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Full Annular Rub in Mechanical Seals, Part I: Experimental Results

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This paper presents the results of recent experimental studies on rotor/seal full annular rub, including forward and reverse precession. It was found that reverse precessional full annular rub (dry whip) could occur repeatedly in small clearance cases without any outside disturbance. The rotor rubbed against the seal with almost constant amplitude and frequency, which was higher than the natural frequency of the original rotor system and lower than of that of the coupled rotor/seal system. Once generated, the reverse precessional rub could be sustained over the whole speed range as long as slippage was maintained. Radius-to-clearance ratio varied from 10 to 40. The experimental studies include forward rub jump phenomena, reverse rub triggering mechanism, and effects of mass unbalance, surface lubrication, shaft speed, and seal stiffness. The results of both forward and reverse precession are discussed in detail, with emphasis on the reverse and the transition from the forward to the reverse.

Keywords: Rotor/seal rub; Full annular rub; Reverse precession; Forward precession; Self-excited vibration

1. INTRODUCTION

Rotor/seal rub is a serious malfunction in rotating machinery that may lead to a machine catastrophic failure. The desire to increase efficiency of fluid-handling machines leads to minimizing rotor/seal clearances, which, in turn, creates a higher hazard of rubbing.

The rub dynamic phenomena in rotating machines result in a very rich variety of occurrences, as during rubbing, several physical factors get involved in different proportions. In the partial rub (Bently, 1974; Bently et al., 1992), contact forces are minimal. The rotor occasionally touches the stationary element and maintains contact during a fraction of its precessional period. In this case, the impacts affected by the rotor/seal contact friction play a major role. This type of rub may lead to relatively stable vibrations with synchronous or subsynchronous frictional frequencies, or to chaotic vibrations, as discussed by Goldman and Muszynska (1994); Horattas and Adams (1998) and Sawicki et al. (1998). The partial rub represents one of the steady-state vibrational regimes caused by the rotor/seal contact. It is usually not too dangerous to the integrity of the machine. The second, and more serious phenomenon in its destructive effects to the machine, is a full annular rub (Muszynska, 1984), which can occur in seals or in auxiliary bearings (Lawen & Flowers, 1999). During full annular rub, the rotor maintains contact with the seal continuously or almost continuously. Depending on dry friction, mass unbalance, and other system parameters, the rotor can rub against the seal in the same direction as it rotates around its centerline (forward or synchronous precession) or in the opposite direction (reverse or backward precession). The latter is a type of self-excited vibration, known as "dry whip", and is more severe than the former. In the reverse precession, the shaft is subjected to high frequency deformation and stressing with large amplitude. The shaft deformation can initiate cracks and fatigue damage. Friction accompanied by high slip velocity can impose demands for power exceeding the driver's capacity, creating thermal gradients that lead to rotor/seal plastic deformations and damage to the surface due to intensive grinding.

Study on the full annular rub of rotors has been of interest to many researchers and engineers as shown in a literature survey done by Muszynska (1989). Black (1968) analyzed synchronous response of a rotor and stator due to contact across a clearance annulus. Ehrich (1969) indicated that ratio of rotor reverse whirl frequency to natural frequency was only related to ratios of stator-to-rotor natural frequency and damping. Muszynska (1984) discussed

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full annular rub with a 2-DOF model and presented measured results of reverse precessional rub triggered by external impacts. Lingener (1990) and Crandall (1990) demonstrated dry-friction whirl and whip for a case when ratio of rotor radius to radial clearance r/C_r reached 2.66. Childs (1993) summarized their work and indicated that dry-friction whirl and whip were only likely to occur at a large clearance and that triggering of contact normally required an outside disturbance.

Therefore, it was often considered that reverse precessional full annular rub (dry whip) seldom occurred and that triggering of it required an outside disturbance along with a large rotor/seal clearance. However, recent experiments show that this type of self-excited reverse precession can be ensured to occur in a rotor kit with an attached mechanical seal, as indicated in experimental and analytical studies conducted by Yu et al. (1998, 2000) and Goldman and Bently (1998). No outside disturbance or impact is needed to trigger the reverse precession. Moreover, it has been found that reverse precessional rub can even occur for small clearance cases where ratio of rotor radius to radial clearance reaches 40.

This paper presents the results of recent experimental studies on rotor/seal full annular rub, especially the detailed new findings of reverse precession occurrence without any outside disturbance. Both forward and reverse precession are described, including the jump phenomena, rub triggering mechanisms, and effects of system parameters and running conditions on the rub.

2. TEST RIG

A shaft with 0.01 m diameter and 0.56 m length was supported by two brass bushing bearings and driven by a 0.1-hp motor (see Figure 1). One or two 0.8-kg disks were attached to the shaft in the midspan. A seal was located at the outboard side of the shaft. The seals used in the experiments were either tightly or flexibly fitted with an "O-ring" in the seal support. The rotor was originally centered in the seal and well balanced. A known mass unbalance was then added to one of the disks at the radius of 0.03 m. Without contact with the seal, the rotor first balance resonance speed of synchronous ($1\times$) motion was around 1500 rpm with two disks attached, and around 2000 rpm with only one disk. Slight system anisotropy existed for both uncoupled (without a seal contact) and coupled (with a seal contact) rotors. Teflon™ or bronze seals with diametral clearances of 250 to 1000 μm were used. The data acquisition and processing system consisted of four *XY* displacement proximity probes (Probes 1 and 2 were located near the seal, and probes 3 and 4 were close to the rotor midspan), one speed probe and one Keyphasor® probe for speed and phase measurement, and ADRE® software.

The friction coefficient between steel and Teflon surfaces, if quoted from a handbook, is usually less than 0.1. During the experiment, however, a high contact force and friction-related heat severely deformed the Teflon seal and changed the normal contact direction on the rotor/seal surface, thus yielding a high equivalent friction coefficient. The reverse precessional full annular rub, accompanied by a higher contact normal force, corresponded to a larger value of the equivalent friction coefficient than the forward precessional rub.

3. FORWARD PRECESSIONAL RUB

In the case that the friction force between the shaft and seal was not high, the rotor $1\times$ forced response to mass unbalance was the major vibration component during startup and shutdown. This regime is also called the "synchronous rub". An interaction with the seal was seen in the first-balance-resonance speed range. When the rotor/

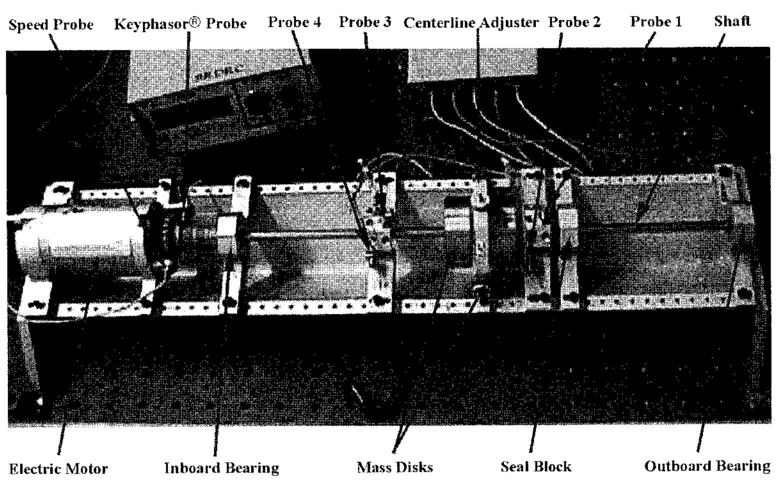


FIGURE 1 Rotor/seal full annular rub test rig.

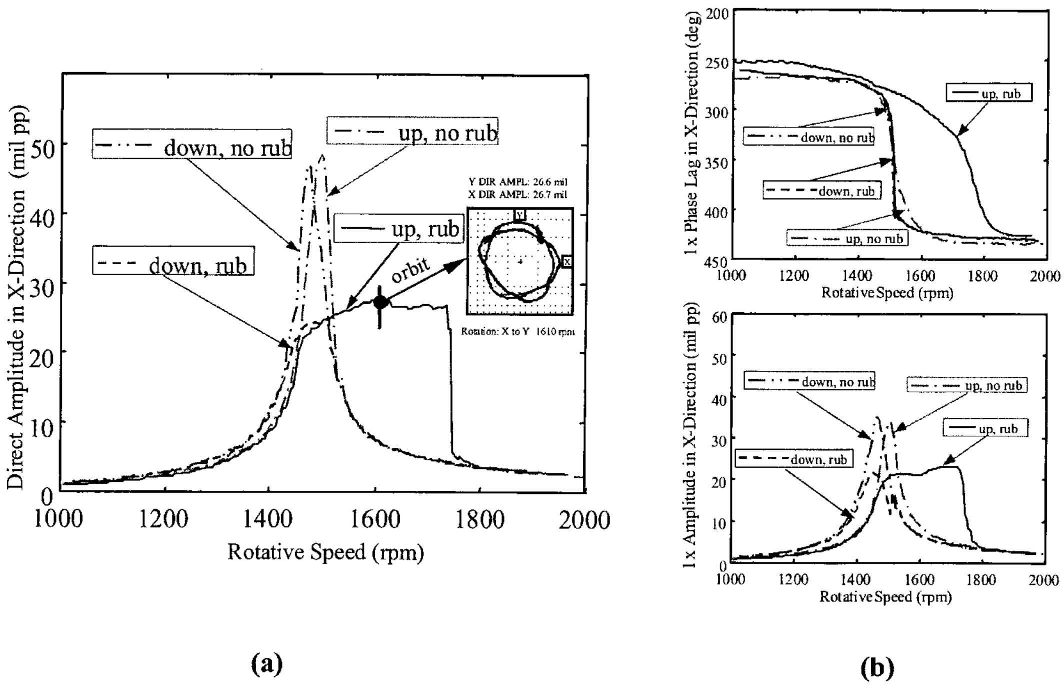


FIGURE 2 Comparison of rotor lateral responses of the two-disk rotor during runup and rundown testing with/without rubbing, respectively. Teflon seal with diametral clearance of 500 μm; two disks with mass unbalance of 1.1 grams at 90 degree ahead of X-probe. (a) Direct responses, and (b) 1 × Bode plots near the seal location. An insert in (a) displays the rotor orbit indicating multi-contact intermittent rub.

seal contact occurred, the seal started acting as an additional bearing, thus yielding a coupled rotor/seal system with increased stiffness and natural frequencies. The rotor slid along the seal surface without rolling. Since the friction force was small, the rub resulted in only minor surface damage.

Figure 2 shows the direct and 1 × Bode plots of rotor horizontal (X-direction) responses during runup and rundown, with and without rubbing. Due to differences in the rotor angular acceleration, the runup and rundown curves differ slightly without contacting the seal. In the range of the original resonance, responses changed when rubbing was involved. The resonance range of speeds was much wider during runup than rundown. Peak response amplitudes were reduced by an amount, which depended on the rotor/seal clearance. In this case, the rotor was not in contact with the seal continuously. The orbit in Figure 2 shows that the rotor bounced inside the seal, but it maintained forward precession. As the rotative speed increased or decreased, the rotor conditions changed from non-contact to contact and then to non-contact again, with jumping amplitude.

During rundown, changes in 1 × phase values were minor, but there was a clear amplitude "jump-up" discontinuity phenomenon, which was often overlooked. The fact that a sudden decrease in 1 × amplitude (see Figure 2), accompanied by occurrence of other frequency components when the rotor was just contacting the seal during rundown, indicates an impact due to amplitude discontinuity. Figure 3 makes it clear that a "jump-up" did exist during rundown. In this case, lubrication oil was dropped onto the contact surface to avoid severe reverse precessional rubbing. Unlike the case in Figure 2 where the contact surface was not fully lubricated, the rotor orbit pattern in Figure 3 was very smooth. The rotor "bounce" pattern depended significantly on surface friction. During rubbing there was a "third

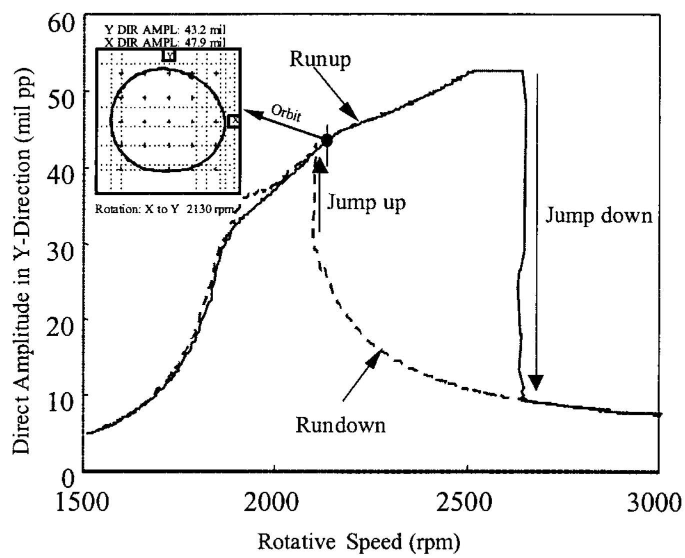


FIGURE 3 Synchronous rub showing that amplitude jumps down during runup and jumps up during rundown. Teflon seal with diametral clearance of 1000 μm; one disk with mass unbalance of 1.6 grams.

bearing" effect, as described by Yu et al. (1998). The rotor mode shape changed due to this physical restriction.

4. REVERSE PRECESSIONAL RUB

Without any outside disturbance, reverse precessional full annular rub occurred repeatedly for the ratio of rotor radius-to-radial clearance r/C_r ranging from 10 to 40. Around the resonance speed, synchronous forward precessional rub developed into reverse precessional full annular rub when rotor/seal surface friction was high for the rotor/seal test rig on a concrete base as shown in Figure 1. Once generated, the reverse precessional rub could be sustained over the whole speed range as long as slippage was maintained between the rotor and the seal. With a hammer impact, the reverse rub could be disengaged at nonresonance speed, and engaged above very low speed (for instance, 10% of the resonance speed).

4.1. Rub Triggering without Outside Disturbance

Figure 4 shows the transition process from synchronous forward precessional rub to reverse precessional full annular rub. When the rotative speed was approaching the first balance resonance, the rotor started bouncing inside the seal, showing elliptical orbits slowly rotating their major axes due to lateral stiffness anisotropy effect. Multi-contact intermittent rub came first with forward precessional orbits. Afterwards, reverse-dominant precession occurred from around 1550 rpm to 1650 rpm. Then suddenly, around 1700 rpm, the full annular rub appeared at a frequency of 2300 cpm. The much higher vibration component in reverse frequency (−2300 cpm) than that in forward frequency (2300 cpm), as shown in Figure 4(b), also indicates an almost circular reverse orbit motion. With the increase in rotative speed, the frequency was kept unchanged, though the amplitude increased slightly due to thermal expansion of the seal.

The reverse precessional full annular rub could also be triggered during rundown after the rotor contacted the seal near the resonance speed (see Figure 5). Figure 5(b) shows the comparison of direct response during rundown and runup. The reverse rub started at around 2035 rpm in either rundown or runup case. After being generated, the rub was maintained till very low speed, replacing the synchronous $1\times$ motion (see Figure 5(a)).

4.2. Mass Unbalance Effect

For the same experimental setup as it is discussed in 4.1, when for the next run the amount of mass unbalance was reduced to 0.4 grams, reverse precessional full annular rub was not generated although the rotor bounced inside the seal around the resonance speed. As shown in Figure 6, the rotor experienced forward rub from 1500 rpm to 1550 rpm, and reverse $1\times$ partial rub around 1600 rpm. At about 1700 rpm, it disengaged from the seal.

In addition to triggering of reverse precessional full annular rub, amount of mass unbalance can also affect the starting point of the rub, as shown in Figure 7 where the seal clearance was 250 μm. The first resonance speed of the original uncoupled rotor system was around 2000 rpm. In this case, for lower mass unbalance ($m = 0.2$ grams), reverse rub occurred after the resonance speed, while for higher mass unbalance ($m = 2.2$ grams) it occurred before the resonance speed. Regardless of difference in mass unbalance amount, the rotor was eventually locked at the same precessional frequency (3120 cpm) once the reverse rub was generated. Although the rotor with higher mass unbalance ($m = 2.2$ grams) started to contact the seal at around 1600 rpm, the reverse rub was not engaged quickly until around 1900 rpm where an increasing contact force, accompanied by increasing amplitude of synchronous $1\times$ rub motion, was able to trigger it.

The location of mass unbalance relative to the probe or Keyphasor is irrelevant to the occurrence of reverse precessional rub. After reverse rub has been triggered, mass unbalance has no effect on the subsequent motion.

4.3. Rotative Speed Effect

Once generated, reverse precessional full annular rub could not be disengaged by changes in rotative speed unless at very low speed where slippage could not be maintained. Figure 8 shows the case of reverse precessional rub for an entire running period, including runup and rundown. Reverse full annular rub started around 1600 rpm during runup. A frequency of 2280 cpm and an amplitude of around 50 mil (1270 μm) were maintained with the increase of speed until 2500 rpm. They were also maintained during almost entire rundown. As the speed decreased to around 500 rpm, the amplitude started to decrease slightly for lack of energy. The vibration frequency was accordingly reduced due to the reduced seal stiffness resulted from the decreased amplitude and contact force.

Figure 9 shows the reverse rub at rotative speed of 170 rpm for the case shown in Figure 8. The reverse full annular rub did not vanish until at speed of 120 rpm, less than 10% of the original no-rub resonance speed (around 1500 rpm in this case). Above 120 rpm, a proper hammer impact could easily generate the reverse rub, and could also release it easily as well.

The ratio of reverse rub frequency to speed (ω/Ω) was always less than the ratio of radius to radial clearance (r/C_r) ($= 13.3$ in this case). Assuming that the seal vibrates

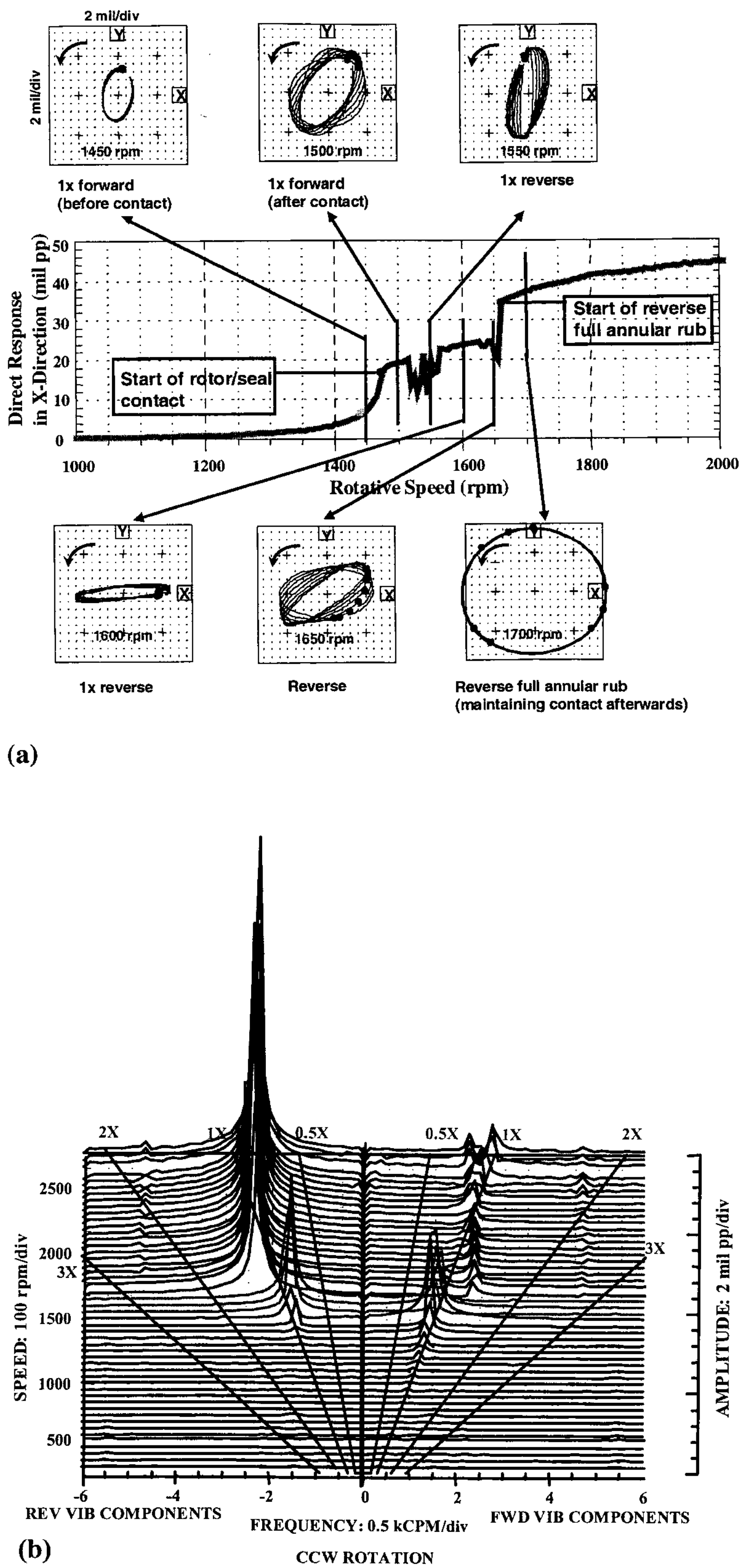


FIGURE 4 Process of generating reverse precessional rub (self-excited vibration, also called "dry whip") without any outside disturbance. Two-disk rotor/Teflon seal diametral clearance of 500 µm with mass unbalance of 0.5 grams. (a) Rotor lateral response and orbits, and (b) Rotor full spectrum response based on both X and Y probe data.

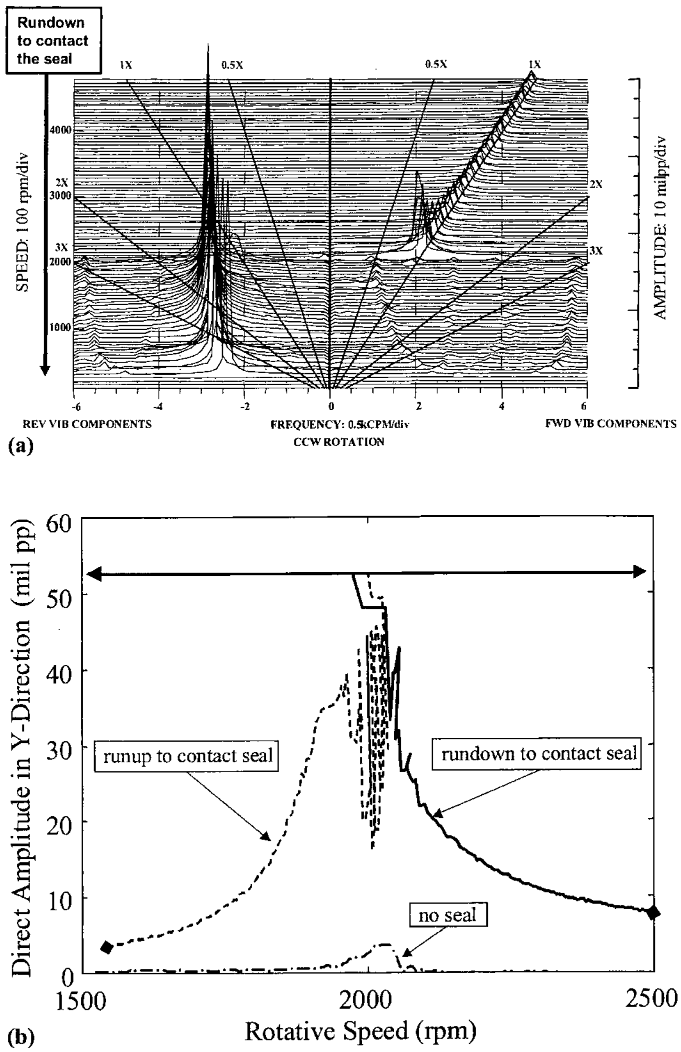


FIGURE 5 Reverse precessional rub triggered during rundown. One-disk rotor/Teflon seal diametral clearance of 1000 μm with mass unbalance of 1.1 grams. (a) Rotor full spectrum response during rundown, and (b) Reverse rub occurred during rundown, compared with that during runup.

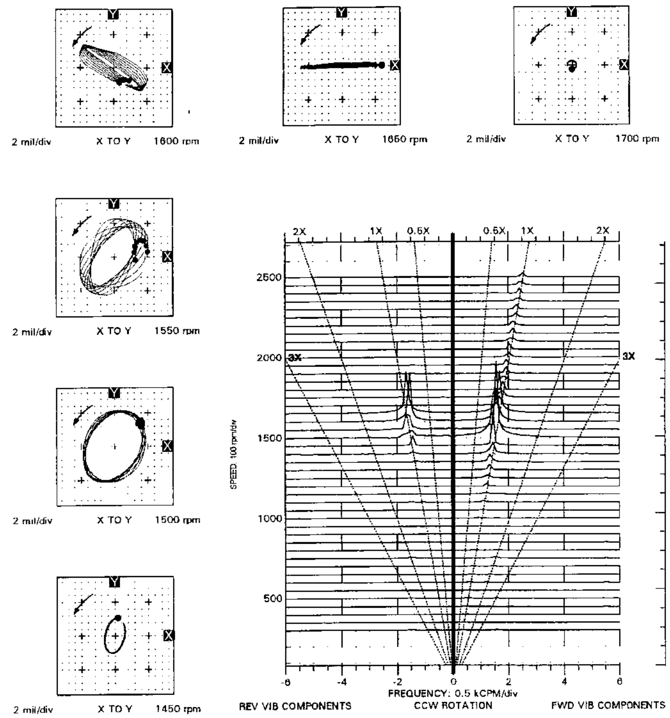


FIGURE 6 Orbits and spectrum for the same setup as in Figure 4, except that mass unbalance was reduced to 0.4 grams. The reverse rub did not occur.

harmonically at the same frequency ω as does the rotor, the relative slip velocity (Yu et al., 1998) can be given by

$$\Delta\nu = r\Omega - C_r\omega = C_r\Omega\left(\frac{r}{C_r} - \frac{\omega}{\Omega}\right) \qquad [1]$$

Figure 10 shows the slippage that occurred for the case shown in Figure 8. It was found that the relief of reverse rub occurred near zero slippage velocity where the friction force was approaching zero and could not balance the damping force. Obviously, the slip velocity was proportional to the

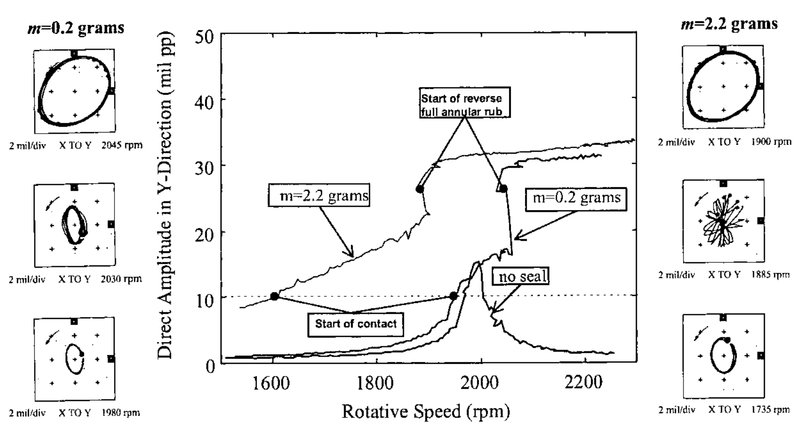


FIGURE 7 Effect of mass unbalance on the starting point where reverse rub occurred with no external impulse. One disk rotor, Teflon seal diametral clearance of 250 μm.

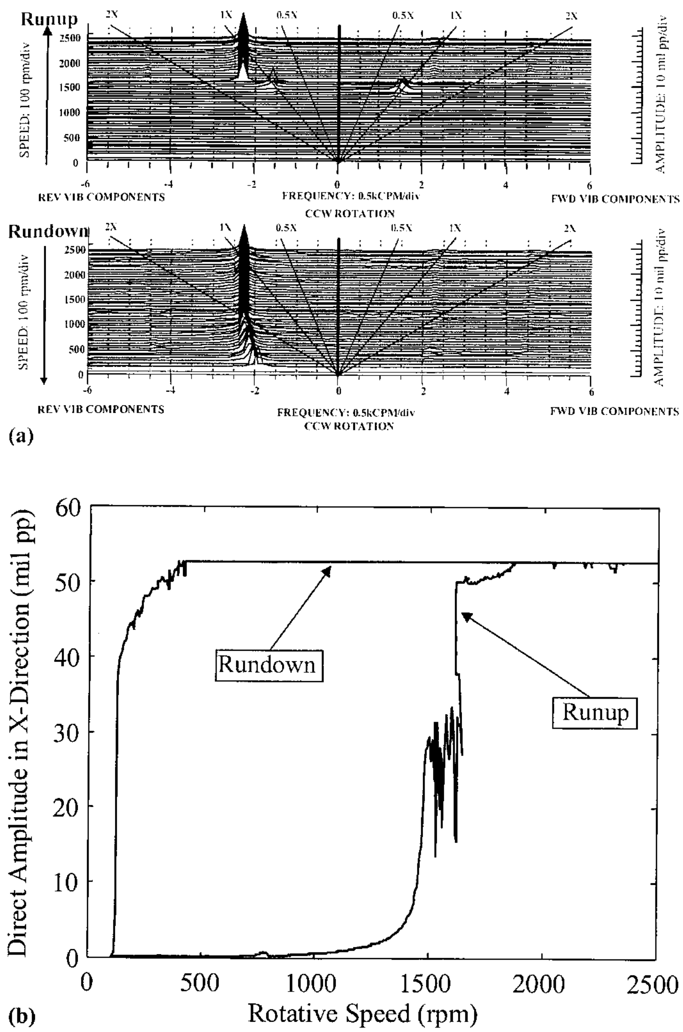


FIGURE 8 Reverse precessional rub during the whole running process including runup and rundown. One disk rotor with Teflon seal diametral clearance of 750 μm (a) Full spectrum, and (b) Direct response.

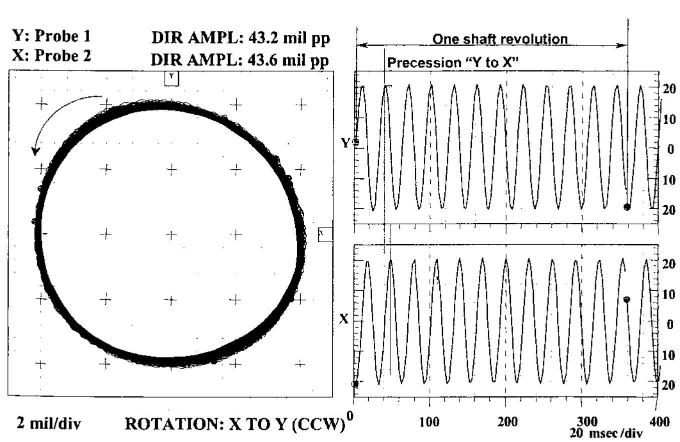


FIGURE 9 Reverse precessional rub at rotative speed of 170 rpm for the case in Figure 8 where no-rub resonance speed was around 1500 rpm.

rotative speed, thus yielding more wear and heat between the rotor and the seal at higher speed. When the slippage could not be maintained as r/C_r approached the ratio ω/Ω at low speed, as shown in Figure 10(b), the rotor disengaged from the seal and the reverse rub then ceased.

4.4. Rub Frequency

The frequency of reverse processional full annular rub was always higher than the lowest natural frequency or resonance speed of the rotor without seal contact. It was also found that the dry-whip frequency was lower than the lowest natural frequency of the rotor/seal coupled system.

Frequencies of reverse rub could be different for the same rotor kit with different seals. In the case of Teflon seals, the rub frequency could be reduced to some extent after several runs with severe grounding, accompanied by increased clearances softening the seals. Seals with the same clearance could also result in different rub frequencies if supported differently (press-fitted or through flexible O-rings) against the seal block. Therefore, besides friction coefficient, seal stiffness has an effect on the rub frequency of the rotor/seal test rig shown in Figure 1.

The rotor/seal test rig in Figure 1 is equivalent to the diagram shown in Figure 11 for the one-disk case. K_1, K_2, and K_3 are values of rotor partial stiffness with consideration of bearing stiffness assumed to be 1000 lb./in (175 kN/m) at the two ends. M and M_s are the rotor and the seal masses, respectively.

Assuming isotropy of the system, the lowest natural frequency of the rotor/seal coupled system can be calculated as follows:

$$\omega_{cpl} = \left\{ \frac{K_2 + K_3}{2M} + \frac{K_1 + K_2 + K_s}{2M_s} - \sqrt{\left(\frac{K_2 + K_3}{2M} - \frac{K_1 + K_2 + K_s}{2M_s} \right)^2 + \frac{K_2^2}{MM_s}} \right\}^{1/2} \quad [2]$$

The lowest natural frequency or resonance speed of the rotor without seal contact is

$$\omega_n = \sqrt{\frac{K_3 + K_1K_2/(K_1 + K_2)}{M}} \quad [3]$$

The calculated resonance speed ω_n using Eq. [3] is close to the measured 2000 rpm.

The seal stiffness was measured at static conditions by applying loads, which enabled the seals to reach the deformations in reverse rub. A stiffness value was defined by the ratio of the equivalent force on the tested seal to

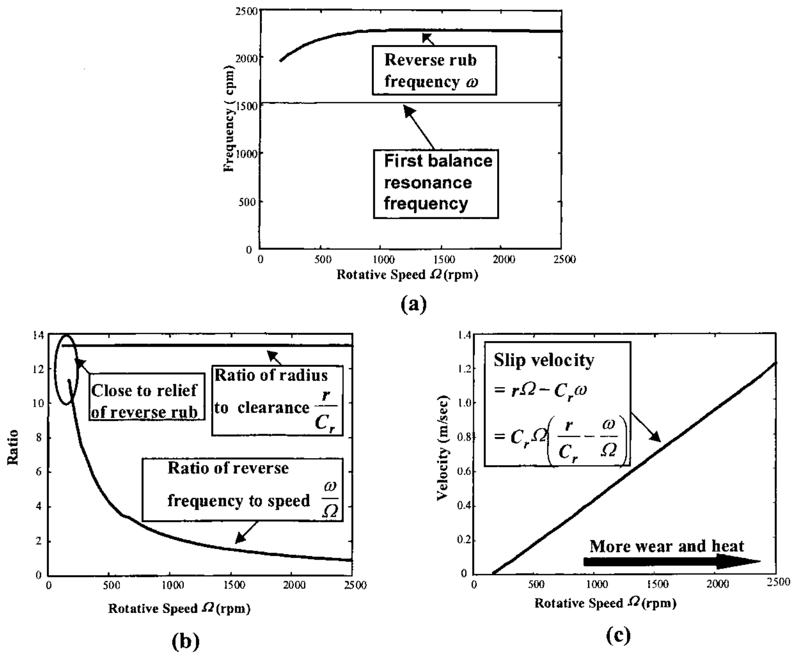


FIGURE 10 Parameters of slippage of rotor against seal with changes in rotative speed for the case in Figure 8. (a) reverse rub frequency, (b) ratio of reverse frequency to speed, and (c) slip velocity.

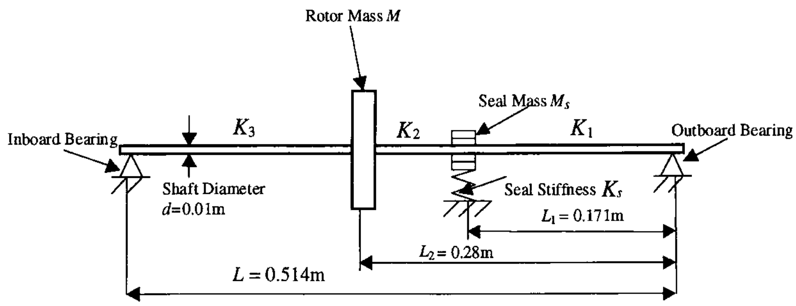
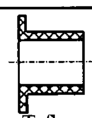
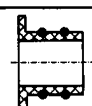
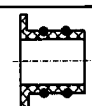
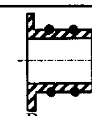


FIGURE 11 Diagram of rotor/seal test rig with one disk.

TABLE I Measured reverse rub frequencies in comparison with seal parameters and natural frequencies of rotor/seal coupled systems

Seal types	 Teflon	 Teflon	 Teflon	 Bronze
Stiffness Ks (kN/m)	699	170	112	122
Mass M_s (grams)	4.7	3.5	3.3	8.8
Diametral Clearance $2C_r$ (μm)	250	250	1000	1000
Natural Frequency of Coupled System ω_{cpl} (cpm)	4698	3728	3394	3389
Measured Reverse Rub Frequency ω (cpm)	4080	3360	2880	2880
$(\omega - \omega_{cpl})/\omega_{cpl}$	− 13%	− 10%	− 15%	− 15%

the desired deformation. Table I gives measured stiffness values of the tested seals along with the corresponding reverse precessional frequencies.

Seals with the same clearance may have different stiffness values, as shown in Table I, thus yielding different reverse precessional frequencies for the same rotor kit. The higher the seal stiffness, the higher the reverse precessional frequency. Since seal mass was so low, compared with rotor mass, the same stiffness resulted in almost the same reverse precessional frequency. Seals with O-rings attached had lower stiffness values, thus yielding lower rub frequencies.

Reverse precessional frequencies, as shown in Table I, were lower than the natural frequencies of rotor/seal coupled systems. This was also confirmed from impacting tests by eliminating the clearance of the contact point between the rotor and the seal.

5. DISCUSSION AND CONCLUSIONS

Full annular rub of rotors in mechanical seals has been investigated experimentally in this paper. When rubbing occurs due to high unbalance-related vibration response amplitudes, two regimes of motion are possible: rotor $1\times$ forward rub or reverse rub ("dry whip") with a constant frequency. Contrary to the common opinion, reverse precessional rub occurs spontaneously due to sufficiently high mass unbalance amount creating large contact forces between the rotor and the seal, without any outside disturbances. When the unbalance is smaller, the full annular reverse rub regime can be forced by a hammer impact. In the experiments, radius-to-clearance ratios for which the reverse precession occurred varied from 10 to 40. A coupled rotor/seal system usually underwent $1\times$ forward, and then very briefly $1\times$ reverse intermittent rubbing just before starting the severe reverse precessional full annular rub. The speed at which the full annular reverse rub started was close (lower or higher depending on the clearance and unbalance amount) to the first balance resonance of the original rotor system without seal contact. Variability of the rotative speed during runup and rundown would not eliminate this wear-causing contact between the rotor and the seal.

When the contact surface was lubricated, no reverse precessional rub would occur. Instead, the forward synchronous motion was maintained during slightly intermittent or continuous rub contact period. A peak response range of speeds would be wider during runup than during the rundown. A sudden decrease of amplitude (jump-down) occurs when the rotor separates from the seal during runup. Likewise a sudden increase of amplitude (jump-up) can be observed when it starts to contact the seal during rundown. In the case when the contact surface is not fully lubricated, low amount of mass unbalance may cause forward synchronous multi-partial intermittent rub.

Higher surface friction and sufficient mass unbalance, which generate a high contact force between the rotor and the seal, trigger the full annular rub reverse precession without additional disturbances. Although not affecting the behavior characteristics of the reverse full annular rub once it starts, the amount of unbalance does affect its rotative speed-related triggering point near the first balance resonance speed. With high mass unbalance, it can occur even before the rotor reaches the first balance resonance speed.

During rundown, the reverse precession can be sustained over the whole speed range until a very low speed (for example, till 120 rpm) when the relative slip velocity between the rotor and the seal approaches zero. Slippage at the rotor/seal surfaces occurs all the time during reverse precessional rub. High rotative speeds, therefore, which result in large relative velocities between the contact surfaces, cause severe surface wear, mainly that of the seal. At low speeds during rundown, reverse precession will be weakened through slight reduction of its amplitude and frequency. The speed at which reverse precessional rub vanishes can be estimated in terms of the radius-to-clearance ratio, and the reverse rub frequency. A hammer impact on the rotor or oil dropping on the contact surface can relieve the reverse full annular rub. Reverse precessional rub can also be generated during rundown with the same rub frequency as during runup.

It has been found that the rotor foundation has a strong effect on the occurrence of reverse precessional full annular rub. For the same rotor kit, reverse precessional rub easily occurred when the rotor was mounted on a stiff concrete base, while only forward synchronous precession could be generated if it was installed on a flexible table or floor. The coupling with additional elastic elements changed rotor/seal system parameters. The higher the seal/support stiffness, the higher the resulting reverse precessional frequency becomes for the same rotor. Though higher than the resonance speed of the rotor system without seal contact, the reverse precessional frequency is lower than the lowest natural frequency of the coupled rotor/seal system. At low rotative speeds, the decreased precession amplitude reduces the equivalent seal stiffness, thus slightly lowering the frequency.

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