

**Fully Decidable Logics,
Automata and Classical
Theories for Defining Regular
Real-Time Languages¹**

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Contents

1	Introduction	2
2	The Continuous Real-Time Models	6
3	The Counter-Free Regular Real-Time ω-Languages	11
3.1	Introduction	11
3.2	Qualitative Formalisms	11
3.2.1	The Temporal Logic of the Reals	11
3.2.2	The First Order Monadic Logic over the Reals	12
3.2.3	Expressiveness Equivalence Result	12
3.3	Two Real-Time Temporal Logics	13
3.3.1	The Metric Interval Temporal Logic	13
3.3.2	The Logic of Event Clocks	15
3.4	A First-Order Classical Theory	16
3.5	Expressiveness Results	18
3.5.1	EventClockTL versus MinMaxML ₁	18
3.5.2	EventClockTL versus MetricIntervalTL	21
3.5.3	Minimal Expressively Complete Fragments	27
4	The Regular Real-Time ω-Languages	28
4.1	Introduction	28
4.2	Propositional Event-Clock Automata	29
4.3	Recursive Event-Clock Automata	32
4.4	Closure Properties of Recursive Event-Clock Automata	38
4.4.1	Closure under Positive Boolean Operations	44
4.4.2	Closure under Negation	47
4.4.3	Closure under Partial Projection	64
4.5	Emptiness and Universality for REventClockTA	65
4.6	Expressiveness: REventClockTA vs EventClockTL	74
5	Adding Counting and Beyond	80
5.1	Adding Ability to Count	81
5.1.1	Adding Automata Operators	81
5.1.2	Adding Second Order Quantification	83
5.1.3	Expressiveness: Equivalence Results	84
5.2	Projected Regular Real-Time Languages	88
5.2.1	Projected Event Clock Temporal Logic	89
5.2.2	Projected (Propositional) Event-Clock Automaton	89
5.2.3	Projected Recursive Event-Clock Automaton	90
5.2.4	Timed Automata	90
5.2.5	Expressiveness Equivalence Result	92
5.3	Undecidable Extensions	97