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'Front propagation into unstable states: Universal algebraic convergence towards uniformly translating pulled fronts

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