorizing based upon constrained optimization. A complex system is defined in the economic context and differentiated from complex systems in physio-chemical and biological settings. It is explained why it is necessary to approach economic analysis from a network, rather than a production and utility function perspective, when we are dealing with complex systems. It is argued that much of heterodox thought, particularly in neo-Schumpeterian and neo-Austrian evolutionary economics, can be placed within a complex systems perspective upon the economy. The challenge is to replace prevailing 'simplistic' theories, based in constrained optimization, with 'simple' theories, derived from network representations in which value is created through the establishment of new connections between elements.*

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## INTRODUCTION

Over the past two decades, a new way of thinking about systems has come to prominence, principally in the natural sciences. Although all the usual physical laws are obeyed, many systems are now viewed as ‘complex’ and, as such, cannot be very well understood using standard approaches to theorizing and modeling. In broad terms, such systems are self-organized structures that absorb and dissipate energy and, despite their apparent complicatedness, can often obey some quite simple behavioural rules in time and space. However, this behaviour cannot be captured by theories that assume away the existence of historical time and, thus, ignore the actual processes that unfold within and beyond complex systems. For example, it can be argued that the outcomes of these historical processes cannot be viewed as the solutions of constrained dynamic optimization problems. This raises some fundamental questions for economists, given that all interesting economic systems and their components can be classified as complex systems.

The acknowledgement that systems are complex has led to a multi-disciplinary literature in which a range of attempts have been made to understand complex systems behaviour through the use of evolutionary computation and artificially intelligent agent models.<sup>1</sup> Contributors to this literature have developed a ‘meta-mathematics’ that can be used to generate models with evolutionary properties, ie a capacity to generate ‘surprises’ (Casti (1994)). However, although these mathematical developments seem to be fundamentally important, they tend to be only loosely connected with less formal ideas and insights in evolutionary and institutional economics that have been around for decades. The goal here will not be to focus upon this meta-mathematical literature but, instead, to offer a view of what a complex system is, specifically in an economic setting, and to connect this view with the rich body of literature in modern evolutionary economics. This involves an emphasis upon gaining an understanding of the structural, or architectural (Simon (1962)), characteristics of economic systems, and how these can be represented analytically, rather than exploring the dynamic paths that quantitative representations of evolutionary economic processes can follow.

<sup>1</sup> Markose (2004) provides a good overview of this literature as it has developed from the seminal contributions of Alan Turing and John von Neumann.

In Section 1, the term ‘complex system’ is defined and the main elements of complex systems theory in an economic setting are presented. In Section 2 it is explained why constrained optimization, which lies at the very foundation of modern economics, is a ‘simplistic system’ conception that cannot, by definition, relate to actual historical events. In complex systems, constrained optimization does occur but it is not the basic driver of economic processes – this is the subject matter of Section 3, in which ‘subjective’ and ‘process’ optimisation are distinguished. In Section 4 it is explained why the relevant spatial construct in analysing complex systems is the network and this is discussed in relation to the conventional concept of the production function. Section 5 is devoted to the discussion of scale free networks and the power laws that can be used to represent them analytically. Section 6 contains some concluding remarks.

## **1. ELEMENTS OF COMPLEX SYSTEMS THEORY IN ECONOMIC SETTINGS**

Complex systems are, at base, dissipative structures that import free energy and export entropy in a way that enables them to self-organise their structural content and configuration, subject to boundary limits. They maintain their boundaries but, at the same time, they are open systems that are irrevocably connected to an environment that contains other systems that can be complementary, competitive, combative, predatory or available as prey. Systems that absorb information from their environment and create stores of knowledge that can aid action are often called ‘complex adaptive systems’. Levin (2003) defines such systems in terms of three properties: (1) diversity and individuality of components, (2) localized interactions among these components and (3) an autonomous process that uses outcomes of those interactions to select a subset of those components for replication or enhancement. However, this definition was constructed with ecological systems in mind. It is contended here that, although physical, chemical, biological, social and economic systems that exhibit ‘organized complexity’ all share common properties, they differ in important ways. How we define a complex system will depend upon the kind of system we are interested in.

What do we mean when we say that an economic system is a complex *adaptive* system?<sup>2</sup> Such a system would appear to have four general properties:

<sup>2</sup> See Foley (2003) for a definition set in the context of classical political economy. This definition is consistent with the more extended one offered here.

- It is a dissipative structure that transforms energy into work and converts information into knowledge for the purpose of creating, maintaining and expanding the organized complexity of the system.
- Such a system is a whole in itself, as well as being a component part of some systems and oppositional to others – it is the connections that are forged between systems that permit the emergence of organized complexity at higher levels of aggregation.
- Such a system must exhibit some degree of structural irreversibility due to the inherent hierarchical and ‘bonding’ nature of the connections between components that are formed as structural development proceeds. It is this that results in the inflexibility and maladaptiveness that precipitates a structural discontinuity of some kind.
- The evolutionary process that such a system experiences can only be understood in an explicit historical time dimension – phases of emergence, growth, stationarity and structural transition can be identified in the historical time domain, leading to theoretical questions concerning the factors that result in the generation of variety, innovation diffusion, selection and system maintenance.

Identification of these properties leads to important questions concerning the relationship between a component and the system into which it is embedded by a particular structure of connections. Complex systems theory is, essentially, a body of theory about connections, distinguishing it from conventional economic theory which is concerned with elements, supplemented by very strong assumptions about connections (Potts(2000)). ‘Ordered complexity’ can be juxtaposed against ‘disordered complexity’. For example, in physics, a state of thermodynamic equilibrium, where there are no systematic connections between elements, is an example of the latter. What are referred to as complex *adaptive* systems, although closed in some respect, are open in others and, thus, capable of reconfiguring their connective structure. Component structures in such systems evolve through a process of specialization and integration, familiar to economists since Adam Smith stressed the importance of the division of labour and

the gains from specialization in trade in his *Wealth of Nations*. However, what has been less recognized by economists is the transitory nature of such systems, despite the influential work of Joseph Schumpeter in the first part of the 20<sup>th</sup> Century. Schumpeter's intuitions concerning the process of "creative destruction" sit very comfortably with modern complex adaptive systems theory (Foster (2000)).

Conventional economic theory often begins with the presumption that a system is in a high state of order, or is capable of attaining such a state of stable equilibrium. Such an equilibrium involves all elements being connected to all other elements, eg, perfect knowledge. This is a force field notion of equilibrium, drawn from physics, that contrasts with the thermodynamic notion of equilibrium which is a state of maximal disorder (Mirowski (1989)). Analysis is then intended to reveal mechanisms (actual or designed) that can be subject to control (Mirowski (2002)). The organized complexity of the system in question is assumed to be so well defined that its mechanisms can be represented in sets of mathematical functions from which equilibrium solutions can be deduced. The problem is that systems with such properties cannot evolve because they are so completely interconnected, ie they lack a sufficient degree of openness, in the sense of potential connectivity. Evolution can only occur when systems can change structurally, both in their internal order and in their relations with the external environment. Order emerges when variety (previously unconnected novel elements) is resolved into organized complexity. Disorder arises when variety, due to breakdowns and disconnections between elements, arises within a system.

Thus, the term 'complexity', as used here, ultimately refers to the connective structure (or lack thereof) of a system. It is different to the 'mathematical complexity' that can be generated by nonlinear dynamical models that are capable of attaining equilibrium curves, eg, limit cycles, or equilibrium regions, eg chaos. Such models can sometimes be used to track the dynamics of non-adaptive complex systems in the physio-chemical domain or in components of higher systems that function in ways that are analogous to the operation of physio-chemical systems, but their usefulness in gaining an understanding of economic evolution is very limited.

Complex adaptive systems are connective structures that exhibit re-entrant connections whereby energy is translated into structure that, in turn, can absorb more energy. This is aided by the absorption of information and the formation of knowledge structures that can be drawn upon in energy seeking. Forces that maintain order co-exist with forces pushing a system towards

disorder, allowing both flexibility and structural integrity. While a system is in balance with respect to these countervailing forces, it is capable of structural development<sup>3</sup>. However, for elemental novelty to continue to translate into connected structure, there must eventually be failure. All economic evolution stems from this process of creative destruction (see Foster and Metcalfe (2001)).

Although we have focussed on complex adaptive systems because they are relevant to understanding the economic system and its components, in no sense are we adopting a biological analogy, despite the fact that complex adaptive systems exist in the biological domain and that many of the theoretical advances made in this field have been by biologists, eg, Brooks and Wiley (1986), Kauffman (1993), Depew and Weber (1995). This has implications for how we define a complex system: no single definition is adequate, instead, we can discern a sequence of homologies, or different orders of complexity, within which we can classify different types of complex systems:

**(1) First order complex systems** (the imposed energy case)

This type of system exists in physio-chemical settings when energy is imposed on chemical elements. Non-adaptive structures or patterns, such as fractals, turbulence and Bernard cells form because they facilitate the dissipation of energy into movement, heat, light, etc. The dynamics exhibited in the presence of this kind of complexity can be modeled with both conventional dynamical mathematics and non-linear dynamical techniques (for example, synergetics).

**(2) Second order complex systems** (the imposed knowledge, acquired energy case)

This type of system can receive information that it can translate into a knowledge structure of some kind that permits some degree of control over the acquisition of energy. Knowledge here is no more than a capacity to make connections between elements in the external world. Acquiring energy, of course, requires the formation of physical structures. This is the level of complexity that operates in the biological domain where much of knowledge is genetically encoded and subject to natural selection. In other words,

<sup>3</sup> Kauffman (1993) conveys a similar idea in his notion that evolution occurs at the “edge of chaos” but caution must be exercised in applying his particular representation of this too literally in economic contexts.

knowledge is imposed by experience. In the case of more advanced species, such as mammals with brains, knowledge is also acquired from experiences such as hunting and can be passed on non-genetically so that brain development becomes important in natural selection. Clearly, such systems are complex and adaptive.

**(3) Third order complex systems** (the acquired knowledge case)

This exists when a biological system does not only interact with its environment as a data field, but also through images formed of possible worlds, ie connections between elements in the knowledge structure that need not have existed in observed reality. Knowledge is no longer just an accumulation of experiences, it is also a source of 'mental models' that can be used to determine aspects of reality. Of course, if this happens then some mental models also become part of the knowledge set. So, with human beings, some knowledge is imposed and some is acquired. There is feedback from reality and feed forward to reality. This is the kind of complex system that we find in the early stages of economic development when the environment is rearranged and exploited advantageously. Such systems are complex and adaptive but adaptation is more than natural selection, it involves creativity.

**(4) Fourth order complex systems** (the interactive knowledge case)

Such systems come into being when mental models interact with each other. My imagination can still mould reality but knowledge that this is so leads others to imagine what my imaginings might be. For example, as Maynard Keynes perceptively observed, participants in financial markets tend to be more interested in the average level of sentiment in the market than the relation of prices to the 'fundamentals'. This kind of complex system becomes prevalent when people form aspirations and commitments into the future and enter into forward contracts and other arrangements with terminal dates in the future. Such complexity constitutes a powerful force in the development of very sophisticated economic systems. It brings together the aspirations of individuals into 'understandings' that enable the creation of otherwise unattainable organized complexity. As ever, this is not a new idea, Adam Smith recognized that the power of capitalism lay in "sentiments" that enabled their to be enough trust and understanding for self-interested economic actors to cooperate with each other into the future in the production and distribution of goods and services (Fitzgibbons (1995)). Such systems contain

interconnected mental models and, thus, interconnected knowledge sets. However, such systems present threats as well as opportunities. If collective imaginings cease to bear a relationship with realistic possibilities in the presence of positive feedback, as Soros (2003) has discussed, severe structural discontinuity can be the result.

As we proceed up through these orders of system complexity, the role of connections and knowledge becomes increasingly apparent and important. The further up we go, the more that knowledge matters, eventually shaping the environment and then becoming part of the environment itself. What is often referred to as the 'knowledge' or 'new' economy is really an acknowledgement that 4<sup>th</sup> order system complexity has become relatively more important than the 3<sup>rd</sup> order system complexity that dominated in less developed economic systems. Also, as we move up these orders, imposed energy gives way to natural selection as the primary driver of change in complex systems which, in turn, is mitigated by both experiential and imaginative knowledge in highly developed brains. Thus, in the human case, little genetic selection has occurred recently and the evolution of the socioeconomic system has lain, increasingly, in the novelty and selection of ideas.

In the main, there has been only limited interest in applying the complex systems perspective in economics, despite the fact that these ideas have been presented in various guises by eminent economists such as Herbert Simon, Friedrich von Hayek, Gunnar Myrdal and Nicholas Georgescu-Roegen, to name only a few. Although two economists, Kenneth Arrow and Brian Arthur, were centrally involved in the founding of the Santa Fe Institute in the late 1980s, interest amongst economists in their contributions has been largely confined to their work on increasing returns (Arthur (1994)). Most of the push for a more general complex systems perspective on the economy has come from evolutionary economists interested in the generation of variety process from the perspective of replicator dynamics (see Foster and Metcalfe (2001)). However, whereas conceptualisations of complex adaptive systems began to be applied in the field of business management in the 1990s (see, for example, McKelvey (1999) and Stacey (1996)), evolutionary economists have tended to think of economic systems, in more limited terms, as dissipative structures engaging in processes of self-organization (or autopoiesis) (see Foster (1997) and Witt (1997)).



Generally speaking, there has not been as much development of the complex systems perspective in evolutionary economics as might have been expected, possibly because of the appeals of complex system researchers, such as Allen (2001), for an interdisciplinary perspective on complex systems in the socioeconomic domain (see also Byrne (1998)). Post-Keynesian economists have engaged in related discussions concerning non-ergodicity in economic systems but, generally, they have been very lukewarm towards the notion that the complex systems perspective on economic behaviour can be enlightening (Davidson (1996)). The dominant concern amongst Post-Keynesians seems to be with the usefulness of ‘complex dynamics’ rather than the development of a complex systems perspective. This boils down to considerations of the properties of dynamical mathematical models (Rosser (1999))<sup>4</sup>. Post Keynesians also been interested in developing dynamical models of the Minsky financial fragility hypothesis (Keen (1995)). Such endeavours are clearly concerned with 4<sup>th</sup> order complexity but little attempt has been made to employ complex systems theory in this kind of research. As we shall discuss more fully below, it is the neo-Austrian economists who have been the heterodox group most open to the complex system perspective. However, despite sharing an economic liberal position with neoclassical economists, the neo-Austrians have had virtually no impact upon mainstream ideas in economics.

## **2. CONSTRAINED OPTIMIZATION**

Constrained optimization lies at the heart of modern economic analysis. Economics journals contain a vast array of optimization exercises and the supply of these is seemingly inexhaustible. Over the years, the optimization techniques used in economic theory have become more and more sophisticated, opening the door to even more challenging journal articles. For example, dynamic optimization has become central in macroeconomics and strategic optimization has become commonplace in microeconomics when the relevant constraint is the action(s) of another economic agent. These more sophisticated variants of optimization are viewed as advances because they offer richer (ie more complicated) theories that seem to be more able to deal with economic behaviour in the real world. The perceived strength of theories based on individual

<sup>4</sup> The context has usually been the macroeconomic modeling of business cycles and economic growth (see Richard Goodwin (1990) and Richard Day (1994)).

optimization is that they stress the cognitive capabilities of humans rather than genetically inherited drivers of behaviour in emotions, such as hope, fear, passion, etc. So, in addition to seeming very scientific to the public because of their mathematical depth, there is a strong rhetorical role played by such theories in that they implore us to behave in considered and logical ways in our relations with each other and the material world that we inhabit. This is, in fact, an example of 4<sup>th</sup> order system complexity, whereby the imaginings of economists are picked up by non-economists and this, in turn, affects the behaviour of the latter. This suggests that there has been a strong reflexive tendency in the process of economic development as people have behaved, increasingly, in ways that mimic neoclassical constrained optimization theory. On the face of it, this seems useful in that it encourages us to behave as ‘economic agents’ rather than the expropriators and exploiters of others. And, indeed, it is quite remarkable just how far some societies have gone, building upon that elemental tendency to “truck and barter” identified by Adam Smith.

There is no doubt that much of Smith’s *Wealth of Nations* is itself rhetoric directed against a semi-feudal society, with its Hobbesian presumptions concerning the need to control an unruly populace, that still existed in his time. It is less obvious that a significant portion of neoclassical economics is also rhetoric cast, not in words but in axiomatic mathematics, expressed in both continuous and discrete forms. Being less obvious, it can constitute a dangerous form of economic liberal rhetoric because it does not highlight the importance of institutional rules, in the form of conventions, customs and laws, as Smith did, in explaining how the capitalist system works. Because the application of deductive logic requires fixity in structural representation, neoclassical economists have tended to simply assume the existence of perfected rules rather than to be concerned with often difficult processes that give rise to such rules. Indeed, there is no real distinction between rules and knowledge in a perfectly connected field. At base, neoclassical economics has a utopian flavour, not because there is much wrong with its basic axioms, but because of the way that optimization is specified. Although, this kind of logic looks scientific, it is more rhetorical in content than Smith’s non-mathematical and open-ended discussions of the capitalist system. Also, we must remember that 4<sup>th</sup> order system complexity can be double edged: collective imaginings can promote a great deal of economic co-operation and creativity but, if it detaches from reality too far, it can result in serious structural instability. Fitzgibbons (1995) perceptively observed that Adam Smith himself was concerned with this possibility in the future (see also Rothschild (2001)).

The utopian character of neoclassical constrained optimization, and its shortcomings as a scientific theory, is betrayed by its references to market *failure*, knowledge *imperfections*, productive *inefficiencies*, states of *disequilibrium*, etc, in attempting to derive hypotheses from theory for empirical application. These references contradict the strong form of field theory used and, without fail, applied econometricians always have to appeal to *ad hoc* ‘translating’ hypotheses, such as the partial adjustment hypothesis, to connect timeless theory with historical data.<sup>5</sup> This has been taken to the limit in ‘vector error correction modelling’ (Hendry (1995)) which simply adopts a statistical procedure to obtain a representation of the data that is presumed to capture a disequilibrium process towards a theoretical equilibrium state.

The unsatisfactory features of constrained optimization in making any convincing connection with history was one of the main reasons for the ontological split between neoclassical and neo-Austrian economists that emerged in the 20<sup>th</sup> Century, despite agreement that the individual is the appropriate level inquiry in economic analysis and that markets offer the best institutional rules for the promotion of economic activity. In essence, Austrians, such as Von Mises and Von Hayek, did not disagree with the rhetoric of neoclassical economics but with its scientific credentials<sup>6</sup>. The sacrifice of Smith’s emphasis on the evolution of the institutions of capitalism in favour of exercises in calculus borrowed from physics is viewed as an entirely unsatisfactory basis for understanding how an economy works (Vanberg (2004)).

Modern neo-Austrians see the economy as a complex system of individuals who, because of this complexity, face uncertainty that they attempt to reduce by adhering to behavioural rules that result in mutual benefits. These rules are an outcome of a process of ‘spontaneous order’ whereby the best rules are selected and, ultimately, gain legitimacy in laws and constitutions. Knowledge and action are not optimized givens but emergent phenomena; it is argued that they must not be interfered with by entities such as governments that are regarded as political entities

<sup>5</sup> Weintraub (2002) provides a compelling account of how axiomatic mathematics came to dominate economics despite its formal disengagement with actual economic processes.

<sup>6</sup> Hayek (1952) had an intuitive grasp of issues raised by Gödel (1931) concerning the incompleteness and impossibility limits, ie non-computability, of the class of fixed point mappings that neoclassical economists characterized as market equilibria. However, he, like most other Austrian economists, did not have the formal skills required to mount a Gödelian attack on neoclassical economics.

‘outside’ the economic system. Of course, this ends up being an entirely rhetorical position because traditional science, that involves the discovery of ways to control systems for the better, are strongly rejected, as is mathematics and almost all forms of econometrics. This neo-Austrian critique of neoclassical economics as scientific analysis provides an early example of the view that complex systems cannot be very well understood using any kind of mathematical analysis.<sup>7</sup> In the light of this critique, how can continued use of neoclassical constrained optimization be justified?

Modern practitioners, such as Robert Lucas, argue that what they call ‘toy’ theoretical models need not be directly connected to reality and that they should be retained if their predictions concur with movements in actual variables. Provided that a toy model contains relationships that are relevant to the system under investigation, it is deemed to be useful in a policy sense. Moving towards reality involves the successive relaxation of assumptions, making the model more complicated, enabling more variables to be incorporated. However, if the toy model is the solution to a constrained optimization problem it cannot, by definition, constitute the essence of the system in question because constrained optimization cannot be a ‘stripped down’ version of the operation of a complex system in the historical time dimension, ie, it has no ontological status. Such a model is *simplistic*, not simple, and it makes no difference if such a model can display movements that mimic movements in actual variables. The timeless character of such simplistic models provides a very large set of possibilities to choose from in achieving this.

However, the intention here is not to deny that optimization is unimportant, either in its strategic or non-strategic forms. As Alfred Marshall stressed a century ago, optimization theory can be useful in short period and local contexts when it is fairly certain that institutions, culture, politics and general economic conditions are not changing very much and there is time and other resources available to engage in optimization exercises. Importantly, Marshall was not positing a simplistic model of an economic system, as the general equilibrium theorists that he opposed would do later on, only an analytical device that could be used in an approximate ‘as if’ way to understand movements in prices. He was aware, in an intuitive sense, of the fact that the

<sup>7</sup> More recently, a similar critique has been made of classical physics by complex system scientists (see Baranger (2002)) but physicists have been difficult to shift in this regard given that, particularly in engineering, classical physics continues to be regarded as very useful.

economy is a complex system which has to be cut up in certain ways before optimization theory can be useful in understanding movements in prices (Foster (1993)).

Economists began to use simplistic optimization theories partly because of a conviction that simplification was the only way to try to understand the economy with only very limited historical data at their disposal. Economic theorists have held to this position determinedly because of a belief that optimization does lie at the core of economic behaviour in all its manifestations, even though other dimensions of behaviour are relevant in the non-economic domain. This is easy to critique but also easy to understand. The problem lies, not with the human capacity to optimize, but with a view of the economic system that dichotomizes theory and history and, in so doing, confuses simplistic theories with simple ones. In the laboratory, controls can enable researchers to examine simple connections between elements. In economics, making assumptions does not isolate simple connections that can be tested but, rather, yields simplistic models that are disconnected from reality.

### **3. SUBJECTIVE AND PROCESS OPTIMIZATION**

Optimization is a technique that is applicable when there is a known set of future outcomes and known probabilities associated with each occurrence. There is a well established literature that deals with the difficulties faced when there is a state of uncertainty, ie an unknown set of outcomes and unknown probabilities. Because economic systems are complex and adaptive, the future is characterized both by a degree of certainty, inasmuch as there is path-dependency in economic structure and processes, and uncertainty, because of the shifting complexity of the internal and external environment. Thus, it is not always possible to conduct optimization exercises, either because of structural rigidities in system connections or because of the presence of uncertainty concerning connective structure. Although this poses difficulties beyond the Marshallian short period, constrained optimization remains important in a subjective sense, ie in planning (or, more generally, in aspiration formation).

What we can refer to as ‘subjective optimisation’ takes place in planning processes. The plans and scenarios in our imaginations can be simplistic since the connections we imagine in our minds are not in actual time and, as such, are affected by our beliefs and prejudices as well as

assumptions about the future state of the world. These simplistic mental models are not intended to be the basis of scientific explanations of historical reality but, rather, they are to aid the design and production of artificial systems and artefacts. So an entrepreneur producing a new kind of product has to imagine how it can be designed, who might buy it and how an effective production strategy can be implemented. This is a 3rd order complex system problem that can be represented in an abstract way in terms of sets of assumptions, profit maximisation strategies and associated cash flows. More than likely, the entrepreneur will have to share this mental model (business plan) with someone who can lend money, ie 4<sup>th</sup> order system complexity may be involved.

If a formed aspiration is novel, such as an entirely new kind of product, the presumption that optimization strategies are adopted in the planning process is unlikely to be a very useful guide to outcomes, even in the short term. In such circumstances, people seem to rely less on cognition and more on emotions to drive their aspiration formation and their quest to achieve targets (Damasio (1995)). This generates a variety of processes and outcomes that are then subject to selection. Complex adaptive systems theory tells us that dissipative structures such as firms cannot enter new niches or escape the stationary states to which they tend through optimisation exercises alone – imagination and creativity are required. So the evolution of economic systems depends on entrepreneurs and inventors/innovators coming up with new products and/or processes.

In addition to seeking optimal solutions in our imagination, we also employ optimization in quite practical ways. What we might call ‘process optimization’, takes place when engaging in processes necessary to achieve aspirations. This is an adaptive form of optimization because it is undertaken sequentially as events unfold in time. Such optimization is simple and relatively costless provided that there is sufficient time to react to events as they unfold and provided that there is a relevant knowledge structure and a set of interactive skills that can be drawn upon.<sup>8</sup> This kind of optimization is constrained by the availability of knowledge concerning threats and opportunities and skills in interacting with the biophysical and socioeconomic environments

<sup>8</sup> Milton Friedman was fond of using examples such as driving a car to argue that humans are capable of solving the vastly difficult optimization problems specified in neoclassical economics. The complex systems perspective tells us

Learning is crucial for both subjective and process optimization. We have to spend time acquiring knowledge from information stocks and flows in order to make plans. This knowledge gathering is also a process in time. There is satisfaction in the discovery of novel connections and there is excitement as aspirations form. Emotional drivers are fundamental in such learning. Similarly, in process contexts, learning by doing also involves satisfaction. Although learning by doing can make us better process optimizers, it does not constitute a disequilibrium path in a behavioural vacuum. Equally, once we have become process optimizers we may not enjoy much satisfaction if our actions become routine in character and we are likely to shift our attention to engagement in other processes. This account of behaviour in complexity turns the standard story on its head. It is variety, creativity and learning that we enjoy (disequilibrium adjustment in the standard story) not an optimized steady state. Enjoyment is an emotional state and optimization is only a means to this end (Steedman (2001)).

What is decisive in complex systems is not optimization but, rather, selection. Subjective optimization will produce a range of strategies to reach an aspirational goal given that assumptions concerning opportunities and constraints (beliefs) will vary across individuals. Processes to achieve an aspiration will depend on particular knowledge structures and the specific interactive skills that exist and these will translate into unique learning by doing and incremental innovation paths. There are two selective dimensions here: there is the best plan (the best car design etc) and the best process (organisation, technology and human capital). Learning is itself a selection process, if we think of knowledge as containing a population of aspirations and skills. Unattainable aspirations will be eliminated in the planning phase and inappropriate processes in the action phase. Selection is crucial to our understanding of complex systems – it eliminates the need to use optimization techniques in the conventional manner. The best aspiration/process mix will emerge over time (all learning and creativity must occur in time). Thus, dominant designs will emerge from production processes that are most apt in the historical/environmental context faced<sup>9</sup>. The process of ‘replicator dynamics’ that is used to analyse the emergence of dominant techniques and products is now well understood in evolutionary economics (see Metcalfe (1997) and Metcalfe, J.S., Foster, J. and Ramlogan, R.

that he was, in fact, discussing sequential process optimization, which is an entirely different matter to outcome optimization, whether specified in objective or subjective terms.

<sup>9</sup> See Dew, Sarasvathy and Venkataraman (2004)) for discussion of how such evolutionary processes need not result in the most efficient outcome in the conventional sense.

(2003)). However, in this literature, although uncertainty and bounded rationality are emphasized, this process is not normally viewed from the more general perspective of complex systems theory (Foster and Metcalfe (2001)).

So, a complex systems perspective suggests that optimization is a subjective device and it is also ever-present in processes, but, in the end, productivity growth and innovation are due to learning new knowledge and new/better skills. Optimization is, ultimately the basis of routine planning (writing the weekly supermarket shopping list) and routine processes (doing the shopping). Such routines, in turn, permit time and attention to be turned to other novel and creative pursuits. Routines are the basis of rules that permit organised complexity to increase in systems. Dopfer, Foster and Potts (2004) build on this notion to argue that systems of rules are the key connections that matter in the economic system and that these are ‘mesoeconomic’ in nature, not microeconomic or macroeconomic. However, rule systems are structures and complexity theory tells us that, as circumstances change, they will become obsolete and, thus, degenerative, if they are not adapted or replaced by new rules. So, if we think of a rule as something static that comes into being because of its useful properties at a point in time, it may turn out to be decidedly useless in other circumstances. Psychologists argue that many emotional responses operate as behavioural rules that are obeyed in a relatively unconscious way and that these evolved because they were useful somewhere in our genetic past. In the complex economy there are many rules embodied in institutions that are of the same character.

This view of optimization in complex systems is not confined to economics. Mature organizational structures facing zero growth in opportunities and degeneration in their internal connections move from being competitive to exploitative, then combative (Foster (2000)). From a complex system perspective, this is important because it leads to eliminations and, thus, access to resources by the emergent. Strategic considerations become dominant in such states. In combat, a plan (subjective optimization) is necessary but, in battles, adaptiveness is crucial and this will depend on the skills of the combatants (process optimization). Thus, optimization need not result in economically beneficial outcomes. This will depend upon the opportunities and threats that systems face. The creation of the ‘wealth of nations’ depends primarily on the emotional disposition of people to create novelty, both in elements and connections, and to



coordinate in production and exchange in increasingly complex and organised ways. All growth involves the creation of complementary structures that produce goods and services and the selection of these through exchange mechanisms such as markets (Metcalf, Foster and Ramlogan (2003)). But what does all this imply for familiar constructs, such as production functions, which play such a central role in both microeconomics and macroeconomics?

#### **4. FROM PRODUCTION FUNCTIONS TO NETWORKS**

Following Potts (2000), we can think of all systems in terms of elements and connections. Neoclassical economists focus on elements (individuals and firms) and largely ignore connections by making very strong and unrealistic assumptions about them in order to conduct deductive analysis. Connections are akin to mathematical operators, which must stay fixed if logical deductions concerning equilibrium outcomes are sought. Of course, various kinds of non-linear mathematical forms can be specified that are intended to reflect the fact that connective structures can be irregular in strength and extent. Both continuous and discrete interval dynamical systems can be specified and, with suitable restrictions, they can yield equilibrium outcomes. However, no amount of mathematical elaboration can override the condition that an equilibrium outcome requires a fixed set of connections in a system. If we let go of the notion that connective structure is fixed and, therefore, the idea that equilibrium solutions, comparative statics and comparative dynamics are generally applicable tools of analysis, what kind of mathematics is required?

Do we have to patch up the mathematics of highly connected elements to deal with connective variation? For, example, such variation can sometimes be captured in nonlinear mathematical forms which have solutions if, for example, numerical methods are applied. Difficulties immediately arise with using nonlinear mathematics to capture connection shift. Aggregation from micro to macro becomes virtually impossible and, in any event, most nonlinear systems do not have meaningful solutions. Thus, it is no surprise that mainstream economists prefer constrained optimization across well-behaved utility and production functions. They particularly love the Cobb-Douglas form because of its easy to handle mathematical properties. And aggregation is elementary if all economic agents are assumed to behave identically so that all

the connective issues raised by Keynes in distinguishing macro from micro are neatly assumed away.

When we think of the economy as a complex system of elements and connections, specifying behaviour in terms of utility and production functions is not invalid but simply inappropriate to the task at hand. The appropriate construct to understand systems at all levels is the *network*. The brain is a network, consumption spending lies in a network of interconnected tastes and interconnected income flows, production is a network, the whole economy is a network. Think of the firm, it is a network and, although firms' networks are similar in many respects because of the presence of higher networks, ie, a state space does exist, they differ in terms of the completeness, strength and particular qualities of their network structures. It is this that determines if a firm can generate value that yields a profit. This value does not just come from the elements contained in the firm – the individuals, the machines, etc. – but from the connections that are forged between them (see Shapiro and Varian (1999)).

There is an emerging literature in economic theory concerning the value of networks. However, the focus of this literature is upon networks as interconnected cooperative games and is primarily concerned with efficiency/stability tradeoffs (see, for example, Jackson and Wolinsky (1996)). However, this omits precisely what we wish to reintroduce, namely, the historical dimension<sup>10</sup>. As networks evolve and produce more and better ranges of products using more productive processes, we observe value increasing. If they are for profit networks, they may begin to make profits, if they are non-profit networks, they may begin to cover their costs. There is little point in summarizing networks as production functions because such functions are always on the move. They are constructs that relate to a point in time or as averages over specified time periods. Accountants will always be interested in building on them to compare the costs and revenues associated with inputs and outputs, but economists should be more interested in how and why change is taking place over time.

<sup>10</sup> Jackson and Watts (2002) provide a “stochastic evolutionary” game theoretic representation of economic networks that is driven by errors in connections. Although this work is a considerable advance on standard utility and production theory, it remains simplistic in orientation, ie, economic agents are not presumed to be operating in complex systems contexts above the 2<sup>nd</sup> order level. For a fuller discussion of this kind of network modeling, see Jackson (2003).

The production function perspective misses the most important dimension of production – the organisation of people and physical objects, ie the network structure of connections (Loasby (1999)). It is the productivity that is enjoyed when machines and workers are connected that matters most for value creation – efficacy is more important than efficiency. There are people who specialize in connecting – they are entrepreneurs and managers. As Williamson (1985) stresses, firms are governance structures, not production functions. What this means is that, although workers provide labour, it is of little value unless it is connected and, thus, the value of a firm hangs on the entrepreneur/manager functions. It is for this reason that entrepreneur/managers are the most highly paid in firms. They do not ‘add value,’ they define what value is. They lay down networks between workers and machines (the division of labour). They increase these inputs and also increase the amount of indirect labour engaged in administration, ie in connecting. Thus, it is possible to produce more sophisticated products using more complex production systems.<sup>11</sup>

We observe the value of the firm’s production growing and the value of the firm itself growing. But this does not occur without limit. Networks, by their very essence, are inflexible in many dimensions. The hierarchical structures that are found in most systems are so to a most pronounced extent. Some bonds in the network become inflexible (but not unbreakable) simply because they hold the network together. Some networks are very stable as they evolve, others are unstable.

Any examination of the new literature on complex systems and networks leads to the conclusion that there is not an easy way to derive ‘normal’ scientific propositions, operationalised in mathematics, from them. However, we do know that diffusion curves exist in time when self-organisation is in evidence (Foster and Wild (1999) and Rycroft and Cash (1999)). And we know that these diffusion curves contain different ‘phases’ of growth. In turn, each phase has a particular network characterization. The emergent phase is ‘radically open’, whereas the saturation phase is a largely closed state – networks are tightly connected within and beyond the

<sup>11</sup> Since Nelson and Winter (1982), many evolutionary economists have viewed firms as bundles of routines. But routines are particular cases of connections and, therefore, should be viewed from a network perspective. Also, the associated ‘capabilities’ view of the firm relates to the formation of skills that involve the development of specialized connections between different people and between people and things.

system. Such systems gradually suffer from connective breakdown and repair costs rise nonlinearly.

The network approach emphasizes that added value is generated by connections between elements, not by elements (such as homogenized labour and capital). The production function perspective was developed with an economy of goods (elements) in mind but the service sector, which constitutes up to 80% of value added in advanced economies, generates products that are purely connections. Firms are bundles of network connections, as are economies. Such networks cannot be fully connected or be maximally efficient – an economic system is not a machine. Networks are constantly being created and destroyed, along with products and organizations. What complex systems theory tells us is that it is more useful to regard the supply side, not as a set of production functions but as a system of interconnecting networks and, equally, the demand side as a network of interconnecting consumption patterns (Earl and Potts (2004))<sup>12</sup>. What are the implications of replacing timeless constructs, such as production and utility functions with networks? To answer this we must understand some aspects of modern network theory.

## **5. SCALE FREE NETWORKS**

In early approaches to networks they were viewed as random in nature (see Erdős and Renyi (1959)). In random network theory, the probability that a node (element) is connected to others decreases exponentially as the number of elements increases, assuming that node formation is completely random. As Barabasi (2002) points out, however, this representation of a network, although relevant in understanding some important networks, such as highway systems and power grids, is not very useful to understand many of the networks that exist in the 3<sup>rd</sup> and 4<sup>th</sup> order complex systems that we observe in the economy (see also Watts (2003)).

This has been brought home in studies of the internet which turns out to be ‘scale free’ since connection is not restricted by location. When connectivity is increased in a network it remains incomplete – so we do not approach the strong field theory assumption of neoclassical

<sup>12</sup> This, of course, does not imply that algorithmic approaches, using, for example, recursion theory cannot also provide useful insights in complex systems contexts (see Velupillai (2000)).

economics, eg a decentralized system of fully informed and fully connected perfectly competitive firms and fully informed and fully connected consumers. What we can get in incomplete networks is the evolution of some nodes that become ‘hubs’ that most connections are routed through. For example, O’Hare Airport or Microsoft qualify as hubs in scale free networks. These constitute core-periphery hierarchies that are very stable since the probability an exogenous shock hitting the hub is small – thus many parts of the internet can have problems but the whole system is relatively unaffected. Of course, if there is a deliberate attack on a hub that is successful, the whole system is in danger.

Why are networks like this? Because people are not random in making connections – people have agendas (aspirations and preferences) in making connections. We do not have random conversations nor do we enter into binding contracts with random individuals. 3rd and 4th order complex systems form selectively and adaptively and, thus, nodes can emerge in unplanned ways. This is, of course, well understood by marketing executives who strive to form connections that yield hub status for their products eg Microsoft’s exclusive deal with IBM to supply an operating system allowed it to evolve into a hub. Marketing can be viewed as the art and science of influencing connection choice (Earl and Potts (2004)). We can think of the Schumpeterian entrepreneur as seeking to evolve from a node to a hub in a particular market that will confer monopoly power of some sort. When an entrepreneur forms a firm, very specific connection choices are made and a governance/productive structure forms. Neo-Schumpeterian theory suggests that such structures are selected depending on the attractiveness of the product and the effectiveness of the productive process devised and that the successful will come to dominate. There are clear parallels between this neo-Schumpeterian process and the evolution of hub-dominated networks. But what is the advantage in thinking of this in network terms?

Scale free networks adhere to power laws and, as such, offer dramatically different representations of value distributions to those presumed in the strong field theory cases preferred in much of neoclassical economics. What is a power law? In a scale free network, the probability that any node being connected to  $k$  other nodes is proportional to  $(1/k)^n$ . For example, the tyre industry in the US started out with hundreds of local firms supplying local needs and, as connections spread, it finally resolved itself into a few producers in one dominant hub (Akron, Ohio). Clearly, the transactions costs associated with networks are important and these have to be lowered to allow a scale free network to emerge. If the power law is sufficiently nonlinear,

then a ‘star’ shape evolves where we have a pure monopoly situation whereby all connections are mediated by one hub.

Power laws across networks provide us with a measurable way of representing the structure of the economy and its components with no need to pretend that all firms are the same or, from a stochastic perspective, differentiated by normally distributed variations. Scale free network theory tells us that we should not expect value to be normally distributed. Thus, an unequal distribution of income and wealth is what we would expect in a well-connected capitalist economy. Also, we would expect industries to become concentrated and to become more so as communications come to involve lower transactions costs. However, not all networks are subject to the same power law and they will range from random networks, often observed in emergence, through to stars in very mature and powerful cases. Thus, as a product moves up a logistic diffusion curve we can predict that there will be a change in network structure associated with its production and sale and, correspondingly, we can measure this (see Cowan and Jonard (2004)). Total value will rise but so will inequality in the distribution of this value. Thus, complex systems theory departs in a fundamental way from neoclassical economics in indicating that the distribution of income and wealth is important in coming to an understanding of how an economic system works.<sup>13</sup>

The fact that the power law exponent across a network evolves as we move up a logistic curve allows us to make precise statements concerning the distribution of value within the system under consideration. The formation of scale free networks involves ‘inequality’ but it can also be associated with rising productivity in some, but not all, cases. The vulnerability of mature systems can be more explicitly identified as the vulnerability of a hub or the apex of a hierarchical network. Furthermore, ‘small world’ analysis (see Watts (1999, 2003)) can be used to show how distant and peripheral innovations can sometimes alter a mature and seemingly immovable complex system radically if a particular kind of connection is established.

<sup>13</sup> Of course, the fact that the distribution of income and wealth adheres to a power law goes back a long way in economics – at least as far back as the work of Vilfredo Pareto. Also, the view that the distribution of income and wealth plays an active role in determining economic growth and development has been a central theme in Cambridge School thinking. So, although complex systems theory is not saying anything new in this regard, it suggests that power laws should be pervasive throughout economic systems, calling into question assumptions made concerning probability distributions in a wide range of contexts.

Complexity theory suggests a ‘connective theory of value’ which looks radically different to conventional theories of value. For example, a conversation may have economic value and, indeed, much that has connective economic value is not measured by a price. Acts of creativity that form connections between elements are often unpriced since they yield intrinsic satisfaction while hubs in networks can generate massive economic returns simply because of their pivotal role. Price determination in markets also has to be qualified in many cases - products become hubs of consumer choice networks and hub firms may sub-contract production to more peripheral firms. Connection with consumers cannot be looked on as simply a matter of pricing but of both marketing and associated contagion. In many cases, this breaks the dichotomy of supply and demand since suppliers can influence the latter.

As has been noted, the transformation of networks from random and local to scale free is part of the cycle of creative destruction. Aggregate measures of economic value cannot always show this very clearly. However, there are two measures we can use to gauge system vitality: one is the tendency for hubs to form. In an increasingly connected system we should observe greater concentration. The second is the degree of turnover of dominant hubs since this will be an indication that competitive networks are emerging and realigning the distribution of value. In countries in which there has been significant turnover of hubs, the distribution of value is much more fluid. In the US, this has been the case with the distribution of income widening, but with greater opportunities for upward mobility than in many other countries. In countries with dominant hubs or even ‘stars’ and low hub turnover, we observe an unequal and ‘frozen’ distribution of income, low growth and a tendency towards political upheaval. Thus, the proposition that ‘more inequality is better for growth’ must be heavily qualified.

Firms, industries and economies are all involved in producing and exchanging products. They do this in complex and interconnected ways, thus, the firm is only a very proximate unit of analysis. Connected systems produce a product and this may be a firm or it may be a firm plus other partner or subcontracting firms and a range of consultants and specialists in the service sector may be involved. Modern production systems are bewildering networks of connections. Networks can be of different types. The random network is prevalent in a barter system where individuals exchange goods locally on a one-to-one basis. New products also tend to start out in being sold in locally based random networks. However, as an industry develops, longer connection chains form and, provided that there are actual and potential variations in the quality

of the product and in the productivity of production processes, then there will be a tendency for nodes to form in scale free networks. Thus, a local tyre producer will become an agent for the sale of tyres produced elsewhere. Dispersed consumers will eventually buy only the products of a few large firms through local agencies. Also, these firms will increasingly disperse aspects of production and distribution to smaller firms.

Enough has been said to suggest that complex systems theory offers a different analytical perspective that is explicit in addressing the temporal and spatial dimensions of economic behaviour. Completely connected networks (fields), as specified in neoclassical general equilibrium economics, can never exist across an economy. Furthermore, actual networks must involve an unequal distribution of income and wealth that cannot be understood in terms of the value of marginal products. Complex systems theory tells us that rents are an inevitable feature of a well-functioning economic system and that it is the extent to which products and firms turn over that is crucial.

## **6. CONCLUDING REMARKS**

The behaviour of complex adaptive systems cannot be captured by constrained optimization models. This is a fundamental departure from the presumptions inherent in conventional economics. Such systems have to be analyzed 'in' time and this limits the way that mathematics can be used. The historical trajectory that the value of an economic network in a complex adaptive system follows can be represented mathematically, eg as a logistic equation, but this is not derivable from a set of axioms set in a timeless context. The isolation of diffusion parameters and capacity limits is an empirical matter. However, conventional deduction can still be used to specify adjunct hypotheses concerning the factors that shift historical trajectories around. It follows that, in complex systems, the stationary states that such trajectories attain are not analytical equilibria but, rather, end states of cumulative historical processes. Although, a journey in this methodological direction contains many landmarks that are familiar, both from a heterodox and orthodox perspective, this approach is fundamentally different to that espoused in the post-Robbins conception of what economics constitutes. Importantly, it suggests a new connective or network theory of value.



Seeking to optimize subject to constraints is a particular kind of creative act and, as such, must be set within an analytical framework that recognizes, in a basic way, the core roles of imagination, aspiration, learning and innovation, both technological and organizational. The economy cannot be viewed in a static way since it involves the continual turnover of products, firms and industries and is characterized by novelty and variety, not similarity and homogeneity. Today, many economists are in denial about this because the implications appear so fundamental, rendering obsolete and irrelevant so much that has been written. In retrospect, simplistic theorizing and associated deduction never played much of a *scientific* role in economics and this is one of the main reasons why the scientific usefulness of economics is challenged so widely today. It seems have played a much more important role as a *rhetorical* device – a kind of mathematical parable. If economics is to have much of a future then economists will have to adopt scientific foundations that are appropriate to the systems that they deal with. Already, the literature on productivity growth is replete with studies that do not fit with neoclassical production theory but do not connect with neo-Schumpeterian evolutionary economics either (eg. Disney, Haskel and Heden (2003)). The gulf between economic theory and historical research is widening and this requires urgent attention if economics is not to bifurcate into, on the one hand, a branch of applied mathematics and, on the other, a business school specialism that deals with aspects of market behaviour and business strategy.

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