From Turner's Logic of Universal Causation to the Logic of GK

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Abstract. Logic of knowledge and justified assumptions, also known as logic of grounded knowledge (GK), was proposed by Lin and Shoham as a general logic for nonmonotonic reasoning. To date, it has been used to embed in it default logic, autoepistemic logic, and general logic programming under stable model semantics. Besides showing the generality of GK as a logic for nonmonotonic reasoning, these embeddings shed light on the relationships among these other logics. Along this line, we show that Turner's logic of universal causation can be naturally embedded into logic of GK as well.

1 Introduction

Lin and Shoham [4] proposed a logic with two modal operators $\mathbf{K}$ and $\mathbf{A}$, standing for knowledge and assumption, respectively. The idea is that one starts with a set of assumptions (those true under the modal operator $\mathbf{A}$), computes the minimal knowledge under this set of assumptions, and then checks to see if the assumptions were justified in that they agree with the resulting minimal knowledge. For instance, consider $\mathbf{A}p \supset \mathbf{K}p$. If one assumes p, then one can conclude $\mathbf{K}p$, thus the assumption that p holds is justified, and one gets a GK model where both $\mathbf{A}p$ and $\mathbf{K}p$ are true. However, there is no GK model of $\neg\mathbf{A}p \equiv \mathbf{K}p$ as one cannot deduce $\mathbf{K}p$ when assuming p but gets $\mathbf{K}p$ when not assuming p.

To date, there have been embeddings from default logic [13] and autoepistemic logic [12] to the logic of GK [4], as well as from general logic programs [2, 3] to logic of GK [5]. Among others, these embeddings shed new lights on nonmonotonic reasoning, and have led to an interesting characterization of strong equivalence in logic programming [7, 5], and helped relate logic programming to circumscription [4] as the semantics of GK is just a minimization together with an identity checking after the minimization. Here we add to this repertoire by providing an embedding from Turner's logic of universal causation [14] to logic of GK.

This paper is organized as follows. Section 2 reviews logic of GK and Turner's logic. Section 3 shows how Turner's logic can be embedded in GK. Finally, Section 4 concludes this paper.

2 Preliminaries

2.1 Propositional languages

We assume a propositional language with two zero-place logical connectives $\top$ for tautology and $\bot$ for contradiction. We denote by *Atoms* the set of atoms, the signature of our language, and *Lit* the set of literals: $Lit = Atoms \cup \{\neg a \mid a \in Atoms\}$. A set I of literals is called *complete* if for each atom a, exactly one of $\{a, \neg a\}$ is in I.

In this paper we identify an interpretation with a complete set of literals. Thus if I is a complete set of literals, we use it as an interpretation when we say that it is a model of a formula, and we use it as a set of literals when we say that it entails a formula. In particular, we denote by $Th(I)$ the logical closure of I (considered to be a set of literals).

2.2 The logic of GK

The language of GK proposed by Lin and Shoham [4] is a modal propositional language with two modal operators, **K**, for knowledge, and **A**, for assumption. GK *formulas* are propositional formulas with **K** and **A**. A GK *theory* is a set of GK formulas.

GK is a nonmonotonic logic, and its semantics is defined using the standard Kripke possible world interpretations. Informally speaking, a GK model is a Kripke interpretation where what is true under **K** is minimal and exactly the same as what is true under **A**. The intuition here is that given a GK formula, one first makes some assumptions (those true under **A**), then one minimizes the knowledge thus entailed, and finally checks to make sure that the initial assumption is justified in the sense that the minimal knowledge is the same as the initial assumption.

Formally, a *Kripke interpretation* M is a tuple $\langle W, \pi, R_K, R_A, s\rangle$, where W is a nonempty set of *possible worlds*, π a function that maps a possible world to an interpretation, R_K and R_A binary relations over W representing the accessibility relations for **K** and **A**, respectively, and $s \in W$, called the *actual world* of M. The *satisfaction relation* $\models$ between a Kripke interpretation $M = \langle W, \pi, R_K, R_A, s\rangle$ and a GK formula F is defined in a standard way as follows:

- $M \not\models \bot$,
- $M \models a$ if $a \in \pi(s)$, where a is an atom,
- $M \models \neg F$ iff $M \not\models F$,
- $M \models F \wedge G$ iff $M \models F$ and $M \models G$,
- $M \models F \vee G$ iff $M \models F$ or $M \models G$,
- $M \models F \supset G$ iff $M \not\models F$ or $M \models G$,
- $M \models \mathbf{K}F$ iff $\langle W, \pi, R_K, R_A, \omega\rangle \models F$ for any $\omega \in W$, such that $(s, \omega) \in R_K$,
- $M \models \mathbf{A}F$ iff $\langle W, \pi, R_K, R_A, \omega\rangle \models F$ for any $\omega \in W$, such that $(s, \omega) \in R_A$.

Note that, for any $\omega \in W$, $\pi(\omega)$ is an interpretation. We say that a Kripke interpretation M is a *model* of a GK formula F if M satisfies F, M is a *model* of a GK theory T if M satisfies every GK formula in T.

The modal logic that we have just defined corresponds to $\mathcal{K}$ [1], with two independent modal operators $\mathbf{K}$ and $\mathbf{A}$. In the following, we write $T \models \varphi$ if the modal formula φ is entailed by T in $\mathcal{K}$.

In the following, given a Kripke interpretation M, we let

$$\mathbf{K}(M) = \{\, \phi \mid \phi \text{ is a propositional formula and } M \models \mathbf{K}\phi \,\},$$
$$\mathbf{A}(M) = \{\, \phi \mid \phi \text{ is a propositional formula and } M \models \mathbf{A}\phi \,\}.$$

Notice that $\mathbf{K}(M)$ and $\mathbf{A}(M)$ are always closed under classical logical entailment.

Given a GK formula T, a Kripke interpretation M is a minimal model of T if M is a model of T and there does not exist another model M_1 of T such that $\mathbf{A}(M_1) = \mathbf{A}(M)$ and $\mathbf{K}(M_1) \subset \mathbf{K}(M)$. We say that M is a GK *model* of T if M is a minimal model of T and $\mathbf{K}(M) = \mathbf{A}(M)$.

We consider only a special kind of GK formulas. An $\mathbf{A}$-*atom* is a formula of the form $\mathbf{A}\phi$ and a $\mathbf{K}$-*atom* is a formula of the form $\mathbf{K}\phi$, where ϕ is a propositional formula. An $\mathbf{A}$-*literal* ($\mathbf{K}$-*literal*) is an $\mathbf{A}$-atom ($\mathbf{K}$-atom) or the negation of it. Both $\mathbf{A}$-atoms and $\mathbf{K}$-atoms are called GK-*atoms*. A GK-*literal* is a GK-atom or the negation of a GK-atom. A GK formula is called a *pure* GK *formula* if it is formed from GK-atoms and propositional connectives. A *pure* GK *theory* is a set of pure GK formulas. Similarly, a $\mathbf{K}$-*formula* is a GK formula formed from $\mathbf{K}$-atoms and propositional connectives and a $\mathbf{K}$-*theory* is a set of $\mathbf{K}$-formulas. Note that there is no nested occurrences of modal operators in pure GK theories or $\mathbf{K}$-theories.

So far in the applications of GK, only pure GK formulas are used. For instance, a (propositional) default theory $\Delta = (W, D)$ is translated into pure GK formulas in the following way:

1. Translate each $\phi \in W$ to $\mathbf{K}\phi$.
2. Translate each default $(\phi : \psi_1, \ldots, \psi_n/\varphi) \in D$ to

$$\mathbf{K}\phi \wedge \neg\mathbf{A}\neg\psi_1 \wedge \cdots \wedge \neg\mathbf{A}\neg\psi_n \supset \mathbf{K}\varphi \in \Delta_{GK}.$$

Similarly, a disjunctive logic program rule

$$p_1 \vee \cdots \vee p_k \leftarrow p_{k+1}, \ldots, p_t, not\, p_{t+1}, \ldots, not\, p_m,$$

where p's are atoms, corresponds to the following pure GK formula:

$$\mathbf{K}p_{k+1} \wedge \cdots \wedge \mathbf{K}p_t \wedge \neg\mathbf{A}p_{t+1} \wedge \cdots \wedge \neg\mathbf{A}p_m \supset \mathbf{K}p_1 \vee \cdots \vee \mathbf{K}p_k.$$

In this paper, we show how Turner's logic of universal causation can be embedded in GK.

2.3 Turner's logic of universal causation

Turner's logic of universal causation [14], called UCL, is a nonmonotonic modal logic that generalizes McCain and Turner's causal action theories [9]. We first briefly review it here and then show how it can be embedded in GK.

The language of UCL is a modal propositional language with one modal operator $\mathbf{C}$. Semantically, a UCL *structure* is a pair $(I, \mathcal{S})$, where $\mathcal{S}$ is a set of interpretations, and $I \in \mathcal{S}$. The satisfaction relation between UCL sentences and UCL structures is defined as follows:

$$(I, \mathcal{S}) \models a \text{ iff } I \models a \quad \text{(for any atom } a\text{)}$$
$$(I, \mathcal{S}) \models \mathbf{C}\phi \text{ iff for all } I' \in \mathcal{S},\ (I', \mathcal{S}) \models \phi$$

and the usual definition of propositional connectives.

Given a UCL theory T, we write $(I, \mathcal{S}) \models T$ to mean that $(I, \mathcal{S}) \models \phi$, for every $\phi \in T$. In this case, $(I, \mathcal{S})$ is said to be a *model* of T. We also say that $(I, \mathcal{S})$ is an I-*model* of T, emphasizing the distinguished interpretation I.

The semantics of UCL is defined by so-called causally explained interpretations: an interpretation I is *causally explained* by a theory T if $(I, \{I\})$ is the unique I-model of T.

Notice that this semantics is language-sensitive. For example, assuming that p is the only atom in the language, $\mathbf{C}p$ has a unique causally explained model $\{p\}$. However, if there is another atom, say q, then $\mathbf{C}p$ has no causally explained models.

It is easy to see that under this definition, for any theory T, I is causally explained by T iff it is causally explained by the theory obtained from T by removing all occurrences of $\mathbf{C}$ that are under the scope of another $\mathbf{C}$. Thus, in the rest of the paper, we consider only UCL formulas that do not have a nested occurrence of $\mathbf{C}$.

3 Embedding Turner's logic of universal causation to GK

Now we show that Turner's logic can be embedded into GK by providing a translation from UCL theories to pure GK theories.

Given a UCL formula F without nested occurrences of $\mathbf{C}$, let $tr_{GK}(F)$ be the pure GK formula obtained from F by replacing every occurrence of $\mathbf{C}$ by $\mathbf{K}$ and adding $\mathbf{A}$ before each atom which is not in the range of $\mathbf{C}$ in F. Given a UCL theory T, let $tr_{GK}(T) = \{\, tr_{GK}(F) \mid F \in T\,\}$. For example, if F is $(a \wedge \neg b) \supset \mathbf{C}(a \wedge \neg b)$, then $tr_{GK}(F)$ is $(\mathbf{A}a \wedge \neg \mathbf{A}b) \supset \mathbf{K}(a \wedge \neg b)$.

Given that causally explained interpretations depend on the underlying language used but GK models do not, it is clear that our translation from UCL to GK needs to have a language dependent component as well. In the following, if *Atoms* is the set of atoms in the language, we let

$$tr_{GK}(Atoms) = \{\mathbf{A}a \vee \mathbf{A}\neg a \mid a \in Atoms\}.$$

Our following result shows that in GK, $tr_{GK}(T) \cup tr_{GK}(Atoms)$ captures causally explained interpretations of T.

Theorem 1. *Let T be a UCL theory. If I is a causally explained interpretation of T, then there exists a GK model M of $tr_{GK}(T) \cup tr_{GK}(Atoms)$ such that*

$\mathbf{A}(M) = Th(I)$. *Conversely, if* M *is a GK model of* $tr_{GK}(T) \cup tr_{GK}(Atoms)$*, then some interpretation* I*,* $\mathbf{A}(M) = \mathbf{K}(M) = Th(I)$*, and* I *is a causally explained interpretation of* T*.*

Proof. Given a pure GK theory T', if M is a model of $T' \cup tr_{GK}(Atoms)$, then $\mathbf{A}(M) = Th(I)$ for some interpretation I.

Given a UCL structure $(I, \mathcal{S})$, we can always create a Kripke interpretation M such that $\mathbf{A}(M) = Th(I)$ and $\mathbf{K}(M) = \bigcap_{I' \in \mathcal{S}} Th(I')$. Note that $I \in \mathcal{S}$, thus $\mathbf{K}(M) \subseteq \mathbf{A}(M)$, if $\{I\} \subset \mathcal{S}$, then $\mathbf{K}(M) \subset \mathbf{A}(M)$. From the definition of $tr_{GK}(T)$, if $(I, \mathcal{S})$ is a model of T, then M is a model of $tr_{GK}(T)$.

Given a Kripke interpretation M such that $\mathbf{A}(M) = Th(I)$ for some interpretation I and $\mathbf{K}(M) \subseteq \mathbf{A}(M)$, we can always create a UCL structure $(I, \mathcal{S})$ such that

$$\mathcal{S} = \{\, I' \mid \text{interpretation } I' \text{ satisfies every propositional formula in } \mathbf{K}(M) \,\}.$$

Note that $\mathbf{K}(M) \subseteq \mathbf{A}(M)$, thus $I \in \mathcal{S}$, if $\mathbf{K}(M) \subset \mathbf{A}(M)$, then $\{I\} \subset \mathcal{S}$. From the definition of $tr_{GK}(T)$, if M is a model of $tr_{GK}(T) \cup tr_{GK}(Atoms)$ such that $\mathbf{K}(M) \subseteq \mathbf{A}(M)$, then $\mathbf{A}(M) = Th(I)$ for some interpretation I and $(I, \mathcal{S})$ is a model of T.

From the above results, if I is a causally explained interpretation of T, then $(I, \{I\})$ is a model of T and there does not exist another model $(I, \mathcal{S})$ such that $\{I\} \subset \mathcal{S}$. We can create a Kripke interpretation M such that $\mathbf{A}(M) = \mathbf{K}(M) = Th(I)$. As $(I, \{I\})$ is a model of T, then M is a model of $tr_{GK}(T)$. There does not exist another model $(I, \mathcal{S})$ such that $\{I\} \subset \mathcal{S}$, then there does not exists another model M' of $tr_{GK}(T)$ such that $\mathbf{A}(M') = Th(I)$ and $\mathbf{K}(M') \subset \mathbf{A}(M')$, thus M is a GK model of $tr_{GK}(T)$, M is a GK model of $tr_{GK}(T) \cup tr_{GK}(Atoms)$.

If M is a GK model of $tr_{GK}(T) \cup tr_{GK}(Atoms)$, then $\mathbf{A}(M) = \mathbf{K}(M) = Th(I)$, where $I = (\mathbf{A}(M) \cap Atoms) \cup (\mathbf{A}(M) \cap \neg Atoms)$. Clearly, $(I, \{I\})$ is a model of T and there does not exist another model $(I, \mathcal{S})$ such that $\{I\} \subset \mathcal{S}$, thus I is a causally explained interpretation of T.

Let us call a GK model M *unary* if for some interpretation I we have that $\mathbf{A}(M) = Th(I)$. Thus by the above theorem, GK theories translated from UCL theories have only unary GK models.

Consider again the simple UCL theory $\{\mathbf{C}p\}$. If $Atoms = \{p\}$, then its GK translation is $\{\mathbf{K}p, \mathbf{A}p \vee \mathbf{A}\neg p\}$. This GK theory has a GK model and if M is such a GK model, then $\mathbf{A}(M) = \mathbf{K}(M) = Th(\{p\})$. However, if $Atoms = \{p, q\}$, then the GK translation is $\{\mathbf{K}p, \mathbf{A}p \vee \mathbf{A}\neg p, \mathbf{A}q \vee \mathbf{A}\neg q\}$, and there is no GK model for this theory as one could not deduce $\mathbf{K}q$ if $\mathbf{A}q$ is assumed, neither could one deduce $\mathbf{K}\neg q$ when $\mathbf{A}\neg q$ is assumed.

4 Conclusion

Logic of GK was proposed as a general framework for nonmonotonic reasoning. Like circumscription [10, 11], it is based on minimization. To date, it has been shown to be able to embed fixed-point nonmonotonic logics such as default logic. In this paper, we show that it can embed Turner's logic of universal causation as well. One potential use of this result is to work as a bridge to connect fixed-point based causal action theories such as McCain and Turner's with minimization-based ones such as that in [6, 8]. However, this remains future work.

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