

Full-range stress-strain curves for aluminum alloys

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Abstract: Aluminum alloys are being increasingly used in a wide range of construction applications owing to their sound mechanical properties, lightness in weight, strong corrosion resistance, ability to be formed into complex and efficient cross-section shapes and natural aesthetics. Aluminum alloys are characterized by a rounded stress-strain response, with no sharply-defined yield point. Such behavior can be accurately represented using Ramberg-Osgood-type equations. In the present study, use of a two-stage Ramberg-Osgood model to describe the full-range stress-strain behavior of aluminum alloys is proposed and, following careful analysis of a comprehensive database of aluminum alloy coupon test data assembled from the literature, standardized values or predictive expressions for the required input parameters are derived. The experimental database includes over 700 engineering stress-strain curves obtained from 56 sources and covers five common aluminum alloy grades, namely 5052-H36, 6061-T6, 6063-T5, 6082-T6 and 7A04-T6. The developed model is shown to be more accurate in predicting the full-range stress-strain response of aluminum alloys than existing expressions, and is suitable for use in the analytical modeling, numerical simulation and advanced design of aluminum alloy structures.

Keywords: Aluminum alloys, Constitutive modeling, Material modeling, Ramberg-Osgood

32 model, Stress-strain curves, Numerical modeling

33

34 **Introduction**

35 There are a wide variety of aluminum alloys with a broad range of mechanical properties. The
36 different alloys are created through the addition of different levels of alloying elements, such
37 as copper, magnesium, silicon and zinc, to the base aluminum metal. Depending on their
38 chemical composition, aluminum alloys are grouped into seven series, the general
39 characteristics of which have been discussed by [Dwight \(1998\)](#). The 5××× and 6××× series
40 alloys, particularly grades 5052, 6061, 6063 and 6082, are well suited to applications in
41 construction, with a good combination of strength, weldability, formability and corrosion
42 resistance. Some 7××× series alloys (e.g. grade 7A04), offering higher strengths but reduced
43 corrosion resistance and formability compared to the 6××× series alloys ([Dwight 1998](#); [CEN](#)
44 [2007](#)) are also emerging in the structural field ([Wang et al. 2020](#)). The 6××× and 7××× series
45 alloys are heat-treatable alloys that gain their strength by means of heat treatment, while the
46 non-heat treatable 5××× series alloys can be enhanced in strength through cold-working during
47 their manufacturing process.

48

49 The stress-strain behavior of aluminum alloys is characterized by a rounded response with no
50 sharply defined yield point, which differs significantly from that of hot-rolled carbon steels
51 ([Yun and Gardner 2017](#)). In addition, the stress-strain curves of aluminum alloys with different
52 grades display differing degrees of nonlinearity, roundedness in the “knee” region (i.e. the
53 region of the yield strength) and strain hardening, due primarily to their different chemical
54 compositions and tempers.

55

56 With the increasing use of advanced analysis in the design of metallic structures ([Gardner et al.](#)

57 2019), it is imperative to improve current code provisions (Aluminum Association 2010; CEN
58 2007; Standards Australia 1997) and to develop an accurate and practical material model to
59 describe the full-range stress-strain behavior of aluminum alloys; this is the focus of the present
60 study.

61

62 For predicting the stress-strain characteristics of cold-formed steel and stainless steel, which
63 exhibit a similar form of rounded response to aluminum alloys, the modified two-stage
64 Ramberg-Osgood model has become the formulation of choice, providing both accuracy and
65 practicality (Mirambell and Real 2000; Rasmussen 2003; Arrayago et al. 2015; Gardner and
66 Yun 2018). An assessment of the applicability of the two-stage Ramberg-Osgood model to
67 aluminum alloys and the derivation of standardized values or predictive equations for the key
68 input parameters are presented herein. Focus is placed on five common structural aluminum
69 alloys – 5052-H36, 6061-T6, 6063-T5, 6082-T6 and 7A04-T6. The developments are based on
70 the analysis of an assembled experimental database comprising over 700 tensile stress-strain
71 curves collected from 56 sources from around the world.

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73 **Existing stress-strain models**

74 The engineering (nominal) stress-strain response of aluminum alloys is characterized by a
75 continuous rounded curve with an absence of a sharply defined yield point, as shown in Fig. 1.
76 More specifically, the curve features an initial linear-elastic region up to the proportional stress
77 f_p , which is generally taken as the 0.01% proof stress, followed by a nonlinear “knee” region
78 up to the conventionally defined yield strength f_y (i.e. the 0.2% proof stress) and strain
79 hardening, the extent of which varies between grades, before reaching the ultimate tensile
80 strength f_u and corresponding ultimate strain ϵ_u . The initial slope of the stress-strain curve and
81 the tangent slope at the 0.2% proof stress are denoted E and $E_{0.2}$, respectively.

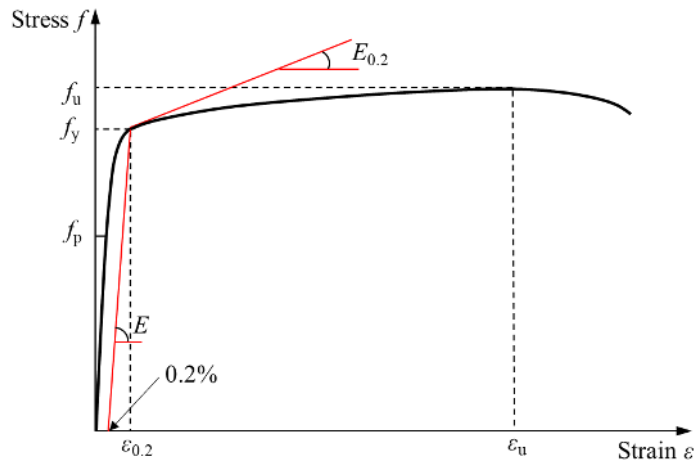
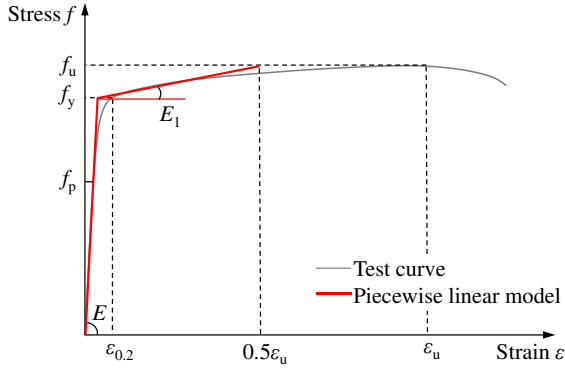


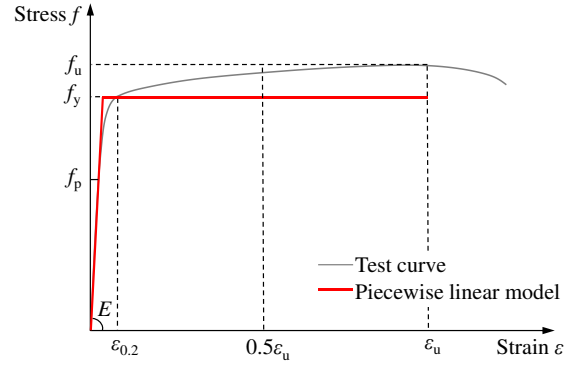
Fig. 1. Typical engineering (nominal) stress-strain curve for aluminum alloys

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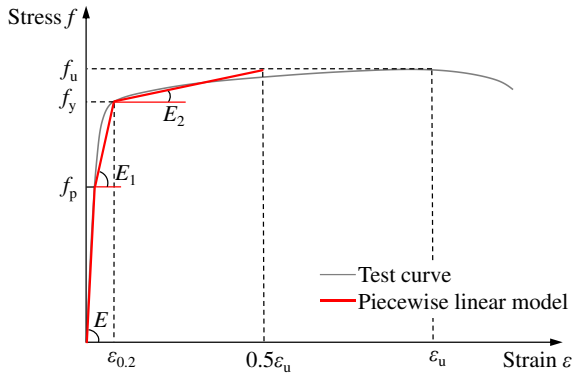
A number of material models have been developed to describe the nonlinear stress-strain behavior of aluminum alloys, with the simplest being piecewise linear models. The piecewise linear models defined in EN 1999-1-1 (CEN 2007) consist of two or three straight lines (corresponding to a bi-linear or a tri-linear material model, respectively) with each line representing a certain region of the stress-strain curve, with or without allowance for strain hardening, as shown in Fig. 2. In the piecewise linear models where strain hardening beyond the 0.2% proof stress is ignored – see Figs. 2(b) and 2(d), strains up to ϵ_u are allowed, while in cases where strain hardening is considered and represented by a sloped line – see Figs. 2(a) and 2(c), a cut-off strain equal to $0.5\epsilon_u$ is defined to avoid over-predictions of strength in the strain hardening range. It can be seen from Fig. 2 that the piecewise linear models, particularly the idealized bi-linear model (Fig. 2(b)), fail to capture the roundedness of the stress-strain response that is characteristic of aluminum alloys. For sophisticated numerical simulations and advanced inelastic design methods (Fieber et al. 2020; Gardner et al. 2019; Walport et al. 2019), a more accurate and continuous full-range stress-strain model is required.



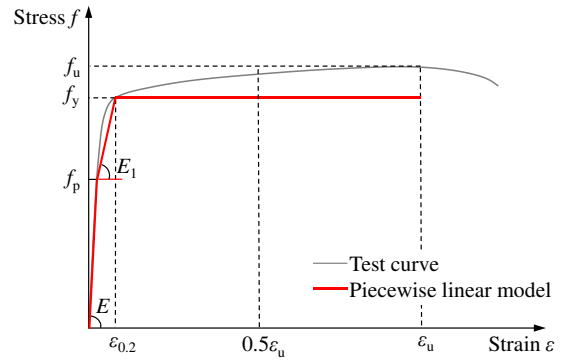
(a) Bi-linear model with hardening



(b) Bi-linear model without hardening



(c) Tri-linear model with hardening



(d) Tri-linear model without hardening

Fig. 2. Piecewise linear stress-strain models for aluminum alloys

Although other models exist (Baehre 1966; Mazzolani 1972, 1995), the most widely used continuous function to describe the rounded stress-strain behavior of metallic materials is the basic Ramberg-Osgood formulation (Ramberg and Osgood 1943), as modified by Hill (1944), or extensions thereof. The Ramberg-Osgood formulation, given by Eq. (1), has three basic input parameters – the Young’s modulus E , the yield (0.2% proof) strength f_y and the strain hardening exponent n , and is adopted in the European standard EN 1999-1-1:2007 (CEN 2007).

$$\varepsilon = \frac{f}{E} + 0.002 \left(\frac{f}{f_y} \right)^n \quad (1)$$

116 The determination of n requires, in addition to the conventional yield stress (i.e. 0.2% proof
 117 stress), the choice of a second reference point on the stress-strain curve. According to Annex E
 118 of EN 1999-1-1:2007 (CEN 2007), the second reference point may be taken as the 0.1% proof
 119 stress $\sigma_{0.1}$, located between f_p and f_y as illustrated in Fig. 3(a), for applications where only
 120 moderately small strains are expected to occur (e.g. in a buckling analysis); this results in Eq.
 121 (2) for the determination of the strain hardening exponent n .

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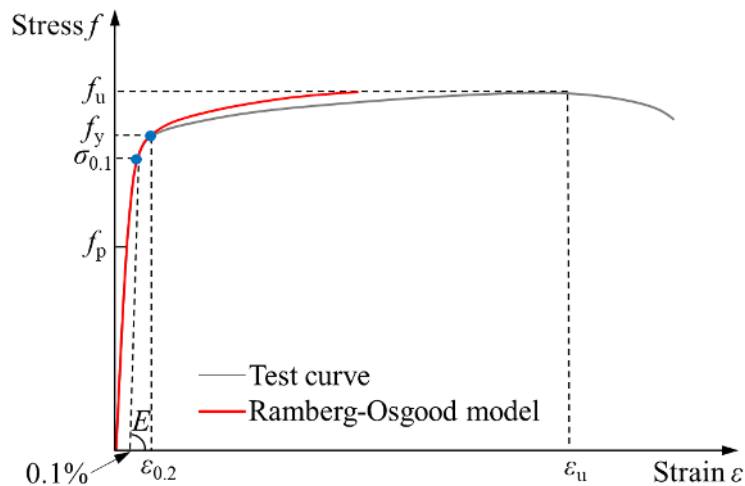
$$123 \quad n = \frac{\ln(2)}{\ln(f_y/\sigma_{0.1})} \quad (2)$$

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125 For analyses in which large strains are encountered (e.g. the simulation of manufacturing
 126 processes or connections), the strain hardening exponent n may be determined from Eq. (3),
 127 whereby the stress-strain curve passes through the point corresponding to the ultimate strength
 128 f_u , as shown in Fig. 3(b). In Eq. (3), $\varepsilon_{u,pl}$ is the plastic strain at f_u , which is equal to $(\varepsilon_u - f_u/E)$;
 129 since the term f_u/E is relatively small in comparison to ε_u , $\varepsilon_{u,pl} \approx \varepsilon_u$.

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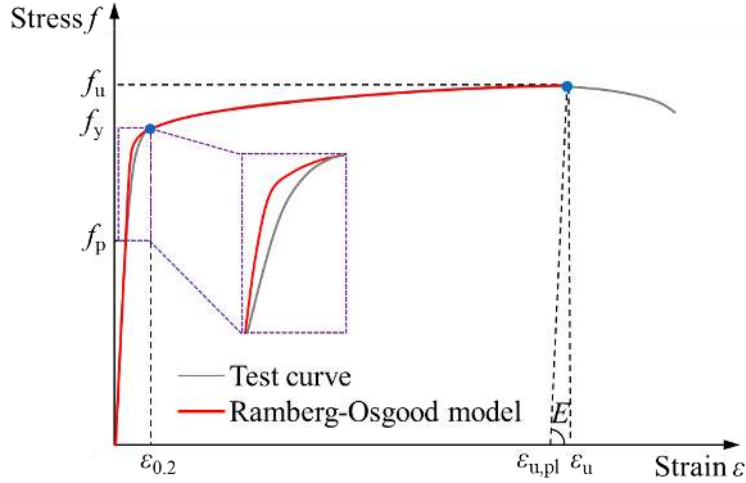
$$131 \quad n = \frac{\ln(0.002 / \varepsilon_{u,pl})}{\ln(f_y/f_u)} \approx \frac{\ln(0.002 / \varepsilon_u)}{\ln(f_y/f_u)} \quad (3)$$



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(a) Reference point corresponding to 0.1% proof stress $\sigma_{0.1}$



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(b) Reference point corresponding to ultimate strength f_u

136 **Fig. 3.** Choice of reference point for determining strain hardening exponent n in Ramberg-Osgood
 137 model

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139 While the Ramberg-Osgood formulation (Eq. (1)) provides an accurate representation of the
 140 degree of nonlinearity of certain regions of the stress-strain curve depending on the choice of
 141 the strain hardening exponent n , it does not, in general, provide an accurate representation of
 142 the full stress-strain curve, as highlighted in Fig. 3. This has led to the development of a number
 143 of two-stage Ramberg-Osgood models for stainless steels at room (Mirambell and Real 2000;
 144 Rasmussen 2003; Gardner and Ashraf 2006; Arrayago et al. 2015; Gardner 2019) and elevated
 145 temperatures (Gardner et al. 2010; Gardner et al. 2016) and cold-formed carbon steels (Gardner
 146 and Yun 2018). Three-stage models have also been proposed (Quach et al. 2008; Hradil et al.
 147 2013).

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149 The basic formulation of the two-stage Ramberg-Osgood models is given by Eq. (4), in which
 150 the nonlinear stress-strain curve is divided into two regions: below and above the yield (i.e.
 151 0.2% proof) strength f_y . In Eq. (4), $E_{0.2}$ is the tangent modulus at the yield strength, illustrated
 152 in Fig. 1 and defined by Eq. (5), $\epsilon_{0.2}$ is the total strain at the yield strength, equal to $(f_y/E +$
 153 $0.002)$, and m is the second strain hardening exponent, reflecting the degree of nonlinearity of

154 the second region of the stress-strain curve (i.e. the region with strains ranging from $\varepsilon_{0.2}$ to ε_u).
155 Note that a modified version of Eq. (4) was proposed by Rasmussen (2013), in which the term
156 relating to the ultimate strain was simplified by setting $\left(\varepsilon_u - \left(\varepsilon_{0.2} + \frac{f_u - f_y}{E_{0.2}} \right) \right) = \varepsilon_u$; while this
157 was appropriate for the studied (Rasmussen 2003) austenitic and duplex stainless steels, which
158 have very high ductility (rendering $\varepsilon_{0.2}$ and $(f_u - f_y)/E_{0.2}$ small in comparison to ε_u), it is less
159 suitable for less ductile materials, such as cold-formed steels (Gardner and Yun 2018) and
160 ferritic stainless steels (Arrayago et al. 2015) studied previously and aluminum alloys studied
161 herein. Hence, Eq. (4) is recommended for aluminum alloys and is used as the basis of the
162 present study. Further information on the two-stage Ramberg-Osgood models proposed by
163 different authors for stainless steels can be found in the review paper by Dundu (2018).

164

$$165 \quad \varepsilon = \begin{cases} \frac{f}{E} + 0.002 \left(\frac{f}{f_y} \right)^n, & \text{for } 0 < f \leq f_y \\ \frac{f - f_y}{E_{0.2}} + \left(\varepsilon_u - \varepsilon_{0.2} - \frac{f_u - f_y}{E_{0.2}} \right) \left(\frac{f - f_y}{f_u - f_y} \right)^m + \varepsilon_{0.2}, & \text{for } f_y < f \leq f_u \end{cases} \quad (4)$$

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$$167 \quad E_{0.2} = \frac{E}{1 + 0.002n \frac{E}{f_y}} \quad (5)$$

168

169 In the present paper, the full-range engineering stress-strain relationship for aluminum alloys
170 is modeled using the two-stage Ramberg-Osgood model proposed by Mirambell and Real
171 (2000) (see Eq. (4)). A comprehensive study is presented to determine values and predictive
172 equations for the key input parameters, based on the analysis of a large database of
173 experimentally obtained stress-strain curves on aluminum alloys collected from the literature
174 and assembled in the following section.

175 **Experimental database**

176 In this section, engineering stress-strain curves obtained from tensile coupon tests on aluminum
177 alloys are collected and analysed. A total of over 700 experimental stress-strain curves from 56
178 sources have been assembled, covering five grades that are commonly used in structural
179 applications, namely 5052-H36, 6061-T6, 6063-T5, 6082-T6 and 7A04-T6, though the
180 developed model is considered to be more broadly applicable. Note that the designation of
181 aluminum alloys starts with a digit number that indicates the series to which the alloy belongs,
182 and includes a letter after the hyphen that denotes the condition, or temper, of the alloy: “H”
183 represents alloys whose strength is enhanced by cold-working while “T” signifies alloys that
184 are thermally treated by different combinations of the following processes – solution annealing,
185 tempering, quenching and artificial or natural ageing. More details about the designation
186 system of aluminum alloys can be found in the American aluminum design manual ([Aluminum
187 Association 2010](#)), the European standard EN 1999-1-1:2007 ([CEN 2007](#)) and [Mazzolani
188 \(1995\)](#). With regards to the manufacturing method, extrusion is the most commonly used
189 process to form aluminum alloy structural components, allowing complex cross-section
190 geometries to be produced. Extrusion is especially suitable for aluminum alloys with good
191 extrudability, e.g. the 6xxx and 7xxx series alloys; for the non-heat treatable 5xxx series alloys,
192 their high magnesium content limits their extrudability and hence cold-rolling is the principle
193 production route ([Huynh et al. 2019](#)). Table 1 summarizes the key information relating to the
194 assembled database of coupon test results, including the source, the material grade, the
195 production method and cross-section profile of the members from which the coupons were
196 extracted, the material thickness and the number of tests performed. It can be seen from Table
197 1 that the tested tensile coupons were extracted from a wide range of extruded or cold-rolled
198 aluminum alloy profiles, including T-stubs, square, rectangular and circular hollow sections
199 (SHS, RHS and CHS respectively), angle sections, plates, cruciform sections, channel sections,

200 I-sections and irregular sections. Note that, although the coupon tests were conducted in
 201 accordance with different specifications, including AS (2007), CEN (2009) and ASTM (2013),
 202 the employed strain rates were all sufficiently low to be considered quasi-static (i.e. normally
 203 no higher than 0.00025 s^{-1}), and therefore to have little influence on the resulting stress-strain
 204 behavior (Huang and Young 2014). The mechanical properties of aluminum alloys at higher
 205 strain rates are out of the scope of the present study, but there remains scope for development
 206 of rate-dependent constitutive models for aluminum alloys considering the effect of high
 207 (dynamic) strain rates. It is also worthwhile to note that all stress-strain curves collected in the
 208 present study are from tensile coupon tests performed at room temperature. The deterioration
 209 of material properties of aluminum alloys 6063-T5 and 6061-T6 at elevated temperatures has
 210 been investigated by Su and Young (2019) while the effects of elevated temperatures on the
 211 material properties of other aluminum alloy grades need to be further investigated.

212

213 **Table 1.** Summary of key information relating to assembled database of aluminum alloy coupon tests

Source	Aluminum alloy grade	Production method	Thickness (mm)	Profiles of specimens from which coupons were extracted	Number of tests where E , f_y and f_u (ϵ_u) were provided ^b	Number of full stress-strain curves
Aalberg (2015)	6082-T6	Extrusion	4.6	I-sections	3 (0)	-
Alsanat et al. (2019)	5052-H36	Cold-rolling	2.5/3	Lipped channel sections	5 (5)	-
Brando et al. (2015)	6082-T6	Extrusion	3.5/5/6	I-section/plate	3 (3)	-
Chen et al. (2017)	6061-T6	Extrusion	-	CHS	3 (0)	-
Chen et al. (2018)	6061-T6	Extrusion	-	I-section	1 (0)	-
Chen et al. (2020)	6061-T6	Extrusion	2	Plate	1 (0)	-
Cho and Kim (2016)	6061-T6	Extrusion	3	Plates	3 (0)	-
Davies and Roberts (1999)	6082-T6	Extrusion	-	I-sections	11 (0)	-
De Matteis et al. (2000)	6061-T6/ 6082-T6	Extrusion	-	T-stubs	2 (2)	-
Đuričić et al. (2017)	6082-T6	Extrusion	2	CHS	4 (0)	-
Faella et al. (2000)	6060-T6/ 6082-T6	Extrusion	2-15.1	SHS/RHS	38 (0)	-
Feng et al. (2017)	6061-T6/ 6063-T5	Extrusion	2.5/3	SHS/RHS	2 (2)	2
Feng et al. (2018)	6061-T6/ 6063-T5	Extrusion	1/2/2.5/3	SHS/RHS	6 (6)	6
Feng et al. (2020)	6061-T6/ 6063-T5	Extrusion	7/7.5	CHS	2 (2)	2
Feng and Young (2015)	6061-T6	Extrusion	1.7/2/3/3.2/ 5	CHS	5 (0)	-

Guo (2006)	6061-T6	Extrusion	-	SHS/RHS/CHS/A ngle sections /T-stubs	53 (36)	-
Guo et al. (2020)	6061-T6/ 6082-T6	Extrusion	4	Plates	9 (9)	-
He et al. (2019)	6061-T6	Extrusion	3/4	SHS	6 (0)	-
Hopperstad et al. (1999)	6082-T6	Extrusion	2.5	Cruciform sections	1 (0)	-
Huynh et al. (2019)	5052-H36	Cold-rolling	2.5/3	Channel-sections	218 (146)	146
Islam and Young (2012)	6061-T6	Extrusion	1.6/2.3/3/3. 2/5	SHS/RHS	8 (0)	-
Jiang et al. (2018)	6061-T6	Extrusion	-	RHS	1 (0)	-
Jiang et al. (2020)	6061-T6	Extrusion	6/7/10	RHS/plate	3 (0)	-
Kim and Cho (2014)	6061-T6	Extrusion	2/3	Plates	6 (0)	-
Liu et al. (2015)	6063-T5	Extrusion	-	Irregular sections	2 (2)	2
Liu et al. (2019a)	6061-T6	Extrusion	8/10	I-sections/plates	6 (0)	-
Liu et al. (2019b)	6082-T6	Extrusion	-	Plate	1 (0)	-
Mazzolani et al. (2011)	6000 series ^a	Cutting from extruded hollow profiles	1.85-6.35	Channel-sections	16 (0)	-
Ma et al. (2020)	6061-T6	Extrusion	-	I-section	1 (0)	-
May and Menzemer (2005)	6061-T6	Extrusion	-	Tee/Channel/ Angle sections	3 (0)	-
Rønning et al. (2010)	6082-T6	Extrusion	3/4/4.4/5	Plates	8 (8)	-
Rouholamin et al. (2020)	5052-H36	Cold-rolling	2.5/3	Lipped channel sections	10 (0)	-
Shi et al. (2018)	6061-T6	Extrusion	8/12	I-sections/plates	3 (0)	-
Su et al. (2014)	6061-T6/ 6063-T5	Extrusion	2.81-10.45	SHS/RHS	15 (15)	15
Su et al. (2015)	6061-T6/ 6063-T5	Extrusion	2.85-10.42	SHS/RHS	15 (15)	15
Su and Young (2019)	6061-T6/ 6063-T5	Extrusion	4.5	RHS	2 (2)	2
Tajeuna et al. (2015)	6061-T6	Extrusion	3.2/6.4/9.5	Plates	18 (17)	-
Tryland et al. (1999)	6082-T6	Extrusion	3/5/8	Tee section/SHS	3 (3)	-
Wang et al. (2019)	6061-T6	Extrusion	8/10/12	T-stubs	9 (9)	9
Wang et al. (2016a)	7A04-T6	Extrusion	8	Angle sections	8 (8)	8
Wang et al. (2020)	7A04-T6	Extrusion	24	Angle sections	4 (4)	4
Wang et al. (2018a)	6061-T6	Extrusion	-	I-sections	9 (9)	9
Wang et al. (2018b)	6061-T6	Extrusion	10.5/11/12 /14	RHS/I-sections	12 (12)	12
Wang and Wang (2016)	7A04-T6	Extrusion	-	Plates	2 (2)	2
Wang et al. (2013)	6082-T6	Extrusion	4/5/6/7/8/10 /12	SHS/RHS/I- /Angle sections	90 (90)	90
Yalçın and Genel (2019)	6063-T5	Extrusion	-	CHS	1 (0)	-
Yuan et al. (2015)	6061-T6/ 6063-T5	Extrusion	3.59-10.89	I-sections	48 (48)	48
Zha and Moen (2003)	6082-T6	Extrusion	5/6/8.5	Plates	3 (0)	-
Zhao et al. (2019)	6082-T6	Extrusion	4/5/6/7/8	SHS/CHS	5 (0)	-
Zhu and Young (2006)	6061-T6/ 6063-T5	Extrusion	1.6/3	CHS	4 (4)	4

Zhou and Young (2009)	6061-T6	Extrusion	2/2.5/3/4/5	CHS	10 (10)	10
Zhou and Young (2018)	6061-T6	Extrusion	2/2.5/3/4/5	CHS	6 (0)	-
Zhou and Young (2019)	6061-T6/ 6063-T5	Extrusion	1.4/1.9	Channel sections	4 (4)	4
Zhu et al. (2018)	6063-T5	Extrusion	4/5	I-sections	4 (0)	-
Zhu et al. (2019)	6061-T6/ 6063-T5	Extrusion	1.6/1.9	Lipped channel sections/Channel sections	4 (0)	4
Zhu et al. (2020)	6061-T6	Extrusion	10/14	I-sections/plate	2 (0)	-
Total					722 (473)	394

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Table note: a. Specific aluminum alloy grade not provided; b. Values in brackets represent the number of cases in which the ultimate strain ε_u were provided.

218 The collected tensile coupon test results were reported to different levels of completeness, as
219 summarized in Table 1, where the values in brackets represent the number of cases in which
220 the ultimate strains ε_u were provided. Among the total of 722 tensile coupon tests, 394 tests
221 were reported (or provided upon request) with their full stress-strain curves; these curves have
222 been employed to derive appropriate values or predictive expressions for the strain hardening
223 exponents n and m for the different aluminum alloy grades. The process by which all the
224 required input parameters for the two-stage Ramberg-Osgood model (Mirambell and Real 2000)
225 given by Eq. (4) were derived from the collected stress-strain data is described below, with
226 particular attention given to the determination of the strain hardening exponents n and m .

227

228 Ordinary least squares (OLS) regression analysis, in which the sum of the squares of the
229 differences between the measured strains $\varepsilon_{\text{mea},i}$ and those predicted by the two-stage Ramberg-
230 Osgood model $\varepsilon_{\text{R-O},i}$ is minimized, was employed to determine the best fit n and m values for
231 the 394 full stress-strain curves. The objective functions for n and m are given by Eqs. (6) and
232 (7), respectively.

233

$$234 \quad S(n) = \text{Min} \sum_{i=1}^k (\varepsilon_{\text{mea},i} - \varepsilon_{\text{R-O},i})^2 = \text{Min} \sum_{i=1}^k \left[\varepsilon_{\text{mea},i} - \left(\frac{f_i}{E} + 0.002 \left(\frac{f_i}{f_y} \right)^n \right) \right]^2 \quad \text{for } 0 < \varepsilon_{\text{mea},i} \leq \varepsilon_{0.2} \quad (6)$$

$$S(m) = \text{Min} \sum_{i=1}^k (\varepsilon_{\text{mea},i} - \varepsilon_{\text{R-O},i})^2$$

$$= \text{Min} \sum_{i=1}^k \left[\varepsilon_{\text{mea},i} - \left(\frac{f_i - f_y}{E_{0.2}} + \left(\varepsilon_u - \varepsilon_{0.2} - \frac{f_u - f_y}{E_{0.2}} \right) \left(\frac{f_i - f_y}{f_u - f_y} \right)^m + \varepsilon_{0.2} \right) \right]^2 \quad \text{for } \varepsilon_{0.2} < \varepsilon_{\text{mea},i} \leq \varepsilon_u$$

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237

238 Since the strain rates are typically varied during coupon testing (Huang and Young 2014), the

239 recorded data points are often not evenly distributed along the stress-strain curve, with higher

240 concentrations of data lying in the region where a lower strain rate was applied (typically in

241 the strain range between 0 and $\varepsilon_{0.2}$). Performing regression using the original stress-strain data

242 points may, thus, result in biased estimates of the strain hardening exponents n and m towards

243 the regions with higher concentrations of data. A further consideration is that anomalous results

244 can be obtained in cases where the test stress-strain curves feature a high degree of

245 experimental noise. To avoid the aforementioned problems, two polynomials, up to seventh

246 order, were first fitted to the test stress-strain curves. The polynomial given by Eq. (8) with

247 regression coefficients a_1 to a_7 was fitted to the initial part of the stress-strain curves, up to the

248 yield strength $\varepsilon_{0.2}$, while Eq. (9), with regression coefficients b_1 to b_7 was used beyond this

249 point.

250

$$f(\varepsilon) = \sum_{k=1}^7 a_k \varepsilon^k \quad \text{for } 0 \leq \varepsilon \leq \varepsilon_{0.2} \quad (8)$$

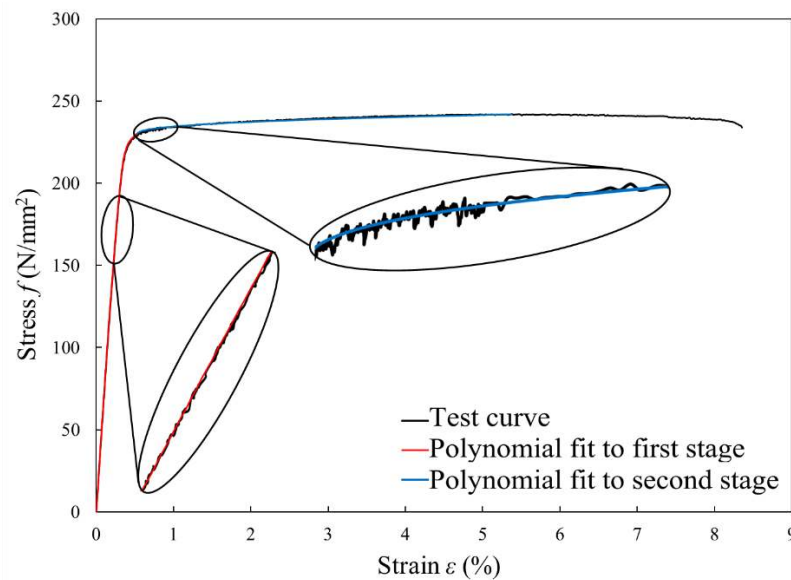
252

$$f(\varepsilon) = f_y + \sum_{k=1}^7 b_k (\varepsilon - \varepsilon_{0.2})^k \quad \text{for } \varepsilon_{0.2} < \varepsilon \leq \varepsilon_u \quad (9)$$

254

255 The regression coefficients were determined using the software package MATLAB (MATLAB

256 2016); this process, as demonstrated in Fig. 4, enables the representation of the test stress-strain
 257 curves by continuous and smooth curves with explicit, yet complicated, functions (i.e. Eqs. (8)
 258 and (9)). The Young's modulus E , as well as other parameters of stresses and strains (including
 259 $f_y, f_u, \varepsilon_{0.2}$ and ε_u) employed in the two-stage Ramberg-Osgood model, can then be determined
 260 from the fitted polynomial curves, following the recommendations by Huang and Young (2014),
 261 and the strain hardening exponents n and m can be accurately captured by performing OLS
 262 regression analysis on the evenly distributed data points extracted from the polynomials.
 263



264 **Fig. 4.** Polynomial fitting to typical measured stress-strain curve
 265
 266

267 **Results and discussion**

268 ***Young's modulus E***

269 The average Young's modulus values E determined from the collected data are presented in
 270 Table 2 and compared with those given in the European standard EN 1999-1-1:2007 (CEN
 271 2007) and the American aluminum design manual (AA 2010). It can be seen that the code
 272 values represent the average measured values well, and that the measurements are generally
 273 very consistent with low coefficients of variation (COV). The results indicate a slightly higher
 274 Young's modulus for the grade 7A04-T6, though this is based on a relatively small number of

275 coupon tests and requires further verification. Given the consistency of the results, the
 276 observation of no clear trend between the different aluminum alloys and considering simplicity
 277 and ease of use, a single Young's modulus value of 70,000 MPa, as adopted in EN 1999-1-
 278 1:2007 (CEN 2007) is recommended for all the investigated grades.

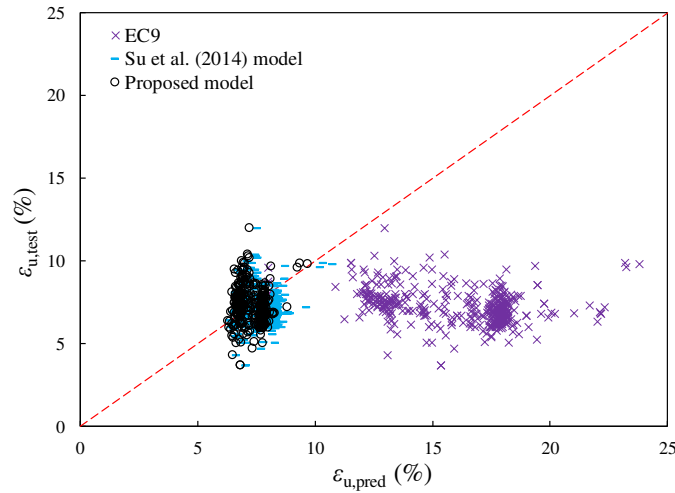
279
 280 **Table 2.** Comparison of the average measured Young's modulus values with those provided in CEN
 281 (2007) and AA (2010) for different aluminum alloy grades
 282

Aluminum alloy grade	No. of coupons with E provided	Average E MPa	COV	E (CEN 2007) MPa	E (AA 2010) MPa
5052-H36	218	69,500	0.014	70,000	70,300
6061-T6	113	68,800	0.049	70,000	69,600
6063-T5	31	68,400	0.050	70,000	69,600
6082-T6	103	69,700	0.035	70,000	69,600
7A04-T6	8	72,000	0.061	70,000	-

283
 284 ***Strain at ultimate tensile strength ϵ_u***
 285 Annex E of EN 1999-1-1:2007 (CEN 2007), referred to as EC9, provides empirical expressions
 286 for the prediction of the strain at the ultimate tensile strength $\epsilon_{u,EC9}$ for aluminum alloys, as
 287 given by Eqs. (10a) and (10b); the predictive expression for $\epsilon_{u,EC9}$ depends only on the material
 288 yield strength f_y , which should be input in N/mm². The accuracy of the EC9 predictive model
 289 is assessed by comparing the 473 test results in which the ultimate strain $\epsilon_{u,test}$ was reported
 290 (see Table 1) with their corresponding predicted values $\epsilon_{u,EC9}$; the comparisons are presented in
 291 Fig. 5. It can be seen from Fig. 5 that the EC9 model generally yields significant over-
 292 predictions of the test values, with the mean value of the ratio of $\epsilon_{u,test}/\epsilon_{u,EC9}$ being 0.48 and the
 293 corresponding coefficient of variation (COV) being 0.303, as reported in Table 3.

294
 295
$$\epsilon_u = 0.3 - 0.22 \left(f_y / 400 \right) \quad \text{for } f_y < 400 \text{ N / mm}^2 \quad (10a)$$

296
 297
$$\epsilon_u = 0.08 \quad \text{for } f_y \geq 400 \text{ N / mm}^2 \quad (10b)$$



299
300
301
302

Fig. 5. Comparisons between test values of strain at ultimate strength $\varepsilon_{u,\text{test}}$ and those determined using different predictive models $\varepsilon_{u,\text{pred}}$

303
304

Table 3. Statistical evaluation of the accuracy of different predictive models for determining ε_u

	$\varepsilon_{u,\text{test}}/\varepsilon_{u,\text{EC9}}$	$\varepsilon_{u,\text{test}}/\varepsilon_{u,\text{Su}}$	$\varepsilon_{u,\text{test}}/\varepsilon_{u,\text{prop}}$
Mean	0.48	0.96	1.00
COV	0.303	0.178	0.163

305

306 To improve the accuracy of the EC9 predictive model, Su et al. (2014) proposed a new model
307 for determining the ultimate strain $\varepsilon_{u,\text{Su}}$ for aluminum alloys, based on an expression similar in
308 format to that developed for steel (Yun and Gardner 2017; Gardner and Yun 2018) and stainless
309 steel (Rasmussen 2003; Arrayago et al. 2015), as given by Eq. (11).

310

$$\varepsilon_{u,\text{Su}} = 0.13(1 - f_y / f_u) + 0.06 \quad (11)$$

312

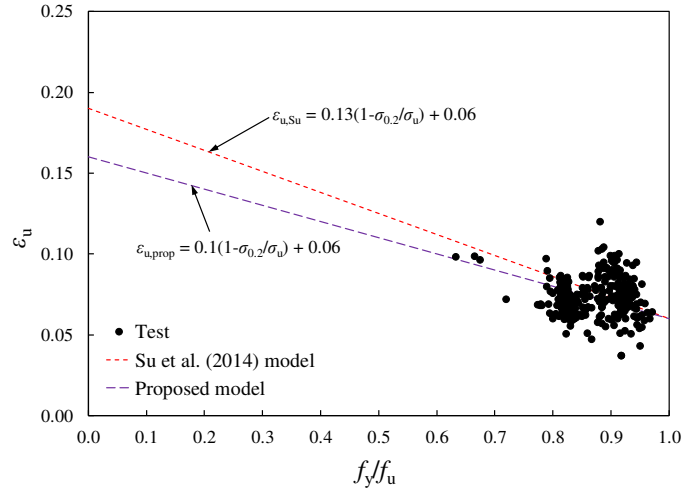
313 The accuracy of the Su et al. (2014) model is demonstrated in Fig. 5 and Table 3, as well as in
314 Fig. 6 where the coupon test values of $\varepsilon_{u,\text{test}}$ are plotted against the corresponding ratios of f_y/f_u .
315 This model can, however, be further improved based on the larger experimental database
316 assembled in the present study, as shown in Fig. 6; the proposed modified predictive expression
317 for $\varepsilon_{u,\text{prop}}$ is given by Eq. (12). As shown in Table 3, the mean value of $\varepsilon_{u,\text{test}}/\varepsilon_{u,\text{prop}}$ is equal to
318 1.00, with a COV of 0.163, revealing an improvement in terms of both accuracy and

319 consistency over the [Su et al. \(2014\)](#) model.

320

321
$$\varepsilon_{u,prop} = 0.1(1 - f_y / f_u) + 0.06 \quad (12)$$

322



323

324

Fig. 6. Assessment of different predictive models for the strain at the ultimate tensile strength ε_u

325

326 *Ultimate strength f_u*

327 For instances in which the yield strength f_y is known (e.g. by measurement) but the ultimate

328 strength f_u is not, a predictive expression for f_u is desirable. By analyzing the 722 collated

329 coupon test results (summarized in Table 1), it was found that the aluminum alloy data follow

330 a general trend of reducing ratios of f_u/f_y with increasing yield strength f_y , as shown in Fig. 7.

331 Note that a similar trend between f_u/f_y and f_y has also been observed for carbon steels ([Fukumoto](#)

332 [1996; Gardner and Yun 2018](#)). The relationship between f_u/f_y and f_y for aluminum alloys can be

333 represented by Eq. (13), which is a power law model with its coefficients of 60 and 1.5 chosen

334 to fit the collated test results. The proposed predictive equation (Eq. (13)), in which f_y and f_u

335 must be in N/mm^2 , shows good agreement with the test results, with a mean value of the test-

336 to-predicted ratios of f_u being 1.02 and a low corresponding COV of 0.044. It should be

337 emphasized that the correlation between f_y and f_u depends largely on the work-hardened

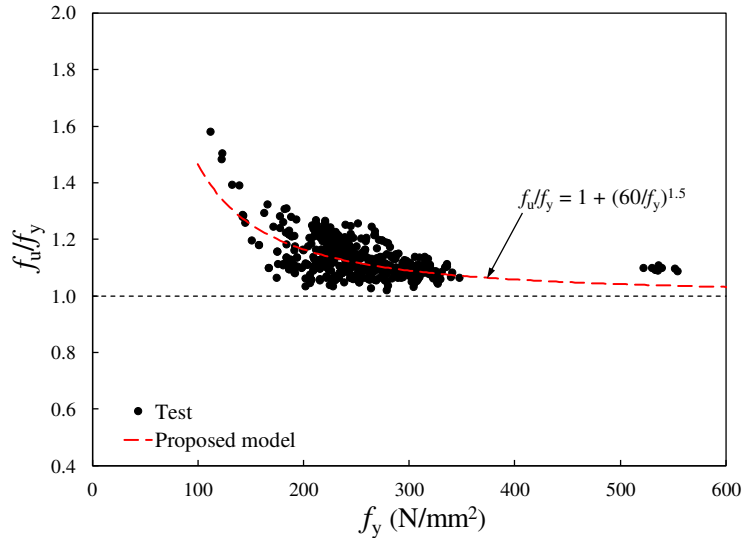
338 conditions (for non-heat-treatable aluminum alloys) or the heat treatment conditions (for heat-

339 treatable aluminum alloys), thus the applicability of Eq. (13) for other aluminum alloy grades
 340 with different manufacturing conditions is yet to be further investigated.

341

$$342 \quad f_u / f_y = 1 + (60/f_y)^{1.5} \quad (13)$$

343



344

345

346

Fig. 7. Assessment of proposed predictive model for the ultimate strength f_u

347 **Strain hardening exponent n**

348 The first strain hardening exponent n employed in the two-stage Ramberg-Osgood model is
 349 conventionally determined for aluminum alloys using Eq. (2), forcing the curve to pass through,
 350 in addition to the 0.2% proof stress, a second reference point corresponding to the 0.1% proof
 351 stress $\sigma_{0.1}$, as shown in Fig. 3(a). It has been shown for cold-formed carbon steel (Gardner and
 352 Yun 2018) and stainless steel (Arrayago et al. 2015) that use of the 0.05% proof stress $\sigma_{0.05}$
 353 yields more accurate estimates of n . Use of this alternative second reference point, which
 354 results in the definition of n given by Eq. (14), is now considered for aluminum alloys. The
 355 accuracy of the two predictive equations (Eqs. (2) and (14)) for aluminum alloys is assessed by
 356 comparing the test values n_{test} , determined from the collected full stress-strain curves by means
 357 of OLS regression analysis, as described above, with those predicted using Eqs. (2) and (14)

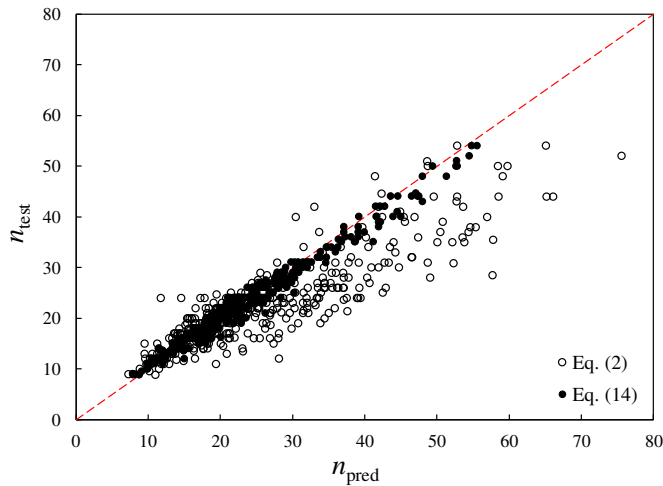
358 (i.e. n_{pred}); the comparisons are shown in Fig. 8. It can be seen from Fig. 8 that Eq. (14) provides
 359 more accurate predictions of the n_{test} values than Eq. (2); this is further demonstrated by the
 360 statistical results presented in Table 4. The use of Eq. (14) is therefore recommended for the
 361 determination of the first strain hardening exponent n , provided that the 0.05% proof stress $\sigma_{0.05}$
 362 is known. It is further recommended that this value is routinely reported in future experimental
 363 studies.

364

365

$$n = \frac{\ln(4)}{\ln(f_y / \sigma_{0.05})} \quad (14)$$

366



367

368

Fig. 8. Assessment of different predictive equations for the first strain hardening exponent n

369

370

371

Table 4. Statistical assessment of different predictive equations for the first strain hardening exponent n

	$n_{\text{test}}/n_{\text{pred}}$	
	Eq. (2)	Eq. (14)
Mean	0.92	1.00
COV	0.22	0.06

372

373 The degree of roundedness of the stress-strain curve approaching the yield strength is reflected
 374 by the value of the first strain hardening exponent n , with lower values signifying a more
 375 rounded response. The degree of nonlinearity depends mainly on the chemical composition of
 376 the material and the production process. Thus, as expected, the measured stress-strain curves

377 for the same aluminum alloy grade display a similar degree of nonlinearity, making the
 378 parameter n grade specific. A summary of the average n values for the five investigated
 379 structural aluminum alloy grades is provided in Table 5. It can be observed that the fully heat-
 380 treated aluminum alloys (i.e. 6061-T6, 6082-T6 and 7A04-T6) tend to exhibit the higher values
 381 of n , corresponding to the stress-strain curves displaying a sharper yield point. Adoption of
 382 these average n values is recommended when the value of $\sigma_{0.05}$ is not available and hence Eq.
 383 (14) cannot be applied. The average values from the collected coupon test results of the other
 384 key material parameters, including E , f_y , f_u and ϵ_u , are also listed in Table 5.

385

386 **Table 5.** Summary of average measured values of the basic material parameters used for two-stage
 387 Ramberg-Osgood model for studied aluminum alloy grades
 388

Aluminum alloy grade	No. of full stress-strain curves	E N/mm ²	f_y N/mm ²	f_u N/mm ²	ϵ_u	n	m
5052-H36	146	69,700	220	270	0.07	16	2.5
6061-T6	103	69,100	250	280	0.07	24	2.5
6063-T5	40	69,000	160	200	0.08	15	2.6
6082-T6	97	69,700	300	330	0.08	26	2.2
7A04-T6	8	72,000	540	590	0.08	33	2.3

389

390 ***Strain hardening exponent m***

391 Analogous to the approach used to determine the first strain hardening exponent n , the second
 392 strain hardening exponent m can be calculated by forcing the second stage of the two-stage
 393 Ramberg-Osgood model to pass through an intermediate reference point between $(\epsilon_{0.2}, f_y)$ and
 394 (ϵ_u, f_u) . Considering the point corresponding to either the 1% proof stress $\sigma_{1.0}$ or the 2% proof
 395 stress $\sigma_{2.0}$ as the intermediate reference point results in Eq. (15) and Eq. (16), respectively, for
 396 the determination of m (Quach and Huang 2011; Gardner and Yun 2018). Both equations
 397 provide good estimates of the test values m_{test} , which were determined using the
 398 aforementioned data processing approach, as shown in Fig. 4. Of the two equations, Eq. (15)
 399 is recommended because $\sigma_{1.0}$ is more likely to be reported by researchers and manufacturers,
 400 but in many cases, the measured values of neither $\sigma_{1.0}$ nor $\sigma_{2.0}$ will be available, preventing the

401 use of either Eq. (15) or Eq. (16). It is thus desirable to provide representative average values
 402 for m to capture the degree of nonlinearity of the second stage of stress-strain curves for
 403 aluminum alloys.

404

$$405 \quad m = \frac{\ln\left(0.008 + \frac{\sigma_{1.0} - f_y}{E} - \frac{\sigma_{1.0} - f_y}{E_{0.2}}\right) - \ln\left(\varepsilon_u - \varepsilon_{0.2} - \frac{f_u - f_y}{E_{0.2}}\right)}{\ln(\sigma_{1.0} - f_y) - \ln(f_u - f_y)} \quad (15)$$

406

$$407 \quad m = \frac{\ln\left(0.018 + \frac{\sigma_{2.0} - f_y}{E} - \frac{\sigma_{2.0} - f_y}{E_{0.2}}\right) - \ln\left(\varepsilon_u - \varepsilon_{0.2} - \frac{f_u - f_y}{E_{0.2}}\right)}{\ln(\sigma_{2.0} - f_y) - \ln(f_u - f_y)} \quad (16)$$

408

409 The second strain hardening exponent m has been found to be related to the ratio of yield to
 410 ultimate strength f_y/f_u for cold-formed carbon steel (Gardner and Yun 2018) and stainless steel
 411 (Rasmussen 2003; Arrayago et al. 2015), but this correlation is not seen for aluminum alloys,
 412 as shown in Fig. 9, where the values of m_{test} are plotted against their corresponding ratios of
 413 f_y/f_u , grouped by aluminum alloy grade. This is because, for heat-treatable aluminum alloys,
 414 the strain hardening level (reflected by the second strain hardening exponent m) may also vary
 415 with temper of the alloy, which, for the same aluminum alloy grade, the higher tempered alloy
 416 generally shows a less pronounced of strain hardening compared to that of the lower tempered
 417 alloy. This is also reflected by the characteristic values of the exponent n EC9 (CEN 2007).
 418 Although there are no clear trends in the m_{test} data, the range of values is small, with the
 419 majority of data falling between 2.0 and 3.0. The average m_{test} values for each aluminum alloy
 420 grade are reported in Table 5, while, for simplicity, an overall average value of 2.4 may be used.

421

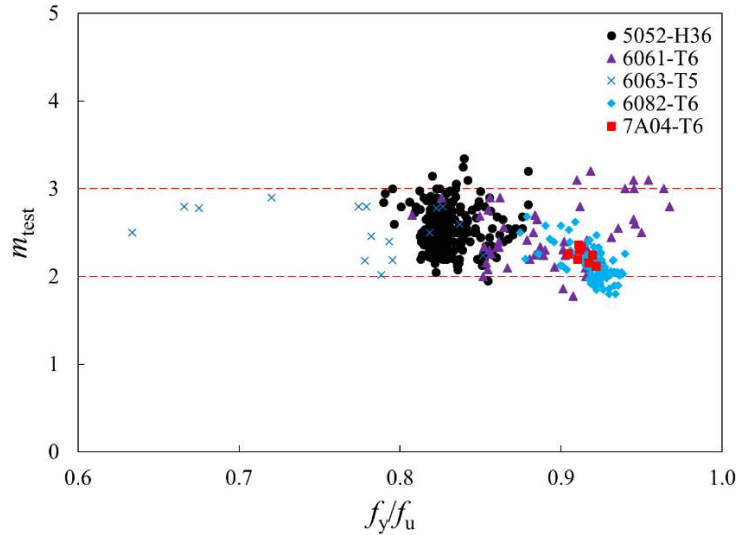


Fig. 9. Relationship between m_{test} and f_y/f_u

422
423

424

425 Comparison with experimental stress-strain curves

426 The accuracy of the proposed two-stage Ramberg-Osgood model for aluminum alloys, as
 427 described above, is assessed by comparing a series of predicted curves with the corresponding
 428 experimental stress-strain curves. Five representative comparisons, one for each aluminum
 429 alloy grade, are presented in Figs. 10-14. The two-stage Ramberg-Osgood curves illustrated in
 430 Figs. 10-14 utilize the measured values of the key input parameters – E , f_y , f_u , ϵ_u , n and m for
 431 each respective aluminum alloy coupon test, as summarized in Table 6. Stress-strain curves
 432 generated from the single-stage Ramberg-Osgood model, with the strain hardening exponent n
 433 determined from either Eq. (2) or Eq. (3), are also plotted in Figs. 10-14 for comparison
 434 purposes. Note that the predicted curves from the single-stage Ramberg-Osgood model, with n
 435 calculated from Eq. (2), are terminated when the stress reaches the measured ultimate strength
 436 f_u . It can be seen from Figs. 10-14 that the single-stage Ramberg-Osgood model with n
 437 calculated using Eq. (2) fails to capture the full-range stress strain response of aluminum alloys,
 438 except for the 7A04-T6 grade, while the single-stage Ramberg-Osgood model with n calculated
 439 using Eq. (3) provides a good overall description of the experimental stress-strain curves but
 440 loses accuracy in the important initial yielding region, as highlighted in Figs. 10(b) to 14(b).

441 The two-stage Ramberg-Osgood model, on the other hand, provides an accurate description of
 442 the experimental stress-strain curves over the full range of strains up to ϵ_u .

443

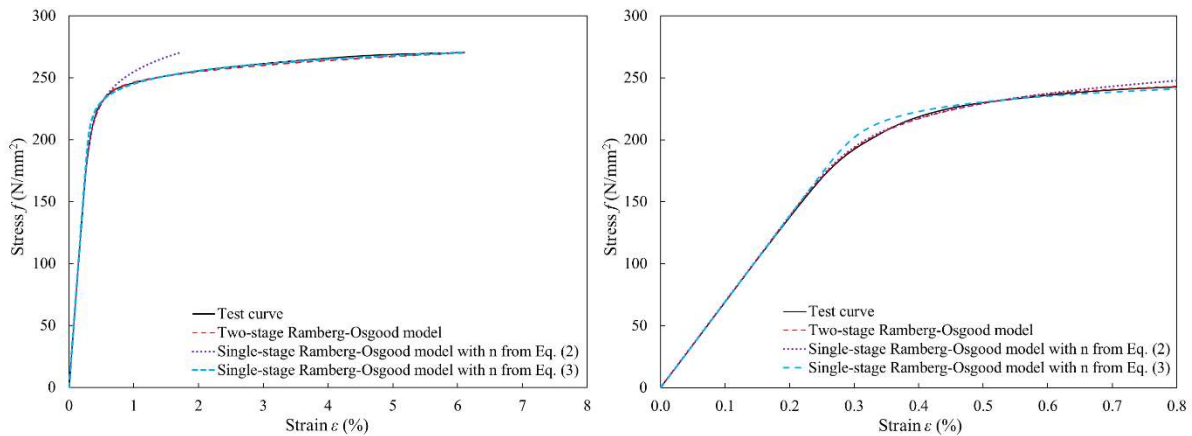
444

445 **Table 6.** Key measured material properties of the selected aluminum alloy coupon tests used for
 446 comparisons with predicted stress-strain curves
 447

Source	Aluminum alloy grade	Coupon label	E	f_y	f_u	ϵ_u	n_{test}	m_{test}
			N/mm ²	N/mm ²	N/mm ²	%		
Huynh et al. (2019)	5052-H36	C40030_LT_F04_1	69,400	232	270	6.10	12	2.8
Su et al. (2015)	6061-T6	H50×95×10.5B5II	70,200	192	222	8.54	15	2.9
Su et al. (2014)	6063-T5	+N95×50×10.5C	70,400	151	181	7.32	10	2.6
Wang et al. (2013)	6082-T6	H3-2	66,500	322	349	7.49	25	2.0
Wang et al. (2016a)	7A04-T6	L100-8-2	69,550	533	582	6.98	33	2.2

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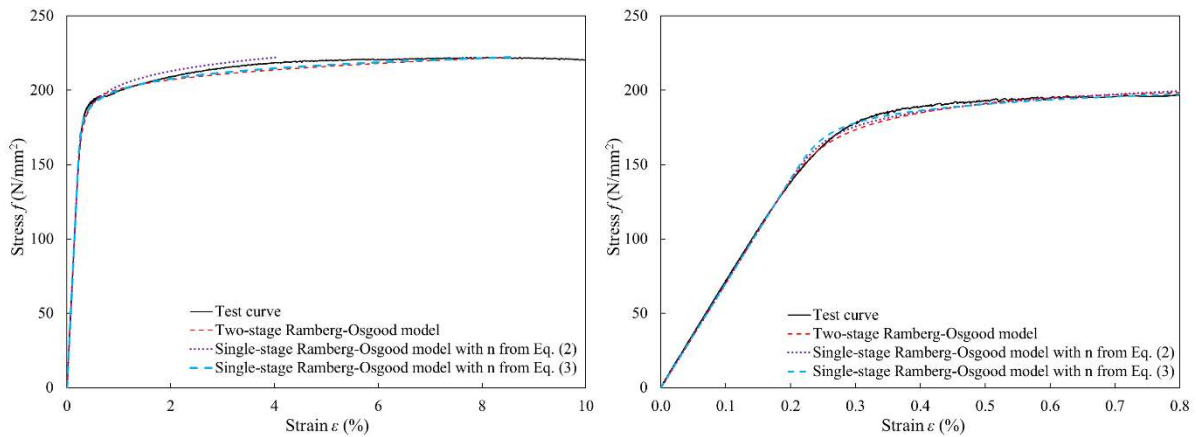


450

451

452 **Fig. 10.** Comparison of different material models with an experimental stress-strain curve on grade
 453 5052-H36 aluminum alloy reported by Huynh et al. (2019)
 454

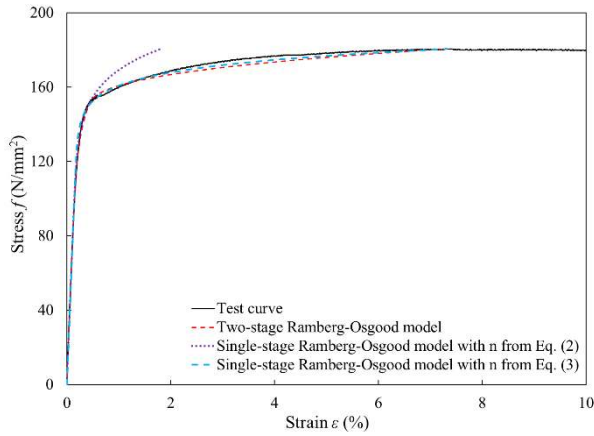
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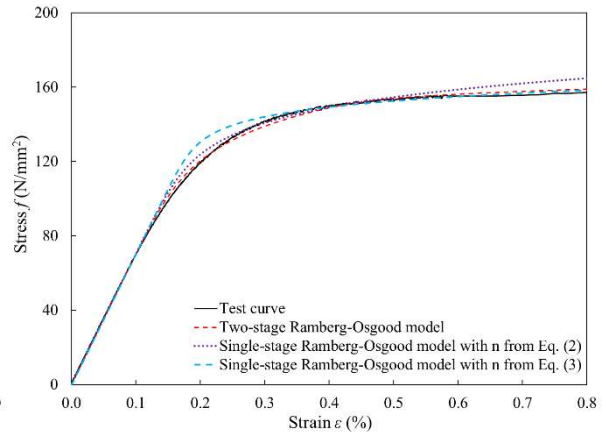
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456

457 **Fig. 11.** Comparison of different material models with an experimental stress-strain curve on grade
 458 6061-T6 aluminum alloy reported by Su et al. (2015)



(a) Full stress-strain curves

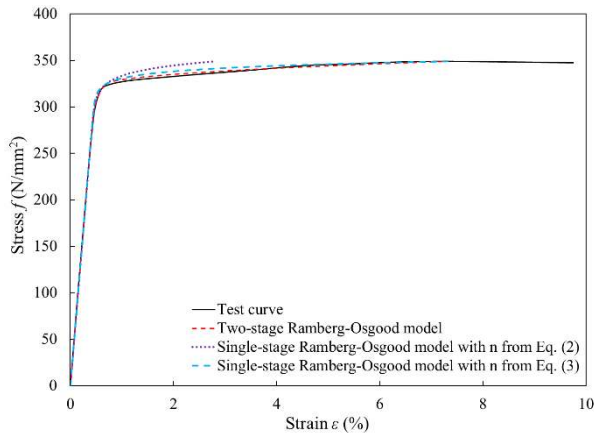


(b) Stress-strain curves up to 0.8% strain

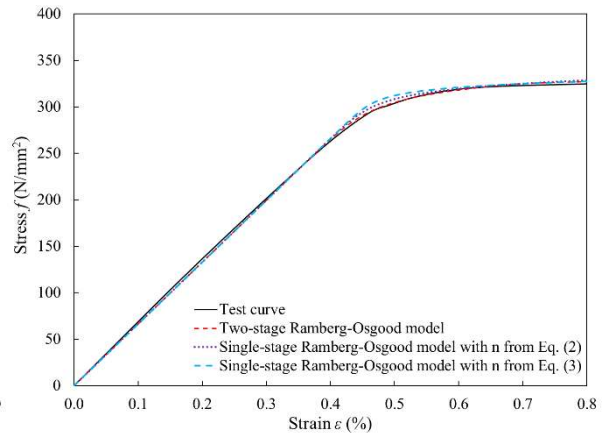
459
460

461 **Fig. 12.** Comparison of different material models with an experimental stress-strain curve on grade
462 6063-T5 aluminum alloy reported by [Su et al. \(2014\)](#)

463



(a) Full stress-strain curves

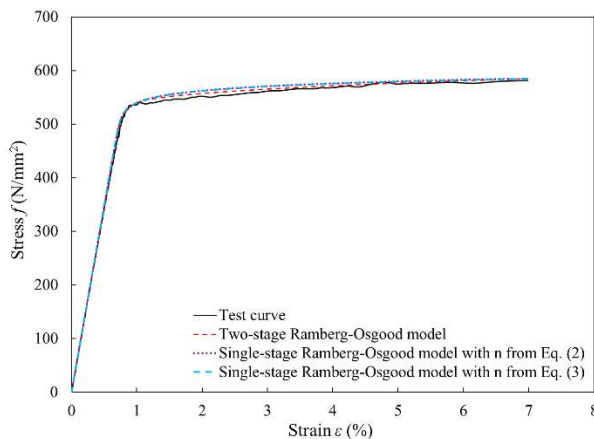


(b) Stress-strain curves up to 0.8% strain

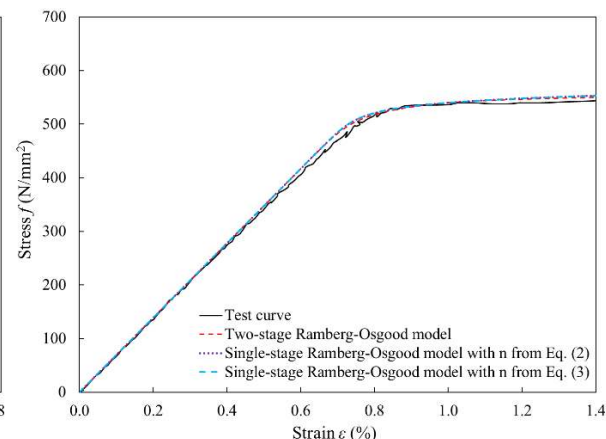
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465

466 **Fig. 13.** Comparison of different material models with an experimental stress-strain curve on grade
467 6082-T6 aluminum alloy reported by [Wang et al. \(2013\)](#)

468



(a) Full stress-strain curves



(b) Stress-strain curves up to 1.4% strain

469
470

471 **Fig. 14.** Comparison of different material models with an experimental stress-strain curve on grade
472 7A04-T6 aluminum alloy reported by [Wang et al. \(2016a\)](#)

473

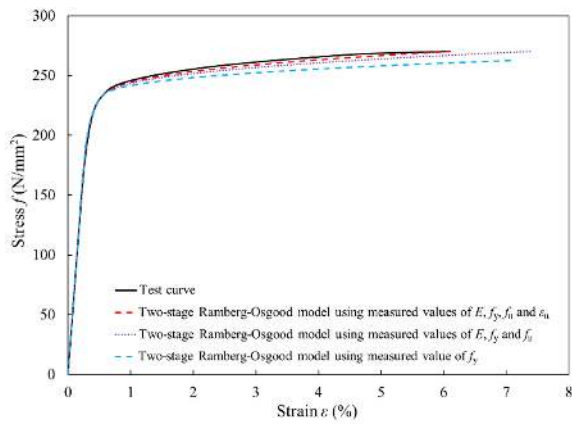
474 The accuracy of the proposed two-stage Ramberg-Osgood model with different levels of
475 assumed knowledge of availability of the key input parameters is also assessed. Three cases
476 are considered. In Case 1, it is assumed that the measured values of E , f_y , f_u and ε_u are known
477 and n and m are taken as the average values given in Table 5. In Case 2, it is assumed that the
478 measured values of E , f_y and f_u are known, ε_u is predicted using Eq. (12), and the average values
479 of n and m from Table 5 are used. In Case 3, it is assumed that only the measured value of f_y is
480 known, E is assigned the recommended value of 70,000 N/mm², ε_u and f_u are predicted using
481 Eqs. (12) and (13), respectively, and the average values of n and m from Table 5 are used. The
482 five previously selected coupon tests (see Table 6) are used for this demonstration; the
483 comparisons are shown in Figs. 15-19.

484

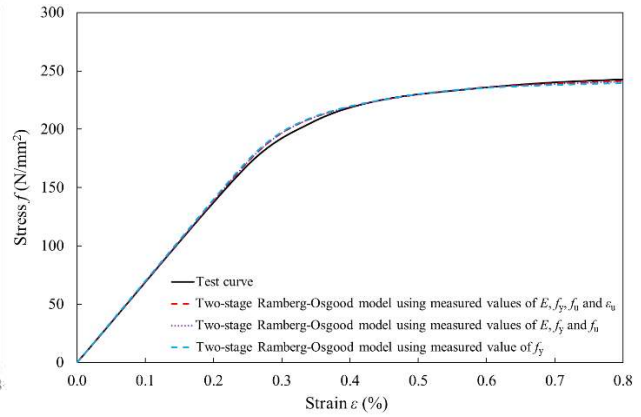
485 The proposed two-stage Ramberg-Osgood model can be seen to generally provide an accurate
486 representation of the experimental stress-strain curves, especially when greater knowledge of
487 the material input parameters is assumed (i.e. moving from Case 3 to Case 1). Typically, the
488 nominal or measured values of the three key material properties E , f_y and f_u are readily available
489 e.g. from material specifications, design standards or experimental reports; this corresponds to
490 Case 2 of the described comparisons. For this scenario, the predicted two-stage Ramberg-
491 Osgood curves are shown to yield consistently accurate representations of the experimental
492 stress-strain curves, as depicted by the purple dotted lines in Figs. 15-19.

493

494



(a) Full stress-strain curves

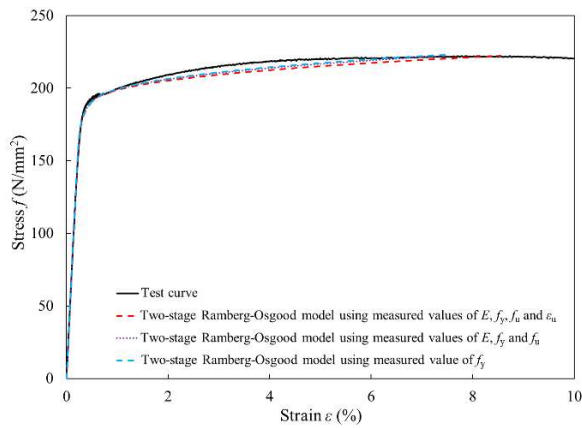


(b) Stress-strain curves up to 0.8% strain

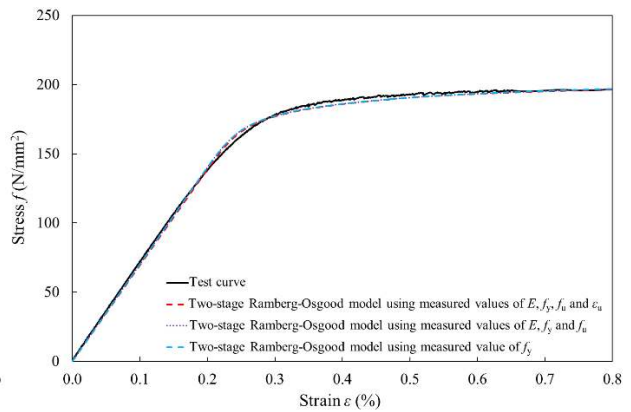
495
496

497 **Fig. 15.** Comparison of the predicted two-stage Ramberg-Osgood curves based on different measured
498 material parameters with an experimental stress-strain curve on grade 5052-H36 aluminum alloy
499 reported by [Huynh et al. \(2019\)](#)

500



(a) Full stress-strain curves

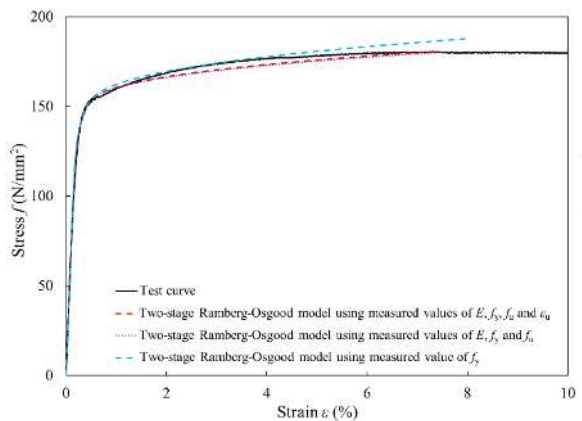


(b) Stress-strain curves up to 0.8% strain

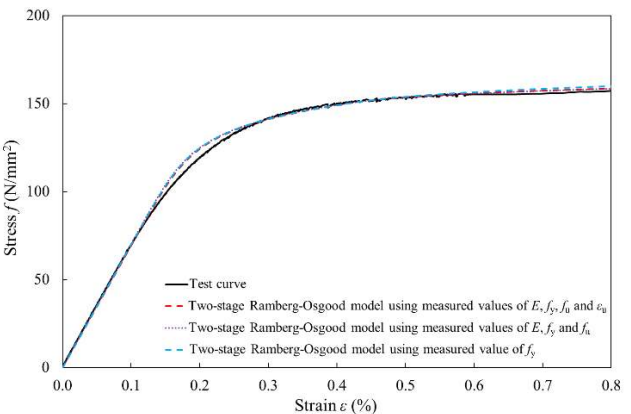
501
502

503 **Fig. 16.** Comparison of the predicted two-stage Ramberg-Osgood curves based on different measured
504 material parameters with an experimental stress-strain curve on grade 6061-T6 aluminum alloy reported
505 by [Su et al. \(2015\)](#)

506



(a) Full stress-strain curves

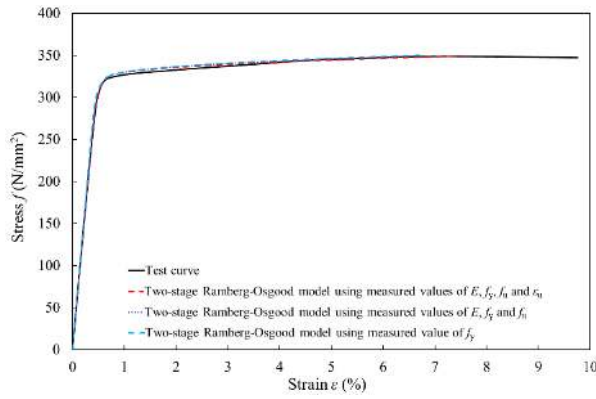


(b) Stress-strain curves up to 0.8% strain

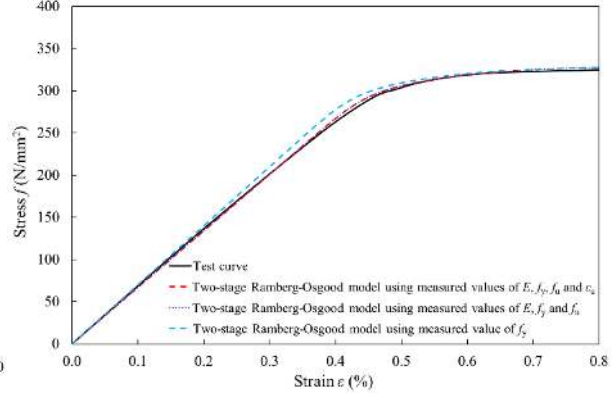
507
508

509 **Fig. 17.** Comparison of the predicted two-stage Ramberg-Osgood curves based on different measured
510 material parameters with an experimental stress-strain curve on grade 6063-T5 aluminum alloy reported
511 by [Su et al. \(2014\)](#)

512



(a) Full stress-strain curves

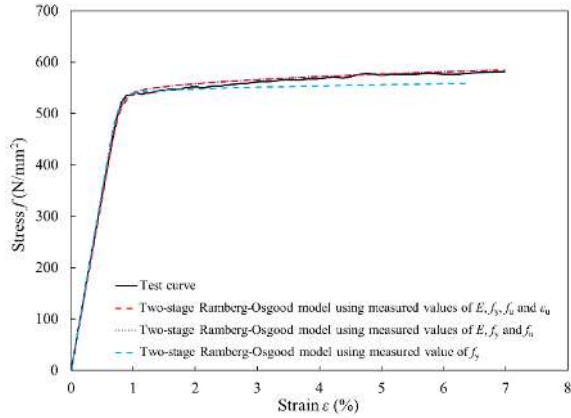


(b) Stress-strain curves up to 0.8% strain

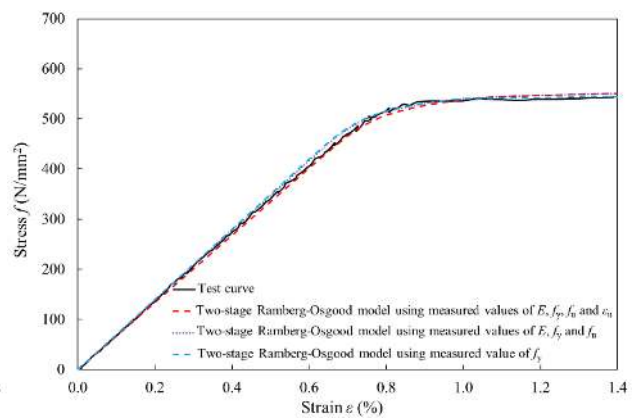
513
514

515 **Fig. 18.** Comparison of the predicted two-stage Ramberg-Osgood curves based on different measured
516 material parameters with an experimental stress-strain curve on grade 6082-T6 aluminum alloy reported
517 by Wang et al. (2013)

518



(a) Full stress-strain curves



(b) Stress-strain curves up to 1.4% strain

519
520

521 **Fig. 19.** Comparison of the predicted two-stage Ramberg-Osgood curves based on different measured
522 material parameters with an experimental stress-strain curve on grade 7A04-T6 aluminum alloy
523 reported by Wang et al. (2016a)

524

525 Summary of the proposed model

526 The proposed two-stage Ramberg-Osgood model, along with the recommended values and
527 predictive expressions for the key input parameters, for describing the stress-strain relationship
528 of aluminum alloys is summarized in this section. The general form of the model is given as
529 follows:

$$530 \quad \varepsilon = \begin{cases} \frac{f}{E} + 0.002 \left(\frac{f}{f_y} \right)^n, & \text{for } 0 < f \leq f_y \\ \frac{f - f_y}{E_{0.2}} + \left(\varepsilon_u - \varepsilon_{0.2} - \frac{f_u - f_y}{E_{0.2}} \right) \left(\frac{f - f_y}{f_u - f_y} \right)^m + \varepsilon_{0.2}, & \text{for } f_y < f \leq f_u \end{cases} \quad (4)$$

531 where:

532 E is the Young's modulus that may be taken as 70,000 N/mm² or as the average value
533 per grade from Table 5;

534 $E_{0.2}$ is the tangent modulus of the stress-strain curve at the yield strength f_y defined by
535 Eq. (5);

$$536 \quad E_{0.2} = \frac{E}{1 + 0.002n \frac{E}{f_y}} \quad (5)$$

537 $\varepsilon_{0.2}$ is the total strain at the yield strength which equals to $f_y/E + 0.002$;

538 ε_u is the strain at the ultimate tensile strength f_u that can be determined by Eq. (12); for
539 cases where f_u is not available, it may be estimated from Eq. (13), in which f_y and f_u are
540 in N/mm²;

541

542

$$543 \quad \varepsilon_u = 0.1 \left(1 - f_y / f_u \right) + 0.06 \quad (12)$$

544

$$545 \quad f_u / f_y = 1 + \left(60 / f_y \right)^{1.5} \quad (13)$$

546

546 n is the first strain hardening exponent that may be taken from Table 5 or determined
547 from Eq. (14) when the measured 0.05% proof stress $\sigma_{0.05}$ is available;

548

$$549 \quad n = \frac{\ln(4)}{\ln(f_y / \sigma_{0.05})} \quad (14)$$

550

551 m is the second strain hardening exponent that may be taken from Table 5 or calculated
552 from Eq. (15), which requires knowledge of the measured 1% proof stress.

553

$$m = \frac{\ln\left(0.008 + \frac{\sigma_{1.0} - f_y}{E} - \frac{\sigma_{1.0} - f_y}{E_{0.2}}\right) - \ln\left(\varepsilon_u - \varepsilon_{0.2} - \frac{f_u - f_y}{E_{0.2}}\right)}{\ln(\sigma_{1.0} - f_y) - \ln(f_u - f_y)} \quad (15)$$

554

555 It should be noted that the two-stage Ramberg-Osgood model is developed to describe the
 556 engineering (nominal) stress-strain curve, while, up to the ultimate tensile strength f_u , the curve
 557 can be converted to true stress-strain curve using the following two equations:

558

559

$$f_{\text{true}} = f(1 + \varepsilon) \quad (17)$$

560

561

$$\varepsilon_{\text{true}} = \ln(1 + \varepsilon) \quad (18)$$

562

563 where f_{true} and $\varepsilon_{\text{true}}$ are true stress and true strain respectively.

564

565 **Conclusions**

566 A comprehensive study into the constitutive modeling of aluminum alloys has been presented.

567 The two-stage Ramberg-Osgood model, originally proposed by [Mirambell and Real \(2000\)](#) for

568 stainless steels, has been adopted in the present study to describe the nonlinear stress-strain

569 behavior of aluminum alloys. Values and predictive expressions for the key input parameters

570 of the two-stage Ramberg-Osgood expression have been developed based on the analysis of an

571 assembled experimental database comprising a total of 722 coupon test results, with a focus on

572 five aluminum alloy grades that are commonly used in structural applications, namely 5052-

573 H36, 6061-T6, 6063-T5, 6082-T6 and 7A04-T6. It has been shown that the proposed model

574 provides a very accurate representation of experimental stress-strain curves over the full range

575 of tensile strains up to the ultimate tensile strain, especially when at least the three fundamental

576 material parameters – the Young's modulus E , the yield strength f_y and the ultimate strength f_u

577 – are known. The proposed model presented herein is considered suitable for use in advanced
578 numerical simulations and design methods, particularly in instances where large plastic strains
579 are encountered.

580

581 **Data Availability Statement**

582 Some or all data, models, or code that support the findings of this study are available from the
583 corresponding author upon reasonable request.

584

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590

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