

FUGLEDE–PUTNAM THEOREM AND QUASISIMILARITY OF CLASS p - $wA(s, t)$ OPERATORS

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Abstract. We show that p - $wA(s, t)$ operators S, T^* ($s + t \leq 1$, $0 < p \leq 1$) with $\ker(S) \subseteq \ker(S^*)$ and $\ker(T^*) \subseteq \ker(T)$ satisfy Fuglede–Putnam theorem, i.e., $SX = XT$ for some X implies $S^*X = XT^*$. Also, we show that two quasisimilar p - $wA(s, t)$ operators S, T ($s + t \leq 1$, $0 < p \leq 1$) with $\ker(S) \subseteq \ker(S^*)$ and $\ker(T) \subseteq \ker(T^*)$ have equal spectra and essential spectra.

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