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FRW and domain walls in higher spin gravity

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Abstract: We present exact solutions to Vasiliev's bosonic higher spin gravity equations in four dimensions with positive and negative cosmological constant that admit an interpretation in terms of domain walls, quasi-instantons and Friedman-Robertson-Walker (FRW) backgrounds. Their isometry algebras are infinite dimensional higher-spin extensions of spacetime isometries generated by six Killing vectors. The solutions presented are obtained by using a method of holomorphic factorization in noncommutative twistor space and gauge functions. In interpreting the solutions in terms of Fronsdal-type fields in spacetime, a field-dependent higher spin transformation is required, which is implemented at leading order. To this order, the scalar field solves Klein-Gordon equation with conformal mass in $(A)dS_4$. We interpret the FRW solution with de Sitter asymptotics in the context of inflationary cosmology and we expect that the domain wall and FRW solutions are associated with spontaneously broken scaling symmetries in their holographic description. We observe that the factorization method provides a convenient framework for setting up a perturbation theory around the exact solutions, and we propose that the nonlinear completion of particle excitations over FRW and domain wall solutions requires black hole-like states.

Keywords: Cosmology of Theories beyond the SM, Higher Spin Gravity, Higher Spin Symmetry

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1 Introduction

Vasiliev's theory in four dimensions [1] has so far been studied mainly around its maximally symmetric anti-de Sitter vacuum. The perturbations around the anti-de Sitter spacetime describe an unbroken phase of the theory, with spectrum given by infinite towers of massless fields, corresponding to conserved higher spin currents of dual free conformal field theories in three dimensions [2–4]. Higher spin gravity is well known to admit a cosmological term of positive sign and de Sitter vacuum solution as well. It has been proposed that the parity invariant minimal version of higher spin dS_4 gravity is holographically dual to the three dimensional conformal field theory of an Euclidean Sp(N) vector model with anticommuting scalars residing at the boundary of dS_4 at future timelike infinity [5]. For further developments in this direction, see [6–12]. These studies mostly exploit the higher spin symmetries. On the other hand, a detailed bulk description of the early universe physics, including the inflationary era, requires understanding of accelerating solutions of Vasiliev theory and cosmological perturbations around them. Such solutions have isometries forming a subgroup of the de Sitter spacetime symmetries.

Higher spin gauge symmetries can be broken by quantum [13] as well as classical effects. In the latter case, a simple mechanism is to replace the maximally symmetric vacuum by vacua with six Killing symmetries forming a Lie algebra $\mathfrak{g}_6$, as summarized in table 1.[1] These correspond to the isometries of domain walls, FRW-like solutions and quasi-instantons.[2] While we shall leave to a future work an analysis of the the holographic aspects of the exact solutions that we present here, we propose to interpret the domain walls as bulk duals of vacua of three-dimensional massive quantum field theories arising through spontaneous breaking of conformal (higher spin) symmetries; for a relatively recent study of spontaneous breaking of scale invariance in certain CFTs in $D = 3$, see [16].

In this paper, we shall use a solution generating technique [17–19] to build $\mathfrak{g}_6$-invariant solutions to Vasiliev's bosonic theory with non-vanishing (positive or negative) cosmological constant from gauge functions, representing large gauge transformations that alter the asymptotics of the gauge fields, and $\mathfrak{g}_6$-invariant scalar field profiles in the maximally symmetric background. Solutions of Vasiliev's equations with $\mathfrak{g}_3, \mathfrak{g}_4$ and $\mathfrak{g}_6$ symmetries, which are subgroups of the AdS_4 symmetry group, were constructed only at the linearized level in [14] (see [19] for a review) by using a different technique. The fully non-linear solutions presented in this paper are instead obtained by using a different method based on a holomorphic factorization ansatz, and in what we refer to as the holomorphic and L-gauges, described in section 3. In furnishing an interpretation of the solution in terms of Fronsdal-type fields in spacetime, however, a higher spin transformation needs to be implemented order by order in weak fields to reach what we refer to as the Vasiliev gauge, also discussed in section 3. We have implemented this gauge transformation only at leading order in this paper, leaving the computation of higher order terms to a future work. As we shall see in section 5, an important advantage of the method we have used to obtain the exact solutions in the holomorphic gauge is the validity of linear superposition principle in constructing solutions, thus facilitating the description of fluctuations around an exact solution. Even though we leave to future work the analysis of a cosmological perturbation theory around our solutions, an inspection of the star product algebra among the master field will lead us to propose that the nonlinear completion of particle excitations over FRW and domain wall solutions requires black hole-like states (see [18] for the study of scalar particle fluctuations over higher-spin black hole modes).

[1] The various vacua possess unbroken higher spin symmetries; the unbroken symmetry algebra of the $\mathfrak{g}_6$-invariant vacua is the intersection of the enveloping algebra of $\mathfrak{g}_6$ with the unbroken symmetry algebra of the maximally symmetric vacua (see [14] for the case of an $\mathfrak{so}(1,3)$-invariant solution), which is given by the quotient of the enveloping algebra of the $(A)dS_4$ Killing symmetry algebra over the two-sided ideal given by the singleton annihilator.

[2] The quasi-instantons are Lorentzian counter parts of instantons, which can be viewed as the results of gluing together a domain wall and a FRW-like geometry [14, 15].

Among all solutions we have found, we shall, in particular, take a closer look at the FRW-like solution with $\mathfrak{iso}(3)$ symmetry and positive cosmological constant. We will provide a perturbative procedure for obtaining the solutions in the Vasiliev gauge mentioned above, to any order in a suitable perturbation parameter that breaks the de Sitter symmetry to $\mathfrak{iso}(3)$. On the solutions, the scalar field, whose value is vanishing in de Sitter vacuum, is turned on at first order in the symmetry-breaking parameter, and the metric gets corrected at the second order. Moreover, at linear order the fields with spins $s > 2$ vanish in the background solution. Whether they arise in higher orders remains to be determined. At linear order the scalar field behaves similarly to a conformally coupled scalar field in dS_4. In section 5, we shall compare its behaviour with that of the inflaton in the standard cosmological scenarios.

The FRW-like solutions are intriguing because if higher spin fluctuation fields are suppressed by the background, then they may yield cosmologically viable models based on Vasiliev's theory, opening up a new window for embedding the standard models of particles and cosmology into higher spin theory, which may be viewed as the unbroken phase of string theory in which the string is tensionless [2, 21–24]. This setting will inevitably involve the coupling of an infinite number of (massive) higher spin multiplets. One may envisage a scenario in which their presence will play a role in the resolution of the initial singularity, and near the end or after the inflation when the breaking of higher spin symmetry is expected to take place. A much bolder proposal would be the consideration of only massless higher spin theory with its matter couplings furnished through the Konstein-Vasiliev or supersymmetric extension of Vasiliev theory (see [26] for a survey). Such a proposal is motivated by the high degree of symmetry that may yield a UV finite theory, and by the availability of a mechanism [13] for breaking of higher spin symmetries by quantum effects without the need to introduce fields other than those present in the theory, whose spectrum consists of the two-fold product of the singleton representation of the AdS_4 group. Thus it is natural to consider the (matter coupled) higher spin theory as the candidate for a tensionless limit of string theory, in which all the massive trajectories are decoupled completely, and to investigate its consequences for the early universe physics. There are very powerful no-go theorems that forbid accelerating spacetimes in string theory in its tensile phase (see [27] and references therein), inviting the considerations of non-perturbative and string loop effects in a full-fledged formulation of string field theory, and finding its vacuum solutions. On the other hand, higher spin theory can be viewed as a much simpler version of string field theory, in which finding asymptotically de Sitter vacua is a more amenable problem.

The introduction of matter and higher spin symmetry breaking remain a largely uncharted terrain. These aspects are expected to play key roles either for reheating in an inflationary scenario or an analogous mechanisms in non-inflationary scenarios. In the simplest inflation model in standard cosmology, Einstein gravity and a single real scalar field with a suitable potential dominate the early inflationary phase. Here we instead envisage a scenario in which the Einstein plus scalar system is replaced by the bosonic Vasiliev higher spin theory, which consists of a coupled set of massless fields with all integer spins $s = 0, 1, 2, 3, \ldots \infty$. One can then try to employ the well-known mechanism whereby

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rapidly inflated fluctuation modes with wavelengths larger than the Hubble length freeze and subsequently re-enter the cosmological horizon after inflation has ended. Assuming that higher spin symmetry breaking and reheating take place at around the same time, one can compute the effects of higher spin fluctuations on the CMB observations at large scales. In these scenarios, it is important to keep in mind that while the higher spin modes may dissipate in time, their couplings to and mixing with the gravitational field may have observable effects. Some studies have already been done along these lines, see e.g. [28–30], but based on assumptions on higher spin dynamics not born out of Vasiliev's theory. Let us also note that the analog of the $\mathfrak{so}(1,3)$ invariant solution, referred to as the "instanton" solution in table 1, was obtained as an exact solution for $\Lambda < 0$ in [15] and for $\Lambda > 0$ as well in [20]. In the case of $\Lambda < 0$, a cosmological implications of the solution has been discussed in [15] where it has been argued that it leads to a bouncing cosmology, in some respects reminiscent of the work of [31] based on supergravity considerations.[3]

This paper is organized as follows: in section 2, we review Vasiliev's higher spin gravity equations. They are formulated in terms of master one-form A and master Weyl zero-form Φ which live on a base manifold $\mathcal{X}_4 \times \mathcal{Z}_4$ with coordinates $(x^\mu, Z^{\underline{\alpha}})$ where $\mathcal{Z}_4$ is a non-commutative real four manifold. The master fields also depend on the coordinates of the fiber space $\mathcal{Y}_4$ with coordinates $Y^{\underline{\alpha}}$. In section 3, we describe the construction of the exact solutions with $\mathfrak{g}_6$ symmetries. For the reader's convenience we summarize the solutions here. The master fields are the zero-form Φ and one-form A whose components are displayed in (2.11). In holomorphic gauge, Φ' is given in table 1, and A'_α in (2.88) and (2.93). In the L-gauge, $\Phi^{(L)}$ is given in (3.53) and (3.12), $A^{(L)}_\alpha$ is given in (3.54) and (3.61), and $W^{(L)}_\mu$ in (3.64). In Vasiliev gauge, the linear order results for $\Phi^{(G,1)}$ is given in (3.66), $A^{(G,1)}_\alpha$ in (3.78) and $A^{(G,1)}_\mu$ is given by $(A)dS$ connection with a detailed discussion of G-gauge transformations given in 3.4.3 and appendix D. In section 4, we examine the regularity of the Weyl zero-form. The scalar field profiles $\phi(x)$ are described in a unified manner by using stereographic coordinate system. In studying their regularity, one needs to distinguish between singularities that are gauge artifacts and genuine singularities in the full (x, Y, Z) space, sometimes referred to as the correspondence space. To this end, one needs to study the solution $\Phi(x, Y, Z)$ for the Weyl zero-form, and associated higher spin invariant and the on-shell conserved zero-form charges, as we shall discuss further in section 4. In section 5, we take a closer look at the $\mathfrak{iso}(3)$ invariant solution and compare with the standard cosmological backgrounds. In section 6, we summarize our results and comment on future directions. Frequently used symbols and notation are summarized in appendix A. Various coordinates systems used to describe $(A)dS$ and the associated Killing vectors are given in appendix B. The gauge functions used in the construction of the exact solutions are described in appendix C. Details of the passage to Vasiliev gauge in leading order are given in appendix D, and useful formula in the description of twistor space distributions and the star products of relevant projector operators are provided in appendix E.

[3]An analogue Lorentz-invariant instanton solutions, with additional twisted sectors of the theory excited, and the characteristic extra deformation parameter λ that allows to vary the mass of the scalar, was also found for the Prokushkin-Vasiliev theory in $D = 3$ in [32], where also its cosmological interpretation was discussed and its holographic study initiated.

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2 Bosonic Vasiliev model

In what follows, we review the basic properties of Vasiliev's equations [35] and their classical solution spaces, including boundary conditions in spacetime and twistor space suitable for asymptotically (anti-)de Sitter solutions. For a recent review of the exact solutions see [19].

2.1 Review of the full equations of motion

2.1.1 Non-commutative space

Vasiliev's theory is formulated in terms of horizontal forms on a non-commutative fibered space $\mathcal{C}$ with four-dimensional non-commutative symplectic fibers and eight-dimensional base manifold equipped with a non-commutative differential Poisson structure. On the total space, the differential form algebra $\Omega(\mathcal{C})$ is assumed to be equipped with an associative degree preserving product $\star$, a differential d, and an Hermitian conjugation operation $\dagger$, that are assumed to be mutually compatible in the sense that if $f, g, h \in \Omega(\mathcal{C})$, then

$$(f \star g) \star h = f \star (g \star h)\,, \tag{2.1}$$

$$d(df) = 0\,, \qquad d(f \star g) = (df) \star g + (-1)^{|f|} f \star (dg)\,, \tag{2.2}$$

$$(df)^\dagger = d(f^\dagger)\,, \qquad (f \star g)^\dagger = (-1)^{|f||g|}(g^\dagger) \star (f^\dagger)\,, \tag{2.3}$$

where $|f|$ denotes the form degree of f. We shall also assume that[4]

$$(f^\dagger)^\dagger = f\ . \tag{2.4}$$

It is furthermore assumed that $\Omega(\mathcal{C})$ contains a horizontal subalgebra, $\Omega_{\rm hor}(\mathcal{C})$, consisting of equivalence classes defined using a globally defined closed and central hermitian top-form on the fiber space, and whose product, differential and hermitian conjugation operation we shall denote by $\star$, d and $\dagger$ as well.

The base manifold is assumed to be the direct product of a commuting real four-manifold $\mathcal{X}_4$ with coordinates x^μ, and a non-commutative real four-manifold $\mathcal{Z}_4$ with coordinates $Z^{\underline{\alpha}}$; the fiber space and its coordinates are denoted by $\mathcal{Y}_4$ and $Y^{\underline{\alpha}'}$, respectively. The non-commutative coordinates are assumed to obey

$$[Y^{\underline{\alpha}'}, Y^{\underline{\beta}'}]_\star = 2iC^{\underline{\alpha'\beta'}}\,, \qquad [Z^{\underline{\alpha}}, Z^{\underline{\beta}}]_\star = -2iC^{\underline{\alpha\beta}}\,, \qquad [Y^{\underline{\alpha}}, Z^{\underline{\beta}'}]_\star = 0\,, \tag{2.5}$$

and the differential Poisson structure is assumed to be trivial in the sense that

$$[Y^{\underline{\alpha}'}, dY^{\underline{\beta}}]_\star = [Z^{\underline{\alpha}}, dY^{\underline{\beta}'}]_\star = [Z^{\underline{\alpha}}, dZ^{\underline{\beta}}]_\star = [Z^{\underline{\alpha}}, dZ^{\underline{\beta}}]_\star = 0\ . \tag{2.6}$$

The star product is defined in (A.1). The non-commutative space is furthermore assumed to have a compatible complex structure, such that

$$Y^{\underline{\alpha}'} = (y^{\alpha'}, \bar{y}^{\dot{\alpha}'})\,, \qquad Z^{\underline{\alpha}} = (z^\alpha, \bar{z}^{\dot{\alpha}})\,, \tag{2.7}$$

$$(y^{\alpha'})^\dagger = \bar{y}^{\dot{\alpha}'}\,, \qquad (z^\alpha)^\dagger = -\bar{z}^{\dot{\alpha}}\,, \tag{2.8}$$

[4] More generally, $\dagger \circ \dagger$ can be an automorphism of $\Omega(\mathcal{C})$, which is of relevance, for example, in the case of models in de Sitter signature with fermions.

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where the complex doublets obey

$$[y^{\alpha'}, y^{\beta'}]_\star = 2i\epsilon^{\alpha'\beta'}\,, \qquad [z^\alpha, z^\beta]_\star = -2i\epsilon^{\alpha\beta}\,. \tag{2.9}$$

The horizontal forms can be represented as sets of locally defined forms on $\mathcal{X}_4 \times \mathcal{Z}_4$ valued in oscillator algebras $\mathcal{A}(\mathcal{Y}_4)$ generated by the fiber coordinates glued together by transition functions. Assuming the latter to be defined locally on $\mathcal{X}_4$ yields a bundle over $\mathcal{X}_4$ with fibers given by the differential graded associative algebra $\Omega(\mathcal{Z}_4) \otimes \mathcal{A}(\mathcal{Y}_4)$, whose elements can be given represented using symbols defined using various ordering schemes, which correspond to choosing different bases for the operator algebra. In what follows, we shall assume that it is possible to describe the field configurations using symbols defined in the Weyl ordered basis, which is manifestly $\mathrm{Sp}(4;\mathbb{R}) \times \mathrm{Sp}(4;\mathbb{R})'$ invariant, as well as the normal ordered basis consisting of monomials in[5]

$$a^{\underline{\alpha}} := Y^{\underline{\alpha}} + Z^{\underline{\alpha}}\,, \qquad b^{\underline{\alpha}} := Y^{\underline{\alpha}} - Z^{\underline{\alpha}}\,, \tag{2.10}$$

with a- and b-oscillators standing to the left and right, respectively, which breaks $\mathrm{Sp}(4;\mathbb{R}) \times \mathrm{Sp}(4;\mathbb{R})' \to (\mathrm{Sp}(4;\mathbb{R}) \times \mathrm{Sp}(4;\mathbb{R})')_{\mathrm{diag}}$. Equivalently, we shall assume that the elements in $\Omega(\mathcal{Z}_4) \otimes \mathcal{A}(\mathcal{Y}_4)$ have well-defined symbols in normal order, which can be composed using the Fourier transformed twisted convolution formula (A.1), and that they can furthermore be expanded over the Weyl ordered basis of $\mathcal{A}(\mathcal{Y}_4)$ with coefficients in $\Omega(\mathcal{Z}_4)$, using the aforementioned star product.

As for the fiber algebra $\mathcal{A}(\mathcal{Y}_4)$, it is assumed to be an associative algebra closed under the star product and the hermitian conjugation operation $\dagger$ defined above. As we shall describe in more detail in sections 2.4 and 5.3, the algebra $\mathcal{A}(\mathcal{Y}_4)$ will furthermore be assumed to contain certain nonpolynomial elements and distributions playing a role in constructing higher spin background and fluctuation fields.[6]

2.1.2 Master fields

The model is formulated in terms of a zero-form Φ, a one-form

$$A = dx^\mu A_\mu + dz^\alpha A_\alpha + d\bar{z}^{\dot\alpha} A_{\dot\alpha}\,, \tag{2.11}$$

and a non-dynamical holomorphic two-form

$$J := -\frac{ib}{4} dz^\alpha \wedge dz_\alpha \kappa\,, \tag{2.12}$$

with Hermitian conjugate $\overline{J} = (J)^\dagger$, where b is a complex parameter and

$$\kappa := \kappa_y \star \kappa_z\,, \qquad \kappa_y := 2\pi\delta^2(y)\,, \qquad \kappa_z := 2\pi\delta^2(z)\,, \tag{2.13}$$

[5] The normal order reduces to Weyl order for elements that are independent of either Y or Z.

[6] In order to construct higher spin invariants playing a role as classical observables, the algebra needs to be furthermore equipped with a trace operation that provides it with a Hilbert space structure or other suitable inner product structure.

are inner Klein operators obeying

$$\kappa_y \star f \star \kappa_y = \pi_y(f)\,, \qquad \kappa_z \star f \star \kappa_z = \pi_z(f)\,, \tag{2.14}$$

for any zero-form f, where π_y and π_z are the automorphisms of $\Omega(\mathcal{Z}_4) \otimes \mathcal{A}(\mathcal{Y}_4)$ defined in Weyl order by

$$\pi_y : (x; z, \bar{z}; y, \bar{y}) \mapsto (x; z, \bar{z}; -y, \bar{y})\,, \qquad \pi_z : (x; z, \bar{z}; y, \bar{y}) \mapsto (x; -z, \bar{z}; y, \bar{y})\,, \tag{2.15}$$

and $\pi_y \circ d = d \circ \pi_y$ and $\pi_z \circ d = d \circ \pi_z$. It follows that

$$dJ = 0\,, \qquad J \star f = \pi(f) \star J\,, \qquad \pi(J) = J\,, \qquad \pi := \pi_y \circ \pi_z\,, \tag{2.16}$$

for any form f, *idem* $\overline{J}$ and $\bar{\pi} := \pi_{\bar{y}} \circ \pi_{\bar{z}}$.

2.1.3 Kinematic conditions

Higher spin gravities consisting of Lorentz tensor gauge fields can be obtained by imposing the integer-spin projection

$$\pi \circ \bar{\pi}(\Phi) = \Phi\,, \qquad \pi \circ \bar{\pi}(A) = A\;.$$

Models in Lorentzian spacetimes with cosmological constants Λ are obtained by imposing reality conditions as follows [20]:

$$\rho(\Phi^\dagger) = \pi(\Phi)\,, \qquad \rho(A^\dagger) = -A\,, \qquad \rho := \begin{cases} \pi\,, & \Lambda > 0\,, \\ \mathrm{Id}\,, & \Lambda < 0\,, \end{cases} \tag{2.17}$$

that is, the real form of the $\mathfrak{sp}(4;\mathbb{C})$ realized in terms of bilinears in $Y^{\underline{\alpha}}$ is chosen by the Hermitian conjugation operation $\rho \circ \dagger$; the consistency follows from $\bar{\pi}(f) = (\pi(f^\dagger))^\dagger$ and the fact that if $\Lambda > 0$, then $(\rho \circ \dagger)^2 \equiv \pi \circ \bar{\pi}$, which reduces to the identity modulo the integer-spin projection.

2.1.4 Equations of motion

Introducing the curvature and twisted-adjoint covariant derivative defined by

$$F := dA + A \star A\,, \qquad D\Phi := d\Phi + [A, \Phi]_\pi\,, \tag{2.18}$$

respectively, one has the Bianchi identities

$$DF := dF + [A, F]_\star \equiv 0\,, \qquad DD\Phi := d(D\Phi) + [A, D\Phi]_\pi \equiv [F, \Phi]_\pi\,, \tag{2.19}$$

where ordinary and π-twisted star commutators

$$[f, g]_\star := f \star g - (-1)^{|f||g|} g \star f\,, \qquad [f, g]_\pi := f \star g - (-1)^{|f||g|} g \star \pi(f)\,, \tag{2.20}$$

respectively. The Vasiliev equations of motion are given by

$$F + \Phi \star (J - \overline{J}) = 0\,, \qquad D\Phi = 0\,, \tag{2.21}$$

which are compatible with the kinematic conditions and the Bianchi identities, implying that the classical solution space is invariant under the following infinitesimal gauge transformations:

$$\delta A = D\epsilon := d\epsilon + [A,\epsilon]_\star\,, \qquad \delta\Phi = -[\epsilon,\Phi]_\pi\,, \tag{2.22}$$

for parameters obeying the same kinematic conditions as the connection, *viz.*

$$\pi\bar\pi(\epsilon) = \epsilon\,, \qquad \rho(\epsilon^\dagger) = -\epsilon\;. \tag{2.23}$$

2.1.5 Component form

Decomposition of the equations of motion under the coordinate basis $(\vec\partial_\mu, \vec\partial_{\underline\alpha})$ for the tangent space of the base manifold, yields the Vasiliev equations

$$F_{\mu\nu} = 0\,, \qquad D_\mu\Phi = 0\,, \qquad F_{\mu\underline\alpha} = 0\,, \qquad D_{\underline\alpha}\Phi = 0\,, \tag{2.24}$$

$$F_{\alpha\beta} + \frac{ib}{2}\,\Phi\star\kappa\epsilon_{\alpha\beta} = 0\,, \qquad F_{\alpha\dot\beta} = 0\,, \qquad F_{\dot\alpha\dot\beta} + \frac{i\bar b}{2}\,\Phi\star\bar\kappa\epsilon_{\dot\alpha\dot\beta} = 0\,, \tag{2.25}$$

$$D_\mu\Phi = \imath_{\vec\partial_\mu} D\Phi = \partial_\mu\Phi + A_\mu\star\Phi - \Phi\star\pi(A_\mu)\,, \tag{2.26}$$

$$D_\alpha\Phi = \imath_{\vec\partial_\alpha} D\Phi = \partial_\alpha\Phi + A_\alpha\star\Phi - \Phi\star\bar\pi(A_\alpha)\,, \tag{2.27}$$

$$D_{\dot\alpha}\Phi = \imath_{\vec\partial_{\dot\alpha}} D\Phi = \partial_{\dot\alpha}\Phi + A_{\dot\alpha}\star\Phi - \Phi\star\pi(A_{\dot\alpha})\,, \tag{2.28}$$

using $\pi(A) = \bar\pi(A)$, $\bar\pi(dz^\alpha) = dz^\alpha$ and $\pi(d\bar z^{\dot\alpha}) = d\bar z^{\dot\alpha}$, and the one-form components obey the following kinematic conditions:

$$\pi\bar\pi(A_\mu, A_\alpha, A_{\dot\alpha}) = (A_\mu, -A_\alpha, -A_{\dot\alpha})\,, \tag{2.29}$$

$$(A_\mu, A_\alpha, A_{\dot\alpha})^\dagger = \begin{cases} (-\pi(A_\mu), \pi(A_{\dot\alpha}), \bar\pi(A_\alpha))\,, & \Lambda > 0\,,\\ (-A_\mu, A_{\dot\alpha}, A_\alpha)\,, & \Lambda < 0\;. \end{cases} \tag{2.30}$$

2.1.6 Deformed oscillators

Alternatively, introducing

$$S_\alpha := z_\alpha - 2iA_\alpha\,, \qquad S_{\dot\alpha} = z_{\dot\alpha} - 2iA_{\dot\alpha}\,, \tag{2.31}$$

the equations of motion involving twistor-space derivatives can be written as

$$D_\mu S_\alpha = 0\,, \qquad\qquad D_\mu S_{\dot\alpha} = 0\,, \tag{2.32}$$

$$[S_\alpha, S_\beta]_\star = -2i\epsilon_{\alpha\beta}(1 - b\Phi\star\kappa)\,, \quad [S_\alpha, S_{\dot\beta}]_\star = 0\,, \quad [S_{\dot\alpha}, S_{\dot\beta}]_\star = -2i\epsilon_{\dot\alpha\dot\beta}(1 - \bar b\Phi\star\bar\kappa)\,, \tag{2.33}$$

$$S_\alpha\star\Phi + \Phi\star\pi(S_\alpha) = 0\,, \qquad\qquad S_{\dot\alpha}\star\Phi + \Phi\star\bar\pi(S_{\dot\alpha}) = 0\,, \tag{2.34}$$

that is, the master fields $(S_\alpha, S_{\dot\alpha})$ define a covariantly constant set of Wigner-deformed oscillators with deformation parameter given by Φ. The deformed oscillators obey reality conditions and integer-spin conditions as follows:

$$\pi\bar{\pi}(S_\alpha, S_{\dot{\alpha}}) = (-S_\alpha, -S_{\dot{\alpha}})\,, \tag{2.35}$$

$$(S_\alpha, S_{\dot{\alpha}})^\dagger = \begin{cases} (-S_{\dot{\alpha}}, -S_\alpha) & \text{for } \Lambda < 0\,, \\ (-\pi(S_{\dot{\alpha}}), -\bar{\pi}(S_\alpha)) & \text{for } \Lambda > 0\,. \end{cases} \tag{2.36}$$

Besides being useful in constructing exact solutions, observables and exhibiting certain discrete symmetries, the deformed oscillators facilitate the casting of the equations of motion into a manifestly Lorentz covariant form.

2.1.7 Discrete symmetries

The equations of motion and the gauge transformations exhibit the following discrete symmetries:

i) Holomorphic parity transformation

$$(\Phi, A; \epsilon) \mapsto (\pi(\Phi), \pi(A); \pi(\epsilon))\;; \tag{2.37}$$

ii) Deformed oscillator parity transformation

$$(\Phi, A_\mu, S_{\underline{\alpha}}; \epsilon) \mapsto (\Phi, A_\mu, -S_{\underline{\alpha}}; \epsilon)\,, \tag{2.38}$$

which is equivalent to $A_{\underline{\alpha}} \mapsto -iZ_{\underline{\alpha}} - A_{\underline{\alpha}}$;

iii) *Vectorial parity transformation*

$$(\Phi, A; \epsilon) \mapsto (P(\Phi), P(A); P(\epsilon))\,, \tag{2.39}$$

where P is the star product algebra automorphism

$$P(y^\alpha, \bar{y}^{\dot{\alpha}}; z^\alpha, \bar{z}^{\dot{\alpha}}) := (\bar{y}^{\dot{\alpha}}, y^\alpha; \bar{z}^{\dot{\alpha}}, z^\alpha)\,, \qquad d \circ P := P \circ d\,, \tag{2.40}$$

from which it follows that $P \circ \dagger = \dagger \circ P$ and $P \circ \pi = \bar{\pi} \circ P$.

From

$$P(J) = -(b/\bar{b})\,\overline{J}\,, \tag{2.41}$$

it follows that (iii) exchanges a solution to the equations with parameter b to a solution to the equations with parameter $\bar{b}$. In particular, if $\bar{b} = \pm b$, then one can extend P to

$$\widehat{P} = P \circ P'\,, \tag{2.42}$$

where P' is an internal parity map acting on the component fields, and project the spectrum of the theory by demanding

$$\widehat{P}(A, \Phi) = \begin{cases} (A, \Phi) & b = 1 \quad (\text{A model})\,, \\ (A, -\Phi) & b = i \quad (\text{B model})\,, \end{cases} \tag{2.43}$$

which thus correlates the internal parity with the vectorial parity in twistor space.

2.1.8 Manifest Lorentz covariance

To cast the equations on a manifestly Lorentz covariant form, one introduces the field-dependent generators [35, 36]

$$M^{(\mathrm{tot})}_{\alpha\beta} := y_{(\alpha}\star y_{\beta)} - z_{(\alpha}\star z_{\beta)} + S_{(\alpha}\star S_{\beta)}\,, \qquad M^{(\mathrm{tot})}_{\dot\alpha\dot\beta} := \bar y_{(\dot\alpha}\star \bar y_{\dot\beta)} - \bar z_{(\dot\alpha}\star \bar z_{\dot\beta)} + S_{(\dot\alpha}\star S_{\dot\beta)}\,, \tag{2.44}$$

and redefines

$$A_\mu = W_\mu + \frac{1}{4i}\left(\omega_\mu^{\alpha\beta}M^{(\mathrm{tot})}_{\alpha\beta} + \omega_\mu^{\dot\alpha\dot\beta}M^{(\mathrm{tot})}_{\dot\alpha\dot\beta}\right)\,, \tag{2.45}$$

where $(\omega_\mu^{\alpha\beta}(x), \omega_\mu^{\dot\alpha\dot\beta}(x))$ is a bona fide canonical Lorentz connection on $\mathcal{X}_4$, after which the equations of motion involving spacetime derivatives can be re-written on the following manifestly Lorentz covariant form[7] [17, 37, 38]:

$$\nabla W + W\star W + \frac{1}{4i}\left(r^{\alpha\beta}M^{(\mathrm{tot})}_{\alpha\beta} + r^{\dot\alpha\dot\beta}M^{(\mathrm{tot})}_{\dot\alpha\dot\beta}\right) = 0\,, \tag{2.46}$$

$$\nabla\Phi + W\star\Phi - \Phi\star\pi(W) = 0\,, \qquad \nabla S_\alpha + [W, S_\alpha]_\star = 0\,, \tag{2.47}$$

where

$$\nabla W := dW + [\omega^{(0)}, W]_\star\,, \qquad \nabla\Phi := d\Phi + [\omega^{(0)}, \Phi]_\star\,, \tag{2.48}$$

$$\nabla S_\alpha := dS_\alpha - \omega_\alpha{}^\beta S_\beta + [\omega^{(0)}, S_\alpha]_\star\,, \tag{2.49}$$

$$r^{\alpha\beta} := d\omega^{\alpha\beta} - \omega^{\alpha\gamma}\wedge\omega_\gamma{}^\beta\,, \qquad r^{\dot\alpha\dot\beta} := d\omega^{\dot\alpha\dot\beta} - \omega^{\dot\alpha\dot\gamma}\wedge\omega_{\dot\gamma}{}^{\dot\beta}\,, \tag{2.50}$$

with

$$\omega^{(0)} := \frac{1}{4i}\left(\omega^{\alpha\beta}M^{(0)}_{\alpha\beta} + \omega^{\dot\alpha\dot\beta}M^{(0)}_{\dot\alpha\dot\beta}\right)\,, \tag{2.51}$$

$$M^{(0)}_{\alpha\beta} := y_{(\alpha}\star y_{\beta)} - z_{(\alpha}\star z_{\beta)}\,, \qquad M^{(0)}_{\dot\alpha\dot\beta} := \bar y_{(\dot\alpha}\star \bar y_{\dot\beta)} - \bar z_{(\dot\alpha}\star \bar z_{\dot\beta)}\,. \tag{2.52}$$

The Lorentz connection is defined, as usual, up to tensorial shifts, that can be fixed by requiring that the projection of W onto $M^{(0)}_{\alpha\beta}$ vanish at $Z = 0$.

2.2 Vacuum solutions

Flat connections. The equations of motion admit solutions

$$\Phi = 0\,, \qquad A = \Omega\,, \tag{2.53}$$

where Ω is a locally defined one-form on $\mathcal{X}_4\times\mathcal{Z}_4$ valued in $\mathcal{A}(\mathcal{Y}_4)$ that is flat, *viz.*

$$d\Omega + \Omega\star\Omega = 0\,. \tag{2.54}$$

If $\Omega\in\Omega(\mathcal{X}_4)\otimes\mathcal{A}(\mathcal{Y}_4)$, then there exists locally defined gauge functions L on $\mathcal{X}_4$ such that

$$\Omega = L^{-1}\star dL\,, \tag{2.55}$$

[7] The closure of the algebra generated by $M^{(\mathrm{tot})}$ contains additional Lorentz transformations on acting on the component fields; for details, see [37, 39].

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that we shall refer to as vacuum connections, as they preserve higher symmetries with rigid parameters

$$\epsilon = L^{-1} \star \epsilon' \star L\,, \qquad d\epsilon' = 0\,, \qquad \epsilon' \in \mathcal{A}(\mathcal{Y}_4)\;; \tag{2.56}$$

the space $\Omega(\mathcal{Z}_4) \otimes \mathcal{A}(\mathcal{Y}_4)$, on the other hand, contains flat connections constructed from projector algebras that cannot be described using gauge functions and that break some of the vacuum symmetries [14].

Maximally symmetric spaces. The $(A)dS_4$ vacua are described by gauge functions valued in the real form G_{10} of $\mathrm{Sp}(4;\mathbb{C})$ selected by the reality condition introduced above. Thus, G_{10} refers to AdS group for $\lambda^2 > 0$ and dS group for $\lambda^2 < 0$, with the commutation rules for the $\mathfrak{g}_{10}$ algebra given by

$$[M_{AB}, M_{CD}] = 4i\eta_{[C|[B}M_{A]|D]}\,, \qquad \eta_{AB} := \left(\eta_{ab}, -\mathrm{sign}(\lambda^2)\right)\,, \qquad \eta_{ab} = \mathrm{diag}(-+++)\,. \tag{2.57}$$

and they can be realized in terms of the Y-oscillators as

$$-\ell^{-1}M_{a5} \equiv P_a = \frac{\lambda}{4}\,(\sigma_a)_{\alpha\dot\beta}\,y^\alpha \bar y^{\dot\beta}\,, \quad M_{ab} = -\frac{1}{8}\left[(\sigma_{ab})_{\alpha\beta}\,y^\alpha y^\beta + (\bar\sigma_{ab})_{\dot\alpha\dot\beta}\,\bar y^{\dot\alpha}\bar y^{\dot\beta}\right]\,, \tag{2.58}$$

where

$$\lambda = \begin{cases} \ell^{-1} & \text{for } \Lambda < 0 \\ i\ell^{-1} & \text{for } \Lambda > 0\,, \end{cases} \tag{2.59}$$

It follows that

$$[M_{ab}, M_{cd}]_\star = i\eta_{bc}M_{ad} + 3\text{ more}\,, \qquad [M_{ab}, P_c]_\star = 2i\eta_{c[b}P_{a]}\,, \tag{2.60}$$

$$[P_a, P_b]_\star = i\lambda^2 M_{ab} \tag{2.61}$$

with reality conditions

$$\rho((P_a)^\dagger) = P_a\,, \qquad \rho((M_{ab})^\dagger) = (M_{ab})^\dagger = M_{ab}\,. \tag{2.62}$$

Introducing coset elements

$$L : G_{10}/\mathrm{SO}(1,3) \to G_{10}\,, \qquad \rho(L^\dagger) = L^{-1}\,, \tag{2.63}$$

the Maurer-Cartan form decomposes into a frame field and a Lorentz connection as follows:

$$\Omega = \frac{1}{4i}\Omega_{\underline{\alpha\beta}}Y^{\underline{\alpha}}Y^{\underline{\beta}} = \frac{1}{4i}\left(2\Omega_{\alpha\dot\alpha}y^\alpha\bar y^{\dot\alpha} + \Omega_{\alpha\beta}y^\alpha y^\beta + \Omega_{\dot\alpha\dot\beta}\bar y^{\dot\alpha}\bar y^{\dot\beta}\right) = i\Omega_a P^a + \frac{1}{2i}\Omega_{ab}M^{ab} \tag{2.64}$$

where thus

$$\Omega_{\alpha\dot\alpha} = -\frac{\lambda}{2}\,(\sigma_a)_{\alpha\dot\alpha}\,\Omega^a\,, \qquad \Omega_{\alpha\beta} := -\frac{1}{4}\,(\sigma_{ab})_{\alpha\beta}\,\Omega^{ab}\,, \qquad \Omega_{\dot\alpha\dot\beta} = -\frac{1}{4}\,(\bar\sigma_{ab})_{\dot\alpha\dot\beta}\,\Omega^{ab}\,. \tag{2.65}$$

In these bases, the flatness condition reads

$$d\Omega^{\underline{\alpha\beta}} - \Omega^{\underline{\alpha\gamma}} \wedge \Omega_{\underline{\gamma}}{}^{\underline{\beta}} = 0\,, \tag{2.66}$$

that is

$$d\Omega_{\alpha\dot{\alpha}} - \Omega_\alpha{}^\beta \wedge \Omega_{\beta\dot{\alpha}} - \Omega_{\dot{\alpha}}{}^{\dot{\beta}} \wedge \Omega_{\alpha\dot{\beta}} = 0\,, \tag{2.67}$$

$$R_{\alpha\beta} - \Omega_\alpha{}^{\dot{\alpha}} \wedge \Omega_{\dot{\alpha}\beta} = 0\,, \qquad R_{\dot{\alpha}\dot{\beta}} - \Omega_{\dot{\alpha}}{}^{\alpha} \wedge \Omega_{\alpha\dot{\beta}} = 0\,, \tag{2.68}$$

or

$$d\Omega_a + \Omega_a{}^b \wedge \Omega_b = 0\,, \qquad R_{ab} + \lambda^2 \Omega_a \wedge \Omega_b = 0\,, \tag{2.69}$$

where the Riemann two-form

$$\begin{aligned} R^{\alpha\beta} &:= d\Omega^{\alpha\beta} - \Omega^{\alpha\gamma} \wedge \Omega_\gamma{}^\beta = -\frac{1}{4} (\sigma_{ab})^{\alpha\beta} R^{ab}\,, \\ R^{\dot{\alpha}\dot{\beta}} &:= d\Omega^{\dot{\alpha}\dot{\beta}} - \Omega^{\dot{\alpha}\dot{\gamma}} \wedge \Omega_{\dot{\gamma}}{}^{\dot{\beta}} = -\frac{1}{4} (\bar{\sigma}_{ab})^{\dot{\alpha}\dot{\beta}} R^{ab}\,, \end{aligned} \tag{2.70}$$

with $R_{ab} := d\Omega_{ab} + \Omega_a{}^c \Omega_{cb}$.

The full equations of motion can be solved in two dual fashions, one involving normal ordered scheme and perturbatively defined Fronsdal fields, and the other based on a topological field theory approach, which we describe below.

2.3 Normal ordered perturbation scheme

In the normal order, defined by the star product formula (A.1), the inner Klein operators become real analytic in Y and Z space, *viz.*

$$\kappa = \kappa_y \star \kappa_z = \exp(iy^\alpha z_\alpha)\,, \qquad \bar{\kappa} = \kappa_{\bar{y}} \star \kappa_{\bar{z}} = \exp(i\bar{y}^{\dot{\alpha}} \bar{z}_{\dot{\alpha}})\;. \tag{2.71}$$

Assuming that the full field configurations are real-analytic on $\mathcal{Z}_4$ for generic points in $\mathcal{X}_4$, one may thus impose initial conditions

$$\Phi|_{Z=0} = C\,, \qquad A_\mu|_{Z=0} = a_\mu\;. \tag{2.72}$$

Assuming furthermore that $A_{\underline{\alpha}}|_{C=0}$ is a trivial flat connection on $\mathcal{Z}_4$, that one may choose to be $A_{\underline{\alpha}}|_{C=0} = 0$, and choosing a homotopy contractor for the de Rham differential on $\mathcal{Z}_4$, which entails imposing a gauge condition on $A_{\underline{\alpha}}$, one may solve the constraints on $D_{\underline{\alpha}}\Phi$, $F_{\underline{\alpha\beta}}$ and $F_{\underline{\alpha}\mu}$ on $\mathcal{Z}_4$ in a perturbative expansion of the form:

$$\begin{aligned} \Phi &= \sum_{n\geqslant 1} \Phi^{(n)}(C,\ldots,C)\,, & \Phi^{(1)}(C) &\equiv C\,, \\ A_{\underline{\alpha}} &= \sum_{n\geqslant 1} A^{(n)}_{\underline{\alpha}}(C,\ldots,C)\,, & & \\ A_\mu &= \sum_{n\geqslant 0} A^{(n)}(a_\mu; C,\ldots,C)\,, & A^{(0)}(a_\mu) &\equiv a_\mu\,, \end{aligned} \tag{2.73}$$

where $\Phi^{(n)}(C,\ldots,C)$ is an n-linear functional in C *idem* $A^{(n)}_{\underline{\alpha}}(C,\ldots,C)$ and $A^{(n)}(a_\mu; C,\ldots,C)$, and the latter is linear in a_μ. These quantities are real-analytic in $\mathcal{Y}_4 \times \mathcal{Z}_4$ provided that C and a_μ are real analytic in Y-space and all star products arising along the perturbative expansion are well-defined.

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From the Bianchi identities, it follows that the remaining equations, that is, $F_{\mu\nu} = 0$ and $D_\mu \Phi = 0$, are perturbatively equivalent to $F_{\mu\nu}|_{Z=0} = 0$ and $D_\mu\Phi|_{Z=0} = 0$, which form a perturbatively defined Cartan integrable system on $\mathcal{X}_4$ for C and a_μ.

To Lorentz covariantize, one imposes

$$W|_{Z=0} = w\,, \tag{2.74}$$

and substitutes

$$a_\mu = w_\mu + \frac{1}{4i}\,(\omega_\mu^{\alpha\beta} M^{(\text{tot})}_{\alpha\beta} + \omega_\mu^{\dot\alpha\dot\beta} M^{(\text{tot})}_{\dot\alpha\dot\beta})\Big|_{Z=0}\,, \tag{2.75}$$

into $A^{(n)}(a_\mu; C, \dots, C)$. Due to the manifest Lorentz covariance, the quantities $F_{\mu\nu}|_{Z=0}$ and $D_\mu\Phi|_{Z=0}$ depend on the Lorentz connection only via the Lorentz covariant derivative ∇ and the Riemann two-form $(r^{\alpha\beta}, r^{\dot\alpha\dot\beta})$; it follows that

$$\begin{aligned} &\nabla w + \frac{1}{4i}\,(r^{\alpha\beta} M^{(\text{tot})}_{\alpha\beta} + r^{\dot\alpha\dot\beta} M^{(\text{tot})}_{\dot\alpha\dot\beta})\Big|_{Z=0} \\ &+ \sum_{\substack{n_1+n_2 \geqslant 0 \\ n_{1,2}\geqslant 0}} A^{(n_1)}(w; C, \dots, C) \star A^{(n_2)}(w; C, \dots, C) = 0\,, \end{aligned} \tag{2.76}$$

$$\nabla C + \sum_{\substack{n_1+n_2 \geqslant 1 \\ n_1 \geqslant 0,\ n_2 \geqslant 1}} [A^{(n_1)}(w; C, \dots, C), \Phi^{(n_2)}(C, \dots, C)]_\pi = 0\,, \tag{2.77}$$

where w can be chosen to not contain any component field proportional to $y_\alpha y_\beta$ and $\bar y_{\dot\alpha} \bar y_{\dot\beta}$.

Perturbatively defined Fronsdal fields. Expanding the differential algebra around the (anti-)de Sitter vacuum

$$\Phi^{(0)} = 0\,, \qquad A^{(0)} = \Omega\,, \tag{2.78}$$

and assuming that the homotopy contraction in Z-space is performed such that

$$z^{\underline{\alpha}} A^{(1)}_{\underline{\alpha}} = 0\,, \tag{2.79}$$

referred to as the Vasiliev gauge [40], the resulting linearized system on X-space provides an unfolded description of a dynamical scalar field

$$\phi = \Phi\,|_{Y=0=Z}\,, \tag{2.80}$$

and a tower of spin-s Fronsdal fields

$$\phi_{a(s)} = \left(((e^{-1})^\mu_a)^{(0)} \left((\sigma_a)^{\alpha\dot\alpha} \frac{\partial^2}{\partial y^\alpha \partial \bar y^{\dot\alpha}} \right)^{s-1} w_\mu \right)\Bigg|_{Y=0=Z}\,, \tag{2.81}$$

where we use the convention that repeated indices are symmetrized.

At the nonlinear level, the Cartan integrable system on X-space provides a deformation of the equations of motion for these fields, which is consistent as a set of partial differential equations but that depends on the choice of initial data for Φ and W_μ as well as the gauge for A_α (which enters via the homotopy contractor in Z space). Whether there exists a choice that yields a formulation of higher spin gravity in X-space that lends itself to a standard path integral formulation remains an open problem.[8]

2.4 Gauge function method

2.4.1 Topological field theory approach

Alternatively, one may treat the system as an infinite set of topological fields on $\mathcal{X}_4 \times \mathcal{Z}_4$ packaged into master fields valued in $\mathcal{A}(\mathcal{Y}_4)$ represented by symbols in Weyl order, that is, as expansions in terms of the generators of $\mathcal{A}(\mathcal{Y}_4)$ star multiplied by differential forms on $\Omega(\mathcal{X}_4 \times \mathcal{Z}_4)$, referred to as mode forms.

The field configurations are assigned a bundle structure, whereby a projection of A is assumed to define a connection valued in a Lie subalgebra of $\mathcal{A}(\mathcal{Y}_4)$. The complementary projection of A, referred to as the generalized frame field, together with the Weyl zero-form Φ are taken to belong to adjoint and twisted adjoint sections, respectively, over $\mathcal{X}_4 \times \mathcal{Z}_4$, which is treated as a base manifold; the two-forms J and $\overline{J}$ by their definition belong to twisted adjoint sections. The bundle connection is assumed to act faithfully on the sections, and the bundle curvature is assumed to be an adjoint section, as required by the equations of motion.

As for boundary conditions, the base manifold is assumed to be compact, and the sections, i.e. their mode forms, are assumed to be bounded away from a set of marked points representing boundaries. In a generic coordinate chart $U \subset \mathcal{X}_4$, the sections are described by an integration constant for the Weyl zero-form, a flat connection on $\mathcal{Z}_4$ and a gauge function on $U \times \mathcal{Z}_4$. At the marked points, the initial data instead consists of prescribed singularities in the generalized frame field and related fall-off in the Weyl zero-form (including the physical scalar field). For asymptotically (anti-)de Sitter solutions, we take $\mathcal{X}_4$ to have the topology of $S^1 \times S^3$ with a marked S^1, such that, to all orders in classical perturbation theory, the leading terms in the master fields at the marked $S^1 \times \mathcal{Z}_4$ describe a set of free Fronsdal fields, which one may view as a condition at the boundary of $(A)dS_4$ times $\mathcal{Z}_4$. To impose boundary conditions on $\mathcal{Z}_4$, we assume that $\Omega(\mathcal{Z}_4)$

i) is closed under star products, which can be achieved by taking the Fourier transforms of the zero-forms in $\Omega(\mathcal{Z}_4)$ to be L^1 in momentum space (i.e. to be expandable in terms of plane waves that generate a twisted abelian group algebra), which requires the zero-form sections on $\mathcal{Z}_4$ to be bounded at $Z = 0$;

ii) has a graded trace operation given by integration of the top-forms on $\mathcal{Z}_4$, which requires these to fall off at $Z = \infty$ so as to belong to $L^1(\mathcal{Z}_4)$.

Finally, $\mathcal{A}(\mathcal{Y}_4)$ is taken to be a set of operators in a quantum-mechanical system equipped with a (possibly regularized) trace operation that is dual to the boundary conditions at the marked S^1.

[8]To our best understanding, the standard classical Noether procedure breaks down [42], while there exists a quantum effective action in $\mathcal{X}_4$ for asymptotically (anti-)de Sitter boundary conditions. In order to obtain a path integral measure, one may instead follow the approach proposed in [37, 43, 44].

The above geometries can be characterized by functionals, playing the role of classical observables (including on-shell actions), given by combined traces over $\mathcal{A}(\mathcal{Y}_4)$ and integrations over $\mathcal{X}_4 \times \mathcal{Z}_4$ (possibly with insertions of delta functions localized to submanifolds). These gauge transformations that leave these functionals invariant are referred to as proper, or small, gauge transformations, as opposed to large gauge transformations that alter the asymptotics of the fields and hence the value of the observables. The resulting moduli space is thus sliced into (proper) gauge orbits labelled by the observables, each of which defines a microstate of the theory.[9]

2.4.2 Gauge functions

In the topological field theory approach, solution spaces are obtained starting from a reference solution $(\Phi', A') \in \Omega(\{p_0\} \times \mathcal{Z}_4) \otimes \mathcal{A}(\mathcal{Y}_4)$ at a base point $p_0 \in \mathcal{X}_4$, constructed from an integration constant C' for Φ' at, say, $Z = 0$, and an flat connection on $\mathcal{Z}_4$, that we shall trivialize in most of what follows. Moduli associated to the connection and generalized frame field on $\mathcal{X}_4$ are then introduced by means of a large gauge transformation

$$A^{(G)} = G^{-1} \star (A' + d) \star G\,, \qquad \Phi^{(G)} = G^{-1} \star \Phi' \star \pi(G)\,, \qquad G = L \star H\,, \tag{2.82}$$

where L is the vacuum gauge function, and H is a gauge function determined perturbatively by the requirements that

a) in Weyl order, $\Phi^{(G)}$ and the twisted open Wilson lines $V(M) := \exp_\star(iM^{\underline{\alpha}} S^{(G)}_{\underline{\alpha}})$, where $M^{\underline{\alpha}} \in \mathbb{C}^4$ (for details, see [37, 38, 46]), are sections in $\Omega(\mathcal{X}_4 \times \mathcal{Z}_4) \otimes \mathcal{A}(\mathcal{Y}_4)$ in form degree zero; and

b) in normal order, $(\Phi^{(G)}, A^{(G)}_{\underline{\alpha}}, W^{(G)}_\mu)$ asymptote to configurations describing free Fronsdal fields[10] in accordance with the central on mass-shell theorem close to the marked $S^1 \times \mathcal{Z}_4$.

We shall refer refer to *(a)* and *(b)* as dual boundary conditions, as *(a)* requires factorization of the master fields in Weyl order, whereas *(b)* requires normal order. We thus propose to fix $H^{(n)}$ by requiring

i) Manifest Lorentz covariance and real analyticity in Y of the normal ordered symbols of $(\Phi^{(G)}, A^{(G)})$ at the origin of $\mathcal{Y}_4 \times \mathcal{Z}_4$, so that the field configurations are expandable in terms of Lorentz tensorial component fields on $\mathcal{X}_4$ defined by Taylor expansion in Y at $Y = 0 = Z$.

[9] A subset of the classical observables are extensive; keeping these fixed defines a higher spin ensemble consisting of a large number of microstates. In [38, 45], it has been proposed that the extensive variables are the zero-form charges [14, 37], and in [46] it has been proposed that a complete set of classical observables in the case that $\mathcal{X}_4$ has trivial topology is given by the space of twisted open Wilson lines in $\mathcal{Z}_4$. According to these proposals, the rigid symmetries of the vacuum should leave the extensive variables invariant while acting nontrivially on the microscopic variables; for related remarks, see [18, 46].

[10] The full field configurations are thus assumed to contain contain asymptotically (anti-)de Sitter regions where the *full* tensor gauge fields $\phi_{a(s)}$ approach Fronsdal fields give on shell in terms of polarization tensors that are non-linear functionals of the zero-form initial data C.

ii) the Weyl ordered symbols of $(\Phi^{(G)}, V(M))$ to be traceable over $\Omega(\mathcal{Z}_4) \otimes \mathcal{A}(\mathcal{Y}_4)$, for there to exist higher spin invariants playing the role of classical observables;

iii) Perturbatively stable asymptotic Fronsdal fields in weak-coupling regions of $\mathcal{X}_4$ (where the Weyl zero-form goes to zero), for the classical observables to admit perturbative expansions in terms of parameters related to sources for weakly coupled higher spin gauge fields.

The following additional remarks are in order:

1. *Zig-zagging self-consistency:* at nth order, the quantity $\Phi^{(G,n)}$ is a functional of $H^{(n')}$ with $1 \leqslant n' \leqslant n-1$ and initial data $C'^{(n')}$ with $1 \leqslant n' \leqslant n$, which means that condition (a), which must hold for finite Z, is in effect a non-trivial admissibility condition on the Y-dependence of the initial data C', i.e. on $\mathcal{A}(\mathcal{Y}_4)$.

2. *Residual small gauge transformations:* the above conditions do not determine the $\mathfrak{hs}_1(4)$ part of $H^{(n)}$, which is real analytic in Y, and which can thus be used for small gauge transformations inside the bulk.

3. *Deformed oscillators:* although the master fields $S^{(G)}$ are not sections, one can require that $\Phi^{(G)}$ and the twisted open Wilson loops $V(M))$ form an associative algebra with traces, which can be use to construct a complete set of higher spin invariant observables that one may think of as substitutes for the standard ADM-like charges that can be used to define higher spin ensembles in unbroken phases; for further details, see [18].

4. *Residual symmetries:* the full solution $(\Phi^{(G)}, A^{(G)})$ is left invariant under gauge transformations with parameters

$$\epsilon^{(G)} = G^{-1} \star \epsilon' \star G\,, \tag{2.83}$$

where ϵ' are constant parameters stabilizing Ψ, *viz.*

$$[\epsilon', \Psi]_\star = 0\ . \tag{2.84}$$

Conversely, given a set of symmetries forming a Lie algebra $\mathfrak{g}$, spaces of $\mathfrak{g}$-invariant solutions can be found by solving the linear constraint (2.84) on Ψ together with the conditions that Ψ belongs to an associative algebra that is left invariant under star multiplication by the inner Klein operators, i.e. $\Psi \star \kappa_y$ and $\Psi \star \Psi$ should belong to the algebra, which is the approach that we shall employ.

In summary, the dual boundary conditions are physically well-motivated and non-trivial; in this paper, we shall focus on their implementation at the linearized level, leaving higher orders, starting with the issue of whether $\Phi^{(G,2)}$ obeys (a), for a forthcoming publication including various types of boundary conditions.

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2.4.3 A universal particular solution in holomorphic gauge

For all vector fields $\vec{v}$ tangent to X-space, we have $\imath_{\vec{v}} A' = 0$, and hence $\imath_{\vec{v}} dA' = 0$ and $\imath_{\vec{v}} d\Phi' = 0$, i.e.

$$A' = dz^\alpha A'_\alpha + d\bar{z}^{\dot\alpha} A'_{\dot\alpha}\,, \qquad \partial_\mu \Phi' = 0 = \partial_\mu A'_\alpha\,, \tag{2.85}$$

and

$$F'_{\alpha\beta} + \frac{ib}{2}\Phi' \star \kappa\epsilon_{\alpha\beta} = 0\,, \qquad F'_{\alpha\dot\beta} = 0\,, \tag{2.86}$$

$$\partial_\alpha \Phi' + A'_\alpha \star \Phi' - \Phi' \star \bar\pi(A'_\alpha) = 0\,. \tag{2.87}$$

Thus, prior to switching on the gauge function G, we need to find a *particular* solution to the above system subject to a generic zero-form initial datum. To this end, we observe that the Ansatz[11]

$$\Phi' = \Psi(y,\bar y) \star \kappa_y\,, \qquad A'_\alpha = A'_\alpha(z;\Psi) = \sum_{n\geqslant 1} a^{(n)}_\alpha(z) \star \Psi^{\star n}\,, \tag{2.88}$$

where thus both Ψ and $\Psi \star \kappa_y$ are assumed to be elements in $\mathcal{A}(\mathcal{Y}_4)$, and

$$\Psi^\dagger = \rho(\Psi) \star \kappa_y \bar\kappa_{\bar y}\,, \tag{2.89}$$

solves the fully non-linear equations provided that

$$\pi_z(a^{(n)}_\alpha(z)) = -a^{(n)}_\alpha(z)\,, \tag{2.90}$$

and that

$$s_\alpha := z_\alpha - 2ia_\alpha\,, \qquad a_\alpha := \sum_{n\geqslant 1} a^{(n)}_\alpha(z)\nu^n\,, \tag{2.91}$$

obeys the deformed oscillator algebra

$$[s_\alpha, s_\beta]_\star = -2i\epsilon_{\alpha\beta}(1 - b\nu\kappa_z)\,, \qquad \kappa_z \star s_\alpha = -s_\alpha \star \kappa_z\,. \tag{2.92}$$

One class of solutions is given by [17][12]

$$a_\alpha = -\frac{ib\nu}{2} z_\alpha \int_{-1}^{+1} \frac{d\tau}{(\tau+1)^2} \exp\left(i\frac{\tau-1}{\tau+1} z^+ z^-\right) {}_1F_1(\tfrac{1}{2}; 2; b\nu \log \tau^2)\,, \tag{2.93}$$

where we have introduced a spin frame (u^+_α, u^-_α) obeying

$$u^{+\alpha} u^-_\alpha = 1\,, \tag{2.94}$$

[11] Note that this ansatz for A'_α is holomorphic in z, and hence the terminology of *holomorphic gauge*; see [19] for a review.

[12] As is well-known, upon representing Wigner's deformed oscillators s_α in a Fock space for the undeformed oscillators z_α, singular vectors appear for an infinite number of discrete critical values of $b\nu$; for a brief review and references to original works, see [19]. It would be interesting to study the behaviour of the master fields close to these critical points, which we leave for a future study. In what follows, we shall assume that $b\nu$ belongs to the cell in moduli space containing $\nu = 0$.

and $z^{\pm}$ is defined in (A.4). The introduction of this frame is required in order to integrate the delta function in Weyl order without choosing any specific basis for $\Psi(Y)$.

It is worth noting that, in the holomorphic gauge, the separation of the dependence on Y and Z of the twistor space connection A_α leads to a non-analytic z-dependent coefficient $a_\alpha(z)$, as given in eq. (2.93). Indeed, the separated deformed oscillator problem on $\mathcal{Y}\times\mathcal{Z}$ reduces to one on $\mathcal{Z}$ only, with oscillators $s_\alpha(z)$, as given in eq. (2.92), deformed by the delta function κ_z. As a consequence, the perturbative expansion of a_α starts with an abelian connection in two dimensions given by a distribution in $\mathcal{Z}$ whose curl is proportional to κ_z. In the holomorphic gauge, it is given by eq. (2.93) with the hypergeometric function replaced by a constant; the resulting distribution is discussed in appendix E. On the other hand, as we shall show later, once the star products between a_α, the zero-form initial data $\Psi(Y)$ and the gauge function $G = L\star H$ are performed (in normal order), the resulting form of A_α is real analytic on $\mathcal{Z}$ already in the intermediate L-gauge to all orders in perturbation theory, and on $\mathcal{Y}\times\mathcal{Z}$ for generic spacetime points in the Vasiliev gauge at first order in perturbation theory. Thus, in the latter gauge, we recover the standard generating functions for gauge fields and Weyl tensors, at least at the linearized level. The issue of what constitutes a physically meaningful gauge at higher orders will be discussed in the Conclusions, and left for future work.

Thus, in order to construct solution spaces with desired physical properties, we need to expand Ψ over suitable subalgebras of $\mathcal{A}(\mathcal{Y}_4)$; for the cases of particle fluctuation modes and black hole-like generalized Type D modes, see [17, 18]. In what follows, we shall examine a new type of subalgebras related to solutions with six Killing symmetries inside the isometry algebra of $(A)dS_4$.

3 Construction of the exact solutions with six symmetries

In this section, we shall begin by describing the factorization method that will be used to construct the solutions. We shall than construct domain walls (DW), instantons[13] (I) and FRW-like solutions (FRW) given by foliations of a four-dimensional spacetime M_4 with three-dimensional foliates M_3 that are maximally symmetric metric spaces, we shall first choose embeddings of the corresponding six-dimensional isometry algebras $\mathfrak{g}_6$ into the ten-dimensional isometry algebra $\mathfrak{g}_{10}$ of the vacuum solution. We then switch on $\mathfrak{g}_6$-invariant Weyl zero-forms and gauge functions.

3.1 Initial data for Weyl zero-form with six Killing symmetries

3.1.1 Unbroken symmetries

In order to describe foliations with maximally symmetric foliates, we embed $\mathfrak{g}_6$ into $\mathfrak{g}_{10}$ as follows [14]:

$$M_{rs} = L_r{}^a L_s{}^b M_{ab}\,, \qquad T_r = L_r{}^a\left(\alpha M_{ab}L^b + \beta P_a\right)\,, \tag{3.1}$$

[13] The instantons break all transvection isometries of the vacuum solution, i.e. $\mathfrak{g}_6$ coincides with the Lorentz algebra of the vacuum solution.

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where[14]

$$\alpha,\beta\in\mathbb{R}\,,\qquad \alpha,\beta\geqslant 0\,,\qquad (\alpha,\beta)\neq(0,0)\,,\tag{3.2}$$

and the representatives of the cosets $\mathfrak{so}(3,1)/\mathfrak{so}(2,1)$ for $\epsilon=1$, and the coset $\mathfrak{so}(3,1)/\mathfrak{so}(3)$ for $\epsilon=-1$ obey

$$\begin{array}{ccc} L_r{}^a L_s{}^b\eta_{ab}=\eta_{rs}\,, & L^aL_a=\epsilon\,, & L_r{}^aL_a=0\,,\\ \eta_{ab}=\mathrm{diag}(-+++)\,, & \eta_{rs}=\mathrm{diag}(++,-\epsilon)\,, & \epsilon=\pm1 \end{array}\tag{3.3}$$

where we have introduced the parameter ϵ. The resulting symmetry algebra reads as follows:[15]

$$[M_{rs},M_{pq}]=i\eta_{sp}M_{rq}+3\text{ more}\,,\qquad [M_{rs},T_p]=2i\eta_{p[s}T_{r]}\,,\tag{3.4}$$

$$[T_r,T_s]=-i(\epsilon\alpha^2-\lambda^2\beta^2)M_{rs}\,,\tag{3.5}$$

giving rise to the cases listed in table 1.

3.1.2 Invariant Weyl-zero form integration constant

Imposing $\mathfrak{g}_6$-invariance of zero-form initial data, *viz.*

$$[M_{rs},\Phi']_\pi=0\,,\qquad [T_r,\Phi']_\pi=0\,,\tag{3.6}$$

it follows from the first condition that

$$\Phi'=\Phi'(P)\,,\qquad P:=L^aP_a\,,$$

and from the second condition that

$$\left(-\frac{\epsilon\beta\lambda^2}{8}\frac{d^2}{dP^2}+i\epsilon\alpha\frac{d}{dP}+2\beta\right)\Phi'(P)=0\ .\tag{3.7}$$

where we have used

$$L_r{}^aL^b\,[M_{ab},P^n]_\star=in\epsilon L_r{}^aP_aP^{n-1}\,,\tag{3.8}$$

$$L_r{}^a\,\{P_a,P^n\}_\star=L_r{}^aP_a\left(2P^n-\frac{n(n-1)\epsilon\lambda^2}{8}P^{n-2}\right)\ .\tag{3.9}$$

3.1.3 Regular presentation

To solve (3.7), we Laplace transform Φ' as

$$\Phi'=\oint_C\frac{d\eta}{2\pi i}\widetilde{\Phi}'(\eta)\exp(-4\eta\lambda^{-1}P)\equiv\mathcal{O}\exp(-4\eta\lambda^{-1}P)\ .\tag{3.10}$$

This gives a characteristic equation for η solved by

$$\eta_\pm=-\gamma\pm\sqrt{\epsilon+\gamma^2}\,,\qquad \gamma:=\frac{i\alpha}{\lambda\beta}\,,\qquad \eta_+\eta_-=-\epsilon\,,\tag{3.11}$$

[14] One can always choose $\alpha\geqslant0$ by redefining L_r^a, after which one may take $\beta\geqslant0$ using the global $\mathbb{Z}_2$-symmetry generated by the π-map, which exchanges β with $-\beta$.

[15] The symmetry of the full solutions is generated by the parameters given by the conjugation of the linearized symmetry parameters by the gauge function.

Type	M_3	$\mathfrak{g}_6$	ϵ	λ^2	Condition on (α,β) for $\mathfrak{g}_6$ closure	$\gamma := \frac{i\alpha}{\lambda\beta}$ (mod G_{10})	(η_+,η_-) $\eta_\pm \equiv -\gamma \pm \sqrt{\epsilon+\gamma^2}$	Φ', $\mu\in\mathbb{C}$, $\nu,\tilde{\nu},\nu_\pm \in \mathbb{R}$ $P = L^a P_a$, $(\lambda^{-1}P)^\dagger = \lambda^{-1}P$
$\mathrm{DW}_+^{(\mathrm{dS})}$	dS_3	$\mathfrak{so}(1,3)$	$+1$	<0	$\alpha^2-\lambda^2\beta^2>0,\ \beta\neq 0$	$\gamma=0$	$(1,-1)$	$\nu_+e^{-4\lambda^{-1}P}+\nu_-e^{4\lambda^{-1}P}$
FRW_+	S^3	$\mathfrak{so}(4)$	-1	<0	$-\lambda^2\beta^2>\alpha^2$	$\gamma=0$	$(i,-i)$	$\mu e^{-4i\lambda^{-1}P}+\bar{\mu}e^{4i\lambda^{-1}P}$
FRW_0	$Eucl_3$	$\mathfrak{iso}(3)$	-1	<0	$-\lambda^2\beta^2=\alpha^2>0$	$\gamma=1$	$(-1,-1)$	$(\nu-4\tilde{\nu}\lambda^{-1}P)e^{4\lambda^{-1}P}$
$\mathrm{FRW}_-^{(\mathrm{dS})}$	H_3	$\mathfrak{so}(1,3)$	-1	<0	$\alpha^2>-\lambda^2\beta^2$	$\gamma>1$	$\eta_-<-1<\eta_+<0$	$\nu_+e^{-4\eta_+\lambda^{-1}P}+\nu_-e^{-4\eta_-\lambda^{-1}P}$
I	dS_3, H_3	$\mathfrak{so}(1,3)$	±1	$\neq 0$	$\alpha^2>0,\ \beta=0$	$\gamma=\infty$	$(0,\infty)$	ν
$\mathrm{DW}_+^{(\mathrm{AdS})}$	dS_3	$\mathfrak{so}(1,3)$	$+1$	>0	$\alpha^2>\lambda^2\beta^2$	$-i\gamma>1$	$0<i\eta_-<1<i\eta_+$	$\nu_+e^{-4\eta_+\lambda^{-1}P}+\nu_-e^{-4\eta_-\lambda^{-1}P}$
DW_0	$Mink_3$	$\mathfrak{iso}(1,2)$	$+1$	>0	$\lambda^2\beta^2=\alpha^2>0$	$-i\gamma=1$	$(-i,-i)$	$(\nu-4i\tilde{\nu}\lambda^{-1}P)e^{4i\lambda^{-1}P}$
DW_-	AdS_3	$\mathfrak{so}(2,2)$	$+1$	>0	$\lambda^2\beta^2>\alpha^2$	$\gamma=0$	$(1,-1)$	$\mu e^{-4\lambda^{-1}P}+\bar{\mu}e^{4\lambda^{-1}P}$
$\mathrm{FRW}_-^{(\mathrm{AdS})}$	H_3	$\mathfrak{so}(1,3)$	-1	>0	$\alpha^2+\lambda^2\beta^2>0,\ \beta\neq 0$	$\gamma=0$	$(i,-i)$	$\nu_+e^{-4i\lambda^{-1}P}+\nu_-e^{4i\lambda^{-1}P}$

Table 1. $\mathfrak{g}_6$-invariant M_3-foliations arising in the bosonic models, with I standing for instantons, and FRW_k and DW_k, respectively, standing for FRW-like solutions ($\epsilon=-1$) and domainwalls ($\epsilon=+1$) with foliates with curvatures of sign $k=\mathrm{sign}(\epsilon\alpha^2-\lambda^2\beta^2)$. The embeddings of $\mathfrak{g}_6$ into the isometry algebra of the $(A)dS_4$ vacua are governed by a vector L^a with $L^2=\epsilon$ and two real parameters $\alpha,\beta>0$. The last column contains the corresponding $\mathfrak{g}_6$-invariant initial data for the Weyl zero-form. Two families of foliations with $k=-1$ interpolate between the cases with $k=0$ and the instantons.

that are either real or purely imaginary. Thus, for $\beta > 0$ we have

$$\mathfrak{so}(1,3) \;:\; \widetilde{\Phi}'(\eta) = \frac{\nu_+}{\eta-\eta_+} + \frac{\nu_-}{\eta-\eta_-}\,, \tag{3.12a}$$

$$\mathfrak{iso}(1,2)\,,\;\mathfrak{iso}(3) \;:\; \widetilde{\Phi}'(\eta) = \frac{\nu}{\eta+\sqrt{-\epsilon}} + \frac{\sqrt{-\epsilon}\,\widetilde{\nu}}{(\eta+\sqrt{-\epsilon})^2}\,, \tag{3.12b}$$

$$\mathfrak{so}(4)\,,\;\mathfrak{so}(2,2) \;:\; \widetilde{\Phi}'(\eta) = \frac{\mu}{\eta-\eta_+} + \frac{\bar{\mu}}{\eta-\eta_-}\,. \tag{3.12c}$$

The small contours C encircle the poles of $\widetilde{\Phi}'$ counterclockwise. The corresponding solutions for Φ' are listed in table 1 modulo rigid G_{10} transformations, that can be used to set $\alpha = 0$ for $\epsilon k = -1$ and $\mathrm{sign}(\epsilon\Lambda) = +1$, i.e. the FRW$_+$, the DW$_-$, the the DW$_+$ in dS_4 and the FRW$_-$ in AdS_4.

3.1.4 Limits

The flat solutions with $k = 0$ arise from the $\mathfrak{so}(1,3)$-invariant families in the limit

$$\gamma \to \sqrt{-\epsilon}\,, \tag{3.13}$$

keeping

$$\nu := \nu_+ + \nu_-\,, \qquad \widetilde{\nu} := \frac{\gamma - \sqrt{\gamma^2+\epsilon} - \sqrt{-\epsilon}}{\sqrt{-\epsilon}}(\nu_- - \nu_+)\,, \tag{3.14}$$

fixed. In the limit $\beta \to 0$, one has $\eta_+ \to 0$ and $\eta_- \to \infty$, and hence

$$\widetilde{\Phi}' = \frac{\nu}{\eta}\,, \qquad \nu \equiv \nu_+\,, \tag{3.15}$$

and C is a small contour encircling $\eta = 0$ counterclockwise.[16]

3.1.5 Regularization of star products

In order to compute

$$\Psi \star \Psi = \Phi' \star \pi(\Phi')\,, \tag{3.16}$$

we use the lemma

$$e^{-4\eta\lambda^{-1}P} \star e^{-4\eta'\lambda^{-1}P} = \frac{1}{(1-\epsilon\eta\eta')^2}\exp\left(-4\frac{\eta+\eta'}{1-\epsilon\eta\eta'}\lambda^{-1}P\right)\,, \tag{3.17}$$

and the regularization procedure spelled out in [17], *viz.*

$$\begin{aligned} e^{-4s\lambda^{-1}P} \star e^{4\frac{\epsilon}{s}\lambda^{-1}P}|_{\mathrm{reg}} &= \oint_s \frac{d\eta}{2\pi i(\eta-s)} e^{-4\eta\lambda^{-1}P} \star e^{4\frac{\epsilon}{s}\lambda^{-1}P} \\ &= \oint_s \frac{d\eta}{2\pi i(\eta-s)^3} \exp\left[4\frac{\eta s-\epsilon}{\eta-s}\lambda^{-1}P\right] = 0\,, \end{aligned} \tag{3.18}$$

[16] Alternatively, the decoupling of the η_+-mode in the limit $\beta \to 0$ can be achieved using a twisted-adjoint G_{10} conjugation of Φ' that annihilates the η_+-mode using the regularization procedure.

which suffices to handle the cases with $k \neq 0$. In the case of $\mathfrak{g}_6 = \mathfrak{iso}(3)$, we have

$$
\begin{aligned}
&\Psi \star \Psi|_{\mathfrak{iso}(3)}\Big|_{\text{reg}} \\
&= \left[\left(\nu + \tilde{\nu}\lambda^{-1}P\right) e^{4\lambda^{-1}P}\right] \star \left[\left(\nu - \tilde{\nu}\lambda^{-1}P\right) e^{-4\lambda^{-1}P}\right] \\
&= \oint_{-1}\oint_{-1} \frac{d\eta d\xi}{(2\pi i)^2}\left(\frac{\nu}{\eta+1} + \frac{\tilde{\nu}}{(\eta+1)^2}\right)\left(\frac{\nu}{\xi+1} + \frac{\tilde{\nu}}{(\xi+1)^2}\right) e^{-4\eta\lambda^{-1}P} \star e^{4\xi\lambda^{-1}P} \\
&= \oint_{-1}\oint_{-1} \frac{d\eta d\xi}{(2\pi i)^2}\left(\frac{\nu}{\eta+1} + \frac{\tilde{\nu}}{(\eta+1)^2}\right)\left(\frac{\nu}{\xi+1} + \frac{\tilde{\nu}}{(\xi+1)^2}\right) \frac{1}{(1-\eta\xi)^2} e^{4\lambda^{-1}\frac{\xi-\eta}{1-\eta\xi}P} \\
&= \oint_{-1}\frac{d\eta}{2\pi i}\left[\frac{2\tilde{\nu}^2\left(4P-\lambda\right)}{\lambda\,(\eta+1)^5} + \frac{\lambda\left(2\tilde{\nu}^2-\nu\tilde{\nu}\right)+4\left(2\nu\tilde{\nu}-\tilde{\nu}^2\right)P}{\lambda\,(\eta+1)^4} + \frac{\lambda\left(\nu^2+2\nu\tilde{\nu}\right)-4\nu\tilde{\nu}P}{\lambda\,(\eta+1)^3}\right] e^{-4\lambda^{-1}P} \\
&= 0\,;
\end{aligned}
\tag{3.19}
$$

and the case of $\mathfrak{g}_6 = \mathfrak{iso}(1,2)$ is similar.[17] It follows that

$$\Psi \star \Psi|_{\text{reg}} = \mathcal{C}^2\,, \tag{3.20}$$

where $\mathcal{C}^2$ is a constant given by

$$\epsilon k = -1: \qquad \mathcal{C}^2 = \frac{\mu^2 + \bar{\mu}^2}{4}\,, \tag{3.21}$$

$$\epsilon k = 0: \qquad \mathcal{C}^2 = 0\,, \tag{3.22}$$

$$\epsilon k = +1: \qquad \mathcal{C}^2 = \frac{(\nu_+)^2}{(1+\epsilon(\eta_+)^2)^2} + \frac{(\nu_-)^2}{(1+\epsilon(\eta_-)^2)^2}\,, \tag{3.23}$$

where the last case contains the instantons, for which $\mathcal{C}^2 = \nu^2$.

3.2 Twistor space connection in holomorphic gauge

The general solution in integral form. In the expression for the twistor space connection, it is convenient to express the hypergeometric function in an integral representation as follows

$${}_1F_1(a;b;w) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)}\int_0^1 \frac{ds}{s(1-s)} s^a (1-s)^{b-a} e^{ws}\,. \tag{3.24}$$

From (2.88), (2.93), (3.24) and (3.20), one finds that

$$
\begin{aligned}
A'_\alpha &= -\frac{ib}{\pi}\int_{-1}^{+1}\frac{d\tau}{(\tau+1)^2}\int_0^1 ds\sqrt{\frac{1-s}{s}} \\
&\quad \left[z_\alpha \exp\left(i\frac{\tau-1}{\tau+1}z^+z^-\right)\right] \star \left[\Psi\cosh\left(\frac{b\,\mathcal{C}s}{2}\log\tau^2\right) + \mathcal{C}\sinh\left(\frac{b\,\mathcal{C}s}{2}\log\tau^2\right)\right] \\
&= \sum_{n\geq 1} A'^{(n)}\,.
\end{aligned}
\tag{3.25}
$$

[17] Crucial for the regularization is that, after evaluating (any) one of the two integrals, the exponential that results from the star product $e^{-4\eta\lambda^{-1}P} \star e^{4\xi\lambda^{-1}P}$ becomes independent of the other auxiliary contour-integral variable. Supposing, for concreteness, that one evaluates the integral over ξ first, as above, the calculation shows that the only assumption one uses in this kind of regularization is that $|\xi+1| \ll |\eta+1| \ll 1$, in order to keep $\xi = -1$ as the only pole encircled by the contour [17].

Thus, expanding in powers of the deformation parameters, we find that all odd terms are linear in Ψ, while all even terms are $(y^\alpha, \bar{y}^{\dot\alpha})$-independent, *viz.*

$$\left(A'_\alpha\right)^{(2k-1)} = \frac{-i\Gamma(2k-\frac{3}{2})}{\sqrt{\pi}\,(2k-2)!\,(2k-1)!}\left(\frac{b\mathcal{C}}{2}\right)^{2k-2}\left(\frac{b\Psi}{2}\right)\star\left(z_\alpha I_{2k-1}\right), \tag{3.26}$$

$$\left(A'_\alpha\right)^{(2k)} = \frac{-i\Gamma(2k-\frac{1}{2})}{\sqrt{\pi}\,(2k-1)!\,(2k)!}\left(\frac{b\mathcal{C}}{2}\right)^{2k} z_\alpha I_{2k}\,, \tag{3.27}$$

where $k = 1, 2, \ldots$, and

$$I_n = I_n(w) = \int_{-1}^{1}\frac{d\tau}{(\tau+1)^2}\, e^{-w\xi}\left(\log\tau^2\right)^{n-1}, \tag{3.28}$$

where

$$w := iz^+z^-\,, \qquad \xi := \frac{1-\tau}{1+\tau}\,. \tag{3.29}$$

Let us proceed by looking into the internal connection order by order in its perturbative expansion.

First order. The linearized twistor space connection is given by

$$\left(A'_\alpha\right)^{(1)} = a^{(1)}_\alpha(z)\star\Psi \tag{3.30}$$

where

$$a^{(1)}_\alpha = -\frac{ib}{2}z_\alpha\int_{-1}^{+1}\frac{d\tau}{(\tau+1)^2}\exp\left(i\frac{\tau-1}{\tau+1}z^+z^-\right) = -\frac{b}{4z^+z^-}z_\alpha\,. \tag{3.31}$$

For its basic distributional properties, see remark made above. Clearly, in Weyl order, the linearized twistor space connection is not real-analytic at the origin of Z-space; whether it becomes real analytic in normal order depends on the details of Ψ, as we shall analyze in more detail below.

Second order. We have the second order

$$\left(A'_\alpha\right)^{(2)} = -\frac{ib^2\mathcal{C}^2}{16}z_\alpha I_2(w)\,, \qquad I_2(w) = \int_{-1}^{+1}\frac{d\tau}{(\tau+1)^2}e^{-w\xi}\log\tau^2\,. \tag{3.32}$$

Thus we can split the integral into two pieces as follows:

$$\begin{aligned}
I_2(w) &= \frac{1}{2}\int_0^\infty d\xi e^{-w\xi}\log\left(\frac{1-\xi}{1+\xi}\right)^2 \\
&= \frac{1}{2}\left(e^{-w}\int_{-1}^\infty d\xi e^{-w\xi}\log\xi^2 - e^{w}\int_1^\infty d\xi e^{-w\xi}\log\xi^2\right) \\
&= I_2^>(w) + I_2^>(-w)\,,
\end{aligned} \tag{3.33}$$

where

$$I_2^>(w) = -\frac{e^w}{2w}\int_w^\infty d\xi e^{-\xi}\log(\xi/w)^2\,, \tag{3.34}$$

which is convergent for all real w. For $w > 0$, we can integrate by parts and rewrite it as

$$I_2^{>}(w) = -\frac{e^w}{w}\mathrm{E}_1(w)\,, \tag{3.35}$$

where the exponential integral

$$\mathrm{E}_1(w) = \int_w^\infty \frac{dt}{t}e^{-t}\,, \qquad w > 0\ . \tag{3.36}$$

This function can be extended from the positive real axis to a complex function that is analytic away from the negative real axis, where is has a Taylor expansion given by

$$\mathrm{E}_1(w) = -\gamma_E - \log w - \sum_{p=1}^{\infty}\frac{(-w)^p}{pp!}\,, \tag{3.37}$$

where γ_E is the Euler-Mascheroni constant; we note that $w\frac{d}{dw}\mathrm{E}_1(w) = -\exp(-w)$. Thus, continuing $I_2^{>}(w)$ to complex w, and adding $I_2^{<}(-w)$, we find

$$I_2(w) = -\frac{e^w\mathrm{E}_1(w) - e^{-w}\mathrm{E}_1(-w)}{w}\,, \tag{3.38}$$

which can be rewritten as

$$I_2(w) = R_1(w) + R_2(w)\log w\,, \tag{3.39}$$

where $R_{1,2}$ are real analytic at $w = 0$:

$$R_1(w) = \frac{2\gamma\sinh w}{w} - e^w\sum_{p=1}^{\infty}\frac{(-w)^{p-1}}{p\,p!} - e^{-w}\sum_{p=1}^{\infty}\frac{w^{p-1}}{p\,p!}\,, \qquad R_2(w) = \frac{2\sinh w}{w}\ . \tag{3.40}$$

Therefore, in summary the second order correction $(A'_\alpha)^{(2)}$ is independent of Y and bounded in Z, though it is not real analytic at $Z = 0$.

3.3 Master fields in L-gauge

We recall that starting from the particular solution obtained in the the holomorphic gauge, which incorporates the zero-form initial data, gauge inequivalent solutions can be reached by means of large gauge transformations generated by gauge functions G defined locally on patches. In the case of asymptotically (anti-)de Sitter spacetimes, we use $G = L \star H$, where L is the vacuum gauge function, which brings the master fields to what we refer to as the L-gauge, after which H is constructed order by order by imposing the dual boundary conditions (a) and (b) specified in section 2.4.2. Finally, the patches are glued together using transition functions belonging to a structure group.

3.3.1 Weyl zero-form

The Weyl zero-form in L-gauge is given by

$$\Phi^{(L)}(y,\bar{y}) = L^{-1}\star\Phi'\star\pi(L)\ . \tag{3.41}$$

Substituting $\Phi' = \Psi \star \kappa_y$ according to the ansatz (2.88) we get

$$\Phi^{(L)}(y,\bar{y}) = \Psi^L \star \kappa_y\,, \qquad \Psi^L := L^{-1} \star \Psi \star L\,. \tag{3.42}$$

We first compute

$$\Psi \equiv \Phi' \star \kappa_y = \left(\mathcal{O}e^{-4\eta\lambda^{-1}P}\right) \star \kappa_y = 2\pi\mathcal{O}\delta^2\left(y_\alpha - b_a(\sigma^a\bar{y})_\alpha\right)\,, \qquad b^a := i\eta L^a\,, \tag{3.43}$$

where we have used (3.10). The L-conjugate of Ψ is given by

$$\Psi^L = 2\pi\mathcal{O}\delta^2\left(y^L_\alpha - b_a(\sigma^a\bar{y}^L)_\alpha\right)\,, \tag{3.44}$$

where

$$\begin{bmatrix} y^L_\alpha \\ \bar{y}^L_{\dot\alpha} \end{bmatrix} = L^{-1} \star \begin{bmatrix} y_\alpha \\ \bar{y}_{\dot\alpha} \end{bmatrix} \star L = \begin{bmatrix} L_\alpha{}^\beta & K_\alpha{}^{\dot\beta} \\ \overline{K}_{\dot\alpha}{}^\beta & \overline{L}_{\dot\alpha}{}^{\dot\beta} \end{bmatrix} \begin{bmatrix} y_\beta \\ \bar{y}_{\dot\beta} \end{bmatrix}\,, \tag{3.45}$$

form a new set of canonical coordinates in which

$$\rho((y^L_\alpha)^\dagger) = \bar{y}^L_{\dot\alpha}\,, \qquad \rho((\bar{y}^L_{\dot\alpha})^\dagger) = \mathrm{sign}(\lambda^2) y^L_{\dot\alpha}\,, \qquad \kappa_y\bar{\kappa}_{\bar{y}} = \kappa_{y^L}\bar{\kappa}_{\bar{y}^L}\,. \tag{3.46}$$

The matrices K and L are computed in stereographic and planar coordinate systems in appendix C.2 and C.4, respectively. It follows that indeed

$$(\Psi^L)^\dagger = \Psi^L \star \kappa_y\bar{\kappa}_{\bar{y}}\,, \qquad \Psi^L \star \Psi^L|_{\mathrm{reg}} = \mathcal{C}^2\,, \tag{3.47}$$

where $\mathcal{C}$ is the constant in (3.20), as can be seen using the lemma

$$(2\pi)^2\delta^2\left(y^L_\alpha - b_a(\sigma^a\bar{y}^L)_\alpha\right) \star \delta^2\left(y^L_\alpha - \tilde{b}_a(\sigma^a\bar{y}^L)_\alpha\right) = \frac{1}{(1{+}\eta\tilde{\eta}\epsilon)^2}\exp\left[\frac{\eta-\tilde{\eta}}{1{+}\eta\tilde{\eta}\epsilon}L_a(\sigma^a)^{\alpha\dot\alpha}y^L_\alpha\bar{y}^L_{\dot\alpha}\right], \tag{3.48}$$

where $\tilde{b}_a = i\tilde{\eta}L_a$, followed by contour integration. Going back to the original canonical coordinates for $\mathcal{Y}_4$, we have

$$\Psi^L = 2\pi\mathcal{O}\delta^2\left((Ay + B\bar{y})_\alpha\right)\,, \tag{3.49}$$

where

$$A_\alpha{}^\beta := L_\alpha{}^\beta - b_a(\sigma^a\overline{K})_\alpha{}^\beta\,, \qquad B_\alpha{}^{\dot\beta} := K_\alpha{}^{\dot\beta} - b_a(\sigma^a\overline{L})_\alpha{}^{\dot\beta}\,, \tag{3.50}$$

Provided that A is invertible, we can write

$$\Psi^L = \mathcal{O}\left(\frac{2\pi}{\det A}\delta^2(\tilde{y}_\alpha)\right)\,, \qquad \tilde{y}_\alpha := y_\alpha + M_\alpha{}^{\dot\beta}\bar{y}_{\dot\beta}\,, \qquad M = A^{-1}B\,. \tag{3.51}$$

We thus find

$$\Phi^{(L)}(y,\bar{y}) = \Psi^L \star \kappa_y = \mathcal{O}\left(\frac{2\pi}{\det A}\delta^2(\tilde{y}) \star \kappa_y\right)\,. \tag{3.52}$$

which is readily computed with the result

$$\Phi^{(L)}(y,\bar{y}) = \mathcal{O}\left(\frac{1}{\det A}e^{iy^\alpha M_\alpha{}^{\dot\alpha}\bar{y}_{\dot\alpha}}\right) = \oint_C \frac{d\eta}{2\pi i}\frac{\widetilde{\Phi}'(\eta)}{\det A}e^{iy^\alpha M_\alpha{}^{\dot\alpha}\bar{y}_{\dot\alpha}}\,, \tag{3.53}$$

with $\widetilde{\Phi}'(\eta)$ from (3.12). The resulting Weyl zero-forms consist of scalar field profiles, that we shall analyze in more detail in section 4 using stereographic coordinates, and in appendix C using adapted coordinate systems.

3.3.2 Twistor space connection at even orders

The even order terms are the same in the holomorphic gauge and the L-gauge, as they are independent of Y. From (3.27), the sum of all even orders is given by

$$(A_\alpha^{(L)})^{(\text{even})} = -\frac{ib\mathcal{C}}{\pi} z_\alpha I_{\text{even}}(w; b\mathcal{C}) , \tag{3.54}$$

where

$$I_{\text{even}}(w;\mu) = \int_{-1}^{+1} \frac{d\tau}{(\tau+1)^2} \int_0^1 ds \sqrt{\frac{1-s}{s}}\, e^{-w\xi} \sinh\left(\frac{\mu s}{2}\log\tau^2\right) . \tag{3.55}$$

We note that $(A_\alpha^{(L)})^{(\text{even})}$ is independent of X and Y, and bounded in Z-space.

3.3.3 Twistor space connection at odd orders

In the L-gauge, the sum of all odd-order terms from (3.26) is given by

$$(A_\alpha^{(L)})^{(\text{odd})} = -\frac{ib}{\pi} \partial_\alpha^{(\rho)} V(\rho) \,|_{\rho=0} , \tag{3.56}$$

where the generating function

$$V(\rho) = \int_{-1}^{+1} \frac{d\tau}{(\tau+1)^2} \int_0^1 ds \sqrt{\frac{1-s}{s}} \exp\left(-\frac{i\xi}{2} u^{\alpha\beta} z_\alpha z_\beta + \rho^\alpha z_\alpha\right) \star \Psi^L \cosh\left(\frac{b\mathcal{C}s}{2}\log\tau^2\right), \tag{3.57}$$

and $u^{\alpha\beta} := 2u^{+(\alpha}u^{-\beta)}$. Substituting for $\Psi_L = \mathcal{O}\left(e^{-4\eta\lambda^{-1}P_0} \star \kappa_y\right)^L$ gives

$$(A_\alpha^{(L)})^{(\text{odd})} = -\frac{ib}{\pi} \partial_\alpha^{(\rho)} \mathcal{O} V(\eta;\rho) \,|_{\rho=0} , \tag{3.58}$$

where the extended generating function

$$\begin{aligned} V(\eta;\rho) = &\int_{-1}^{+1} \frac{d\tau}{(\tau+1)^2} \int_0^1 ds \sqrt{\frac{1-s}{s}} \times \\ &\times \exp\left(-\frac{i\xi}{2} u^{\alpha\beta} z_\alpha z_\beta + \rho^\alpha z_\alpha\right) \star \left(e^{-4\eta\lambda^{-1}P_0} \star \kappa_y\right)^L \cosh\left(\frac{b\mathcal{C}}{2}\log\tau^2\right) , \end{aligned} \tag{3.59}$$

Next we perform the Gaussian star product

$$\begin{aligned} &\exp\left(-\frac{i\xi}{2} u^{\alpha\beta} z_\alpha z_\beta + \rho^\alpha z_\alpha\right) \star (e^{-4\eta\lambda^{-1}P_0} \star \kappa_y)^L \\ &= \frac{1}{\xi \det A} \exp\left(\frac{i}{2\xi} u^{\alpha\beta} (\rho_\alpha - i\tilde{y}_\alpha)(\rho_\beta - i\tilde{y}_\beta) - i z^\alpha \tilde{y}_\alpha\right) . \end{aligned} \tag{3.60}$$

It follows that $(A_\alpha^{(L)})^{(\text{odd})}$ is real analytic in Z-space, which simplifies the construction of H, and that it has singularities in Y-space, stemming from the divergence at $\tau = +1$ that arises as a result of the above Gaussian integration. Thus, we need to demonstrate that the latter singularities go away upon switching on H. From (3.58), (3.59) and (3.60) we find

$$(A_\alpha^{(L)})^{(\text{odd})} = \mathcal{O} V_\alpha(\eta) , \tag{3.61}$$

where the generating function

$$V_\alpha(\eta) = \frac{ib}{\pi} u_\alpha{}^\beta \tilde{y}_\beta \frac{e^{i\tilde{y}^\alpha z_\alpha}}{\det A} \int_{-1}^{+1} \frac{d\tau}{(\tau-1)^2} \int_0^1 ds \sqrt{\frac{1-s}{s}} \exp\left(\frac{i}{\xi}\tilde{y}^+\tilde{y}^-\right) \cosh\left(\frac{b\mathcal{C}s}{2}\log\tau^2\right) , \tag{3.62}$$

consists of even orders in deformation parameters, and its η dependence enters via A and $\tilde{y}_\alpha$.

3.3.4 Spacetime connection

The spacetime connection is simply given by

$$A^{(L)}_\mu = L^{-1} \star \partial_\mu L , \tag{3.63}$$

while form (2.45) one finds

$$W^{(L)}_\mu = L^{-1} \star \partial_\mu L - \frac{1}{4i}\left(L^{-1} \star M^{(\mathrm{tot})}_{\alpha\beta} \star L + h.c.\right) , \tag{3.64}$$

with $M^{(\mathrm{tot})}_{\alpha\beta}$ from (2.44).

3.3.5 Patching

The expressions given so far are defined in the region of validity of the gauge function L, that is, for $\lambda^2 x^2 < 1$, whereas a global formulation on the the vacuum manifold $M_4^{(0)}$ requires the usage of several coordinate charts. A simple configuration consists of two gauge functions $L_\pm$ defined on two stereographic coordinate charts $U_\pm$ with $1 > \lambda^2 x^2_\pm \geqslant -1$ glued together along $\lambda^2 x^2_\pm = -1$, which implies that the transition function is trivial since

$$L_\pm|_{\lambda^2 x^2_\pm = -1} = L_\mp|_{\lambda^2 x^2_\mp = -1} , \tag{3.65}$$

i.e. this particular configuration can be implemented for any choice of structure group, that is, in any topological phase of the theory. In these types of configurations, it follows from the reflection symmetry that any singularity in the master fields that arises inside $U_\pm$ cannot be removed using patching.

3.4 Reaching Vasiliev gauge at first order

3.4.1 The Weyl zero-form

At first order we observe that

$$\Phi^{(G,1)}(y,\bar{y}) = \Phi^{(L)}(y,\bar{y}) = \mathcal{O}\left(\frac{1}{\det A} e^{iy^\alpha M_\alpha{}^{\dot\alpha}\bar{y}_{\dot\alpha}}\right) , \tag{3.66}$$

3.4.2 Twistor space connection

At the linearized level, the role of $H^{(1)}$ is to ensure that

$$A^{(G,1)}_{\underline{\alpha}} := A^{(L,1)}_{\underline{\alpha}} + \partial_{\underline{\alpha}} H^{(1)} \tag{3.67}$$

is real analytic in $\mathcal{Y}_4 \times \mathcal{Z}_4$ and obeys the Vasiliev gauge condition,

$$z^{\underline{\alpha}} A^{(G,1)}_{\underline{\alpha}} = 0 . \tag{3.68}$$

Since $A^{(L,1)}_{\underline{\alpha}}$ is real analytic in $\mathcal{Z}_4$, it follows that

$$H^{(1)} = H^{(1)}|_{Z=0} - \frac{1}{\mathcal{L}_{\vec{Z}}} \left(z^{\alpha} A^{(L,1)}_{\alpha} + \bar{z}^{\dot{\alpha}} A^{(L,1)}_{\dot{\alpha}} \right) , \tag{3.69}$$

where $\mathcal{L}_{\vec{Z}} = \{q, \imath_{\vec{Z}}\}$ is the Lie derivative along the Euler vector field

$$\vec{Z} = Z^{\underline{\alpha}} \vec{\partial}_{\underline{\alpha}} , \tag{3.70}$$

whose invserse can be represented (on real analytic functions) as

$$\frac{1}{\mathcal{L}_{\vec{Z}}} = \int_0^1 \frac{dt}{t} t^{\mathcal{L}_{\vec{Z}}} , \tag{3.71}$$

where $t^{\mathcal{L}_{\vec{Z}}}$ acting on differential forms implements the diffeomorphism $z^{\underline{\alpha}} \to t z^{\underline{\alpha}}$. Thus,

$$A^{(G,1)}_{\alpha} = \left(\delta_{\alpha}{}^{\beta} - \partial_{\alpha} \frac{1}{\mathcal{L}_{\vec{Z}}} z^{\beta} \right) A^{(L,1)}_{\beta} , \tag{3.72}$$

to which the initial datum $H^{(1)}|_{Z=0}$ does not contribute, and we have taken into account the holomorphicity in Z space. Writing

$$A^{(L,1)}_{\underline{\alpha}} = \mathcal{O} V^{(0)}_{\underline{\alpha}}(\eta) , \qquad V^{(0)}_{\alpha}(\eta) = \frac{ib}{2} u_{\alpha}{}^{\beta} \tilde{y}_{\beta} \frac{e^{i\tilde{y}^{\alpha} z_{\alpha}}}{\det A} \int_{-1}^{+1} \frac{d\tau}{(\tau-1)^2} \exp\left(\frac{i}{2} \frac{\tau+1}{\tau-1} u^{\alpha\beta} \tilde{y}_{\alpha} \tilde{y}_{\beta} \right) , \tag{3.73}$$

it follows that

$$H^{(1)} = H^{(1)}|_{Z=0} - \mathcal{O} \frac{1}{\mathcal{L}_{\vec{Z}}} \left(z^{\alpha} V^{(0)}_{\alpha}(\eta) + \bar{z}^{\dot{\alpha}} V^{(0)}_{\dot{\alpha}}(\eta) \right) , \tag{3.74}$$

and

$$A^{(G,1)}_{\alpha} = \mathcal{O} \left(\delta_{\alpha}{}^{\beta} - \partial_{\alpha} \frac{1}{\mathcal{L}_{\vec{Z}}} z^{\beta} \right) V^{(0)}_{\beta}(\eta) . \tag{3.75}$$

Using

$$\int_0^1 dt t^{\mathcal{L}_{\vec{Z}}} e^{i\tilde{y}^{\alpha} z_{\alpha}} = \frac{e^{i\tilde{u}} - 1}{i\tilde{u}} , \qquad \tilde{u} = \tilde{y}^{\alpha} z_{\alpha} , \tag{3.76}$$

and

$$u^{\alpha\beta} \tilde{y}_{\alpha} \tilde{y}_{\beta} \int_{-1}^{+1} \frac{d\tau}{(\tau-1)^2} \exp\left(\frac{i}{2} \frac{\tau+1}{\tau-1} u^{\gamma\delta} \tilde{y}_{\gamma} \tilde{y}_{\delta} \right) = -i , \tag{3.77}$$

in accordance with eq. (E.3), one finds

$$A^{(G,1)}_{\alpha} = -\frac{ib}{2} z_{\alpha} \mathcal{O} \frac{e^{i\tilde{u}} - 1 - i\tilde{u} e^{i\tilde{u}}}{\tilde{u}^2 \det A} . \tag{3.78}$$

where the dependence on the auxiliary spinor frame $u^{\pm}_{\alpha}$ has dropped out. Indeed, the above result agrees with that found working directly in normal order, *viz.*

$$A^{(G,1)}_{\alpha} = -\frac{ib}{2} z_{\alpha} \int_0^1 dt\ t\ e^{ity^{\alpha} z_{\alpha}} \left[\Phi^{(G,1)}(y, \bar{y}) \Big|_{y \to -tz} \right] , \tag{3.79}$$

as can be seen using $\Phi^{(G,1)}(y, \bar{y}) = \Phi^{(L)}(y, \bar{y})$ and

$$\int_0^1 dt\ t\ e^{ity^{\alpha} z_{\alpha}} \left[\Phi^{(G,1)}(y, \bar{y}) \Big|_{y \to -tz} \right] = \mathcal{O} \frac{e^{i\tilde{u}} - 1 - i\tilde{u} e^{i\tilde{u}}}{\tilde{u}^2 \det A} . \tag{3.80}$$

3.4.3 Spacetime connection

In the Vasiliev gauge, the linearized spacetime connection

$$dx^\mu A^{(G,1)}_\mu = L^{-1}dL + D^{(0)}H^{(1)} \equiv L^{-1}dL + U^{(G,1)}\,, \tag{3.81}$$

where the background covariant derivative

$$D^{(0)} = d + \frac{1}{4i}\Omega^{\underline{\alpha\beta}}\mathrm{ad}^\star_{Y_{\underline{\alpha}}Y_{\underline{\beta}}}\,. \tag{3.82}$$

As the linearized Weyl zero-form consists of a scalar mode, it follows that there exists an initial datum $H^{(1)}|_{Z=0}$ such that $U^{(L,1)}|_{Z=0}$ is a pure (abelian) gauge field on $\mathcal{X}_4$ that is real-analytic on $\mathcal{Y}_4\times\mathcal{Z}_4$. To corroborate this fact, we first use $[\partial_\alpha, D^{(0)}]=0$ and $D^{(0)}A^{(L,1)}_\alpha = 0$ to deduce that

$$\partial_\alpha U^{(G,1)} = \partial_\alpha(D^{(0)}H^{(1)}) = D^{(0)}(\partial_\alpha H^{(1)}) = D^{(0)}(A^{(G,1)}_\alpha - A^{(L,1)}_\alpha) = D^{(0)}A^{(G,1)}_\alpha\,. \tag{3.83}$$

Thus, as $A^{(G,1)}_\alpha$ is real analytic on $\mathcal{Y}_4\times\mathcal{Z}_4$ and independent on the auxiliary spin frame, it follows that these properties hold true as well for the Z-dependent part of $U^{(G,1)}$. As for its Z-independent part, *viz.*

$$U^{(G,1)}|_{Z=0} = D^{(0)}\left(H^{(1)}|_{Z=0}\right) - \mathcal{O}\left(D^{(0)}\frac{1}{\mathcal{L}_{\vec{Z}}}\left(z^\alpha V^{(0)}_\alpha(\eta) + \bar{z}^{\dot\alpha}V^{(0)}_{\dot\alpha}(\eta)\right)\right)\Bigg|_{Z=0}\,, \tag{3.84}$$

the requirement of real-analyticity on $\mathcal{Y}_4$ fixes $H^{(1)}|_{Z=0}$ up to a residual real analytic part (in $\mathfrak{hs}_1(4)$), with the desired result

$$U^{(G,1)}|_{Z=0} = 0\,, \tag{3.85}$$

modulo a pure gauge term; for details, see appendix D.

3.5 Comments on residual symmetries, factorization and Vasiliev gauge

We recall from section 2.4.2 that as far as symmetry considerations are concerned in finding exact solutions, these are facilitated by the the combined use of gauge functions and the holomorphic factorization method employed in (2.88), which ensures that the symmetries of the initial datum $\Psi(Y)$, that can be imposed by means of undeformed generators, remain symmetries of the full master fields. We would like to contrast this approach to that followed in [14], where an exact $\mathfrak{so}(1,3)$-invariant solution was constructed for $\Lambda<0$ using the vacuum gauge function L followed by requiring the primed twistor space configuration to be invariant under the full field-dependent Lorentz generators (instead of using the holomorphic factorization method). In the same paper, the six-parameter symmetry groups considered here were also examined, but as the factorization method was not used, the imposition of symmetry conditions involving translations became problematic at the nonlinear level, and FRW-like and domain wall solutions were given explicitly at the linearized level, in agreement with table 1, and shown to exist at the second order of classical perturbation theory. It would be interesting to pursue the latter construction to the second order and compare it to the second order expressions obtained in the current paper in L-gauge.

The factorization method implies, however, that the linearized master gauge fields are not real analytic in $(y^\alpha, \bar{y}^{\dot{\alpha}})$ in L-gauge, but as we have seen, these singularities can be removed by going to Vasiliev gauge by means of a large gauge transformation. It remains to be shown whether this procedure can be imposed to order by order in weak field perturbative expansion by imposing dual boundary conditions as discussed in section 2.4.2.

We conclude this section by explaining technically the reason for being able to impose equally the first of the conditions (3.6) via the full Lorentz generator (2.44). First, defining $\epsilon_L^{(tot)} := \frac{1}{4i}\Lambda^{\alpha\beta}M_{\alpha\beta}^{(tot)} - \text{h.c.} = \epsilon_L^{(0)} + \epsilon_L^{(\text{extra})} := \frac{1}{4i}\Lambda^{\alpha\beta}M_{\alpha\beta}^{(0)} - \text{h.c.} + \frac{1}{4i}\Lambda^{\alpha\beta}S_\alpha \star S_\beta - \text{h.c.}$, one can show [34–36] that the fully nonlinear completion of the Lorentz generator is exactly such that, on the solutions of the Vasiliev equations, $\delta_L\Phi(Y,Z) = -[\epsilon_L^{(tot)}, \Phi]_\pi = -[\epsilon_L^{(0)}, \Phi]_\pi$. The reason is that (2.34) implies $[\epsilon_L^{(\text{extra})}, \Phi]_\pi = 0$. It is then clear that, on a purely Y-dependent Weyl zero-form, such as that of our Ansatz (2.88), the action of the Lorentz generators reduces to the one of their zeroth-order, purely Y-dependent piece, and we can therefore impose $\mathfrak{so}(3)$-symmetry as in (3.6). In general, however, imposing invariance conditions on the master fields under subalgebras that include translations can only be done perturbatively, as it was suggested in [14], since no fully non-linear completion of the P_a is known. This limitation is not present on the subspace of the full solution space captured by the factorized Ansatz (2.88), where Φ is first-order-exact.

The virtue of the factorization (combined with the gauge function method) is that it gives us the possibility of solving exactly for the Z-dependence of the master fields irrespectively of the initial datum $\Phi'(Y)$, thereby dressing a solution of the linearized twisted-adjoint equation into a full solution of the Vasiliev equations. In particular, due to the factorized form, the equations for S'_α reduce to (2.92), that do not involve Φ' and can be therefore solved once and for all (via the methods developed in [17, 18]). This allows us to impose the $\mathfrak{g}_r$ symmetries at full level, since the action of symmetry parameters $\epsilon(Y)$ is sufficient to impose symmetry conditions on the full solution space (2.88). Indeed, the symmetries $\epsilon(Y)$ of Φ' (that is, the parameters such that $[\epsilon(Y), \Phi'(Y)]_\pi = 0$) are also symmetries of the *full* S'_α, since $[\epsilon(Y), S'_\alpha] = 0$ holds if $[\epsilon(Y), \Psi] = 0$, which in its turn is implied by $[\epsilon(Y), \Phi'(Y)]_\pi = 0$.

4 Regularity of full master fields on correspondence space

In this section, we examine the scalar field profiles and the Weyl zero-form using the stereographic coordinates (see appendix B and C for details), which facilitate a uniform treatment of all cases. We first study the linearized scalar field profile and then turn to the analysis of the regularity of the full Weyl zero-form in the correspondence space, by which we mean the twistor space extended spacetime with coordinates (x, Y, Z).

4.1 Linearized scalar field profile

In stereographic coordinates, the linearized scalar field is given by

$$\phi^{(1)} = \mathcal{O}\frac{1}{\det A} \equiv \mathcal{O}\phi_\eta\,, \qquad \phi_\eta := \frac{h^2}{1 - 2\lambda b^a x_a + \lambda^2 b^2 x^2}\,, \tag{4.1}$$

which can be re-written as

$$\phi_\eta = \frac{1}{h^{-2}(\lambda x - b)^2 + 1 - b^2} = \frac{h^2}{b^2(\lambda x + R(b))^2}\,, \qquad b^a = i\eta L^a\,, \tag{4.2}$$

where R is the reflection map. For $\beta > 0$ and $k \neq 0$, we have

$$|k| = 1\,, \quad \beta > 0 \;:\quad \phi^{(1)} = \sum_{i=\pm} \nu_i \phi_{\eta_i}\,, \tag{4.3}$$

where $\eta_\pm = -\gamma \pm \sqrt{\epsilon + \gamma^2}$, with $\gamma = \frac{i\alpha}{\lambda\beta}$ and $\epsilon = L^a L_a$, such that that $R^a(b_\pm) = -b^a_\mp$, and $\nu_+ := \mu$ and $\nu_- = \bar{\mu}$ for $\epsilon k = -1$.

In the limit $\beta \to 0$, one has $\eta_- \to \infty$ and $\eta_+ \to 0$, and hence

$$\beta = 0 \;:\quad \phi^{(1)} = \nu_+ h^2\;. \tag{4.4}$$

The case of $k = 0$ arises in the limit (3.13) and (3.14). More directly, from

$$\phi^{(1)} = \oint_C \frac{d\eta}{2\pi i}\widetilde{\Phi}'(\eta)\phi_\eta\,, \tag{4.5}$$

using (4.1) and (3.12b) readily gives

$$k = 0: \qquad \phi^{(1)} = \frac{1 - \lambda^2 x^2}{1 + 2i\lambda\sqrt{-\epsilon}\,L^a x_a + \lambda^2 x^2}\left[\nu + \frac{2\widetilde{\nu}(i\lambda\sqrt{-\epsilon}\,L^a x_a + \lambda^2 x^2)}{1 + 2i\lambda\sqrt{-\epsilon}\,L^a x_a + \lambda^2 x^2}\right], \tag{4.6}$$

where $\lambda = i\ell^{-1}$ and $\epsilon = -1$ for $\mathfrak{iso}(3)$, and $\lambda = \ell^{-1}$ and $\epsilon = 1$ for $\mathfrak{iso}(1,2)$. In the planar coordinate system defined in (B.21) and (B.22), this solutions takes the simple form given in (5.4) which will be discussed in more detail in section 5.

The iso-scalar surfaces $S_i(c_i)$ $(i = \pm)$ are defined by

$$\phi_{\eta_i}|_{S_i(c_i)} = c_i\,, \qquad c_i \in \mathbb{C}\,, \tag{4.7}$$

which are $\mathfrak{g}_6$ invariant, and complexified for $\epsilon k = -1$; in particular, $S_i(0)$ is the boundary. Away from the boundary, we have

$$(1 + b_i^2 - 2\lambda b_i^a x_a - (c_i^{-1} + b_i^2)h^2)|_{S_i(c_i)} = 0\,, \qquad c_i\,,\; h^2 \neq 0. \tag{4.8}$$

It follows that

$$S_+(c_+) = S_-(c_-) \quad\Leftrightarrow\quad \frac{\eta_-}{c_+} + \eta_+ = \frac{\eta_+}{c_-} + \eta_-\,, \tag{4.9}$$

as can be seen first by eliminating h^2, which yields

$$(c_+^{-1} + b_+^2)(1 + b_+^2 - 2\lambda b_+^a x_a) = (c_-^{-1} + b_-^2)(1 + b_-^2 - 2\lambda b_-^a x_a)\,, \tag{4.10}$$

that must hold identically for all x^a modulo $\eta_- b_+^a = \eta_+ b_-^a$, $\eta_-(1 + b_+^2) = \eta_+(1 + b_-^2)$, and $\eta_+\eta_- = -\epsilon$, provided that $c_\pm$ obey the relation in (4.9). Taking the limit $|c_\pm| \to \infty$, it follows from (4.9) that if $k \neq 0$, so that $\eta_+ \neq \eta_-$, then $S_+(\infty) \cap S_-(\infty) = \emptyset$ away from the boundary; conversely, requiring $S_+(\infty) \cap S_-(\infty) \neq \emptyset$ implies that

$$h^2 = 0\,, \qquad L^a x_a = \frac{i\epsilon}{2}(\eta_+ + \eta_-)\,, \tag{4.11}$$

i.e. the two singular surfaces coincide on a two-dimensional subspace of the boundary. If $k = 0$, then $\eta_+ = \eta_-$, and it follows from (4.9) that $S_+(c) = S_-(c)$ for all c. Moreover, if $\epsilon k = -1$, then $S_\pm(\infty) \cap M_4^{(0)} = \emptyset$, while if $\epsilon k = 0, 1$, then $S_\pm(\infty) \cap M_4^{(0)}$ is the cone

$$\widetilde{x}^2_\pm \;=\; 0\,, \qquad \widetilde{x}^a_\pm := \lambda x^a + R^a(b_\pm)\;. \tag{4.12}$$

4.2 (Ir)regularity of Weyl zero-form

As the description of the solutions in terms of Fronsdal fields is reliable only at weak coupling, we resort to the full master fields close to the surfaces $S_\pm(\infty)$. The Weyl zero-form (see (3.42), (3.44) and (3.49)) is given by

$$\Phi^{(L)} \;=\; 2\pi\mathcal{O}\,\delta^2(Ay+B\bar{y})\star\kappa_y\,, \tag{4.13}$$

which is regular on $M_4^{(0)}$ for $\epsilon k=-1$, and degenerates on the cones in eq. (4.12) for $\epsilon k=0,1$. From (A.4) and (C.12), at the apexes and for $\epsilon k=1$, we have

$$\begin{aligned} A_\alpha{}^\beta|_{\tilde{x}^a_\pm=0,\;\eta=\eta_\pm} &= h_\pm^{-1}\left(\delta_\alpha{}^\beta+(\not{b}_\pm\bar{\not{R}}(b_\pm))_\alpha{}^\beta\right)=h_\pm^{-1}\left(1-\frac{b_\pm^2}{b_\pm^2}\right)\delta_\alpha{}^\beta=0\,,\\ B_\alpha{}^{\dot\beta}|_{\tilde{x}^a_\pm=0,\;\eta=\eta_\pm} &= h_\pm^{-1}(-\not{R}(b_\pm)-\not{b}_\pm)_\alpha{}^{\dot\beta}=h_\pm^{-1}\left(\frac{1}{b_\pm^2}-1\right)(\not{b}_\pm)_\alpha{}^{\dot\beta}=-\sqrt{1-\frac{1}{b_\pm^2}}\,(\not{b}_\pm)_\alpha{}^{\dot\beta}\,, \end{aligned} \tag{4.14}$$

from which it follows that

$$\epsilon k=+1\;\;:\;\;\Phi^{(L)}_\pm|_{\tilde{x}^a_\pm=0}=\frac{\nu_\pm}{1+\epsilon\eta_\pm^2}\,\kappa_y\bar{\kappa}_{\bar{y}}\,, \tag{4.15}$$

which are regular $\mathfrak{so}(1,3)$-invariant elements in $\mathcal{A}(\mathcal{Y}_4)$. For $k=0$, on the other hand, the Weyl zero-form diverges at the apex, which now resides at the boundary.

At the light cones $\tilde{x}^2_\pm=0$, away from the apexes, we can treat the case $\epsilon k=+1$ by writing

$$\lambda\tilde{\not{x}}_{\pm\alpha\dot\alpha}=\tau u^{(+)}_\alpha\bar{u}^{(+)}_{\dot\alpha}\,,\qquad \not{b}_{\alpha\dot\alpha}=i\eta(u^{(+)}_\alpha\bar{u}^{(+)}_{\dot\alpha}-\epsilon u^{(-)}_\alpha\bar{u}^{(-)}_{\dot\alpha})\,, \tag{4.16}$$

where $\tau=\epsilon\tau^\dagger$ is a linear coordinate along the lightcone and $u^{(\pm)}_\alpha$ is a normalized x-dependent spin frame, *viz.*

$$u^{(+)\alpha}u^{(-)}_\alpha=1\,,\quad \delta^\beta_\alpha=u^{(-)}_\alpha u^{(+)\beta}-u^{(+)}_\alpha u^{(-)\beta}\,,\quad \delta^2(v_\alpha)=\delta(u^{(+)\alpha}v_\alpha)\delta(u^{(-)\alpha}v_\alpha)\,, \tag{4.17}$$

and $\bar{u}^{(\pm)}_{\dot\alpha}=(u^{(\pm)}_\alpha)^\dagger$. It follows from (4.12) that

$$\lambda x_{\alpha\dot\alpha}|_{\tilde{x}^2_\pm=0}=\left(\tau-\frac{i\epsilon}{\eta_\pm}\right)u^{(+)}_\alpha\bar{u}^{(+)}_{\dot\alpha}+\frac{i}{\eta_\pm}u^{(-)}_\alpha\bar{u}^{(-)}_{\dot\alpha}\,,\qquad h^2|_{\tilde{x}^2_\pm=0}\equiv h^2_\pm=\frac{1+\epsilon\eta_\pm^2+i\epsilon\eta_\pm\tau}{\epsilon\eta_\pm^2}\,, \tag{4.18}$$

and hence

$$\begin{aligned} A_\alpha{}^\beta|_{\tilde{x}^2_\pm=0\,,\eta=\eta_\pm} &= -i\epsilon\eta_\pm\tau h_\pm^{-1}u^{(-)}_\alpha u^{(+)\beta}\,,\\ B_\alpha{}^{\dot\beta}|_{\tilde{x}^2_\pm=0\,,\eta=\eta_\pm} &= h_\pm^{-1}\left(\tau u^{(+)}_\alpha\bar{u}^{(+)\dot\beta}-i\left(\eta_\pm+\frac{1}{\epsilon\eta_\pm}\right)\left(u^{(+)}_\alpha\bar{u}^{(+)\dot\beta}-\epsilon u^{(-)}_\alpha\bar{u}^{(-)\dot\beta}\right)\right). \end{aligned} \tag{4.19}$$

Using

$$\int_{\mathbb{R}^2}dsdt\,\exp(iast)=\frac{2\pi}{a}\,, \tag{4.20}$$

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which provides a normalization of the real analytic delta function in two variables, corresponding to the delta sequence

$$\lim_{a\to\infty} a\exp(iast) = 2\pi\delta(s)\delta(t)\,, \tag{4.21}$$

and staying away from the apex, i.e. taking $\tau\neq 0$, we find that

$$\begin{aligned}\Phi^{(L)}_{\pm}|_{\tilde{x}^2_{\pm}=0\,,\tau\neq 0} &= \nu_{\pm}h^2_{\pm}\int d\xi^{(+)}d\xi^{(-)}e^{i(y^{(-)}\xi^{(+)}-y^{(+)}\xi^{(-)})}\delta\left(-i\epsilon\left(\eta_{\pm}\tau\xi^{(+)}-\left(\eta_{\pm}+\frac{1}{\epsilon\eta_{\pm}}\right)\bar{y}^{(-)}\right)\right)\\ &\quad\times\delta\left(\tau\bar{y}^{(+)}-i(\eta_{\pm}+\frac{1}{\epsilon\eta_{\pm}})\bar{y}^{(+)}\right)\,,\\ &= \frac{2\pi\nu_{\pm}}{\epsilon\eta^2_{\pm}\tau}\,\delta(y^{(+)})\delta(\bar{y}^{(+)})\,\exp\left[i\tau^{-1}\left(1+\frac{1}{\epsilon\eta^2_{\pm}}\right)y^{(-)}\bar{y}^{(-)}\right]\,,\end{aligned} \tag{4.22}$$

whose apex limit is indeed in agreement with (4.15), *viz.*

$$\epsilon k=+1\ :\quad \lim_{\tau\to 0}\Phi^{(L)}_{\pm}|_{\tilde{x}^2_{\pm}=0} = \frac{\nu_{\pm}}{1+\epsilon\eta^2_{\pm}}\kappa_y\bar{\kappa}_{\bar{y}}\ . \tag{4.23}$$

In general, one needs to distinguish between singularities that are gauge artifacts and genuine singularities showing up in higher spin invariants. In particular, the zero-form charges of [14, 37] are higher spin invariants built directly from the Weyl zero-form and the twistor space connection; in the present case, the simplest such charges takes the form

$$\mathcal{I}_n = \int_{\mathcal{Z}_4\times\mathcal{Y}_4}[\Phi^{(L)}\star\pi(\Phi^{(L)})]^{\star n}\star J\star\overline{J}\star d^4Y = [\Phi^{(L)}\star\pi(\Phi^{(L)})]^{\star n}|_{Y^{\underline{\alpha}}=0}\,, \tag{4.24}$$

which are formally de Rham closed on $\mathcal{X}_4$ on shell. We have checked that indeed

$$\epsilon k=+1\ :\quad \Phi^{(L)}_{\pm}|_{\tilde{x}^2_{\pm}=0}\star\pi(\Phi^{(L)}_{\pm}|_{\tilde{x}^2_{\pm}=0}) = (\Phi^{(L)}_{\pm}\star\pi(\Phi^{(L)}_{\pm}))|_{\tilde{x}^2_{\pm}=0} = \frac{(\nu_{\pm})^2}{1+\epsilon(\eta_{\pm})^2}\,, \tag{4.25}$$

while it remains to compute $\Phi^{(L)}_{\pm}|_{\tilde{x}^2_{\pm}=0}\star\pi(\Phi^{(L)}_{\mp}|_{\tilde{x}^2_{\pm}=0})$; if the latter quantity vanishes, which is our expectation, then the Weyl zero-form Φ is regular on $M^{(0)}_4$ in the sense that $\mathcal{I}_n$ if well-defined on $M^{(0)}_4$.

As for $k=0$, on the other hand, we can treat the case $\tilde{\nu}=0$ by taking a limit, leading to

$$k=0\ :\quad \Phi^{(L)}_{\pm}|_{\tilde{x}^2_{\pm}=0\,,\tau\neq 0\,,k=0\,,\tilde{\nu}=0} = -\frac{2\pi\nu}{\tau}\,\delta(y^{(+)})\delta(\bar{y}^{(+)})\,, \tag{4.26}$$

that indeed diverges as $\tau\to 0$, in agreement with the separate analysis at the apex given above, and yields divergent values for $\mathcal{I}_n$. Thus, as $\mathcal{I}_n$ vanishes away from the cone $\tilde{x}^2_{\pm}=0$, it follows that if $k=0$, then the zero-form charges are not smooth on $\mathcal{M}^{(0)}_4$.

For $k=0$, one may instead seek to remove the singularities by quotienting $M^{(0)}_4$ by discrete symmetries, as to restrict the exact solution to a subregion $M_4\subset M^{(0)}_4$. In particular, if $\Lambda>0$, this can be done by restriction to the causal patch

$$M_4 = (dS_4\setminus S(\infty))/\mathbb{Z}_2\,, \tag{4.27}$$

where $\mathbb{Z}_2 = \{e, \gamma\}$ is defined by

$$(X^0, X^i, X^5)(\gamma(p)) = (-X^0, X^i, -X^5)(p)\,, \qquad p \in dS_4\,, \tag{4.28}$$

using embedding space coordinates, and $S(\infty)$ is the surface where the Weyl zero-form blows up. As $S(\infty)$ coincides with the set of fixed points of γ, it follows that M_4 is a smooth manifold, on which thus the Weyl zero-form is well-defined. Whether there exists a similar construction for the $k = 0$ domain wall when $\Lambda < 0$, remains to be analyzed.

5 The $\mathfrak{iso}(3)$ invariant solution and cosmology

5.1 The solution at linear order in deformation parameter

In this section we take a closer look at the $\mathfrak{iso}(3)$ invariant solution and compare it with those which arise in standard inflationary cosmologies. Cosmological aspects will be discussed further in the conclusions. The linearized solution for the scalar field $\phi(x) = \Phi(x, y, \bar{y})|_{y=\bar{y}=0}$, can be obtained from (3.66) by setting $y = \bar{y} = 0$, and using (3.10), (3.12b) and (C.12). One thus finds

$$\phi(x) = \frac{1 + x^2}{1 + 2x^0 - x^2}\left[\nu - \frac{2\widetilde{\nu}(-x^0 + x^2)}{1 + 2x^0 - x^2}\right]\,, \tag{5.1}$$

where $x^2 = x^\mu x^\nu \eta_{\mu\nu}$, and we have chosen $L^a = (1, 0, 0, 0)$, $\lambda = i$. The rotational symmetries generated by the Killing vectors K^μ_{rs} are manifest, but not the translational symmetries generated by K^μ_r, which are worked out in appendix B; see (B.45). In planar coordinates defined in (B.43) (with $\varsigma = \sigma =$"$+$" and $(t, y^i) = (t_+, y^i_+)$), the metric becomes

$$ds^2 = -dt^2 + e^{2t} \sum_i (dy^i)^2\,, \tag{5.2}$$

and the scalar field takes the form

$$\phi(x) = (\nu + \widetilde{\nu})\, e^{-t} - \widetilde{\nu} e^{-2t}\ . \tag{5.3}$$

In the conformal coordinate system, obtained by the coordinate transformation $\tau = -e^{-t}$, the solution reads

$$\phi(x) = -\,(\nu + \widetilde{\nu})\,\tau - \widetilde{\nu}\tau^2\ . \tag{5.4}$$

As usual in cosmology, we will call t the cosmic time and τ the conformal time. While the solution for the scalar field given above arises as the solution of the scalar field equation arising in higher spin theory at linear order in deformation parameters, we observe that it is also the solution of Klein-Gordon field equation with conformal mass, i.e. $(\Box^{(0)} + 2\lambda^2)\phi = 0$, subject to the condition that ϕ depends only on τ, by $\mathfrak{iso}(3)$ symmetry. Note that this differs from the vacuum solution of the same equation if we require $\mathfrak{so}(1, 4)$ invariance by which the scalar field vanishes. Since the metric in this case is de Sitter, we expect that the solution for the metric will be a deformation from de Sitter metric starting at second order in the deformation parameter. As for the fields with spins $s > 2$, they vanish in the background solution at lowest order in the deformation parameter. Whether they arise in higher orders remains to be determined.

5.2 Comparison with standard cosmological backgrounds

In order to compare with standard inflationary models, let us quickly summarize the behavior of the inflaton. In standard slow roll inflation, one studies a solution for which the metric is close to de Sitter, with deviations parametrised by the slow-roll parameters[18]

$$\epsilon := -\frac{\dot{H}}{H^2}\,, \quad \eta := -\frac{\ddot{\phi}}{H\dot{\phi}}\,, \tag{5.5}$$

where $H := \dot{a}/a$ is the Hubble parameter, and a dot denotes a derivative with respect to cosmic time t. The metric is de Sitter when H is constant, and by definition inflation happens when $\ddot{a} > 0$ or $\epsilon < 1$. The solutions for the scalar field and the metric to next-to-leading order in slow-roll, and valid for $t - t_* \lesssim \phi/\dot{\phi}$, are given in terms of conformal time τ by

$$a \approx a_* \left(\frac{\tau_*}{\tau}\right)^{1/(1+\epsilon)} \,, \quad \phi \approx \phi_* + 2\sqrt{\epsilon}\ln\frac{\tau}{\tau_*}\,, \tag{5.6}$$

where at some fixed time τ_* we have imposed that the scale factor and the scalar field take on some given values a_* and ϕ_*. For the simplest potential $V(\phi) = m^2\phi^2/2$, the slow-roll parameters are $\epsilon_v = 2\phi^{-2}$ and $\eta_v = 2\phi^{-2}$, with ϕ being the background solution. The slow-roll approximation then requires $\phi \gg 1$ in Planck units. It follows that $\phi_* = 6H/m$, which means that we must have $m \ll H$. If one perturbs around this background, writing $\phi(x) = \bar{\phi}(\tau) + \varphi(x)$, one obtains in Fourier space after choosing the Bunch-Davies vacuum

$$\varphi(k,\tau) = e^{i(2\kappa+1)\pi/4}\left(-\frac{\pi H^4\tau^3}{8\epsilon}\right)^{1/2} H^{(1)}_{\kappa}(-k\tau)\,, \quad \kappa = \frac{3}{2} + 2\epsilon - \eta\,, \tag{5.7}$$

where we see that the slow-roll parameters appear in the index of the Hankel functions. These solutions are long-lived in the sense that $\varphi \sim \tau^{3/2-\kappa}$ at late times $\tau \to 0$. For a recent review, see [49].

In the approaches to cosmology with higher spin fields in [28–30], the scalar field is taken to be an inflaton, whose background behaves as described above, and the background metric field is taken to be de Sitter spacetime; all higher spin fields are taken to vanish in the background. However, these approaches are not derived from a theory with higher spin symmetry.

In comparing the background solutions summarised above with ours, we note that the linearised Vasiliev scalar field satisfies the equation $(\Box^{(0)} + 2\lambda^2)\phi = 0$, subject to $\mathfrak{iso}(3)$ invariance as discussed earlier. As such it has a potential with $m \sim H$ in the notation employed above, which is incompatible with the slow-roll approximation. Indeed, it does not have the shape of the solution for standard slow-roll inflation (5.6), and even goes to zero at late times.

Even if the scalar behaves differently from the standard slow-roll inflaton at the level we are working, our solution is still inflationary in the sense that the metric is close to de

[18]These are the slow-roll parameters defined in terms of the Hubble parameter and the derivatives of the field. The slow-roll parameters defined in terms of the potential $\epsilon_v := \frac{M_{pl}}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2$, $\eta_v := M_{pl}^2\frac{V''(\phi)}{V(\phi)}$ are related to these by $\epsilon \approx \epsilon_v$ and $\eta \approx \eta_v - \epsilon_v$ to leading order in slow-roll.

Sitter. The time-dependent deformation away from de Sitter may in principle lead to an end of the accelerated phase. This would require $\ddot{a} < 0$ which is far from de Sitter, and may in principle be achieved by summing all orders in the deformation parameter.

Though the calculation of the fluctuations in higher spin gravity is beyond the scope of this paper, let us discuss their expected behavior by considering fluctuations of a conformally coupled scalar field around de Sitter. CMB observations have fixed primordial fluctuations sourced by scalar fluctuations, to be nearly scale-invariant (in this context this is defined as their 2-point function in Fourier space behaving as $1/k^3$). They are also observed to have a larger amplitude than fluctuations sourced by the graviton. This is different from what would be generated by a conformally coupled scalar field: the behaviour of its 2-point function in Fourier space in the limit $\tau \to 0$ goes like $1/k$. Furthermore, the amplitude of this 2-point function is suppressed by positive powers of τ, so one can say that they are "short lived" and suppressed with respect to the fluctuations of a massless graviton (which go as τ^0 in that limit, and are thus "long lived"). We expect that the corrections to this behaviour of the scalar field during inflation will be suppressed by the deformation parameter.

We can envisage two mechanisms by which the behavior of the scalar in higher spin theory may be "long lived". One possibility is that an exact FRW-like solution, i.e. to all orders in the deformation parameter, may lead to a behavior of the metric for which it takes an infinite proper time to reach a critical value of the conformal time. Whether this leads to long-lived scalar fluctuations remains to be seen. Another possible mechanism is to consider a coupling with a massive higher spin multiplet that contains a massive scalar with conformal dimension zero. Indeed, this arises in 6-fold product of the fundamental representation of de Sitter group. A long-lived scalar field would arise in this scenario even though the coupling of massive higher spin multiplets with Vasiliev higher spin gravity is a formidable task which has hardly been studied so far. We should also require the scalar two-point function to be approximately, but not exactly, scale invariant. Since our solution is close to de Sitter, but not exactly, such behaviour can emerge.

Assuming that one resolves the problem described above, the amplitude of fluctuations produced should agree with observations, in particular the CMB data. Clearly, since observations are made at very late times, when the characteristic energy scales are small, higher spin symmetry should be broken. In a conservative scenario, one may assume that at such low energies physics is well described by the Standard Model coupled to gravity in a gravitational background inherited from inflation. However, it remains to be seen whether the details of the higher spin symmetry breaking gives rise to novel interactions in the effective action. For inflation to be described by the unbroken phase of higher spin gravity, the symmetry breaking should happen at energy scales smaller than $\sim 10^{15}$ GeV according to the upper bounds on graviton (tensor mode) fluctuations[19] [41]. If the dependence of the graviton two-point function deviates from the H^2/M_{pl}^2 behavior significantly, this scale will change accordingly.

[19]In particular, the amplitude of the two-point function of massless graviton (tensor mode) fluctuations should be smaller than $\sim 10^{-11}$. If the two-point function is similar to that of standard inflation, which goes as H^2/M_{pl}^2, this means that $H \lesssim 10^{-4} M_{pl}$.

5.3 Towards perturbation theory around exact solutions

Given the $\mathfrak{g}_6$-invariant solutions constructed above, it is natural to study fluctuations around them. This can be facilitated using the factorization method, with zero-form initial data

$$\Psi = \Psi_{\rm bg} + \Psi_{\rm fl}\,, \tag{5.8}$$

and treating $\Psi_{\rm bg}$ exactly while keeping only the first order in $\Psi_{\rm fl}$. In what follows, we shall make the stronger assumption that $\Psi_{\rm bg}, \Psi_{\rm fl} \in \mathcal{A}(\mathcal{Y}_4)$, i.e. we assume that both background and fluctuations belong to the same algebra, such that $\Psi^{\star n} \in \mathcal{A}(\mathcal{Y}_4)$, which can then be expanded separately in background as well as fluctuation parameters.

Thus, in order to construct a concrete model, we need to choose $\mathcal{A}(\mathcal{Y}_4)$ in accordance with the dual boundary conditions in twistor space and spacetime. As a concrete example, let us take $\Lambda < 0$, and consider fluctuations around

$$\Psi_{\rm bg} = \Psi_{{\rm FRW}^{(\rm AdS)}_{-}}\,, \tag{5.9}$$

i.e. the (unique) FRW-like solution in the case of negative cosmological constant. On physical grounds, we take $\mathcal{A}(\mathcal{Y}_4)$ to consists of deformations of the cosmological background, which correspond to spacetime mode functions that cannot be localized,[20] and particle and black hole-like states, corresponding to localizable spacetime mode functions. Thus,

$$\mathcal{A}(\mathcal{Y}_4) = \mathcal{A}_{\rm nl} \oplus \mathcal{A}_{\rm pt}(\mathcal{Y}_4) \oplus \mathcal{A}_{\rm bh}(\mathcal{Y}_4)\,, \tag{5.10}$$

given, respectively, by the orbits of the higher spin algebra $\mathfrak{hs}_1(4)$ (obtained by repeated action with constant $\mathfrak{hs}_1(4)$ parameters) of $\Psi_{{\rm FRW}^{(\rm AdS)}_{-}}$, denoted by $\mathcal{A}_{\rm nl}$, and the identity operator; the massless scalar particle ground state (with anti-de Sitter energies ± 1 and vanishing spin); and the black hole-like solution with vanishing anti-de Sitter energy and spin [17, 48]. The higher spin algebra $\mathfrak{hs}_1(4)$ is simply the algebra of even order polynomials in $Y^{\underline{\alpha}}$ with respect to the commutation rule (2.5). Using the regular presentation (see eq. (3.17) and appendix E), the star products of these ground states are well-defined, leading to the following fusion rules:

$$\mathcal{A}_{\rm nl} \star \mathcal{A}_{\rm nl} \subseteq \mathcal{A}_{\rm nl}\,, \qquad \mathcal{A}_{\rm nl} \star \mathcal{A}_{\rm pt} \subseteq \mathcal{A}_{\rm pt}\,, \qquad \mathcal{A}_{\rm nl} \star \mathcal{A}_{\rm bh} \subseteq \mathcal{A}_{\rm bh}\,, \tag{5.11}$$

$$\mathcal{A}_{\rm pt} \star \mathcal{A}_{\rm pt} \subseteq \mathcal{A}_{\rm bh}\,, \qquad \mathcal{A}_{\rm pt} \star \mathcal{A}_{\rm bh} \subseteq \mathcal{A}_{\rm pt}\,, \qquad \mathcal{A}_{\rm bh} \star \mathcal{A}_{\rm bh} \subseteq \mathcal{A}_{\rm bh}\,, \tag{5.12}$$

where we note the interesting facts that the non-localizable modes and the localizable modes form two self-interacting subsystems, and that non-localizable modes undergo stimulated decay to localizable modes. The system of self-interacting system of particles and black holes has been studied in [18], where a fully non-linear solution space was obtained by superposing rotationally invariant scalar particle and black hole-like states, which form a

[20] As for holographic interpretations, while the particle and black hole-like states can be mapped to operators of dual conformal theories, it is natural to associate the non-localizable modes to operators in a phase of the boundary field theory in which conformal invariance is spontaneously broken; for the case of the flat domainwall in anti-de Sitter spacetime, see [16].

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subalgebra of $\mathcal{A}_{\rm pt} \oplus \mathcal{A}_{\rm bh}$ spanned by projectors and twisted projectors. Put into equations, letting E denote the energy operator, one has

$$\Psi_{\rm pt} := \sum_{\mu=\pm1,\pm2,\dots} \mu_n P_n \star \kappa_y \,, \qquad \mu_{-n} = \mu_n^* \,, \tag{5.13}$$

$$\Psi_{\rm bh} := \sum_{\mu=\pm1,\pm2,\dots} \nu_n P_n \,, \qquad \nu_n = \nu_n^* \,, \tag{5.14}$$

where

$$P_n(E) = 2(-1)^{n-1}\varepsilon \oint_{C(\varepsilon)} \frac{d\eta}{2\pi i} \left(\frac{\eta+1}{\eta-1}\right)^n e^{-4\eta E} \,, \qquad \varepsilon := \frac{n}{|n|} \,. \tag{5.15}$$

and

$$\Psi_{FRW_-} = \oint_{C(i)} \frac{d\eta}{2\pi i} \frac{\nu_+}{\eta - i} e^{-4\eta E} \star \kappa_y + \oint_{C(-i)} \frac{d\eta}{2\pi i} \frac{\nu_-}{\eta + i} e^{-4\eta E} \star \kappa_y \,, \tag{5.16}$$

with $C(\varepsilon)$ and $C(\pm i)$ being small contours encircling ε and $\pm i$, respectively. Using the star product lemmas in appendix E and contour integration techniques, it follows that

$$P_n \star P_m = \delta_{m,n} P_n \,, \qquad (P_n)^\dagger = P_n \,, \qquad P_n \star \kappa_y \star \bar\kappa_{\bar y} = (-1)^n P_n \,, \tag{5.17}$$

and indeed

$$\Psi_{\rm pt} \star \Psi_{\rm pt} \in \mathcal{A}_{\rm bh} \,, \quad \Psi_{FRW_-} \star \Psi_{\rm bh} \in \mathcal{A}_{\rm bh} \,, \quad \Psi_{FRW_-} \star \Psi_{\rm pt} \in \mathcal{A}_{\rm pt} \,, \tag{5.18}$$

in accordance with the fusion rules given above.

The following remarks are in order:

1. The particle states form Hilbert spaces with $\mathfrak{hs}_1(4)$-invariant sesqui-linear forms that are isomorphic to direct product of two singletons, the black hole-like states belong to a real vector space with $\mathfrak{hs}_1(4)$-invariant Euclidean bilinear forms that are isomorphic to the direct product of a singleton and an anti-singleton [18, 33]; it remains to be seen whether the non-localizable modes admit any such first-quantized interpretation.

2. The above considerations apply to other $\mathfrak{g}_6$ invariant solutions with $\Lambda < 0$ as well, while for $\Lambda > 0$ the star product realization of particle and black hole-like states need further study.

3. In the case of $\Lambda > 0$, we let $\mathcal{A}_{\rm nl}$ stand for the $\mathfrak{hs}_1(4)$ orbit generated from the $\mathfrak{so}(4)$-invariant and the identity operator, and $\mathcal{A}_{\rm pt}$ for the orbit of the $\mathfrak{iso}(3)$ invariant solutions. Using eq. (3.17) and the regular presentation, it follows that

$$\mathcal{A}_{\rm nl} \star \mathcal{A}_{\rm nl} \subseteq \mathcal{A}_{\rm nl} \,, \quad \mathcal{A}_{\rm nl} \star \mathcal{A}_{\rm pt} \subseteq \mathcal{A}_{\rm pt} \,, \quad \mathcal{A}_{\rm pt} \star \mathcal{A}_{\rm pt} = 0 \,. \tag{5.19}$$

   Thus, if it is possible to equip $\mathcal{A}_{\rm pt}$ with a basis plane waves normalized on Dirac delta functions that permits an interpretation in terms of localizable particle states such that the two first fusion rules remain intact, while possibly $\mathcal{A}_{\rm pt} \star \mathcal{A}_{\rm pt}$ may become nontrivial, then we would have a mechanism in the case of $\Lambda > 0$ analogous to that given above in the case of $\Lambda < 0$.

A comment on the qualitative nature of the fusion rules is in order. The solutions found in this paper are organized in a perturbative expansion in star-power series of the initial datum $\Psi(Y)$ starting from specific choice of basis functions on $\mathcal{Y}$. Clearly, sufficiently large changes of twistor-space basis, by means of large gauge transformations or redefinitions of Ψ, may affect the perturbative organization of the master fields, and, consequently, the description of the solutions in terms of master field configurations on correspondence space, or component field configurations on spacetime. In particular, as already discussed in [18] for solutions involving particle and black hole modes, the fusion rules (5.11)-(5.12), which are direct consequences of working with star-product interactions on the correspondence space, translate into a choice of frame for the effective spacetime field theory with highly non-local vertices. It was recently shown in [50], however, that, at least at the quadratic order (in the equations of motion), there exists a different frame, corresponding to imposing specific gauges and boundary conditions on $\mathcal{Z}$, that leads to vertices that are quasi-local in the sense that they involve finitely many derivatives for a fixed set of Lorentz spins. Whether there exists a quasi-local frame beyond this order (within the context of a Noether-like procedure for obtaining a classical theory), remains an open problem. If it does, then the particle modes should form a closed subsystem within this frame at any finite order in perturbation theory, leaving the possibility that the original star product interactions of master fields in the correspondence space actually describe a quantum effective theory including effects that are non-perturbative in the quasi-local frame. To this extent, it is important that the actual observables of the theory are not the master fields themselves, but rather higher spin invariant functionals thereof, and it is only at the level of such observables that we may expect an agreement between the two schemes.

6 Conclusions

We have constructed classes of exact solutions of Vasiliev's bosonic higher spin gravities with Killing symmetries given the enveloping of six-dimensional subalgebras of the (anti-)de Sitter symmetry algebras. In order to construct the solutions we have used the fact that Vasiliev's equations form a integrable system on an enlargement of spacetime by an internal non-commutative twistor space. As the integrability is of Cartan type, we can solve the integrable system transforming a particular holomorphic solution $(\Phi', A'_{\underline{\alpha}})$ in twistor space into a physical solution $(\Phi^{(G)}, A^{(G)}_{\underline{\alpha}}, A^{(G)}_{\mu}) := G^{-1} \star (\Phi', A'_{\underline{\alpha}} + \partial^{(Z)}_{\underline{\alpha}}, \partial_\mu) \star G$ using a gauge transformation generated by a gauge function $G = L \star H$ that is large in the sense that it alters the asymptotic behaviour of the master fields in both spacetime and twistor space. The resulting chain of maps take the following form:

$$(\Phi', A'_{\underline{\alpha}}) \overset{L}{\mapsto} (\Phi^{(L)}, A^{(L)}_{\underline{\alpha}}, A^{(L)}_{\mu}) \overset{H}{\mapsto} (\Phi^{(G)}, A^{(G)}_{\underline{\alpha}}, A^{(G)}_{\mu}) \,, \tag{6.1}$$

where L is a vacuum gauge function and H is a field dependent gauge transformation. The role of L is to switch on the dependence of the fields on the spacetime coordinates and to create a finite region of spacetime in which

$$(\Phi^{(L)}, A^{(L)}_{\underline{\alpha}}, A^{(L)}_{\mu}) := L^{-1} \star \left(\Phi', A'_{\underline{\alpha}} + \partial^{(Z)}_{\underline{\alpha}}, \partial_\mu\right) \star L \tag{6.2}$$

are real analytic in the twistor Z space. The latter property permits the perturbative construction of H, whose role is to create asymptotic Fronsdal fields. The symmetries of the solution are encoded into the particular solution, which is chosen to be invariant under parameters in the enveloping algebra generated from the six-dimensional symmetry Lie algebra $\mathfrak{g}_6$, *viz.*

$$D'\epsilon' = 0\,, \qquad [\epsilon', \Phi']_\pi = 0\,, \tag{6.3}$$

where ϵ' are constants built from star products of the generators of $\mathfrak{g}_6$. As a result, the solutions in L-gauge and the physical gauge are invariant under gauge transformations generated by the rigid gauge parameters $\epsilon^{(L)} = L^{-1} \star \epsilon' \star L$ and $\epsilon^{(G)} = H^{-1} \star \epsilon^{(L)} \star H$, respectively. In the holomorphic and L-gauges, we have given the master fields to all orders, involving an expression for the twistor space connection given by two parametric integrals. In the physical gauge, we have given the solution to first order, and proposed a perturbative scheme for continuing to higher orders based on dual boundary conditions in spacetime and twistor space. It remains to push the gauge function method to higher orders of perturbation theory in the physical gauge, which we hope to report on in a future work. We expect this to generate physically interesting domain wall solutions and FRW-like solutions.

A strong motivation for this work has been the prospects for a higher spin cosmology by a direct approach based on finding its accelerating solutions and studying the cosmological perturbations around them. As a first step in this direction, we have constructed the FRW-like solutions and described a framework for studying the fluctuations around them, with the unusual feature that they involve black hole like states as well. Our solutions are exact in holomorphic and L-gauges. While the higher spin transformations that put these solutions in Vasiliev gauge are implemented at the leading order here, nontrivial consequences can still be extracted by studying the cosmological perturbations around solution, just as the study of such perturbations in standard cosmology where slow roll approximation is made for the background.

In a realistic higher spin cosmology, matter couplings and internal symmetry will need to be introduced. The requirement of higher spin symmetry puts severe constraints in doing so. The Vasiliev higher spin theory we have considered here is a universal sector of any higher spin theory, just as the graviton, dilaton and Kalb-Ramond two-form potential form a universal sector of any string theory. Assuming that the universal higher spin gravity sector dominates the physics of the inflation, it has the advantage of being unique, thereby avoiding the excessive freedom in choosing field content, interactions and parameters. For example, in the favored approach to standard inflationary scenario, Einstein gravity is coupled to a real scalar with a potential that is picked by hand to satisfy suitable 'slow-roll' conditions. Moreover, the origin of the scalar field in a fundamental theory is not known. In the higher spin theory based inflation scenario envisaged here, however, the scalar field is necessarily part of the spectrum for the consistency of the higher spin theory, and the inflation is not driven solely by the energy stored in a slowly varying scalar field. Indeed, there is a frame in which the only contact term for the scalar field is a mass term [4]. Note, however, that the theory comes with infinite derivative couplings even at a given order in weak fields. Given that there is no mass scale at our disposal to argue that those couplings will be suppressed, they are all equally important. Thus, the inflationary solution to the higher spin theory will be driven by the higher spin invariant, higher derivative couplings.

While the problem of matter couplings and breaking of higher spin symmetry will need to be ultimately attended to, at present the more pressing problems to tackle seem to be the determination of the higher order terms in the FRW background in Vasiliev gauge, carrying out the cosmological perturbation theory along the line described in section 5.3 and seeking possible holographic interpretation of the results.

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A Conventions and definitions

Using conventions in which $(z_\alpha)^\dagger = -\bar{z}_{\dot\alpha}$, the star product can be realized using a normal ordering scheme as follows:

$$\begin{aligned}
&f(y,\bar{y},z,\bar{z}) \ \star\ g(y,\bar{y},z,\bar{z}) \qquad\qquad (A.1)\\
&= \int \frac{d^2\xi d^2\eta d^2\bar{\xi} d^2\bar{\eta}}{(2\pi)^4}\ e^{i\eta^\alpha\xi_\alpha + i\bar{\eta}^{\dot\alpha}\bar{\xi}_{\dot\alpha}} f(y+\xi,\bar{y}+\bar{\xi},z+\xi,\bar{z}+\bar{\xi}) g(y+\eta,\bar{y}+\bar{\eta},z-\eta,\bar{z}-\bar{\eta})\ .
\end{aligned}$$

Defining

$$\partial_\alpha^{s_1 s_2} := s_1\frac{\partial}{\partial z^\alpha} + s_2\frac{\partial}{\partial y^\alpha}\,, \qquad \partial_{\dot\alpha}^{s_1 s_2} := s_1\frac{\partial}{\partial \bar{z}^{\dot\alpha}} + s_2\frac{\partial}{\partial \bar{y}^{\dot\alpha}}\,, \tag{A.2}$$

where s_1 and s_2 are $+1$ or -1, and given a function $f(y,\bar{y},z,\bar{z})$, one finds

$$\begin{aligned}
y_\alpha \star f &= y_\alpha f + i\partial_\alpha^{-+} f\,, & f\star y_\alpha &= y_\alpha f + i\partial_\alpha^{--} f\,,\\
z_\alpha \star f &= z_\alpha f + i\partial_\alpha^{-+} f\,, & f\star z_\alpha &= z_\alpha f + i\partial_\alpha^{++} f\,,\\
\bar{y}_{\dot\alpha} \star f &= \bar{y}_{\dot\alpha} f + i\partial_{\dot\alpha}^{-+} f\,, & f\star \bar{y}_{\dot\alpha} &= \bar{y}_{\dot\alpha} f + i\partial_{\dot\alpha}^{--} f\,,\\
\bar{z}_{\dot\alpha} \star f &= \bar{z}_{\dot\alpha} f + i\partial_{\dot\alpha}^{-+} f\,, & f\star \bar{z}_{\dot\alpha} &= \bar{z}_{\dot\alpha} f + i\partial_{\dot\alpha}^{++} f\,,
\end{aligned} \tag{A.3}$$

Frequently used quantities in the body of the paper are defined as follows:

$$
\begin{aligned}
z^{\pm} &= u^{\pm\alpha} z_\alpha \,, & w &= i z^+ z^- \,, & \xi &= (1-\tau)/(1+\tau)\,, \\
u^{\alpha\beta} &= 2u^{+(\alpha} u^{-\beta)}\,, & b^a &= i\eta L^a \,, & \not{b} &= b^a \, (\sigma_a)_{\alpha\dot\alpha} \;. \\
h &= \sqrt{1-\lambda^2 x^2}\,, & x^2 &= x^a x_a \,, & \not{x}_{\alpha\dot\alpha} &= x^a \, (\sigma_a)_{\alpha\dot\alpha} \,,
\end{aligned}
\tag{A.4}
$$

$$
\begin{bmatrix} y^L_\alpha \\ \bar y^L_{\dot\alpha} \end{bmatrix} = L^{-1} \star \begin{bmatrix} y_\alpha \\ \bar y_{\dot\alpha} \end{bmatrix} \star L = \begin{bmatrix} L_\alpha{}^\beta & K_\alpha{}^{\dot\beta} \\ \overline{K}_{\dot\alpha}{}^\beta & \overline{L}_{\dot\alpha}{}^{\dot\beta} \end{bmatrix} \begin{bmatrix} y_\beta \\ \bar y_{\dot\beta} \end{bmatrix}, \qquad \widetilde{y}_\alpha = y_\alpha + M_\alpha{}^{\dot\beta} \bar y_{\dot\beta}\,,
$$

$$
A_\alpha{}^\beta = L_\alpha{}^\beta - b_a (\sigma^a \overline{K})_\alpha{}^\beta\,, \qquad B_\alpha{}^{\dot\beta} := K_\alpha{}^{\dot\beta} - b_a(\sigma^a \overline{L})_\alpha{}^{\dot\beta}\,, \qquad M = A^{-1} B\;.
$$

The matrices A, B, M and $\det A$ are given in sterographic coordonates in (C.12), and in planar coordinates in (C.29).

We use the convention in which $(\sigma^a)_{\alpha\dot\alpha} = (1, \vec\sigma)$ where $\vec\sigma$ are the Pauli matrices, and $(\bar\sigma)_{\dot\alpha\alpha}$ is complex conjugate of $(\sigma^a)_{\alpha\dot\alpha}$. Furthermore we define $(\sigma_{ab})_{\alpha\beta} = \left(\sigma_{[a}\right)_\alpha{}^{\dot\gamma} \left(\bar\sigma_{b]}\right)_{\dot\gamma\beta}$ and $(\bar\sigma_{ab})_{\dot\alpha\dot\beta} = \left(\bar\sigma_{[a}\right)_{\dot\alpha}{}^{\gamma} \left(\sigma_{b]}\right)_{\gamma\dot\beta}$. The spinor-indices are raised or lowered by $\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = \epsilon_{\dot\alpha\dot\beta} = \epsilon^{\dot\alpha\dot\beta} = i\sigma^2$, using the NW-SE convention.

B Coordinate systems and Killing vectors

B.1 Embedding space

The metric space $(M^{(0)}_4, (ds^2_4)^{(0)}) \equiv (A)dS_4$ with inverse radius $|\lambda^{-1}|$ can be defined as the surface in the five-dimensional plane $(\mathbb{R}^5, ds^2_5)$ with global Cartesian coordinates X^M and constant metric $ds^2_5 = dX^M dX^N \eta_{MN}$, where $\eta_{MN} = (\eta_{ab}, -\mathrm{sign}(\lambda^2))$, described by the constraint

$$
E := X^M X^N \eta_{MN} + \lambda^{-2} \approx 0\;. \tag{B.1}
$$

Formally, this amounts to a smooth map $f : M^{(0)}_4 \hookrightarrow \mathbb{R}^5$ that obeys $E \circ f \equiv 0$ and that is invertible on $f(M^{(0)}_4)$, that is, there exists an inverse $f^{-1} : f(M^{(0)}_4) \to M^{(0)}_4$ that is a diffeomorphism. The intrinsic metric on $M^{(0)}_4$ is defined by

$$
(ds^2_4)^{(0)}) := f^* ds^2_5 \equiv (dX^M dX^N \eta_{MN})|_{E\approx 0}\,, \tag{B.2}
$$

where f^* denotes the pull-back operation, that is, in terms of coordinates x^μ on $M^{(0)}_4$, one has

$$
g^{(0)}_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N \eta_{MN}\;. \tag{B.3}
$$

B.2 Killing vector fields

The symmetry algebra $\mathfrak{g}_6$ of the solutions under consideration is embedded via (3.1) into the isometry algebra $\mathfrak{g}_{10}$ of $(M^{(0)}_4, (ds^2_4)^{(0)})$. The latter is inherited from the isometry algebra of $(\mathbb{R}^5, ds^2_5)$, namely as its subalgebra

$$
\mathfrak{l}_{10} := \left\{ \vec L :\ \mathcal{L}_{\vec L} ds^2_5 = 0\,, \quad \vec L E \approx 0 \right\} \cong \mathfrak{g}_{10}\,, \tag{B.4}
$$

with Lie bracket induced from the Schouten bracket, and a well-defined action on the ring of equivalence classes $[\Phi] = \{\Phi' \in C^\infty(\mathbb{R}^5) : (\Phi' - \Phi) \circ f \equiv 0\}$ with product $[\Phi_1][\Phi_2] := [\Phi_1\Phi_2]$, given by $\vec{L}[\Phi] := [\vec{L}\Phi]$. This ring is isomorphic to $C^\infty(M_4^{(0)})$ via $\phi_{[\Phi]} := \Phi \circ f$. Thus, each $\vec{L} \in \mathfrak{l}_{10}$ induces an intrinsic Killing vector field $\vec{K}_{\vec{L}}$ on $M_4^{(0)}$ defined by $\vec{K}_{\vec{L}}\phi_{[\Phi]} := \phi_{\vec{L}[\Phi]} \equiv (\vec{L}\Phi)\circ f$. Hence, letting $p \in M_4^{(0)}$ and assuming that Φ is smooth close to $f(M_4^{(0)})$), we have the Killing vector relation

$$f_*(\vec{K}_{\vec{L}}|_p)\Phi \equiv \vec{K}_{\vec{L}}|_p(\Phi \circ f) = \vec{K}_{\vec{L}}|_p\phi_{[\Phi]} = [\vec{L}\Phi]|_{f(p)} = (\vec{L}\Phi)|_{f(p)}\,, \tag{B.5}$$

where f_* denotes the push-forward operation, that is

$$f_*(\vec{K}_{\vec{L}}|_p) = \vec{L}|_{f(p)}\,, \tag{B.6}$$

or $K^\mu_{\vec{L}}|_p f_*(\vec{\partial}_\mu|_p) = L^M\vec{\partial}_M|_{f(p)}$, in terms of an intrinsic coordinate x^μ at p. In the global coordinate basis,

$$\vec{L}_{MN} = X_M\vec{\partial}_N - X_N\vec{\partial}_M\,, \tag{B.7}$$

inducing the intrinsic Killing vectors fields

$$\vec{K}_{MN} = K^\mu_{MN}\vec{\partial}_\mu\,, \qquad K^\mu_{MN}\partial_\mu X^P = 2X_{[M}\delta^P_{N]}\,, \tag{B.8}$$

with components

$$K^\mu_{MN} = 2g^{\mu\nu}X_{[M}\partial_\nu X_{N]}\,. \tag{B.9}$$

It follows that the intrinsic Killing vector fields associated with the $\mathfrak{g}_6$ generators M_{rs} and T_r defined in (3.1) are given by

$$K^\mu_{rs} = 2g^{\mu\nu}L_{[r}{}^aL_{s]}{}^bX_a\partial_\nu X_b\,, \tag{B.10}$$

$$K^\mu_r = g^{\mu\nu}L_r{}^a\left(\alpha L^bX_a\partial_\nu X_b - \alpha L^bX_b\partial_\nu X_a - \beta\ell^{-1}X_a\partial_\nu X_5 + \beta\ell^{-1}X_5\partial_\nu X_a\right). \tag{B.11}$$

Thus, under the $\mathfrak{g}_6$ transformations defined in (3.1), we have

$$\delta x^\mu = \xi^{rs}K^\mu_{rs} + \xi^rK^\mu_r\,, \tag{B.12}$$

with constant parameters (ξ^{rs}, ξ^r).

B.3 Global dS_4/S^3 and AdS_4/AdS_3 foliations (iso-scalar leafs for $\epsilon k = -1$)

Coordinates adapted to the solutions with $\epsilon k = -1$, that is, the $\mathrm{FRW}_+^{(\mathrm{dS})}$ and $\mathrm{DW}_-^{(\mathrm{AdS})}$ solutions, can be obtained by foliating dS_4 and AdS_4 with S^3 and AdS_3 iso-scalar leafs, respectively, *viz.*

$$dS_4:\ \eta_{AB} = (-,\delta_{IJ})\,,\quad X^0 \approx \ell\tau\,,\quad X^I \approx \ell\sqrt{1+\tau^2}\,n^I\,,\quad n^In^J\delta_{IJ} = 1\,, \tag{B.13}$$

$$AdS_4:\ \eta_{AB} = (\eta_{RS},+)\,,\quad X^3 \approx \ell\sigma\,,\quad X^R \approx \ell\sqrt{1+\sigma^2}\,n^R\,,\quad n^Rn^S\eta_{RS} = -1\,, \tag{B.14}$$

where $\delta_{IJ} = \mathrm{diag}(+,+,+,+)$ and $\eta_{RS} = \mathrm{diag}(-,-,+,+)$. Here we have used the labeling $I = 1,2,3,5$ and $R = 0,5,1,2$. The resulting global parametrizations of the induced metrics are given by

$$dS_4:\ (ds_4^2)^{(0)} = \ell^2\left(-\frac{d\tau^2}{1+\tau^2} + (1+\tau^2)dn^2|_{n^In^J\delta_{IJ}=1}\right), \qquad \tau \in \mathbb{R}\,, \tag{B.15}$$

$$AdS_4:\ (ds_4^2)^{(0)} = \ell^2\left(\frac{d\sigma^2}{1+\sigma^2} + (1+\sigma^2)dn^2|_{n^Rn^S\eta_{RS}=-1}\right), \qquad \sigma \in \mathbb{R}\,. \tag{B.16}$$

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B.4 Bifurcating $(A)dS_4/\{dS_3, H_3\}$ foliations (iso-scalar leafs for instantons)

Coordinates adapted to the instanton solutions can be obtained by decomposing dS_4 and AdS_4 into subregions foliated by dS_3 and H_3 leafs as follows:

$$dS_4: \; X^a \approx \ell\xi_\pm n^a\,, \quad n^a n^b \eta_{ab} = \pm 1\,, \quad |X^5| \approx \ell\sqrt{1 \mp \xi_\pm^2}\,, \tag{B.17}$$

$$AdS_4: \; X^a \approx \ell\xi_\pm n^a\,, \quad n^a n^b \eta_{ab} = \pm 1\,, \quad |X^5| \approx \ell\sqrt{1 \pm \xi_\pm^2}\,, \tag{B.18}$$

where $a = 0,1,2,3$ and $\eta_{ab} = \mathrm{diag}(-+++)$. The resulting induced metrics are

$$dS_4 : (ds_4^2)^{(0)} = \ell^2 \left(\pm \frac{d\xi_\pm^2}{1 \mp \xi_\pm^2} + \xi_\pm^2 dn^2|_{n^a n^b \eta_{ab} = \pm 1} \right), \quad 0 \leqslant \xi_+ < 1 < \xi_-\,, \tag{B.19}$$

$$AdS_4 : (ds_4^2)^{(0)} = \ell^2 \left(\pm \frac{d\xi_\pm^2}{1 \pm \xi_\pm^2} + \xi_\pm^2 dn^2|_{n^a n^b \eta_{ab} = \pm 1} \right), \quad 0 \leqslant \xi_- < 1 < \xi_+\,, \tag{B.20}$$

where the upper (lower) sign corresponds to $dS_3(H_3)$ foliations.

B.5 Planar coordinates for $(A)dS_4$ (iso-scalar leafs for $\epsilon k = 0$)

Coordinates adapted to the solutions with $\epsilon k = 0$ can be obtained by foliating dS_4 $(\epsilon = -1)$ and AdS_4 $(\epsilon = +1)$ using Euclidean and Lorentzian three-planes, respectively, as follows:

$$\begin{aligned} dS_4 : X^0 &\approx \sigma\ell\left(\sinh t + \tfrac{1}{2}r^2 e^t\right)\,, & X^5 &\approx \sigma'\ell\left(\cosh t - \tfrac{1}{2}r^2 e^t\right)\,, && \text{(B.21)} \\ X^i &\approx \ell e^t y^i\,, \quad r^2 := y^i y^j \delta_{ij}\,, & i,j &= 1,2,3 && \\ AdS_4 : X^5 &\approx \sigma\ell\left(\cosh t + \tfrac{1}{2}r^2 e^t\right)\,, & X^3 &\approx \sigma'\ell\left(\sinh t - \tfrac{1}{2}r^2 e^t\right)\,, && \text{(B.22)} \\ X^i &\approx \ell e^t y^i\,, \quad r^2 := y^i y^j \eta_{ij}\,, & i,j &= 0,1,2\,, && \end{aligned}$$

where $\eta_{ij} = \mathrm{diag}(-++)$, $(\sigma, \sigma') = (\pm 1, \pm 1)$ and $(y^i, t) \in \mathbb{R}^4$, which provides four charts $U_{\sigma,\sigma'} \cong \mathbb{R}^4$, referred to as Poincaré patches, on which the intrinsic metrics

$$(ds_4^2)^{(0)}|_{U_{\sigma,\sigma'}} = \ell^2\left(\epsilon dt^2 + e^{2t} dy^2\right)\,, \tag{B.23}$$

and an embedding space light-cone coordinate has a definite sign, *viz.*

$$dS_4 : (\sigma X^0 + \sigma' X^5)|_{U_{\sigma,\sigma'}} = \ell e^t > 0\,, \tag{B.24}$$

$$AdS_4 : (\sigma X^3 + \sigma' X^5)|_{U_{\sigma,\sigma'}} = \ell e^t > 0\,, \tag{B.25}$$

such that

$$dS_4 = U_{+,+} \cup U_{-,-} \cup (\mathbb{R} \times S^2)\,, \qquad AdS_4 = U_{+,+} \cup U_{-,-} \cup (\mathbb{R} \times AdS^2)\,, \tag{B.26}$$

where $U_{+,+} \cap U_{-,-} = \emptyset$, with equivalent expressions using $U_{\pm,\mp}$. Adding $U_{\pm,\mp}$ one obtains an atlas with nontrivial transition functions given by

$$U_{\sigma,\pm} \cap U_{\sigma,\mp} : \; e^{\tilde{t}} = (r^2 e^{2t} + \epsilon)e^{-t}\,, \quad \tilde{y}^i = \frac{e^{2t}}{r^2 e^{2t} + \epsilon} y^i\,, \quad r^2 > -\epsilon\, e^{-2t}\,, \tag{B.27}$$

$$U_{\pm,\sigma} \cap U_{\mp,\sigma} : \; e^{\tilde{t}} = -(r^2 e^{2t} + \epsilon)e^{-t}\,, \quad \tilde{y}^i = -\frac{e^{2t}}{r^2 e^{2t} + \epsilon} y^i\,, \quad r^2 < -\epsilon\, e^{-2t}\,. \tag{B.28}$$

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In the case of dS_4, each Poincaré patch provide a geodesically complete spacetime, with time flowing in the direction of X^0 and $-X^0$ on $U_{+,\pm}$ and $U_{-,\pm}$, respectively. If one introduces conformal time

$$\tau = -e^{-t} \in \mathbb{R}_- \,, \tag{B.29}$$

then $\tau \to 0^-$ at the future (or past) boundary, and the metric takes the form

$$dS_4 \;:\quad (ds_4^2)^{(0)}|_{U_{\sigma,\sigma'}} = \ell^2 \frac{-d\tau^2 + dy^2}{\tau^2} \,. \tag{B.30}$$

In terms the inverse conformal time, which can be extended from $\mathbb{R}_-$ to $\mathbb{R}$, the transition function between $U_{+,+}$ and $U_{-,-}$ is given by

$$(\tau^{-1} + \widetilde{\tau}^{-1})|_{U_{+,+}\cap U_{-,-}} = 0 \,. \tag{B.31}$$

In the case of AdS_4, the conformal radius

$$z = e^{-t} \in \mathbb{R}_+ \,, \tag{B.32}$$

obeys $z \to 0^+$ at the boundary, and the metric takes the form

$$AdS_4 \;:\quad (ds_4^2)^{(0)}|_{U_{\sigma,\sigma'}} = \ell^2 \frac{dz^2 + dy^2}{z^2} \,, \tag{B.33}$$

in each Poincaré patch; the two Poincaré patches can be glued together using

$$(z^{-1} + \widetilde{z}^{-1})|_{U_{+,+}\cap U_{-,-}} = 0 \,. \tag{B.34}$$

B.6 Stereographic coordinates

A convenient set of coordinates, that facilitate a unified description of all solutions, are the stereographic coordinates $\{x^a_\pm\}_{a=0,1,2,3}$ that arise via the parameterization

$$X^M|_{U_\pm} \approx \left(\frac{2x^a_\pm}{1-\lambda^2 x^2_\pm}, \pm\ell \frac{1+\lambda^2 x^2_\pm}{1-\lambda^2 x^2_\pm} \right) \,, \qquad -1 < \lambda^2 x^2_\pm < 1 \,, \qquad x^2 = x^a x^b \eta_{ab} \,, \tag{B.35}$$

with inverse

$$x^a_\pm = \left. \frac{X^a}{1+\sqrt{1+\lambda^2 X^b X_b}} \right|_{U_\pm} \,, \tag{B.36}$$

where $U_\pm$ denotes the two stereographic coordinates charts. Each chart covers one half of $(A)dS_4$, and can be continued smoothly into $\lambda^2 x^2_\pm < -1$; the resulting transition function is given by

$$x^a_\pm = \lambda^{-2} R^a(x_\mp) \,, \qquad \lambda^2 x^2_\pm < 0 \,, \tag{B.37}$$

where the reflection map

$$R^a(v) := -\frac{v^a}{v^2} \,. \tag{B.38}$$

The charts can also be extended (non-smoothly) into $\lambda^2 x^2_\pm > 1$ as follows: as a point $p \in (A)dS_4$ approaches a point p_0 the subspace $\lambda^2 x^2_\pm(p_0) = 1$ from the inside, i.e. $\lambda^2 x^2_\pm(p) < 1$, the reflected image $R(p)$ approaches the point $R(p_0)$ with $x^\mu_\pm(R(p_0)) = -x^\mu_\pm(p_0)$ from the

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outside, i.e. $\lambda^2 x_\pm^2(R(p)) > 1$. Thus, one may cover all of $(A)dS_4$ using a single stereographic coordinate, that we shall take to be $x^a \equiv x_+^a$, defined on four-dimensional Minkowski space minus the subspace $\lambda^2 x^2 = 1$. The boundary is given two copies of the surface $\lambda^2 x^2 = 1$; an outer sheet with normal pointing inwards and an inner sheet with normal pointing outwards.

In the AdS_4 case, the surface $\lambda^2 x_\pm^2 = 1$ has the topology of $dS_3 \cong \mathbb{R} \times S^2$, while its two-sheeted counterpart can be glued together using the reflection map into a single surface with $S^1 \times S^2$ topology, i.e.

$$\partial(AdS_4) \cong S^1 \times S^2 \,, \tag{B.39}$$

as can be seen by taking a tour around the boundary using reflection maps as follows: start at a point p_1 on the outer sheet at large negative (stereographic) time; move up to a point p_2 on the same sheet at large positive times; cross over to $R(p_2)$, which is a point at the inner sheet at large negative times; move up to a point p_3 on the same sheet at large positive times; finally, cross back to $R(p_3) = p_1$, thereby closing a time-like curve.

In the dS_4 case, the boundary consists of two two-sheeted surfaces; one with $x^0 > 0$ and another one with $x^0 < 0$. Using the reflection map, these four sheets, each of which thus has the topology of an hyperbolic three-plane, form two pairs, each of which can be glued together into a three-sphere, i.e.

$$\partial(dS_4) \cong S_-^3 \cup S_+^3 \,, \tag{B.40}$$

where $x^0 < 0$ on S_-^3 and $x^0 > 0$ on S_+^3.

In stereographic coordinates, the metric takes the form

$$(ds_4^2)^{(0)} = \frac{4dx^2}{(1-\lambda^2 x^2)^2} \,. \tag{B.41}$$

Using the plus-branch, where $X^5 = \ell(1+\lambda^2 x^2)/(1-\lambda^2 x^2)$, the $\mathfrak{g}_6$ Killing vector fields read

$$\begin{aligned} K^\mu_{rs} &= 2L_{[r}{}^a L_{s]}{}^b (x_a\, \delta_b{}^\mu)\,, \\ K^\mu_r &= L_r{}^a \left\{ \alpha L^b (x_a \delta^\mu_b - x_b \delta^\mu_a) - \beta\left[-\tfrac{1}{2}(1+\lambda^2 x^2)\delta^\mu_a + \lambda^2 x_a x^\mu\right]\right\} \,. \end{aligned} \tag{B.42}$$

Comparing (B.21) with (B.35), one finds the following transition functions on $U_\varsigma \cap U_{\sigma,\sigma}$ $(\varsigma, \sigma = \pm)$:

$$\begin{aligned} dS_4:\ x^0_\varsigma &= \sigma\ell \frac{\sinh t + \frac{1}{2}r^2 e^t}{1+\varsigma\sigma(\cosh t - \frac{1}{2}r^2 e^t)}\,, \qquad x^i_\varsigma = \ell\frac{e^t\, y^i}{1+\varsigma\sigma(\cosh t - \frac{1}{2}r^2 e^t)}\,, \qquad i=1,2,3\,, \\ AdS_4:\ x^3_\varsigma &= \sigma\ell \frac{\sinh t - \frac{1}{2}r^2 e^t}{1+\varsigma\sigma(\cosh t + \frac{1}{2}r^2 e^t)}\,, \qquad x^i_\varsigma = \ell\frac{e^t\, y^i}{1+\varsigma\sigma(\cosh t + \frac{1}{2}r^2 e^t)}\,, \qquad i=0,1,2\,, \end{aligned} \tag{B.43}$$

and

$$\lambda^2 x_\varsigma^2 = 1 - \frac{2}{1+\varsigma\sigma\left[\cosh t - \frac{1}{2}\mathrm{sign}(\lambda^2)\, r^2 e^t\right]} \,. \tag{B.44}$$

In particular, in checking that the solution (5.1) is invariant under the translations with parameter ξ^r, it is useful to note that (taking $\alpha = \beta, (L^a, L_r{}^a) = (\delta^a_0, \delta^a_r)$ and setting $\lambda = i$)

$$\delta x^0 = \alpha \xi^r x_r (1+x^0)\,, \qquad \delta x^2 = \alpha\xi^r x_r(1+x^2) \,. \tag{B.45}$$

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C Gauge functions

C.1 Stereographic coordinates

Given the Lie algebra $\mathfrak{so}(p,q)$ in the basis over $\mathbb{R}$ spanned by P_I, $I=1,\dots,p+q-1$, and $M_{IJ}=-M_{JI}$ obeying

$$[M_{IJ},M_{KL}]_\star = 4i\eta_{[K|[J}M_{I]|L]}\,,\quad [M_{IJ},P_K]_\star = 2\eta_{K[J}P_{I]}\,,\quad [P_I,P_J]=i\lambda^2 M_{IJ}\,,\tag{C.1}$$

where η_{IJ} has signature (p',q') and $\lambda^2\in\mathbb{R}\setminus\{0\}$, the gauge function

$$L := \exp_\star(i\xi^I P_I)\,,\qquad \xi^I\in\mathbb{R}\,,\tag{C.2}$$

yields a Maurer-Cartan form

$$L^{-1}\star dL = ie^I P_I + \frac{1}{2i}\omega^{IJ}M_{IJ}\,,\tag{C.3}$$

with components

$$e^I = 2h^{-2}dx^I\,,\qquad \omega^{IJ}=4h^{-2}x^{[I}dx^{J]}\,,\tag{C.4}$$

where

$$\xi^I = 4\Upsilon x^I\,,\qquad \Upsilon = \frac{1}{\sqrt{1-h^2}}\tanh^{-1}\sqrt{\frac{1-h}{1+h}}\,,\tag{C.5}$$

$$h=\sqrt{1-\lambda^2x^2}\,,\qquad x^2 = x^I x^J\eta_{IJ}\,,\tag{C.6}$$

which is defined for $\lambda^2x^2<1$.

C.2 Stereographic coordinates for $(A)dS_4$

The gauge function

$$L=\exp_\star(4i\Upsilon x^a P_a) = \frac{2h}{1+h}\exp\left(\frac{i\lambda \not{x}_{\alpha\dot\alpha}y^\alpha\bar y^{\dot\alpha}}{1+h}\right)\,,\qquad \not{x}_{\alpha\dot\alpha} = x^a(\sigma_a)_{\alpha\dot\alpha}\,,\tag{C.7}$$

which is defined for $\lambda^2x^2<1$, yields the Maurer-Cartan form for $(A)dS_4$ in the stereographic coordinates, with components

$$e^a = 2h^{-1}dx^a\,,\qquad \omega^{ab}=4h^{-1}x^{[a}dx^{b]}\,;\tag{C.8}$$

which can be extended into $\lambda^2x^2>1$. Thus, whereas the Maurer-Cartan form can be described globally on $(A)dS_4$ using a single stereographic coordinate, the usage of vacuum gauge functions requires patching. Using the two charts $U_\pm$ defined above, with $1>\lambda^2x^2_\pm|_{U_\pm}\geqslant -1$, and letting $L_\pm$ denote the corresponding locally defined gauge functions, it follows from

$$L_\pm|_{\lambda^2x^2_\pm=-1} = L_\mp|_{\lambda^2x^2_\mp=-1}\,,\tag{C.9}$$

that the transition function

$$T^\mp_\pm := (L_\pm\star(L_\mp)^{-1})|_{\lambda^2x^2_\pm=-1} = 1',\tag{C.10}$$

i.e. the procedure of gluing together U_+ and U_- into $(A)dS_4$ does not refer to any choice of structure group.

From (3.45) one finds

$$\begin{bmatrix} L_\alpha{}^\beta & K_\alpha{}^{\dot\beta} \\ \overline{K}_{\dot\alpha}{}^\beta & \overline{L}_{\dot\alpha}{}^{\dot\beta} \end{bmatrix} = h^{-1} \begin{bmatrix} \delta_\alpha^\beta & \lambda \not{x}_\alpha{}^{\dot\beta} \\ \lambda \bar{\not{x}}_{\dot\alpha}{}^\beta & \delta_{\dot\alpha}^{\dot\beta} \end{bmatrix} , \tag{C.11}$$

From (3.50) it then follows that

$$A = h^{-1}(1 - \lambda \not{b} \bar{\not{x}}) , \qquad B = h^{-1}(\lambda \not{x} - \not{b}) , \tag{C.12a}$$

$$M = A^{-1}B = \frac{1}{\det A}\left(\frac{1 - 2\lambda b^a x_a + b^2}{h^2}\lambda\not{x} - \not{b}\right) = \lambda\not{x} + \frac{\lambda b^2 \not{x} - \not{b}}{\det A} , \tag{C.12b}$$

$$\det A = h^{-2}(1 - 2\lambda b_a x^a + b^2\lambda^2 x^2) , \tag{C.12c}$$

where $b^a = i\eta L^a$; on the poles of $\mathcal{O}$ it is imaginary if $\epsilon k = -1$ and real if $\epsilon k = 0$ or 1.

C.3 Global foliation coordinates for $\epsilon k = -1$

The gauge function

$$L = \exp_\star(i\xi^r T_r) \star \exp_\star(i\ell\rho P) , \qquad (\xi^r, \rho) \in \mathbb{R}^4 , \tag{C.13}$$

yield the Maurer-Cartan form

$$L^{-1} \star dL = i\ell d\rho P + i\check{e}^r \left(\cosh(\sqrt{\epsilon}\lambda\ell\rho)T_r + \frac{\lambda}{\epsilon}\sinh(\sqrt{\epsilon}\lambda\ell\rho)B_r\right) + \frac{1}{2i}\check{\omega}^{rs}M_{rs} , \tag{C.14}$$

where $B_r = L^a L^b_r M_{ab}$ and

$$\check{e}^r = 2\check{h}^{-2}dx^r , \qquad \check{\omega}^{rs} = 4\check{h}^{-2}x^{[r}dx^{s]} , \qquad \check{h} = \sqrt{1-\lambda^2 x^2} , \qquad x^2 = x^r x^s \eta_{rs} , \tag{C.15}$$

with $\lambda^2 x^2 < 1$. The global foliation coordinates in dS_4 and AdS_4 are obtained by taking

$$dS_4 : \ \epsilon = -1 , \qquad \tau = \sinh^{-1}\rho , \qquad n^I = \left(\frac{1+\lambda^2x^2}{1-\lambda^2x^2}, \frac{2\ell^{-1}x^r}{1-\lambda^2x^2}\right) , \tag{C.16}$$

$$AdS_4 : \ \epsilon = +1 , \qquad \sigma = \sinh^{-1}\rho , \qquad n^R = \left(\frac{1+\lambda^2x^2}{1-\lambda^2x^2}, \frac{2\ell^{-1}x^r}{1-\lambda^2x^2}\right) . \tag{C.17}$$

C.4 Planar coordinates for $(\epsilon, k) = (\pm 1, 0)$

If $k = 0$, then

$$\epsilon = \mathrm{sign}(\lambda^2) , \qquad \beta = \ell\alpha , \qquad \gamma \equiv \frac{i\alpha}{\lambda\beta} = i\epsilon\ell\lambda = \begin{cases} 1 \text{ if } \epsilon = -1 , \\ i \text{ if } \epsilon = +1 , \end{cases} \tag{C.18}$$

and the gauge function

$$L = \exp_\star\left(i\alpha^{-1}y^r T_r\right) \star \exp_\star(-i\epsilon\ell t P) , \qquad [P, T_r]_\star = -i\epsilon\frac{1}{\ell}T_r , \tag{C.19}$$

gives rise to the Maurer-Cartan form

$$L^{-1} \star dL = i\ell(-\epsilon dt P + \beta^{-1} e^t dy^r T_r)\,, \tag{C.20}$$

with components

$$e^a = \ell(-\epsilon L^a dt + e^t L^a_r dy^r)\,, \qquad \omega^{ab} = 2\ell e^t L^{[a} L^{b]}_r dy^r\ . \tag{C.21}$$

As for the corresponding adjoint action, we have

$$L^{-1} \star Y_{\underline{\alpha}} \star L = \underline{L}_{\underline{\alpha}}{}^{\underline{\beta}} Y_{\underline{\beta}}\,, \qquad \underline{L} = \underline{L}_T \underline{L}_P\,, \tag{C.22}$$

where

$$(\underline{L}_T)_{\underline{\alpha}}{}^{\underline{\beta}} Y_{\underline{\beta}} := e^{-i\alpha^{-1} y^r \mathrm{Ad}_\star T_r} Y_{\underline{\alpha}}\,, \qquad (\underline{L}_P)_{\underline{\alpha}}{}^{\underline{\beta}} Y_{\underline{\beta}} := e^{i\epsilon\ell t \mathrm{Ad}_\star P} Y_{\underline{\alpha}}\ . \tag{C.23}$$

From[21]

$$T_r = \frac{1}{8} (\underline{T}_r)_{\underline{\alpha\beta}} Y^{\underline{\alpha}} Y^{\underline{\beta}}\,, \qquad \underline{T}_r = L^a_r(-\alpha \Gamma_{ab} L^b + \beta\lambda\Gamma_a)\,, \qquad \underline{T}_r \underline{T}_s = 0\,, \tag{C.24}$$

$$P = \frac{1}{8} (\underline{P})_{\underline{\alpha\beta}} Y^{\underline{\alpha}} Y^{\underline{\beta}}\,, \qquad \underline{P} = \lambda L^a \Gamma_a\,, \qquad \underline{P}^2 = \epsilon\lambda^2 \mathbf{1}\,, \tag{C.25}$$

it follows that

$$\underline{L}_T = 1 + \frac{1}{2}\alpha^{-1} y^r \underline{T}_r = 1 + \frac{1}{2} y^a (-\Gamma_{ab} L^b + i\gamma^{-1}\Gamma_a)\,, \qquad y^a := y^r L^a_r\,, \tag{C.26}$$

$$\underline{L}_P = \cosh\frac{t}{2} - \epsilon\ell \underline{P} \sinh\frac{t}{2} = \cosh\frac{t}{2} + i\gamma L^a \Gamma_a \sinh\frac{t}{2}\,, \tag{C.27}$$

and hence

$$\underline{L} = \cosh\frac{t}{2} + i\gamma L^a \Gamma_a \sinh\frac{t}{2} + \frac{1}{2} e^{t/2} y^a (-\Gamma_{ab} L^b + i\gamma^{-1}\Gamma_a)\ . \tag{C.28}$$

From the definitions made in (4.14) and (3.51), it follows that

$$A = \cosh\frac{t}{2} + \epsilon\gamma\eta \sinh\frac{t}{2} - \frac{1}{2}\left(1 + \frac{\eta}{\gamma}\right) e^{t/2} y^a L^b \sigma_{ab}\,, \tag{C.29a}$$

$$B = i\left(\gamma \sinh\frac{t}{2} - \eta\cosh\frac{t}{2}\right) L^a \sigma_a + \frac{i}{2} e^{t/2} (\gamma^{-1} - \epsilon\eta) y^a \sigma_a\,, \tag{C.29b}$$

$$M = A^{-1}B = \frac{i\gamma}{\det A}\left\{\left[\det A + \frac{\epsilon}{2}\left((\eta+\gamma)^2 e^{-t} - \eta^2 - \epsilon\right)\right] L^a \sigma_a + \frac{1}{2}(\eta+\gamma)^2 y^a \sigma_a\right\}, \tag{C.29c}$$

$$\det A = \left(\cosh\frac{t}{2} + \epsilon\gamma\eta\sinh\frac{t}{2}\right)^2 - \frac{1}{4}(\eta+\gamma)^2 y^a y_a e^t\ . \tag{C.29d}$$

The resulting form of the scalar field ϕ is given by

$$\phi = \left(\nu + \gamma\widetilde{\nu}\frac{\partial}{\partial\eta}\right) \frac{1}{\det A}\bigg|_{\eta=-\gamma} = (\nu + \widetilde{\nu})\, e^{-t} - \widetilde{\nu} e^{-2t}\,, \tag{C.30}$$

which in the $\mathfrak{iso}(3)$ case agrees with (5.3).

[21] It also follows that $T_{r_1} \star \cdots \star T_{r_n} = T_{r_1} \cdots T_{r_n}$.

D Analysis of integrability condition on $H^{(1)}|_{Z=0}$

From $D^{(0)}V^{(0)}_\alpha(\eta)=0$ it follows that

$$\begin{aligned}\left(D^{(0)}\frac{1}{\mathcal{L}_{\vec{Z}}}z^\alpha V^{(0)}_\alpha(\eta)\right)\Big|_{Z=0} &= \left(\left[D^{(0)},\frac{1}{\mathcal{L}_{\vec{Z}}}z^\alpha\right]V^{(0)}_\alpha(\eta)\right)\Big|_{Z=0}\\ &= \frac{1}{4i}\Omega^{\underline{\alpha\beta}}\left(\left[\mathrm{ad}^\star_{Y_{\underline{\alpha}}Y_{\underline{\beta}}},\frac{1}{\mathcal{L}_{\vec{Z}}}z^\alpha\right]V^{(0)}_\alpha(\eta)\right)\Big|_{Z=0}\,, \end{aligned}\tag{D.1}$$

where

$$\left(\left[\mathrm{ad}^\star_{y_\alpha y_\beta},\frac{1}{\mathcal{L}_{\vec{Z}}}z^\gamma\right]V^{(0)}_\gamma(\eta)\right)\Big|_{Z=0}=4\partial^{(y)}_{(\alpha}V^{(0)}_{\beta)}(\eta)|_{z=0}\,,\tag{D.2}$$

$$\left(\left[\mathrm{ad}^\star_{y_\alpha \bar{y}_{\dot\alpha}},\frac{1}{\mathcal{L}_{\vec{Z}}}z^\gamma\right]V^{(0)}_\gamma(\eta)\right)\Big|_{Z=0}=2\partial^{(\bar y)}_{(\dot\alpha}V^{(0)}_{\alpha)}(\eta)|_{z=0}\,,\tag{D.3}$$

$$\left(\left[\mathrm{ad}^\star_{\bar y_{\dot\alpha} \bar{y}_{\dot\beta}},\frac{1}{\mathcal{L}_{\vec{Z}}}z^\gamma\right]V^{(0)}_\gamma(\eta)\right)\Big|_{Z=0}=0\,,\tag{D.4}$$

using the holomorphicity properties of $V^{(0)}_\alpha(\eta)$. Thus, adding also the contributions from the anti-holomorphic connection, we find

$$U^{(G,1)}|_{Z=0}=D^{(0)}\left(H^{(1)}|_{Z=0}\right)+i\mathcal{O}\,\Omega^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline{\alpha}}V^{(0)}_{\underline{\beta}}(\eta)\Big|_{Z=0}\,.\tag{D.5}$$

From $(D^{(0)})^2=0$ it follows that the singularities of the second term at $Y=0$ can be cancelled by the first term only if

$$D^{(0)}(U^{(G,1)}|_{Z=0})=i\mathcal{O}D^{(0)}\left(\Omega^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline{\alpha}}V^{(0)}_{\underline{\beta}}(\eta)\Big|_{Z=0}\right)\tag{D.6}$$

is real analytic at $Y=0$, which is thus a necessary condition for the existence of $H^{(1)}|_{Z=0}$. To demonstrate this, we use once more $(D^{(0)})^2=0$, on the form $R^{\underline{\alpha\beta}}:=d\Omega^{\underline{\alpha\beta}}-\Omega^{\underline{\alpha\gamma}}\wedge\Omega_{\underline{\gamma}}{}^{\underline{\beta}}=0$, and also $D^{(0)}V^{(0)}_{\underline{\beta}}(\eta)=0$, to compute

$$\begin{aligned}&D^{(0)}\left(\Omega^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline{\alpha}}V^{(0)}_{\underline{\beta}}(\eta)\Big|_{Z=0}\right)\\ &=\left(d+\Omega^{\underline{\gamma\delta}}Y_{\underline{\gamma}}\partial^{(Y)}_{\underline{\delta}}\right)\left(\Omega^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline{\alpha}}V^{(0)}_{\underline{\beta}}(\eta)\Big|_{Z=0}\right)\\ &=d\Omega^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline{\alpha}}V^{(0)}_{\underline{\beta}}(\eta)\Big|_{Z=0}+\Omega^{\underline{\gamma\delta}}\wedge\Omega^{\underline{\alpha\beta}}\left[Y_{\underline{\gamma}}\partial^{(Y)}_{\underline{\delta}},\partial^{(Y)}_{\underline{\alpha}}\right]V^{(0)}_{\underline{\beta}}(\eta)\Big|_{Z=0}\\ &\quad-\Omega^{\underline{\alpha\beta}}\wedge\partial^{(Y)}_{\underline{\alpha}}D^{(0)}\left(V^{(0)}_{\underline{\beta}}(\eta)\Big|_{Z=0}\right)\\ &=R^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline{\alpha}}V^{(0)}_{\underline{\beta}}(\eta)\Big|_{Z=0}-\Omega^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline{\alpha}}\left(D^{(0)}V^{(0)}_{\underline{\beta}}(\eta)-i\Omega^{\underline{\gamma\delta}}\wedge\partial^{(Y)}_{\underline{\gamma}}\partial^{(Z)}_{\underline{\delta}}V^{(0)}_{\underline{\beta}}\right)\Big|_{Z=0}\\ &=i\Omega^{\underline{\alpha\beta}}\wedge\Omega^{\underline{\gamma\delta}}\partial^{(Y)}_{\underline{\alpha}}\,\partial^{(Y)}_{\underline{\gamma}}\left(\partial^{(Z)}_{[\underline{\delta}}V^{(0)}_{\underline{\beta}]}\right)\Big|_{Z=0}\,.\end{aligned}\tag{D.7}$$

Using also $\mathcal{O}V^{(0)}_{\underline{\alpha}}=A^{(L,1)}_{\underline{\alpha}}$ (see eq. (3.61)), we arrive at

$$D^{(0)}(U^{(G,1)}|_{Z=0})=-\Omega^{\underline{\alpha\beta}}\wedge\Omega^{\underline{\gamma\delta}}\partial^{(Y)}_{\underline{\alpha}}\,\partial^{(Y)}_{\underline{\gamma}}\left(\partial^{(Z)}_{[\underline{\delta}}A^{(L,1)}_{\underline{\beta}]}\right)\Big|_{Z=0}\,,\tag{D.8}$$

where $\partial^{(Z)}_{[\underline{\delta}} A^{(L,1)}_{\underline{\beta}]}$ is a linear combination of $\epsilon_{\alpha\beta}\Phi^{(L,1)}\star\kappa$ and its Hermitian conjugate, which are real analytic in Y space. In the space of forms $f(x,dx,Y)$ in X-space that are functions of Y, the background exterior derivative $D^{(0)}$ commutes to the Euler derivative

$$\vec{E}_Y := Y^{\underline{\alpha}}\vec{\partial}^{(Y)}_{\underline{\alpha}}\,, \tag{D.9}$$

viz. $(\vec{E}_Y D^{(0)} - D^{(0)}\vec{E}_Y)f(x,dx,Y) = 0$. Moreover, the spectrum of $\vec{E}_Y$ in this space is given by $\{0,1,2,\dots\}$. Thus, letting

$$P^{(1)} := i\mathcal{O}\Omega^{\underline{\alpha\beta}}\partial^{(Y)}_{\underline{\alpha}}V^{(0)}_{\underline{\beta}}|_{Z=0}\,, \tag{D.10}$$

we have

$$(\vec{E}_Y+2)P^{(1)} = 0\,, \qquad (\vec{E}_Y+2)D^{(0)}P^{(1)} = 0\,, \tag{D.11}$$

which together with the already established real analyticity of $D^{(0)}P^{(1)}$ in Y space implies that

$$D^{(0)}P^{(1)} = 0\,, \tag{D.12}$$

as can also be seen directly using the fact that $\partial^{(Z)}_{[\underline{\delta}} A^{(L,1)}_{\underline{\beta}]}|_{Z=0}$ is a linear function of Y, hence annihilated by $\Omega^{\underline{\alpha\beta}}\Omega^{\underline{\gamma\delta}}\partial^{(Y)}_{\underline{\alpha}}\partial^{(Y)}_{\underline{\gamma}}$. Finally, conjugation by L yields

$$d(L\star P^{(1)}\star L^{-1}) = 0 \quad\Rightarrow\quad L\star P^{(1)}\star L^{-1} = dQ^{(1)}\,, \tag{D.13}$$

that is

$$P^{(1)} = L^{-1}\star dQ^{(1)}\star L = D^{(0)}(L^{-1}\star Q^{(1)}\star L)\,. \tag{D.14}$$

Hence, choosing

$$H^{(1)} = -L^{-1}\star Q^{(1)}\star L\,, \tag{D.15}$$

we arrive at $U^{(G,1)}|_{Z=0} = 0$, which is the desired result in view of the fact that the Weyl zero-form consists of scalar modes. As for the explicit form of $H^{(1)}$ we refer to a more general analysis including solutions with four and two Killing symmetries to appear elsewhere.

E Lemmas

E.1 A twistor space distribution

At the first order in the ν-expansion of a_α given above, one encounters the integral (see (3.31))

$$I^{\pm}(z) \;:=\; 2z^{\pm}\int_{-1}^{1}\frac{d\tau}{(\tau+1)^2}\,\exp\left(i\frac{\tau-1}{\tau+1}z^+z^-\right)\,. \tag{E.1}$$

Using the delta sequence

$$\lim_{\epsilon\to 0^+} e^{-\frac{i}{\epsilon}z^+z^-} = 0\,, \tag{E.2}$$

one finds

$$I^{\pm}(z) \;=\; \frac{1}{iz^{\mp}}\,. \tag{E.3}$$

The linearized equations of motion require

$$\partial_\pm I^\pm = \kappa_z \ . \tag{E.4}$$

In order to differentiate $I^\pm(z)$, we must first rewrite it as a distribution that is differentiable at $z^\mp = 0$, for which we use

$$\partial_\pm I^\pm = \partial_\pm \left(\int_0^{z^\pm} dz'^\pm \lim_{\epsilon \to 0^+} \frac{1}{\epsilon} e^{-\frac{i}{\epsilon} z'^\pm z^\mp} \right) = 2\pi \partial_\pm \left(\int_0^{z^\pm} dz'^\pm \delta(z'^\pm) \delta(z^\mp) \right) = 2\pi \delta(z^\pm)\delta(z^\mp) \, . \tag{E.5}$$

E.2 Fusion rules

Denoting the generators of the complexified Weyl algebra $\mathcal{W}$ by (I, u, v), where I is central and $[u, v]_\star = I$, we factor out the ideal generated by $I - \hbar \mathrm{Id}_{\mathcal{W}}$, and set $\hbar = 1$, leading to a graded associative algebra degree map given by the monomial degree and $\mathfrak{osp}(1|2)$ subalgebra $(u, v; u \star u, \frac{1}{2}\{u, v\}_\star, v \star v)$. Letting

$$w := \frac{1}{2}\{u, v\}_\star \, , \qquad g_\xi := \exp_\star(\xi w) \, , \qquad \xi \in \mathbb{C} \, , \tag{E.6}$$

one has $g_\xi \star g_{\xi'} = g_{\xi+\xi'}$. Going to Weyl order, one finds the symbols

$$g_\xi = \frac{1}{\cosh \xi} \exp[\tanh(\xi w)] \, , \tag{E.7}$$

which are real analytic except for $\xi \in (\mathbb{Z} + \frac{1}{2})\pi i$ in which case they are phase space delta functions defined using delta sequences. It follows that

$$E_\eta \star E_{\eta'} = \frac{1}{1 + \eta\eta'} E_{\frac{\eta+\eta'}{1+\eta\eta'}} \, , \qquad E_\eta := \exp(-2\eta w) \, , \tag{E.8}$$

whose star product we extend to all values of η using the closed contour regularization scheme defined in section 3.1.5. In particular, for $\eta = \pm 1$ we recover the Fock space and anti-Fock space ground state projectors $P_\sigma := 2E_\sigma$, $\sigma = \pm 1$, thus obeying $u \star P_+ = v \star P_- = 0$ and

$$(P_\sigma \star P_{\sigma'})|_{\mathrm{reg}} = \delta_{\sigma,\sigma'} P_\sigma \ . \tag{E.9}$$

The $\mathfrak{g}_6$-invariant solutions make use of the elements $E_{\pm i}$, which thus obey

$$(E_{\sigma i} \star E_{\sigma' i})|_{\mathrm{reg}} = \frac{1}{2} \delta_{\sigma,-\sigma'} \, , \tag{E.10}$$

that is, they close on the identity. We note that $E_{\pm i} = \frac{1}{2} g_{\pm\pi/4}$, that is, the regularization amounts to discarding the non-real analytic group elements $g_{\pm\pi/2}$. We also remark that E_η gives rise to an $\mathrm{Env}(\mathfrak{osp}(1|2))$ orbit obtained by left- and right-action by polynomial elements in $\mathfrak{osp}(1|2)$, that is, by u and v; for $\eta = \pm 1$ these are simply the algebras of endomorphisms of the Fock and anti-Fock spaces. Taking instead $\eta = \pm i$ and restricting to even elements, the resulting $\mathrm{Env}(\mathfrak{sp}(1|2))$ orbits of $E_{\pm i}$ are of use in considering fluctuations around the $\mathfrak{g}_6$-invariant solutions; see section 5.3 for an outline.

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